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ETHIOPIA

MATHEMATICS GRADE 9
UNIT 5 SUMMARY
Geometry and Measurement

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Main contents

- congruent
- Similarity
- Theorems on similarity and congruent

Further on congruency and similarity

- Congruency
  - When two figures have same size and shape, they are called congruent (if they are exact copies of each other)
- congruency of triangle

Triangles ABC and DEF are congruent, if six parts of triangle (three sides and three angles) are correspondingly congruent

The sides and angles with then match up like this:-

Corresponding angles

$<A \equiv\ <D$

$<B \equiv <D$

$<C \equiv <F$

corresponding sides

$\overline{AB} \equiv \overline{DE}$

$\overline{BC} \equiv \overline{EF}$

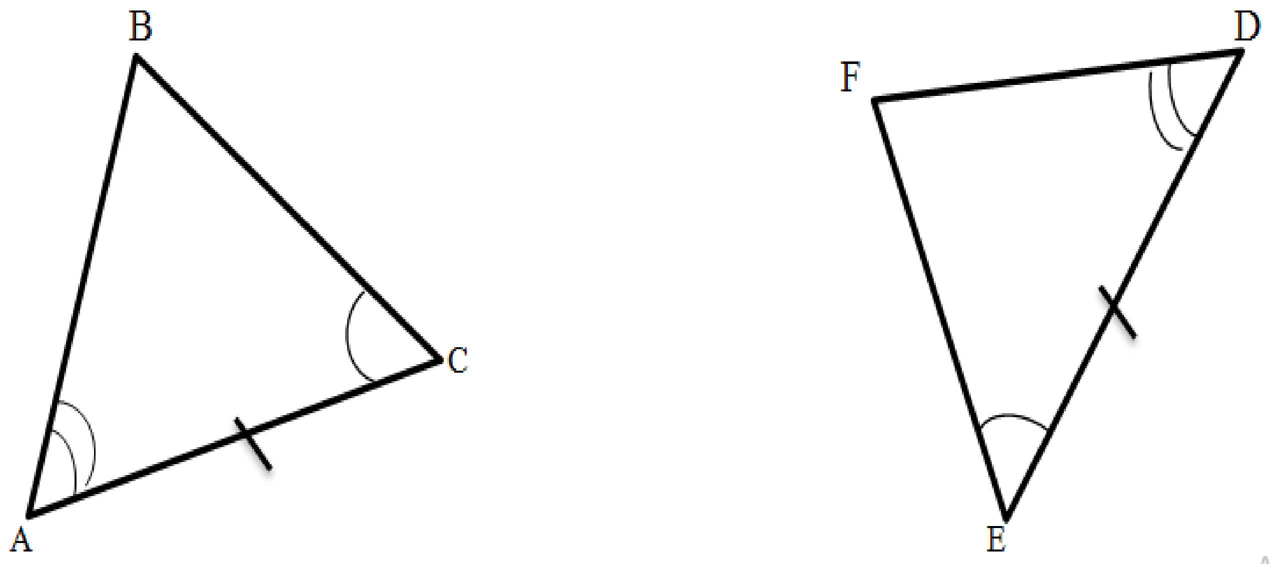
$\overline{AC} \equiv \overline{DF}$

Therefor $\Delta ABC\ \equiv \Delta DEF$

Some ways to prove congruency of triangles

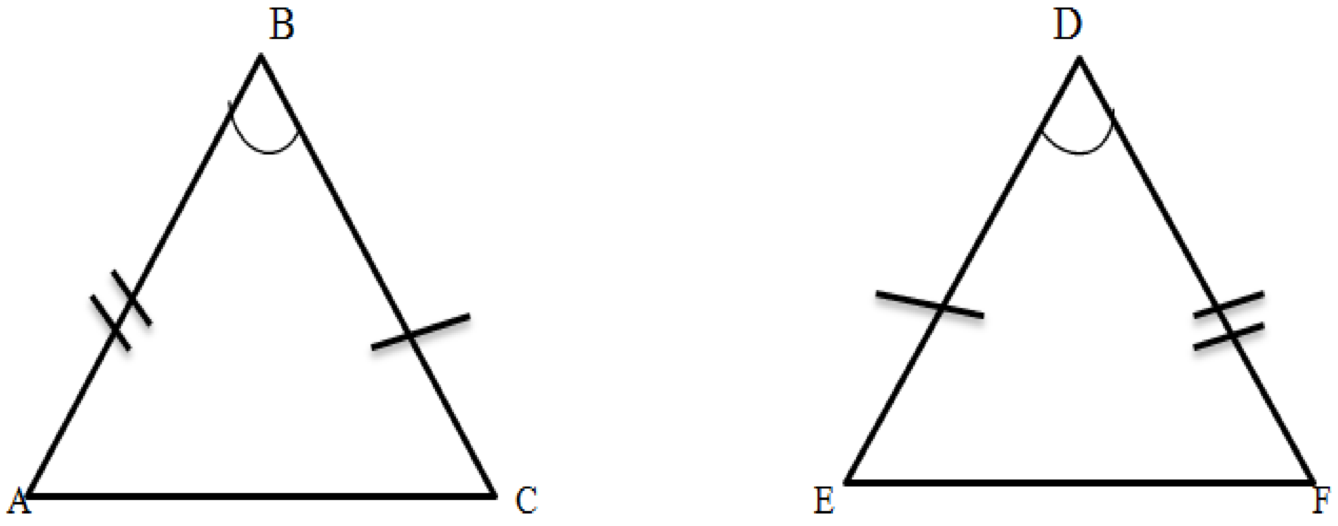
- **Congruent**
- The following postulates give you three different ways to show that two triangles are congruent by comparing three pairs of angles and sides.
- **<u>SSS theorem</u>**
- If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

ASA theorem: - if two angles of one triangle are congruent to two sides of another triangle and the inclined of another triangle is congruent to, then the triangles are congruent.



$\Delta ABC \equiv \Delta DEF$ by ASA

SAS theorem: - if two sides and the included angle of one triangle are congruent to the two sides and the included angles of another triangle, then the triangles are congruent



$\Delta ABC \equiv \Delta DEF$ by SAS

similarity

- Figures that have the same shape but that might have different sizes are called similar .
- In similar figures
- One is the enlargement of the other
- Angles in corresponding position are congruent.
- Corresponding sides have the same ratio (proportional)

Similarities on triangles

- SAS similarity theorem
- If the two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

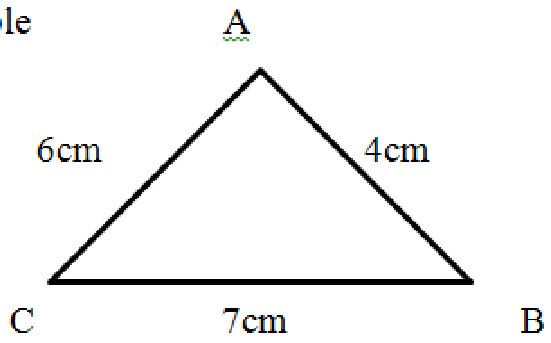
- <u>AA similarity theorem</u>

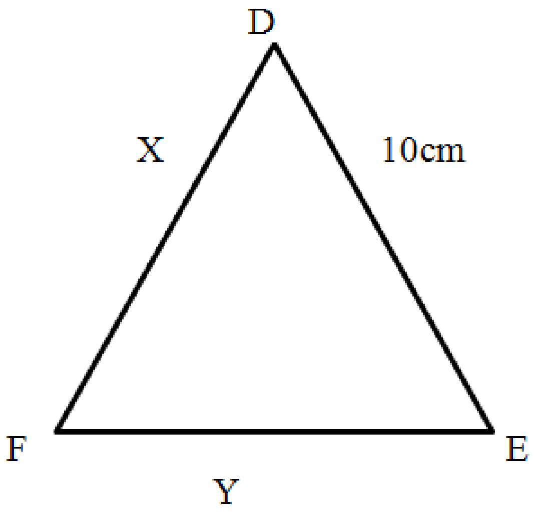
If two angles of one triangle are equal in measures to two angles of another triangle, then the two triangles are similar

- <u>SSS similarity theorem</u>

If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar

Example





Find the length of DF and FE if triangle ABC is similar to DEF

Solution

Since the corresponding sides of similar triangle are proportional (have the same ratio)

That is, $\Delta ABC \sim \Delta DEF$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

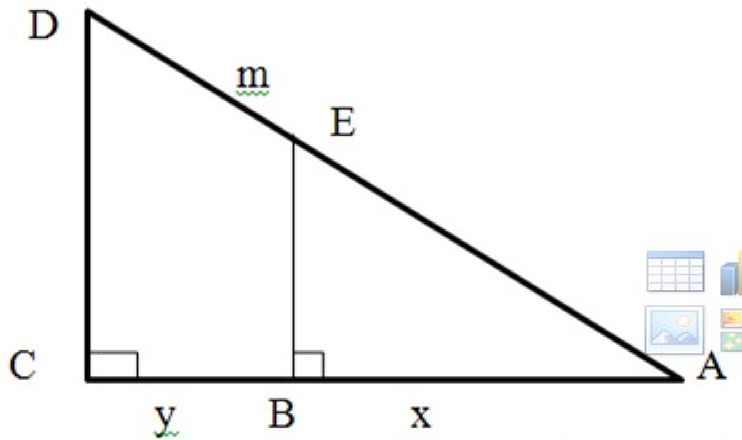
$$\frac{4cm}{10cm} = \frac{7cm}{y} = \frac{6cm}{x}$$

$$\frac{4cm}{10cm} = \frac{7cm}{y} \qquad \text{and} \qquad \frac{4cm}{10cm} = \frac{6cm}{x}$$

$$y = \frac{70cm}{4} \text{ and } x = \frac{60cm}{4}$$

$$y = 17.5\ cm \text{ and } x = 15cm$$

Example 2:- find x, y and m using the triangle below given DC=9cm, EB=3cm and AE=5cm



Here you have two triangles $\Delta ABE \equiv \Delta ACD$

- <A is common angle <B congruent right angles

Therefore by AA similarity theorem

- <A is common angle $<B \equiv <C$ congruent right angles

Therefore $\Delta ABE \equiv \Delta ACD$ by AA similarity theorem

- $\frac{AB}{AC} = \frac{BE}{CD} = \frac{AE}{AD}$

- $\frac{x}{x+y} = \frac{3cm}{9cm} = \frac{5cm}{m+5}$
- A triangle ABE is the right angle triangle and we can find the measure of AB using Pythagoras theorem
- $(5cm)^2 = (3cm)^2 + (x)^2$

  x^2=16
- $x = 4cm$

- A triangle ABE is the right angle triangle and we can find the measure of AB using Pythagoras theorem
- =16

- $\frac{4cm}{4+y} = \frac{3m}{9cm}$ and and $\frac{3cm}{9cm} = \frac{5cm}{m+5}$
- $y = 8cm$ and $m = 10cm$ and

Theorems on similar plane figures

- Ratio of perimeter and ratio of areas of similar plane figures
- Theorem

If the ratio of the lengths of the corresponding sides of two similar polygons is K

- That is ($\frac{s_1}{s_2} = k$), then
- The ratio of their perimeter is K ($\frac{p1}{p_2} = k$)
- The ratio of their area is k^2 ($\frac{A1}{A2} = K^2$)

- Example 1 :- The area of two similar triangles are $25cm^2$ and $16cm^2$,then what is the ratio of their perimeter

- Example 1 :- The area of two similar triangles are 25and ,then what is the ratio of their perimeter

- $\frac{A1}{A2} = K^2$ where k is ratio of their corresponding sides
- where k is ratio of their corresponding sides
- $\frac{25cm^2}{16cm^2} = K^2$
- $\frac{5}{4} = k = \frac{p1}{p2}$
- =

Example 2:- Two triangles are similar. A side of one is 2cm long. The corresponding sides of the other is 6cm long. What is the ratio of

a) Their perimeter? b) Their areas?

S1=2cm S2=6cm

=

Example 2:- Two triangles are similar. A side of one is 2cm long. The corresponding sides of the other is 6cm long. What is the ratio of

a) Their perimeter? b) Their areas?

S1=2cm S2=6cm

$$\frac{s_1}{s_2} = \frac{p_1}{p_2} = \frac{1}{3} = k$$

$$\frac{A_1}{A_2} = \left(\frac{s1}{s2}\right)^2 = k^2$$

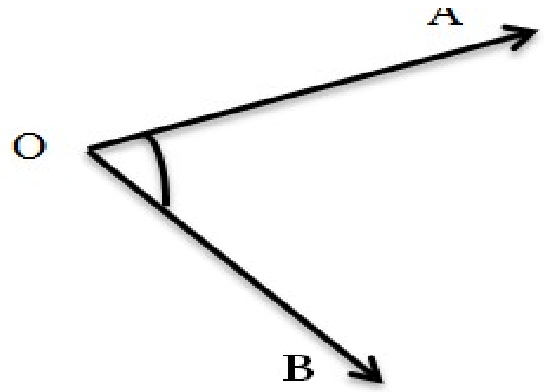
$$\frac{A_1}{A_2} = \frac{1}{9} = k^2$$

Unit 5
Geometry and Measurement

Further on trigonometry

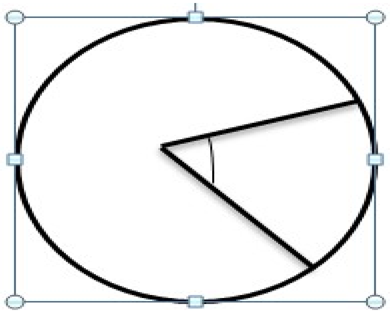
- Radian and degree measure of angles

An angle is formed by two rays with a common end point



Radian measure of angles

- Radian measure
- 1 radian is the central angle subtended by an arc equals to the radius



- 1 revolution in terms of radian measure is equal to a central angle subtended by $2\pi r$
- That is ,1 revolution $=360^0$
- $1 \text{ rad } = \frac{180^0}{\pi} \approx 57.3^0$
- $1\pi rad = 180^0$
- One can convert radian measure into degree and degree measure to radian
- To convert degree to radian multiply with $\frac{\pi}{180^0}$
- To convert radian into degree measure multiply with $\frac{180^0}{\pi}$

- 1 revolution in terms of radian measure is equal to a central angle subtended by 2
- That is ,1 revolution =
- 1 rad =
- 1
- One can convert radian measure into degree and degree measure to radian
- To convert degree to radian multiply with
- To convert radian into degree measure multiply with

- Example : convert into radian

a) 90^0

$= 90^0 \, x \, \frac{\pi}{180}$

$= \frac{\pi}{2}$

b) 270^0 c) 120^0

- Example : convert into degree

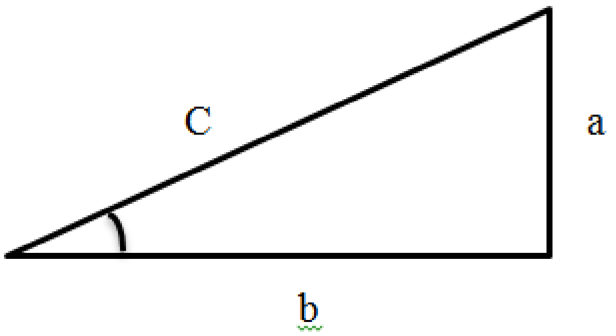
a) $\frac{5}{6}\pi \, rad$ b) $\frac{3\pi}{4} \, rad$

$= \frac{5}{6}\pi \, rad \, \frac{180^0}{\pi}$

$= 150^0$

Trigonometric ratio to solve right angled triangle

Using the figure below



a is opposite to the angle
b is adjacent to the angle
c is hypotenuse to the angle

sine $=\dfrac{opposite}{hypotenuse}$ cosine $=\dfrac{adjacent}{hypotenuse}$ tangent $=\dfrac{opposite}{adjacent}$

$sin\theta = \frac{a}{c}$ $cos\theta = \frac{b}{c}$ $tan\theta = \frac{a}{b}$

- Note the trigonometric values of an acute angle is between 0 and 1

Every angles can be expressed in terms of an acute angle

- Trigonometric values of obtuse angle is given as

if θ is obtuse angle

$$sin\theta = \sin(180 - \theta)$$

= $$cos\theta = -\cos(180 - \theta)$$

=) $$tan\theta = -\tan(180 - \theta)$$

- From the above relationship cosine and tangent of an obtuse angle is negative

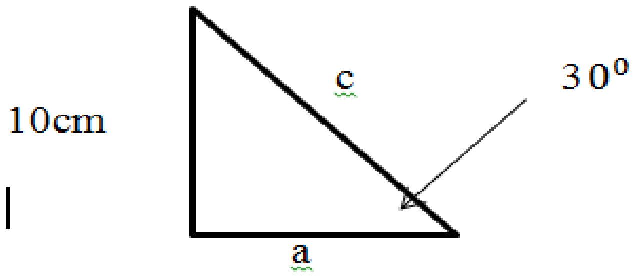
Example 1: if $cos15^0 = x$, what is $\cos 165^0$

coscos $$\cos 165^0 = -\cos(180^0 - 165^0)$$

= $$= -cos15^0$$

= $$= -x$$

Example 2: find the missing sides of the right angled triangle



$$\text{sine} = \frac{opposite}{hypotenuse}$$

$$\sin 30^0 = \frac{10}{c}$$

$$c = \frac{10}{\frac{1}{2}}$$

c=20cm

$$\cos 30^0 = \frac{adjacent}{hypotenuse}$$

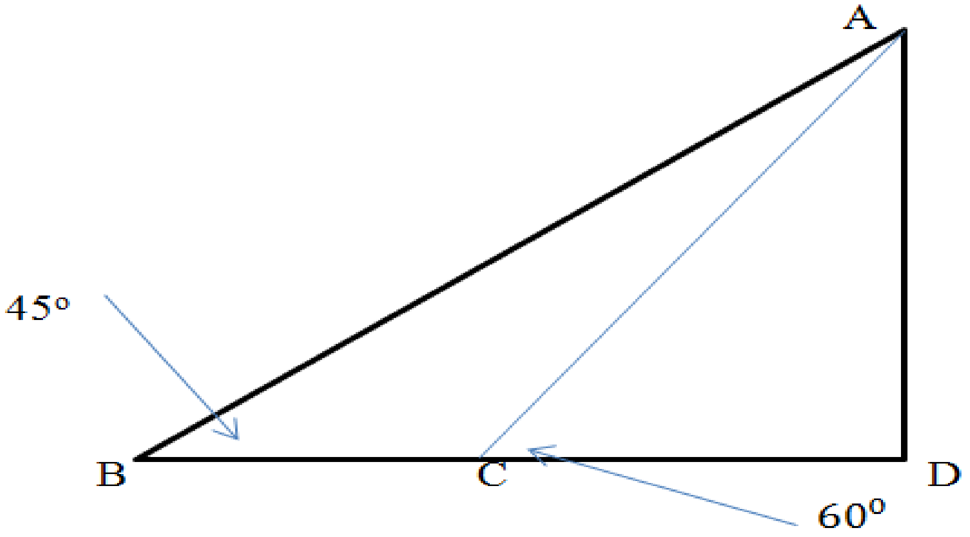
$$\cos 30^0 = \frac{a}{c}$$

$$a = \cos 30^0 x c$$

$$a = \frac{\sqrt{3}}{2}\ x 20cm$$

$$a = 10\sqrt{3}\ \text{cm}$$

Example 3: using the firure below find CD,AD,ACand AB



Given the right angle triangle ABC right angled at D
BC=12cm, then find the length of AD?

Let CD = xcm and AD = ycm

Let CD = xcm and AD = ycm

$tan45^0 = \frac{AD}{BD}$ $\qquad$ $tan45^0 = \frac{Ycm}{12cm+X}$

$1 = \frac{Ycm}{12cm+X}$ $\qquad$ $12cm + Xcm = Ycm$

$tan60^0 = \frac{AD}{CD}$ $\qquad$ $tan60^0 = \frac{Ycm}{Xcm}$ $\qquad$ $\sqrt{3} = \frac{Ycm}{Xcm}$

$\sqrt{3}Xcm =$Ycm

Ycm

Using the two equations

Using the two equations

$\sqrt{3}Xcm$=Ycm and $12cm + Xcm = Ycm$

=Ycm and

$$12cm + Xcm = \sqrt{3}Xcm$$

$\sqrt{3}Xcm -$ Xcm=12cm

Xcm=12cm

X($\sqrt{3} - 1$)=12cm

X(=12cm

X=$\frac{12}{\sqrt{3}-1}$ cm

X=cm

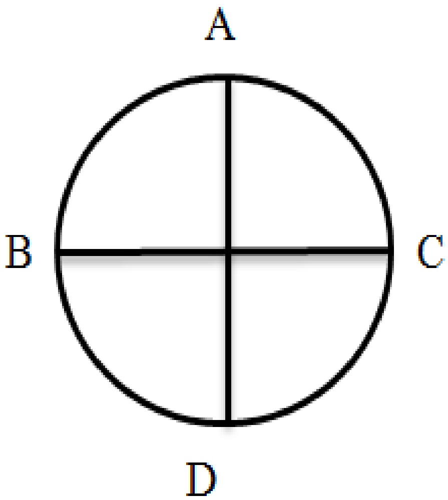
Circles

- Symmetrical properties of circles

If the figure can be folded or divided into half so that the two halves match exactly then such a figure is called symmetrical figure

If you recall regular polygon is also symmetrical figure and line of symmetry is equal to the number of sides

Symmetrical figure is the figure that has at least one line of symmetry .Thus a circle has infinitely many line of symmetry



From the above figure AD and BC are diameter of the circle if you fold along these lines one half overlaps with the other half and we call it is a line of symmetry

For a circle

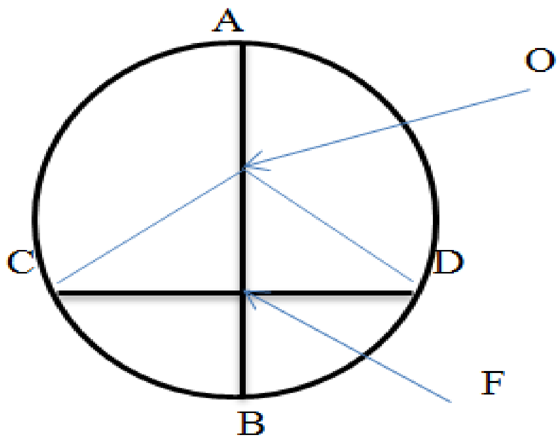
Chord is the line segment whose end point is on the line

If two chords in the given circle are equal,then they are equidistance from the center and the perpendicular distance is also a perpendicular bisector

Note distance from a point to a line is a perpendicular distance

Example

In the figure below,the circle has radius 5cm and the length of the chord is 8cm and AB is a diameter ,then what is distance of A from the chord (the length of the segment AF)



AB is perpendicular bisector of CD

Thus CF=FD=4cm

- Using the figure above triangle CFO is a right angle triangle

$$CF^2 + FO^2 = CO^2$$

$$4^2 + FO^2 = 5^2$$

$$5^2 - 4^2 = FO^2$$

FO=3cm

Therefore distance of A from CD is equal to

OF+AO=AF

3cm+5cm=AF

AF=8cm

Using the figure above triangle CFO is a right angle triangle

FO=3cm

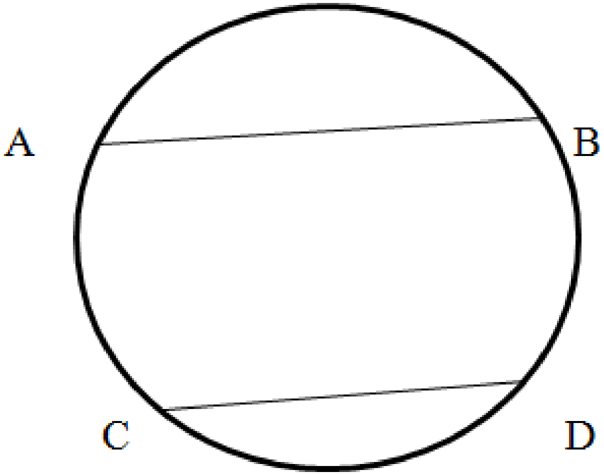
Therefore distance of A from CD is equal to

OF+AO=AF

3cm+5cm=AF

AF=8cm

Example 2:in the figure below chord AB is 12cm and CD is 15cm,then which one is nearest to the center

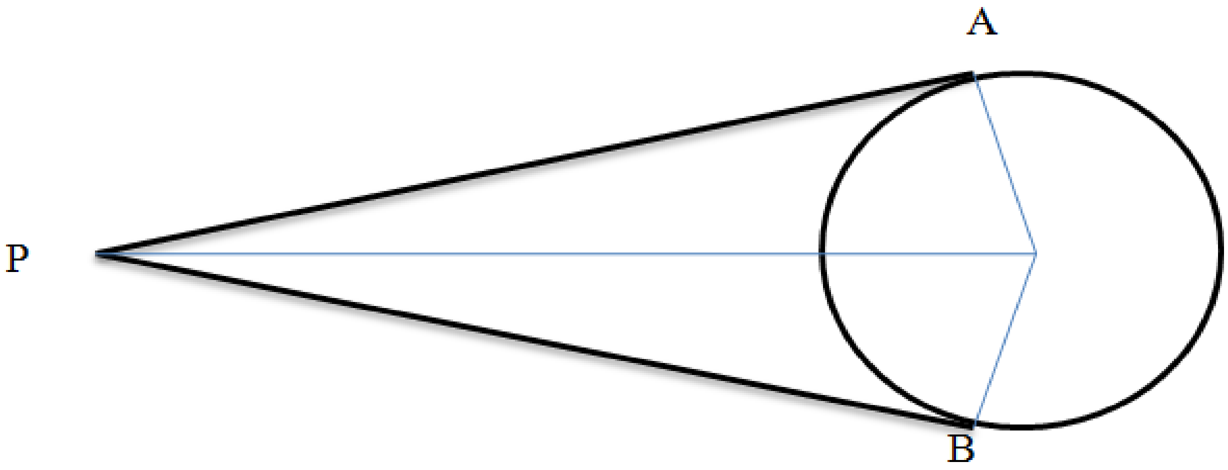


The largest chord of the circle is always nearest to the center .therefore AB is nearest to the center
Note the largest chord of the circle is diameter

Note if two tangents are drawn from to a circle from an external point,then

i. The tangents are equal in length

ii. The line segment joining the center to the external point bisects the angle between the tangents



AP=PB