GRADE – 12 PHYSICS

HANDOUT FOR SECOND SEMESTER LESSONS

OROMIA EDUCATION BUREAU

In Collaboration with

EXCEL ACADEMY

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UNIT 5

STEADY ELECTRIC CURRENT AND CIRCUIT PROPERTES

5.1. **Electric current (I)**

Electric current is the rate at which charge flows perpendicular a surface area (This area could be the cross- sectional area of a wire). If ΔQ is the amount of charge that passes through this area in a time interval Δt, the average current I_{av} is equal to the charge that passes through A per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

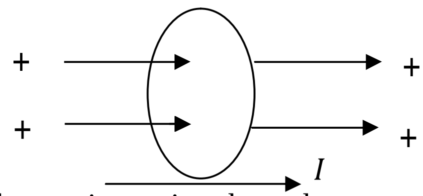

Figure 5.1 Charges in motion through an area *A*.

If the rate at which charge flows varies in time, then the current varies in time; we define the instantaneous current *I* as the differential limit of average current:

$$I = \frac{dQ}{dt}$$

The SI unit of current is the ampere (A): 1A $=\frac{1C}{1S}$

The charges passing through the surface can be positive or negative or both. **It is conventional to assign to the current the same direction as the flow of positive charge.** In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, the direction of the current is opposite the direction of flow of electrons.

Microscopic Model of Current

We can relate current with the motion of charge carriers by describing microscopic model of conduction in a metal.

N- number of charge carriers in the part of the conductor

$n = \frac{N}{V}$ - charge carrier density

Δx -the length of the part of the conductor.

Δt - the time taken for the charge carriers to move the distance Δx.

V - the volume of the part of the conductor is given as

$$V = A\,\Delta x$$

v_d -the average drift speed of the charge carriers.

Thus, the distance covered by charge carriers in time Δt is:

$$\Delta x = v_d\,\Delta t$$

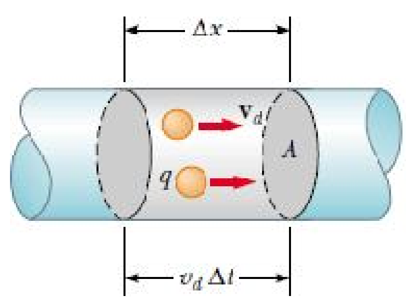

Figure 5.2 A section of a uniform conductor of cross-sectional area *A*.

Inserting Eq. (1) into (2), we have

$$V = A\,v_d$$

The total number of charge in the part of the conductor is

$$N = nV = nAv_d$$

The total charge that cross the right cross-sectional area of part of the conductor in time in Δt is

$$\Delta Q = Nq = nAqv_d \, \Delta t \,,$$

where q is the charge of each charge carrier. The average current in the conductor is thus

$$I = \frac{\Delta Q}{\Delta t} = nAqv_d \,.$$

Example 5.1

A wire with a circular cross-sectional area 10^{-6} m^2 carries current of 1A. Find the drift speed of electrons if the number of mobile charge carriers is 10^{28} electrons per cubic meter.

Solution

$A=10^{-6}\ m^2$ $\quad$ $I=1A$ $\quad$ $n=10^{28}$ electrons per cubic meter $\quad$ $q=e=1.6 \times 10^{-19} C$

$$v_d = \frac{I}{nAe} = \frac{1}{10^{28} \times 10^{-6} \times 1.6 \times 10^{-19}} = 6 \times 10^{-4} m/s$$

Resistance and Ohm's law

Consider a conductor of cross-sectional area A carrying a current I. The current density **J** in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density

$$\boldsymbol{J} = \frac{I}{\boldsymbol{A}} = nq\mathbf{v}_d,$$

where **J** has SI units of A/m^2 and in general, current density is a vector quantity.
From this equation, we see that current density is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

Ohm's law

A current density **J** and an electric field **E** are established in a conductor whenever a potential difference is maintained across the conductor. In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E},$$

where the proportionality constant σ is called conductivity of the conductor. σ is independent of the field producing the current. Materials that obey Ohm's law are said to be ohmic.

Consider a straight wire of uniform cross sectional area A and length L as shown. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the potential c through the relationship

$$\Delta V = El$$

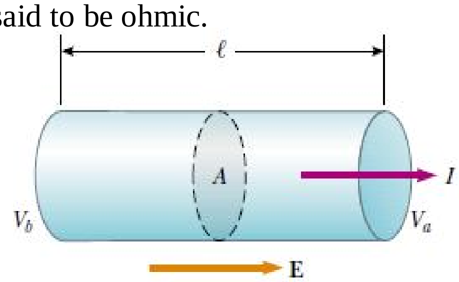

Figure 5.3 A uniform conductor of length l and cross-sectional area *A*.

Therefore, we can express the magnitude of the current density in the wire as

$$J=\sigma E=\sigma\frac{\Delta V}{l}$$

Applying the relation J=I/A, we have

$$\frac{I}{A}=\frac{\sigma}{l}\Delta V$$

Or

$$\Delta V=\left(\frac{l}{A\sigma}\right)I=RI$$

The quantity $R=\frac{l}{A\sigma}$ is called the resistance of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

$$R=\frac{\Delta V}{I}$$

This is also Ohm's law used mostly for practical purpose. Resistance has SI units of volts per ampere. One volt per ampere is defined to be one ohm (Ω').

$$1\Omega=\frac{1V}{1A}$$

The inverse of conductivity is resistivity ρ

$$\rho=\frac{1}{\sigma},$$

where ρ has the units ohm-meters (Ω.m). Because $R=\frac{l}{\sigma A}$, we can express the resistance of a uniform block of material along the length l as

$$R=\frac{\rho l}{A}$$

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. The resistance of a sample depends on geometry (a given cylindrical conductor such as a wire is proportional to its length and inversely proportional to its cross-sectional area.) as well as on resistivity.

Example 5.2

What is the resistance of a cubical block of iron having sides of length 1.2cm?

Solution:

$L=1.2\times10^{-2}$m,

$A=L^2=1.44\times10^{-4}m^2$

$$R=\frac{\rho l}{A}=\frac{9.68\times10^{-8}\Omega m\times1.2\times10^{-2}m}{1.44\times10^{-4}m^2}=8.07\times10^{-6}\,\Omega$$

Resistance and Temperature

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho=\rho_0(1+\alpha\Delta T)$$

where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the temperature coefficient of resistivity. We see that the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

We can write the variation of resistance as

$$R = R_0(1 + \alpha\Delta T)$$

Note that the value of α is positive for conductors and negative for semiconductors.

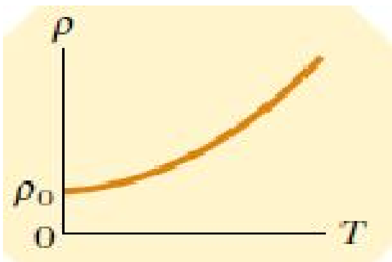

Figure 5.4 Resistivity versus temperature for a metal such as copper

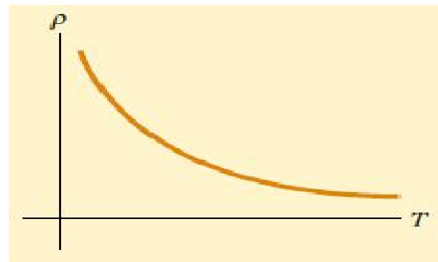

Figure 5.5 Resistivity versus temperature for a pure semiconductor, such as silicon or germanium

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Superconductors

There is a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature T_c, known as the **critical temperature**. These materials are known as **superconductors**.

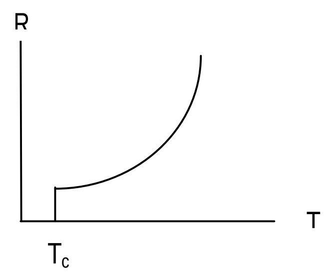


Figure 5.6 Resistance versus temperature superconductor

Electrical Energy and Power

Consider a simple circuit shown in the Fig. As the charge moves from a to b through the battery, it gains electric potential energy.

$$\Delta U = \Delta V \Delta Q = \Delta V(I\Delta t)$$

However as the charge moves through the resistor from c to d it losses potential energy. The power delivered to the resistor (the rate at which the charge ΔQ losses potential energy in going through the resistor) is

$$P = \frac{W}{\Delta t} = \frac{\Delta U}{\Delta t} = \Delta V\frac{\Delta Q}{\Delta t} = \Delta V I$$

This can also be put in the forms

$$P = I^2 \quad \text{or} \quad P = \frac{(\Delta V)^2}{R}$$

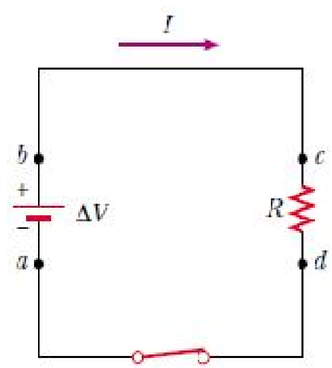


Figure 5.7 A circuit consisting of a resistor of resistance *R* and a battery

Example 5.3

A (400, 200V) bulb is connected to a 120V source of potential difference. Find the current drawn by the bulb.

Solution:

P=200W $\Delta V = 120V$ from theses values we get the resistance $R = \frac{\Delta V^2}{P}$=100Ω.

Now the bulb is connected to a $\Delta V' = 120$

Applying Ohm's law $I = \frac{\Delta V'}{R} = 1.2A$

Electromotive Force (*emf*)

A mechanism that maintains current in a dissipative medium is called emf. It is the energy source that maintains current in a circuit. The common characteristic of all sources of emf is to effect a charge separation. This separation is accomplished against electrostatic force created by the charge separation. The common sources of emf include: electric cell, generator and solar cell.

Consider an electric cell shown in the Fig.

E is the electrostatic field due to the charge separation.

E$_n$ is the non electrostatic field. It exists only in a very restricted space (inside a battery).

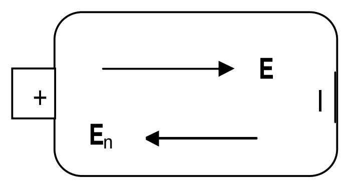


Figure 5.8 electric cell

Emf (ε) is the work done per unit positive charge by **E**$_n$ from the **–** to the + plate.

Terminal voltage (ΔV) is the work done per unit charge by **E** from the + to the **–** plate.

- For an open circuit or for a battery with no internal resistance terminal voltage and emf are equal ($\Delta V = \varepsilon$).
- For a source of emf that has internal resistance (r) and connected to external resistor (load) R, we have $\Delta V = \varepsilon - Ir$
  Using the relation $\Delta V = IR$, we get

$$\varepsilon = IR + Ir$$

The current flowing in the circuit is give as

$$I = \frac{\varepsilon}{R+r}$$

One can also readily see that

$$\varepsilon I = I^2R + I^2r$$

The term in the left hand side is the total power, the first term in the right hand side is the useful power (dissipated in R) and the second term in the right hand side is the wasted power (dissipated in r)2.

Example 5.4.

Show that the maximum power delivered to the load resistor R occurs when the load resistance matches to the internal resistance.

Solution:

$$P=I^2R=\frac{\varepsilon^2 R}{(R+r)^2}$$

$$\Rightarrow \frac{dP}{dR} = 0$$

$$\Rightarrow \frac{\varepsilon^2 - \frac{2\varepsilon^2 R}{R+r}}{(R+r)^2}=0$$

$$\Rightarrow R=r$$

Example 5.5

An electric cell with $\varepsilon = 12V \; and \; r = 1\Omega$ is connected with a 5 Ω load resistor. Calculate the current in the circuit.

Solution:

$$I=\frac{\varepsilon}{R+r}=\frac{12V}{5\Omega+1\Omega}=2A$$

Resistors in series and in parallel

Series connection

For a series combination of two resistors show in Fig. 5.9, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the sar

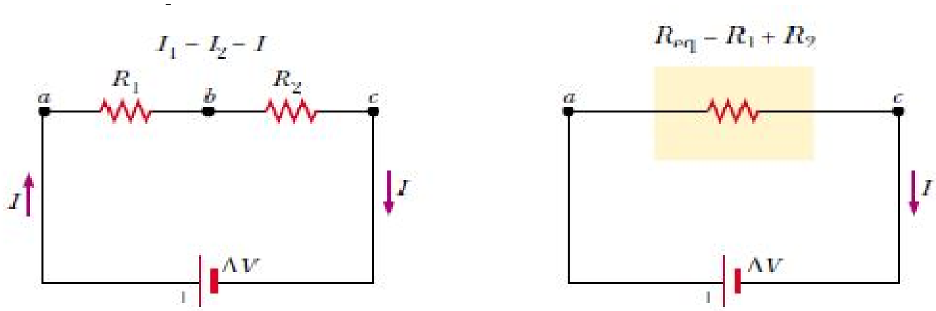


Figure 5.9 Resistors in series

The potential difference applied across the series combination of resistors will divide between the resistors.

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

The resistors can be replaced with a single resistor having an equivalent resistance

$$R_{eq} = R_1 + R_2$$

The potential difference across the battery is also applied to the **equivalent resistance** Req

$$\Delta V = IR_{eq}$$

The equivalent resistance of three or more resistors connected in series is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Parallel connection

For a parallel connection of two resistors shown in Fig. 5.10 because electric charge is conserved, the current I that enters point *a* must equal the total current leaving that point:

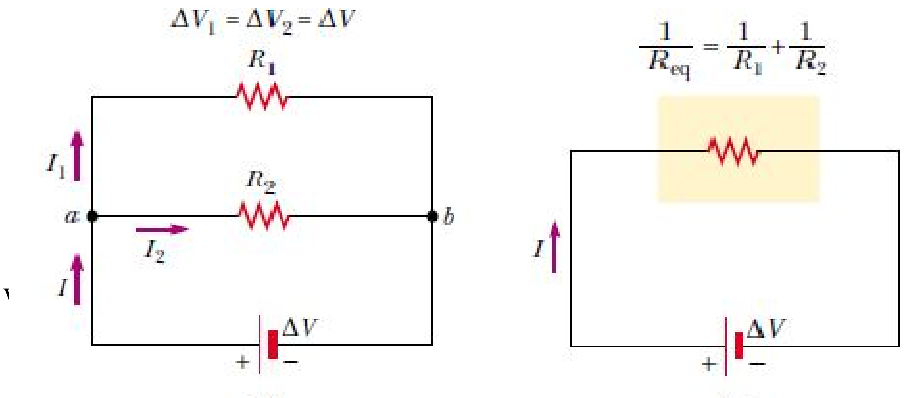


Figure 5.10 Parallel connection of resistors

As can be seen from Fig. 5.10, both resistors are connected directly across the terminals of the battery. Therefore, when resistors are connected in parallel, the potential differences across the resistors is the same. Because the potential differences across the resistors are the same, the expression $\Delta V = IR$ gives

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

where Req is an equivalent single resistance which will have the same effect on the circuit as the two resistors in parallel. For two resistors in parallel their equivalent is given as

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

For three or more resistors in parallel, we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

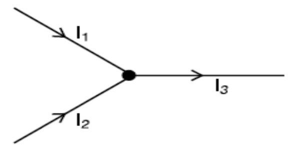
5.2. Kirchhoff's Rules

As we saw in the preceding section, simple circuits can be analyzed using the expression $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop. The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff 's rules:

1. **Kirchhoff's point (junction) rule**. The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out}$$

It is a statement of conservation of charge

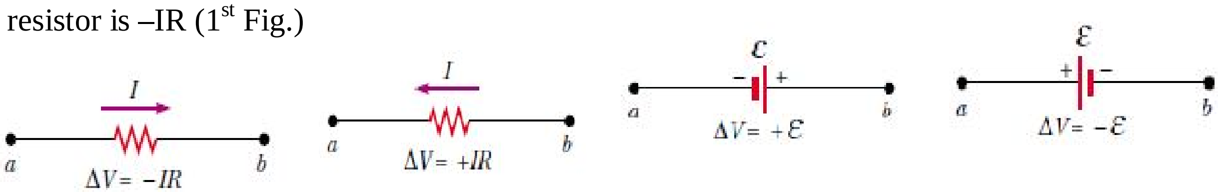


2. **Kirchhoff's loop rule**. The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{closed \\ loop}} \Delta V = 0$$

It follows from conservation of energy. You should note the following sign conventions when using the second rule:

• Because charges move from the high-potential end of a resistor toward the low potential end, if a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is –IR (1st Fig.)



• If a resistor is traversed in the direction *opposite* the current, the potential difference ΔV across the resistor is +IR (2nd Fig.).

• If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from – to+), the potential difference ΔV is $+\varepsilon$ (3rd Fig.). The emf of the battery increases the electric potential as we move through it in this direction.

• If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from + to -), the potential difference ΔV is $-\varepsilon$ (4th Fig.). In this case the emf of the battery reduces the electric potential as we move through it.

In all cases, it is assumed that the circuits have reached steady-state conditions (left for enough long time)—that is, the currents in the various branches are constant. Any capacitor acts as an open branch in a circuit; that is, the current in the branch containing the capacitor is zero under steady-state conditions.

Example 5.6
Find the current in the circuit shown in Fig. 5.11

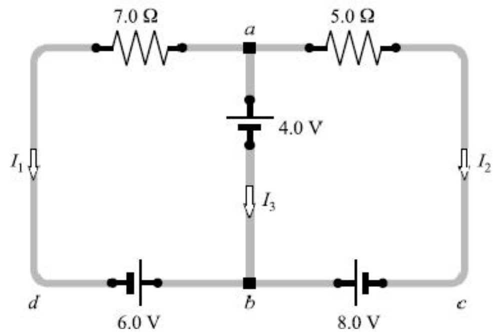


Figure 5.11

No special care be taken in assigning the current directions, since those chosen incorrectly will simply give negative value

$$\text{Current into } b = \text{current out of } b$$
$$I_1 + I_2 + I_3 = 0$$

Next we apply the loop rule to loop *adba*. In volts,

$$-7.0\,I_1 + 6.0 + 4.0 = 0 \qquad \text{or} \qquad I_1 = \frac{10.0}{7.0}\text{A}$$

(Why must the term $7.0\,I_1$ have a negative sign?) We then apply the loop rule to loop *abca*. In volts,

$$-4.0 - 8.0 + 5.0\,I_2 = 0 \qquad \text{or} \qquad I_2 = \frac{12.0}{5.0}\text{A}$$

(Why must the signs be as written?)
Now we return to Eq. (*1*) to find

$$I_3 = -I_1 - I_2 = -\frac{10.0}{7.0} - \frac{12.0}{5.0} = \frac{-50 - 84}{35} = -3.8\text{ A}$$

The minus sign tells us that I_3 is opposite in direction to that shown in the figure.

RC Circuits

In DC circuits containing capacitors, the current is always in the same direction but may vary in time. A circuit containing a series combination of a resistor and a capacitor is called an RC circuit.

Charging a Capacitor

Let us assume that the capacitor in this circuit is initially uncharged. There is no current while switch S is open (Fig. 5.19b). If the switch is closed at t=0, however, charge begins to flow and the capacitor begins to charge. Current exist in the wire connected to the capacitor until the capacitor is fully charged.

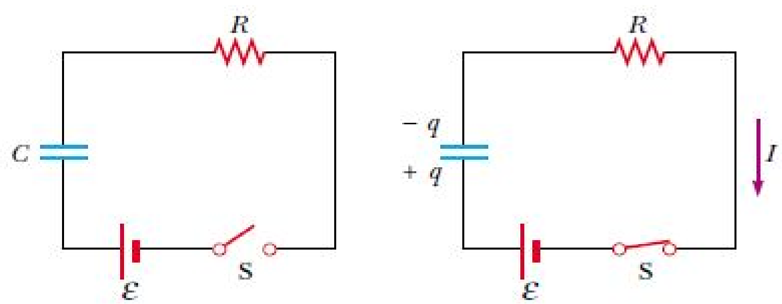


Figure 5. 12 RC circuit

Applying Kirchhoff's loop rule, we have

$$\varepsilon - \frac{q}{c} - IR = 0 \qquad \text{(I and q are instantaneous values)}$$

Now employing this equation, the initial current and the maximum charge in the capacitor are: Since the capacitor is initially has no charge, we have

$$\varepsilon - 0 - I_0 R = 0 \qquad \Rightarrow \quad I_0 = \frac{\varepsilon}{R} \text{ is the initial current}$$

When the capacitor is fully charged the current is zero, then we have

$$\varepsilon - \frac{Q}{c} - 0 = 0 \qquad \Rightarrow Q = \varepsilon c \text{ is the maximum charge on the capacitor}$$

To find q(t) and I(t), inserting $I(t) = \frac{dq}{dt}$, we have

$$\varepsilon - \frac{q}{c} - R\frac{dq}{dt} = 0$$

To find an expression for q, we solve this separable differential equation. We first combine the terms on the right-hand side:

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{RC} - \frac{q}{RC} = -\frac{q - C\mathcal{E}}{RC}$$

Now we multiply by dt and divide by $q - C\mathcal{E}$ to obtain

$$\frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that $q = 0$ at $t = 0$, we obtain

$$\int_0^q \frac{dq}{(q - C\mathcal{E})} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

From the definition of the natural logarithm, we can write this expression as

$$q(t) = C\mathcal{E}(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

One can get the current by taking the derivative of q with respect to time

$$I(t) = \frac{dq}{dt} = \frac{Q}{Rc}e^{-\frac{t}{Rc}} = I_0 e^{-\frac{t}{Rc}}$$

The quantity Rc is called time constant (τ) of the circuit. It represents the time it takes the current to decrease to 1/e of its initial value. In a time τ; $I = I_0/e = 0.368 I_0$ and in time τ the charge increases from 0 to $Q(1-e^{-1}) = 0.632Q$

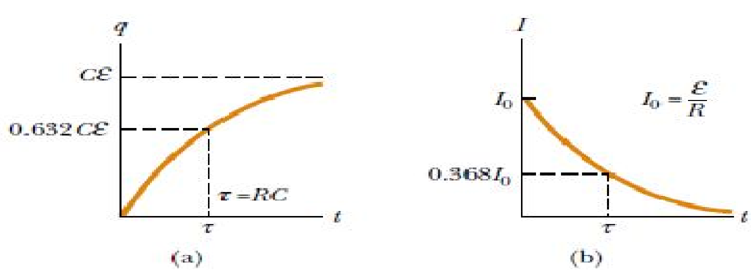


Figure 5. 13 Plots of q and I versus time for charged RC circuit

The energy output of the battery as the capacitor is fully charged is $Q\varepsilon = C\varepsilon^2$. After the capacitor is fully charged, the energy stored in the capacitor is $\frac{1}{2}Q\varepsilon = \frac{1}{2}C\varepsilon^2$, which is just half the energy output of the battery.

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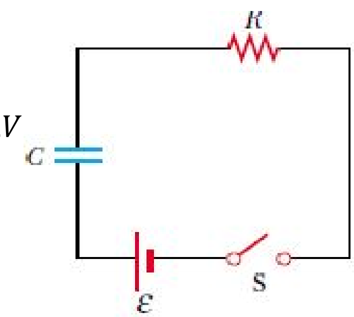


Example 5.7

For the RC circuit shown in Figure, R=8x10^{5}Ω, C=5μF and $\varepsilon = 12V$

If the switch S is closed at t=0, find

a) τ

b) Q_{max} and I_{max}

c) q(t) and I(t)

Solution:

a) τ=Rc=4se

b) $Q_{max}=\varepsilon c=60\mu C$ and $I_{max}=I_0=\frac{\varepsilon}{R} = 15\mu A$

c) q(t)=Q(1-$e^{-\frac{t}{\tau}}$) =60μC(1-$e^{-\frac{t}{4s}}$) and $I = I_0 e^{-\frac{t}{\tau}} = 15\mu A e^{-\frac{t}{4s}}$

Discharging a Capacitor

Now consider the circuit shown in Figure 5. 14, which consists of a capacitor carrying an initial charge Q , a resistor, and a switch. If the switch is closed at t =0, the capacitor begins to discharge through the resistor. At some time t the Kirchhoff's loop equation for the circuit can be written as

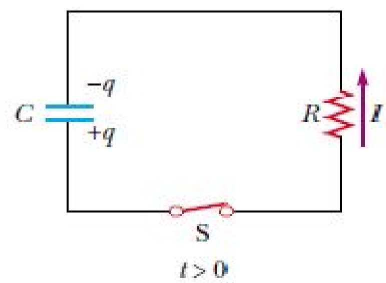


Figure 5.14 Discharged RC circuit

$$-\frac{q}{C} - IR = 0$$

When we substitute $I = dq/dt$ into this expression, it becomes

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}dt$$

Integrating this expression, using the fact that $q = Q$ at $t = 0$ gives

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$q(t) = Qe^{-t/RC}$$

Ot

$$q(t)=\varepsilon c e^{-t/RC}$$

Differentiating this expression with respect to time gives the instantaneous current as a function of time

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right) = -\frac{Q}{RC}\,e^{-t/RC}$$

where $\frac{Q}{Rc} = I_0$ is the initial current. The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged. In the time constant $\tau = Rc$: q=Q/e=0.37Q and I= $\frac{-I_0}{e}$ =0.37I_0.

The time taken for the capacitor to lose half its charge is known as half-life ($t_{1/2}$) of the decay process. In this case q=Q/2, then we have

$$\ln {}^{q}/_{Q} = -{}^{t}/_{Rc}$$

Or

$$\ln {}^{Q/2}/_{Q} = -{}^{t_{1/2}}/_{Rc}$$

$$\ln 2 = {}^{t_{1/2}}/_{Rc}$$

Thus, the half life is $t_{1/2} = Rc \ln 2 = 0.693Rc$

Example 5. 8

A 2nF capacitor with initial charge $5.1\mu C$ is discharged through a 1.3KΩ resistor.

a) Calculate the half life and the time constant of the circuit.

b) What charge remains on the capacitor after 8 μs.

c) Calculate the current in the resistor 9 μsafter the resistor is connected across the terminals of the capacitor.

d) What is the maximum current?

Solution:

a) $t_{1/2}$=Rc ln 2=(1.3KΩx2nF) ln2=

b) $\tau = Rc$ ==(1.3KΩx2nF)=

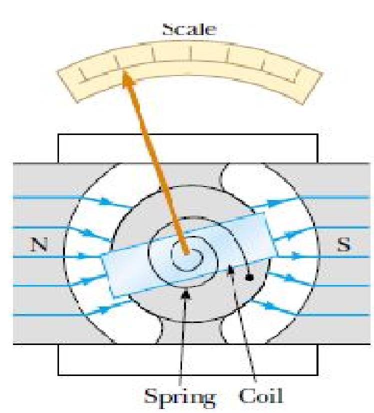
c) I=-$I_0e^{-t/Rc}$= $\frac{-Q}{Rc}e^{-t/Rc} = \frac{-5.1\mu C}{1.3\text{K}\Omega\text{x2nF}}e^{-9\mu s/1.3\text{K}\Omega\text{x2nF}}$

d) $I_0 = \frac{Q}{Rc}$

Exercise

A $50\mu F$ capacitor is discharged through 10 KΩ resistor. How long will it take for the potential difference across the capacitor to fall 40% of its initial value?

Measuring Instruments (Electrical Meters)

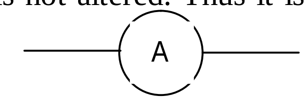


The galvanometer

The main component in analog meter for measuring current l galvanometer. It make use of magnetic effect of current for i movement only.

The Ammeter

A device that measures current is called ammeter. It must be connected in series with other elements in the circuit. It is necessary to observe the polarity of the instrument. Ideally an ammeter should have zero resistance so that the current measured is not altered. Thus it is very sensitive.



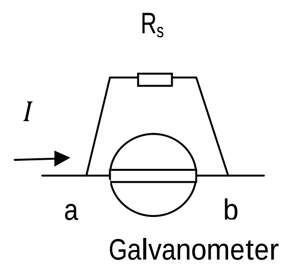
A typical galvanometer is often not suitable for use as an ammeter, because:

- It has resistance about 50Ω. (10Ω − 100Ω)
- It gives full-scale deflection for currents on the order of 5mA ($10\mu A$-10mA) is very sensitive.

A galvanometer can be converted to a useful ammeter by placing a small resistance, shunt resistor (R_s), in parallel with the galvanometer so that the current to be measured is directed to the shunt resistor.

Example 5.9

What is the value of Rs to modify a galvanometer (R_g=20 and I_g=1mA) to an ammeter with range of 0 to 10A.



$I=I_g+I_s$ $\Rightarrow I_s=I-I_g=10A-0.001=9.999A$

$V_{ab}=I_gR_g=I_sR_s$

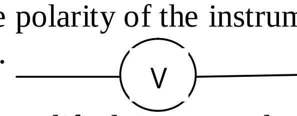
$$R_s = R_g\frac{I_g}{I_s} = 20\Omega\frac{0.001A}{9.999A} = 0.002\Omega$$

The equivalent resistance R_{eq} of the instrument is

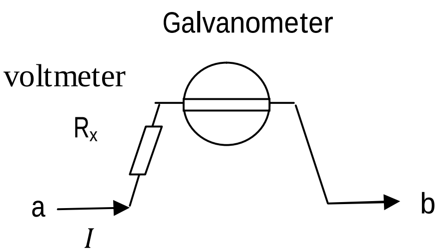
$$\frac{1}{R_{eq}} = \frac{1}{R_g} + \frac{1}{R_s} = 0.002\Omega$$

The voltmeter

A device that measures potential difference is called a **Voltmeter**. The voltmeter should be connected in parallel with a resistor. It is necessary to observe the polarity of the instrument. An ideal volt meter has infinite resistance so that no current exist in it.



A galvanometer can measure voltage up to 20mv. It can be modified into a voltmeter by connecting a high resistance R_x in series with the moving coil.



Suppose we wish to modify the galvanometer (20 Ω, 1mA) for use as voltmeter with range of 0 to 10V.

When Vab=10V and the current is 1mA,we have

$V_{ab}=I(R_g+R_x)$

Thus, the value of R_x is

$$R_x = \frac{V_{ab}}{I} - R_g = \frac{10V}{0.001A} - 20\Omega = 9980\Omega$$

The equivalent resistance R_{eq} is

$R_{eq}=R_g+R_x=10{,}000\ \Omega$

A voltmeter and an ammeter can be use together to measure **resistance and power**.

Exercise

1. A galvanometer of full-scale deflection 5mA is to be converted into a 0-10a Ammeter. If the coil has a resistance of 50 Ω, what value of shunt must be fitted?
2. How can the galvanometer in the above problem converted to a 0-10V voltmeter?

The ohmmeter

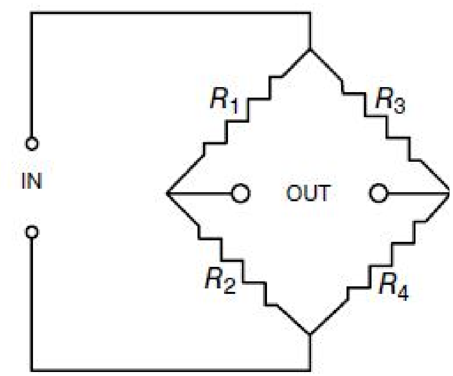
You can use ammeter and voltmeter to find to find out the resistance of a component plotting its V-I characteristic.

Although not a precision instrument, the ohmmeter is a useful device for rapid measurement of resistance. It consists of ammeter, a resistor and a source.

The Wheatstone bridge and the Potentiometer

The Wheatstone bridge

The fundamental concept of the bridge circuit is the two voltage dividers (R_1, R_2 and R_3 , R_4) are both supplied by the same input. A galvanometer is connected between the output terminals and is used to monitor the current flowing from one voltage divider to the other. If the two voltage dividers have exactly the same ratio ($\frac{R_1}{R_2} = \frac{R_3}{R_4}$), then the brige is said to be balanced and no current flows in either direction through the galvanometer.



In its basic application, an unknown resistor, R_x, is connected as the fourth side of the circuit and voltage(ε) is applied.

Figure 5. 16 Wheatstone bridge

R2 (variable) is adjusted until the galvanometer G., reads zero current. At this point

$$V_L = V_R$$

$$V_L = \varepsilon - V_1 = \varepsilon - IR_1$$

On account of the relation $I = \frac{\varepsilon}{R_1+R_2}$, we have

$$V_L = \varepsilon - \left(\frac{R_1}{R_1+R_2}\right)\varepsilon$$

$$V_R = \varepsilon - V_3 = \varepsilon - I'R_3$$

On account of the relation $I' = \frac{\varepsilon}{R_3+R_x}$, we get

$$V_R = \varepsilon - \left(\frac{R_3}{R_3+R_x}\right)\varepsilon$$

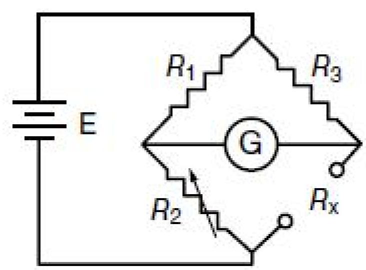


Figure 5. 17

Equating these equations, we get

$$\frac{R_1}{R_1+R_2} = \frac{R_3}{R_3+R_x}$$

$$R_3+R_x = \frac{R_3}{R_1}(R_1 + R_2)$$

Finally, we find

$$R_x = R_2\frac{R_3}{R_1}$$

Example 5.10

In the bridge circuit shown above the galvanometer G shows zero deflection when R_1=10Ω, R_2= 1000 Ω and R_3=27.49 Ω . What is the unknown resistor R_x?

Solution:

Employing the relation

$R_x = R_2 \frac{R_3}{R_1}$, we get

$R_x = 2749\ \Omega$

The potentiometer

A potentiometer has a sliding contact and acts as an adjustable potential divider. It permits measurements of potential difference without drawing current from the circuit being measured and hence acts as an infinite- resistance voltmeter.

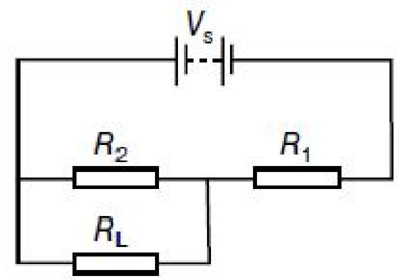


Figure 5. 18 The potentiometer

The equivalent resistance of the parallel-connection is

$$\frac{R_2 R_L}{R_2+R_L}$$

The equivalent resistance of the network is

$$R_{eq} = \frac{R_2 R_L}{R_2+R_L} + R_1 = \frac{R_L R_1 + R_L R_2 + R_1 R_2}{R_2+R_L}$$

The current drawn from the source is

$$I = \frac{V_s}{R_{eq}} = \frac{R_2+R_L}{R_L R_1 + R_L R_2 + R_1 R_2} V_s$$

Thus, the voltage across R_L is

$$V_L = I\left(\frac{R_2 R_L}{R_2+R_L}\right)$$

Plugging the expression for I, we obtain

$$V_L = \frac{R_2 R_L}{R_L R_1 + R_L R_2 + R_1 R_2} V_s$$

However, if R_L is large in comparison with R_1 and R_2 then the equation can be simplified to

$$V_L = \frac{R_2}{R_1+R_2} V_s$$

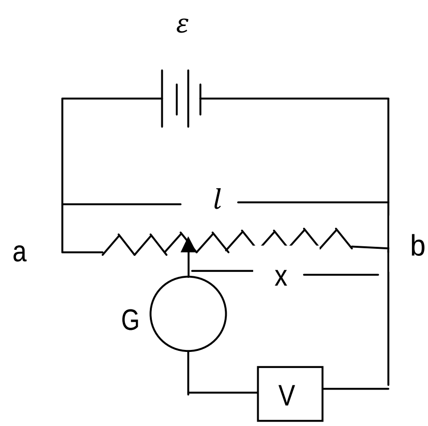
Similarly, if R_L is in parallel with R_1, the potential difference V_L is

$$V_L = \frac{R_1}{R_1+R_2} V_s$$

You can see that the supply voltage (ε) is divided by the circuit in proportion ti the values of R_1 and R_2.

Example 5.11

The resistor between a and b is a wire of length l, with a sliding contact at a distance x from b. An unknown potential difference V_x is measured by sliding the contact until the galvanometer reads zero.



a) show that under this condition the unknown potential difference is given by $V_x = \frac{x}{l}\varepsilon$

Solution: Using the relation

$$V_x = \frac{R_1}{R_1+R_2}\varepsilon = \frac{\rho x/A}{\rho l/A}\varepsilon$$

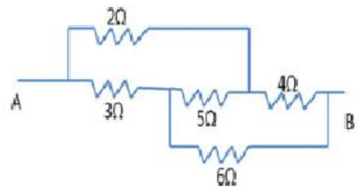
It then gives us $V_x = \frac{x}{l}\varepsilon$

b) If $\varepsilon = 12V$, $l = 1m$ and the galvanometer G reads zero, what is the potential difference V_x?

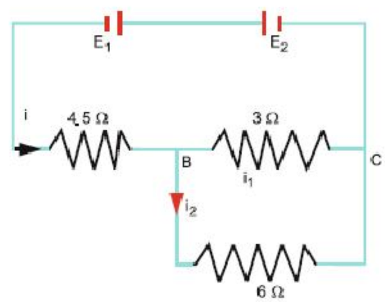
$$V_x = \frac{x}{l}\varepsilon = 9.52V$$

Problems

1. If a copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance?
2. The equivalent resistance of the series combination of two resistors is p. When they are joined in parallel, the equivalent resistance is q. If $p = nq$, find the minimum possible value of n.
3. Five resistors are arranged as in Fig.
   Find the effective resistance between A and B.



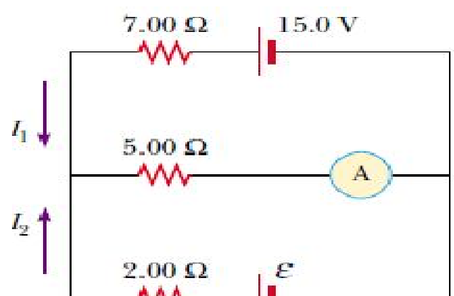
4. Three resistors of 4, 6 and 12Ω are connected together in parallel. This parallel arrangement is then connected in series with a 1 and 2 Ω resistors. If a potential difference of 120V is applied across the end of the circuit, what will be the potential drop across the part of the circuit connected in parallel?
5. A battery having an emf 24V and a resistance 2 Ω is connected to two resistances arranged (a) in series and (b) in parallel. If the resistances are 4 and 6 Ω, respectively, calculate the watts expended in each resistance, in each of the two cases.
6. A moving coil meter has a full scale reading of 1mA and a resistance of 80 Ω. How could the meter be used to measure (a) 100mA full scale and (b) 80V full scale?
7. In the circuit shown in Fig., the cells E_1 and E_2 have emfs 4 and 8V and internal resistances 0.5 and 1 Ω, respectively. Calculate the current in each resistor and the potential difference across each cell.



8. A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor *R*. (a) What is the value of *R*? (b) What is the internal resistance of the battery?

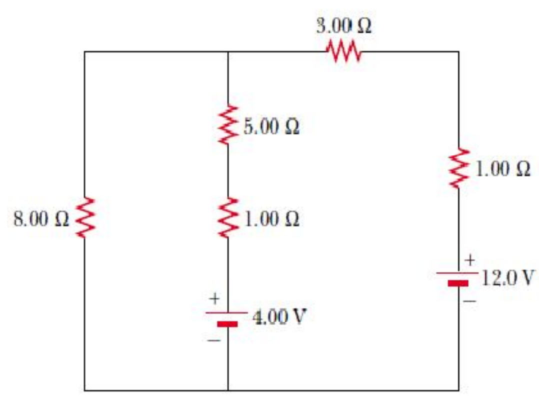
9. A 2.00-nF capacitor with an initial charge of 5.10)C is discharged through a 1.30-k' resistor. (a) Calculate the current in the resistor 9.00)s after the resistor is connected across the terminals of the capacitor. (b) What charge remains on the capacitor after 8.00)s? (c) What is the maximum current in the resistor?

10. The ammeter shown in Figure reads 2.00 A. Find I_1, I_2 and ∈.



11. A capacitor in an *RC* circuit is charged to 60.0% of its maximum value in 0.900 s. What is the time constant of the circuit?

12. Determine the current in each branch of the circuit shown in Figure



Unit 6

MAGNETISM

6.1 Concepts of a Magnetic Field

Magnetic field

Magnetic field is a region around moving charges. A magnetic field also surrounds a magnetic substance making up a permanent magnet. The symbol **B** is used to represent a magnetic field. The direction of the magnetic field **B** at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with *magnetic field lines*.

Note that the magnetic field lines outside the magnet point away from north poles and toward south poles. One can display magnetic field patterns of a bar magnet using small iron filings.

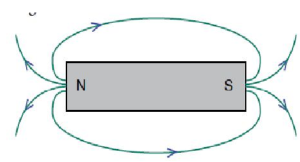


Figure 6.1 Magnetic field lines

We can define a magnetic field **B** at some point in space in terms of the magnetic force $\mathbf{F}_B$ that the field exerts on a charged particle moving with a velocity **v**, which we call the test object. (Assume that no electric or gravitational fields are present at the location of the test object). Experiments on various charged particles moving in a magnetic field give the following results:

- The magnitude $\mathbf{F}_B$ of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- The magnitude and direction of $\mathbf{F}_B$ depend on the velocity of the particle and on the magnitude and direction of the magnetic field **B**.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle $\theta \neq 0$ With the magnetic field, the magnetic force acts in a direction perpendicular to both v and B; that is, FB is perpendicular to the plane formed by **v** and **B** (Fig. 6.2).

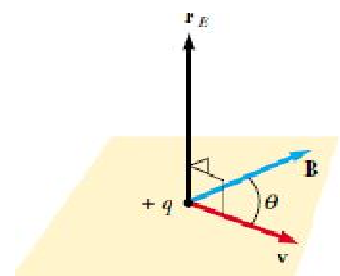


Figure 6.2 The magnetic force

- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin\theta$, where θ is the angle the particle's velocity vector makes with the direction of **B**.

We can summarize these observations by writing the magnetic force in the form

$$\boldsymbol{F}_B = q\mathbf{V} \times \mathbf{B}$$

The magnetic force is perpendicular to both **v** and **B.** Its direction can be determined using right-hand rule. We recall from definition of vector product that the magnitude of the magnetic force is given as

$$F_B = |q| vB\sin\theta$$

where θ is the smaller angle between **v** and **B**. From this expression, we see that F_B is zero when **v** is parallel or antiparallel to **B** ($!\theta = 0$ or 180°) and maximum when **v** is perpendicular to **B** (θ =90°).

There are several important differences between electric and magnetic forces:

- The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.

We can regard this equation as an operational definition of the magnetic field at some point in space. That is, the magnetic field is defined in terms of the force acting on a moving charged particle as

$$B=\frac{F_B}{|q|Vsin\theta}$$

The SI unit of the magnetic field B is Tesla (T)

$$1T = 1\frac{N}{Cm/s}$$

Example 6.1

An electron moves to the positive x-axis with a speed of $8x10^6$m/s inside a magnetic field 0.25T directed at angle of 60^0 to the +x-axis and lying in xy-plane. Find the magnetic force on the electron.

Solution:

V= $8x10^6$m/s $\boldsymbol{i}$ **B=** B$cos\theta\boldsymbol{i}$+B$sin\theta\boldsymbol{j}$= and q=$-1.6x10^{-19}$C

The magnetic force becomes

$\mathbf{F}_B$= q**V**x**B**==$-1.6x10^{-19}$C($8x10^6$m/s $\boldsymbol{i}$)**x** (0.13 $\boldsymbol{i}$+ 0.22 $\boldsymbol{j}$)T

$\mathbf{F}_B$=$2.8x10^{-14}$N($-\boldsymbol{k}$)

Magnetic Force Acting on a Current-Carrying Conductor

To indicate the direction of **B** in illustrations, we may use symbol X (into the plane) and · (out of the plane)

Consider a straight segment of wire of length L and cross-sectional area A, carrying a current I in a uniform magnetic field B is directed perpendicular into the page as shown. The magnetic force exerted on a charge q moving with a drift velocity V_d is

$$q\mathbf{V}_d XB$$

The total force acting on the wire can be found by multiplying the force exerted on a charge by the total number of charges in the segment (nAL),

$$\mathbf{F}_B=(q\mathbf{V}_d XB)nAL,$$

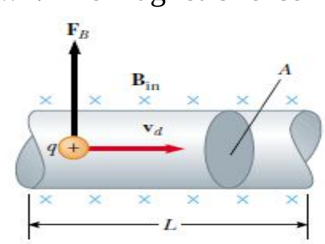


Figure 6.3 current carrying wire

where n is the number of charges per unit volume and L is the length of the segment. Using the relation $I=nqV_dA$, we have

$$\mathbf{F}_B=I\ (\mathbf{L}X\mathbf{B}),$$

where **L** is a vector that points in the direction of the current. The direction is determined using right-hand rule.

Example 6.2

A straight, horizontal length of copper wire has a current I=28 A to the right through it. What are the magnitude and direction of the magnetic field needed to exert an upward magnetic force of 0.56N on 1m of the wire?

Solution

I=28A F_B=0.56N L=1m

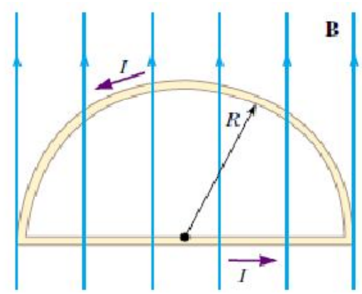
Applying the relation $B = \frac{F_B}{IL}$, we get B=0.02T

Using the right-hand rule the direction of **B** is into the page.

Note that the magnetic force on a current-carrying wire in a uniform magnetic field is equal to that on a straight wire connecting the end points and carrying the same curve.

Example 6.3

A wire bent into a semicircle of radius *R* forms a closed circuit and carries a current *I*. The wire lies in the *xy* plane, and a uniform magnetic field is directed along the positive *y* axis, as shown in Fig. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.



Solution The magnetic force F1 acting on the straight portion has a magnitude F_1= ILB =2IRB because L =2R and the direction is out of the page based on the right-hand rule

The magnetic force on the curved portion is the same as that on a straight wire of length 2R carrying current I to the left. Thus, F_2 = ILB =2IRB. The direction of F**2** is into the page based on the right-hand rule.

Because the two force have the same magnitude but opposite in direction the net force on the wire is

F=F1-F2=2IRB-2IRB=0

Note that the net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

Torque on a current loop in a uniform magnetic field

Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field parallel to the plane of the loop. No magnetic forces act on sides 1 and 3 because these wires are parallel to the field; hence, L X B =0 for these sides. However, magnetic forces do act on sides 2 and 4 because these sides are oriented perpendicular to the field. The magnitude of these forces is

F2=F4=IaB

The forces are couple forces, if the loop can rotate about point O, the magnitude torque τ_{max} is

$$\tau_{max} = F_2\, {}^{b}/_{2} + F_4\, {}^{b}/_{2} = \frac{(IaB)b}{2} + \frac{(IaB)b}{2} = \text{IabB}$$

$$\tau_{max} = IAB,$$

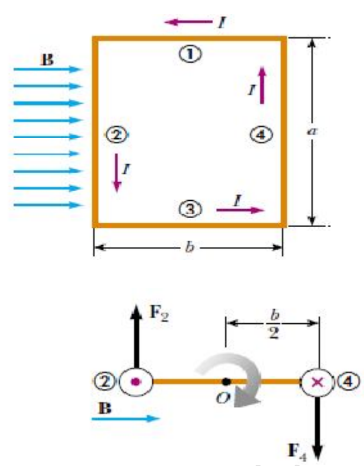


Figure 6.4 A rectangular loop in uniform magnetic field.

where ab=A is the area. The maximum torque is valid when **B** is parallel to the plane of the loop. Now suppose the uniform field **B** makes angle $\theta > 90^0$ with a line perpendicular to the plane of the loop. The magnitude of the net torque about O is

$$\tau_{max} = F_2\, {}^{b}\!/_{2}\, sin\theta + F_4\, {}^{b}\!/_{2}\, sin\theta = \frac{(IaB)b}{2} sin\theta + \frac{(IaB)bsin\theta}{2} = \text{IabB } sin\theta$$

$$\tau_{max} = IABsin\theta$$

This result shows that the torque has its maximum value IAB when **B** is perpendicular to the normal to the plane of the loop and is zero when **B** is parallel to the normal to the plane of the loop ($\theta = 0$).

The torque can be expressed as

$$\boldsymbol{\tau} = \mathrm{I}\mathbf{A} \times \mathbf{B}$$

The loop tends to rotate in the direction of decreasing θ (**A** rotates towards **B**). The product I**A** is defined to be the magnetic dipole moment $\boldsymbol{\mu}$ (magnetic moment) of the loop.

$$\boldsymbol{\mu} = I\boldsymbol{A}$$

Its SI unit is Am^2.

Thus the torque is given as

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}.$$

If a coil contains N turns of wire, each carrying the same current and enclosing the same area the total magnetic moment of the coil is

$$\boldsymbol{\tau} = \mathrm{N}\boldsymbol{\mu}_{\mathbf{loop}} \times \mathbf{B} = \boldsymbol{\mu}_{\mathbf{coil}} \times \mathbf{B}.$$

The potential energy of a system of a magnetic dipole in a magnetic field depends on the orientation of the dipole in the magnetic field and is given by

$$\mathrm{U} = -\boldsymbol{\mu} \cdot \mathbf{B}$$

The system has its lowest energy $U_{min} = -\mu B$ when $\boldsymbol{\mu}$ points in the same direction as **B.** The system has its highest energy $U_{max} = +\mu B$ when $\boldsymbol{\mu}$ points in the direction opposite to **B.**

Example 6.4

A rectangular coil of dimensions 5.40 cm X 8.50 cm consists of 25 turns of wire and carries a current of 15.0 mA. A 0.350-T magnetic field is applied parallel to the plane of the loop.

A) Calculate the magnitude of its magnetic dipole moment.

B**)** What is the magnitude of the torque acting on the loop?

Solution

A) $\mu_{coil} = NIA$= (25)(15x 10^3 A)(0.054 0 m)(0.085 0 m)=1.72x10^{-3} Am^2

B) Because B is perpendicular to coil, we have

$\tau = \mu_{coil}B$=6.02x10^{-4} N.m

Motion of charged particles in electric and magnetic fields

Motion of charged particles in uniform Magnetic field

Consider a charged particle moving in a uniform magnetic field with initial velocity perpendicular to the field.

The magnetic force is perpendicular to the velocity and the path of the particle is circle. (F_B does no work) and performs uniform circular motion.

The magnetic force provides centripetal force

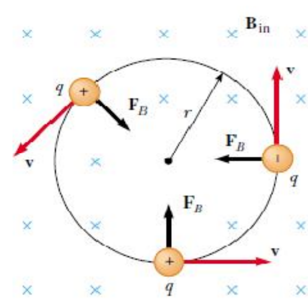


Figure 6.5 a charge projected perpendicular to the **B** field.

$$F_B = F_c$$

$$qvB = \frac{mV^2}{r}$$

$$r = \frac{mV}{qB}$$

The radius is proportional to the momentum of the particle mv.

The angular speed of the particle is

$$\omega = \frac{V}{r} = \frac{qB}{m}$$ is referred as the cyclotron frequency

The period of the motion is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

The angular speed (ω) and the period (T) do not depend on the linear speed of the particle or the radius.

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to **B**, its path is helix**.**

Example 6.5

A proton (m=1.67x10^{-23}Kg, q=+e) is moving in uniform 0.35T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton

Solution:

$$V = \frac{qBr}{m_p} = 4.7 \times 10^6 m/s$$

Applications Involving Charged Particles Moving in a Magnetic Field

A charge moving with a velocity **v** in the presence of both an electric field **E** and a magnetic field **B** experiences both an electric force q**E** and a magnetic force q **v** x **B**. The total force (called the Lorentz force) acting on the charge is

$$\mathbf{F} = q\mathbf{E} + q\mathbf{V} \times \mathbf{B}$$

Velocity selector

A uniform electric field is directed vertically downward (in the plane of the page as shown in Fig.) and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page).

If q is positive and the velocity **v** is to the right, the magnetic force q **v** x **B** is upward and the electric force q**E** is downward. When the magnitudes of the two fields are chosen so that qE = qvB, the particle moves in a straight horizontal line through the region of the fields.

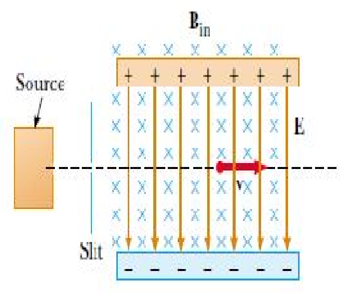


Figure 6.6 Velocity selector

From the expression qE = qvB, we find that

$$V = \frac{E}{B}$$

Particles having this speed pass undeflected.

Exercise

What happen to particles moving at speed greater than this speed?

J.J. Thompson's experiment of charge to mass ratio

J.J. Thompson used balanced electric and magnetic fields to measure the charge to mass ratio for an electron. His apparatus is shown in Figure 6.7. We recall that

$$V = \frac{E}{B}$$

The electron beam is accelerated by the potential difference between the cathode and anode. We know that

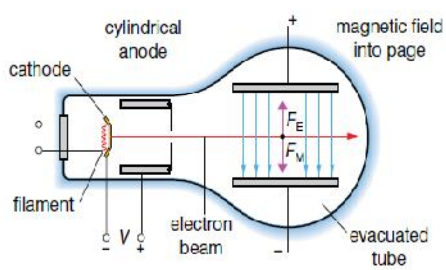


Figure 6.7. Charge to mass ratio

energy of the electrons is given by

$$\frac{1}{2}mV^2 = q\Delta V$$

or $$V = \sqrt{\frac{2q\Delta V}{m}}$$

Equating the two equations and squaring both sides, we get

$$\frac{E^2}{B^2} = \frac{2q\Delta V}{m}$$

Rearranging, we obtain

$$\frac{q}{m} = \frac{E^2}{2\Delta V B^2}$$

Example 6.6

a) Find the charge mass ratio for an electron accelerated through 600 V in a magnetic field of strength 45 mT where the speed of the electron is 1.4×10^7 m/s.

b) What is the percentage difference between your result and the accepted ratio with the values $q = 1.6 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg?

Solution:

V=1.4×10^7 m/s; B=45mT; ΔV= 600V; q= $q = 1.6 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg

a) Employing

$$V = \sqrt{\frac{2q\Delta V}{m}} \Rightarrow V^2 = \frac{2q\Delta V}{m}$$

Rearranging

$$\frac{q}{m} = \frac{V^2}{2\Delta V} = 1.63 \times 10^{11} Kg$$

b) Accepted value= q/m =$1.76\text{x}10^{11}$

Percentage difference=$\frac{accepted\ value - Calculated\ value}{Accepted\ value} \times 100\% = \frac{1.76\times10^{11} - 1.63\times10^{11}}{1.76\times10^{11}} 100\%$

$$= 7.4\%$$

The mass spectrometer

A mass spectrometer is a machine that allows chemicals to be separated according to their mass. A simplified diagram of a mass spectrometer is shown in Figure 6.8.

The chemical enters the machine and is ionized (charged). It is then accelerated by an electric field and then its direction is changed when it enters a magnetic field. In the last section, we learnt that in a magnetic field, a charged particle experiences a force as a result of the magnetic field, The radius of the circular path is

$$r = \frac{mV}{qB}$$

Rearranging, we have

$$\frac{q}{m} = \frac{V}{Br}$$

The values of B and r are known from the calibration of the machine. We need to know the speed of the particles when they enter the electromagnet. From J. J. Thompson experiment, we know that

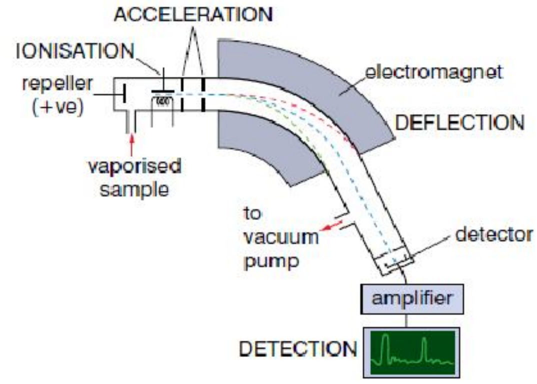


Figure 6.8 A mass spectrometer

$$V = \sqrt{\frac{2q\Delta V}{m}}$$

Inserting this in place of the speed, we get

$$\frac{q}{m} = \frac{\sqrt{2q\Delta V}}{Br\sqrt{m}}$$

Squaring both sides, we find

$$\frac{q^2}{m^2} = \frac{2q\Delta V}{B^2 r^2}$$

Thus, the charge to mass ration is found to be

$$\frac{q}{m} = \frac{2\Delta V}{B^2 r^2}$$

If the charge is known, the mass of the particle is

$$`m = \frac{B^2 r^2 q}{2\Delta V}$$

Sources of magnetic field

The Biot–Savart Law

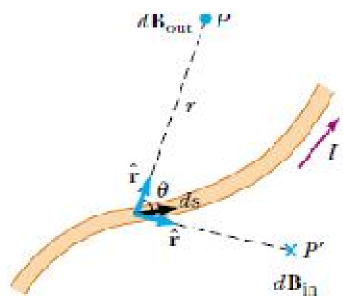


Figure 6.9 Magnetic field d**B**

Biot and Savart performed quantitative experiments on the force exerted by an electric current on a nearby magnet and arrived at a mathematical expression that gives the magnetic field at some point in space. Experimental observations for the magnetic field d**B** at a point P associated with a length element d**s** of a wire carrying a steady current I Fig. 6. 9

- The vector d**B** is perpendicular both to ds (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from d**s** toward P.
- The magnitude of d**B** is inversely proportional to r^2, where r is the distance from d**s** to P.
- The magnitude of d**B** is proportional to the current and to the magnitude ds of the length element d**s**.
- The magnitude of d**B** is proportional to sinθ , where θ is the angle between the vectors d**s** and $\hat{r}$

These observations are summarized in the mathematical expression known today as the Biot–Savart law:

$$d\boldsymbol{B} = \frac{\mu_0 I}{4\pi}\frac{d\boldsymbol{l}\times\hat{\boldsymbol{r}}}{r^2},$$

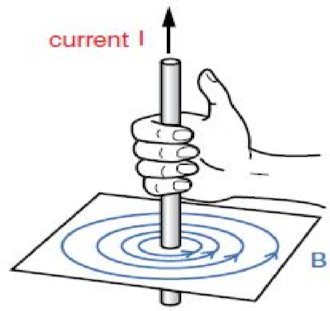
Where $\mu_0 = 4\pi \times 10^{-7} T\frac{m}{A}$ is called permeability of free space.

The total magnetic field B created at some point by a current of finite size is

$$\boldsymbol{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\boldsymbol{l}\times\hat{\boldsymbol{r}}}{r^2}$$

Please be informed that application of Biot-Savart law is beyond the scope of this course.

The magnetic field produced by an electric current in long straight conductor



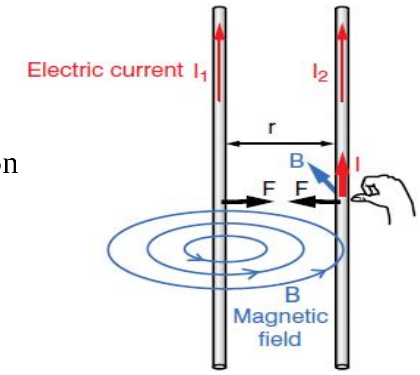
Using BIot-Svart law the magnetic field produced by a long straight conductor can be found to be

$$B = \frac{\mu_0 I}{2\pi r} \quad or \quad B = \frac{kI}{r},$$

where the constant k is given as $\frac{\mu_0}{2\pi} = 2 \times 10^{-7} T\frac{m}{A}$.

Figure 6. 10 Magnetic field of long straight current carrying wire

The magnetic force between two parallel wires



Consider two long, straight, parallel wires separated by a distance a and carrying currents I_1 and I_2 in the same direction

The magnetic field in wire 2 produced by I_1 is given by

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

The force due to wire 1 on length Δl of wire 2 is given by

$$F_{12} = I_2 \Delta l\ B_1$$

Inserting the expression for B_1, we get

Figure 6. 11 Magnetic force between two parallel wires

$$F_{12} = \frac{\mu_0 I_1 I_2 \Delta l}{2\pi r}$$

The force per unit length in terms of the currents is therefore

$$\frac{F_{12}}{\Delta l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Wire 2 also exert a force F_{21} equal in magnitude to that of F_{12} but opposite in direction. The forces are attractive and they are action and reaction ($\mathbf{F}_{12}=-\mathbf{F}_{21}$).

Note that when the two wires carry currents in opposite direction the expression for the force is the same as the above one but the forces are repulsive.

The force between two parallel wires is used to define the ampere as follows:

Definition of the Ampere

When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by 1 m is $2\text{x}10^{-7}$ N/m, the current in each wire is defined to be 1 A.

Example 6.7

For I_1=2 A and I_2= 6 A in Figure 6.11, if they are 50cm apart, what is the force on unit length of the wires?

Solution

$$\frac{F_{12}}{\Delta l} = \frac{\mu_0 I_1 I_2}{2\pi r} = 4.8 \text{ x}10^{-7}\text{N/m}$$

Ampère's law

Ampere's law is useful relation used to find magnetic field it is closely analogous to Gauss' law. Consider first a long straight wire carrying a current I. we know that the lines **B** form circles around the wire and by symmetry the magnitude of B is the same everywhere on a circular path (Amperian loop) and is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

Its direction at each point is tangent to the circle. Δl is an element of the circle in this special case the tangent component $B_{||}$ of **B** is equal to B itself. Now we multiply $B_{||}$ by the length of the arc Δl and sum these products for all the line elements Δl_1, Δl_1,⋯, which together make up the circle

$$\sum B_{||}\,\Delta l = B_{||}\Delta l_1 + B_{||}\Delta l_2 + \cdots,$$

Since $B_{||}$ is the same it can be factored out and we have

$$\sum B_{||}\,\Delta l = B_{||}(\Delta l_1 + \Delta l_2 + \cdots)$$

The term in the bracket is the circumference of the circle. Thus we get

$$\sum B_{||}\,\Delta l = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

If the angle between the parallel component and the path is θ then the expression becomes

$$\sum B_{||}\,\Delta l \cos\theta = \mu_0 I$$

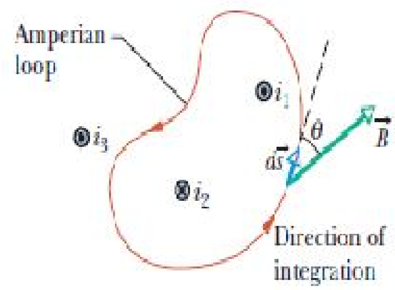


Figure 6.12 Ampere's law applied to an arbitrary Amperian loop.

Ampere's law states that in any loop path (*amperian loop*), the length element times the magnetic field in the direction of the length element is equal to the permeability of free space time the electric current enclosed in the loop. If the loop contains any nember of current carrying conductor, Ampere's law takes the form

$$\sum B_{||}\,\Delta l = \mu_0 \sum \mathrm{I}$$

Application of Ampere's law

Magnetic field produced by a long-straight current carrying wire

Take a circular amperian loop of radius r

$$\sum B_{||}\,\Delta l = \mu_0 \mathrm{I}$$

Since the magnetic field is uniform along the circle, on can write

$$B_{||}\sum \Delta l = \mu_0 \mathrm{I}$$

Using the circumference of a circle, we have

$$B_{||}(2\pi r) = \mu_0 \mathrm{I}$$

Finally, taking into account that the magnetic field is always tangent to the circle, we obtain

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field of a solenoid

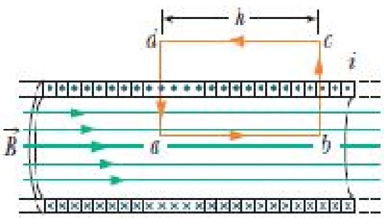


Figure 6.13 A long solenoid carrying current i. The amperian loop is the rectangle abcda

A solenoid is a long straight coil of wire. If the length of the solenoid L is large compared with its cross-sectional diameter, the internal field near its center is very nearly uniform and parallel to the axis and the external field near the center is very small. The field along the center can be found using Ampere's law The field along side ab is parallel to this side and is constant. $(B_{||} = B)$ and $\sum B_{||} \Delta h = Bh$

Along side ad and bc B perpendicular to h

$$\sum B_{||} \Delta h = 0$$

And along side cd

$$\sum B_{||} \Delta h = 0$$

If n is the number of turns per unit length, the number of turns in the length h is nh. Each of these turns passes once through the rectangle abcda and carries current I. the total current through the rectangle is then nhI and from Ampere's law

$$Bh = \mu_0 nIh$$

Hence, the magnetic field turns out to be

$$B = \mu_0 nI = \frac{\mu_0 IN}{L}$$

Field of a toroid

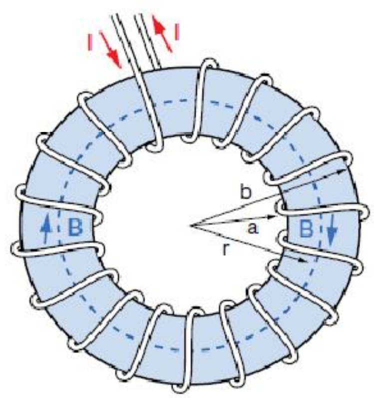


Figure 6. 14 A toroid

The field B is tangent to the path and has constant magnitude B. Thus

$$\sum B_{||} \Delta l = B(2\pi r)$$

The total current through the loop is NI,

where N is the total number of loops. Then, from Ampere's law

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

Magnetic properties of Matter

The magnetic state of a substance is described by a quantity called the magnetization vector **M** (the magnetic moment per unit volume of the substance $\boldsymbol{M} = \frac{\boldsymbol{\mu}}{V} = \frac{NI\mathbf{A}}{V}$). When a substance is placed in a magnetic field, the total magnetic field in the region is

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_\mathrm{m} = \mathbf{B}_0 + \mu_0\mathbf{M} = \mu_0(\mathbf{H} + \mathbf{M}),$$

H is the magnetic field strength within the substance. It is related to the magnetic field due to the current in the wire

Classification of Magnetic substances

Substances can be classified as belonging to one of the three categories, depending on their magnetic properties; paramagnetic and ferromagnetic materials are those made of atoms that have permanent magnetic moments (due to unpaired electrons). Diamagnetic materials are those made of atoms that donot have permanent magnetic moments.

For paramagnetic and diamagnetic substances ($\mathbf{M} \sim \mathbf{H}$)

$$\mathbf{M} = \chi\mathbf{H},$$

where χ is magnetic susceptibility. It is a measure of how susceptible a material is to being magnetized. For paramagnetic substances, χ is positive and **M** is the same direction as **H**. (Al, Li, Mg, Ca, $O_2 \cdots$). For diamagnetic substances, χ is negative and **M** is opposite to **H.** (Bismuth, Cu, diamond, Au, Pb,Hg, $N_2 \cdots$) .

$$\mathbf{B} = \mu_0(\mathbf{H} + \chi\mathbf{H}) = \mu_0(1 + \chi)\mathbf{H} = \mu_\mathrm{m}\mathbf{H} ,$$

where $\mu_m = \mu_0(1 + \chi)$. χ is very small for paramagnetic and diamagnetic materials$\Rightarrow \mu_m \approx \mu_0$. For Ferromagnetic materials μ_m is typically severalthousands times greater than μ_0 (χ *is very large*). Nickel, iron, cobalt and their alloys are ferromagnetic.

Earth's Magnetism

Circulating electric currents in the molten core of the earth produce the magnetic field. The rotation of the earth may play a part in generating the currents that are presumed to be the source of the magnetic field.

The magnetic north pole is near the geographic south pole of the earth. They are about 1000Km apart.

Horizontal and vertical components of the Earth's magnetic field

Except near the equator, the field lines of the Earth's magnetic field are at an angle to the Earth's surface. At the magnetic poles, the field lines pass through the Earth's surface vertically. However, at any other point on the Earth's surface the Earth's magnetic field has a vertical and a horizontal component.

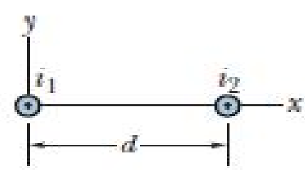
The Earth's magnetic field is a vector quantity; at each point in space it has a strength and a direction.

The strength of the field at the Earth's surface ranges from less than 30,000 nT in South America and South Africa to over 60,000nT around the magnetic poles in northern Canada and south of

Australia and in part of Siberia. Near the poles, the field strength diminishes with the inverse square of the distance, i.e. at a distance of *R* Earth radii it only amounts to 1/*R*2 of the surface field in the same direction, whereas at greater distances, such as in outer space, it diminishes with the cube of the distance. Where the prime meridian intersects with the equator, the field strength is about 31 μT.

Problems

1. An electric field of 1500 V/m and a magnetic field act on an electron moving with a speed of 3000 m/s. If the resultant field is to be zero what should be the strength of the magnetic field (in Wb/m2).
2. An electron moves in a circle of radius 1.9m in a magnetic field of 3×10^{-5}T. Calculate (a) the speed of electrons and (b) time taken to move round the circle.
3. The magnetic field at 40 cm from a long straight wire is 10^{-6} T. What current is carried by the wire?
4. Two parallel wires 20 cm apart attract each other with a force of 10^{-5} N/m length. If the current in one wire is 10 A, find the magnitude and direction of current in the other wire?
5. Three long parallel wires, each carrying 20A in the same direction, are placed in the same plane with the spacing of 10 cm. What is the magnitude of net force per metre on (a) an outer wire and (b) central wire?
6. A proton moves with a velocity of V= (2***i*** + 4**j**-**k**) m/s in a region in which the magnetic field is **B** = (**i** -2**j** + 3***k***) T. What is the magnitude of the magnetic force this charge experiences?
7. A wire carries a steady current of 2.40 A. A straight section of the wire is 0.750 m long and lies along the *x* axis within a uniform magnetic field, B =1.60$\hat{k}$ T. If the current is in the -*x* direction, what is the magnetic force on the section of wire?
8. A current of 17.0 mA is maintained in a single circular loop of 2.00 m circumference. A magnetic field of 0.800 T is directed parallel to the plane of the loop. (a) Calculate the magnetic moment of the loop. (b) What is the magnitude of the torque exerted by the magnetic field on the loop?
9. In the Fig. , two long straight wires at separation *d* =16.0 cm carr currents i_1=3.61 mA and i_2 =3.00i_1 out of the page.
 (a) Where on the *x* axis is the net magnetic field equal to zero?
 (b) If the two currents are doubled, is the zero-field point shifted toward wire 1, shifted toward wire 2, or unchanged?



10. A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?
11. A proton moving in a circular path perpendicular to a constant magnetic field takes 1.00 μs to complete one revolution. Determine the magnitude of the magnetic field.
12. A wire is formed into a circle having a diameter of 10.0 cm and placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A. Find the maximum torque on the wire.

Unit 7

ELECTROMAGNETIC INDUCTION AND AC CIRCUIT

7.1 Phenomenon of Electromagnetic Induction

Magnetic flux (Φ)

Magnetic flux is the number of magnetic field lines that pass through given area and is given by

$$\Phi = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta.$$

If B and A are parallel, the flux is max and is equal to $\Phi = BA$.
Its unit is Tm^2 and is named as Weber (Wb). The magnetic field strength B is given by

$$B = \frac{\Phi}{A}$$

Consequently, **B** is named as magnetic flux density and its unit is Wb/m^2. It is also named as magnetic induction vector.

Induced emf

We next seek to discuss two experiments:

First Experiment (moving a bar magnet toward the loop**)**

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases
2. Faster motion produces a greater current.
3. If moving the magnet's north pole toward the loop causes, say, current, then moving the north pole away causes counterclockwis
Moving the south pole toward or away from the loop also causes in the reversed directions.

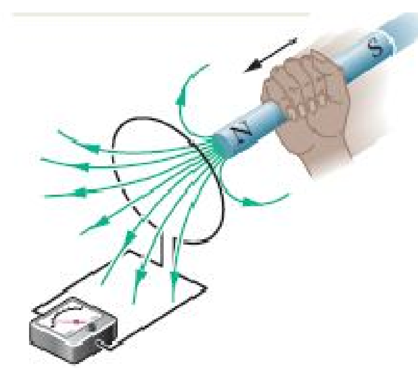


Figure 7.1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

Second experiment (Closing and opening switch)

If we close switch S, to turn on a current in the right-hand loop, The Meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left-hand loop, but in the opposite direction

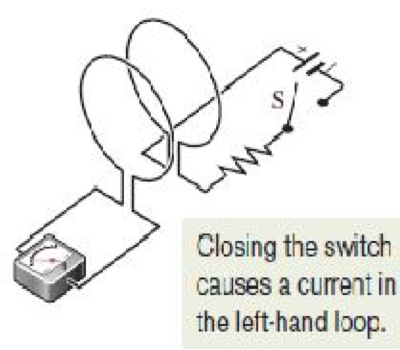


Figure 7. 2 An ammeter registers a current in the left-hand wire loop just as switch S is closed or opened.

The two experiments have one thing in common: in each case, an emf is induced in the circuit when the magnetic flux through the circuit changes with time and this emf produce an induced current.

Flux linkage

The amount of magnetic flux that interacts with a coil of wire is called the magnetic flux linkage. Magnetic flux linkage is the product of the number of turns of wire N and the flux in the region, Φ, so one can write

$$\text{Flux linkage} = N\,\Phi = BAN\cos\theta$$

The laws of electromagnetic induction

Faraday's law of electromagnetic induction

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t},$$

where $\Phi_B = \mathbf{B}\cdot\mathbf{A}$. If the circuit consists of N loops the magnetic flux through the circuit, on can write

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}.$$

The negative sign indicates that the effect of ε is always to oppose the $\Delta\Phi$ that produce it.

Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.

2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).

3. Change the angle between the direction of the magnetic field **B** and the plane of the coil (for example, by rotating the coil so that field **B** is first perpendicular to the plane of the coil and then is along that plane

Example 7.1

A 50turns rectangular coil has dimensions 5cmx10cm. If a magnetic field directed perpendicular to the plane of the coil changes from B=0 to B=0.5T in 0.25s, calculate the magnitude of the average emf induced in the coil.

Solution:

$B_0=0$; B=0.5T ; A=ab=5cmx10cm=5×10^{-3}m^2 ; $\Delta t = 0.25s$

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{(B-B_0)A}{\Delta t} = 500\text{mV}$$

Lenz's Law

Soon after Faraday proposed his law of induction, Heinrich Friedrich Lenz devised a rule for determining the direction of an induced current in a loop:

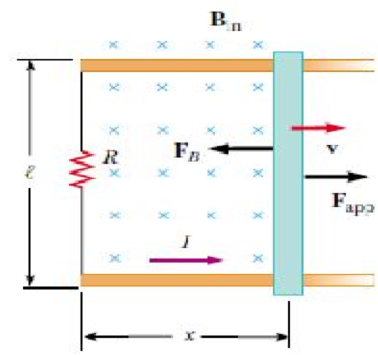
An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.

Motional emf

Motional emf is the emf induced in a conductor moving through a constan magnetic field.

Example 7.2

A conductor of length l is moving on two parallel rails with velocity **V** as shown in Fig. find the expression for the emf induced in the circuit.



In this case $\frac{\Delta\Phi_B}{\Delta t}\,\alpha\,\Delta A \quad (\varepsilon\,\alpha\,\Delta A)$

$$\Phi_B = BA = Blx$$

Using Faraday's law

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{Bl\Delta x}{\Delta t}$$

Using the fact that the velocity $V = \frac{\Delta x}{\Delta t}$, we obtain

Figure 7.4 Conductor moving

$$\varepsilon = -BlV$$

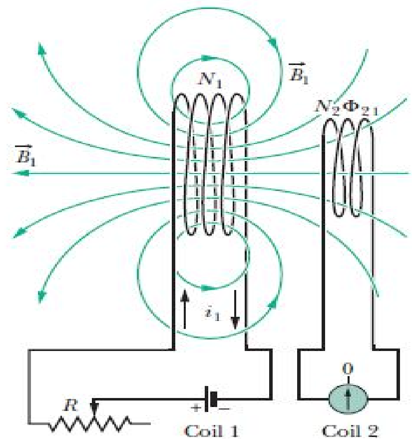
This is called motional emf. The current in the conductor is

$$I = \frac{|\varepsilon|}{R} = \frac{BlV}{R}$$

Inductance

Mutual inductance

A steady current *i* in one coil will set up a magnetic flux Φ through the other coil (*linking* the other coil). If we change *i* with time, an emf ε given by Faraday's law appears in the second coil; we called this process *induction*. We could better have called it **mutual induction.**



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From Faraday's law the emf in the second coil is given as

$$\varepsilon_2 = -N_2\frac{\Delta\Phi_2}{\Delta t}$$

The flux linkage in the second coil is proportional to the current in the first coil(i_1), thus one can write

Figure 7.5 Two circuits

$$N_2\Phi_2 = Mi_1$$

From this one can write

$$\varepsilon_2 = -N_2\frac{\Delta\Phi_2}{\Delta t} = M\frac{\Delta i_1}{\Delta t}.$$

The quantity $M = \frac{\varepsilon_2}{\Delta i/\Delta t} = N_2\frac{\Phi_2}{i}$ is called the mutual inductance. Its unit is Wb/A= Vs/A and is named as Henry (H) 1H=1W/A=1Vs/A

Eddy currents

If there is a changing magnetic flux in a solid metallic objet, then there will also be an induced voltage and current in the metallic object. This induced current is called **eddy current**. The current circulate in complete loop. They produced heating effect used in induction welding. They are also used in security checks.

Self Inductance

The changing flux through a circuit produce induced emf in itself. This effect is called self- induction. The emf ε_L set up in this way is called self-induced emf.

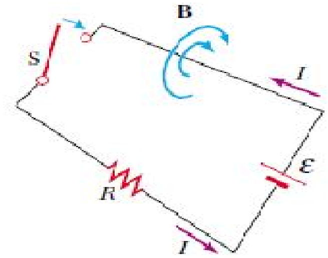


Figure 7.6

From Faraday's law the emf in the second coil is given as

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

The flux linkage in the coil is proportional to the current in it (i), thus one can write

$$N\Phi = Li$$

From this one can write

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -L\frac{\Delta i}{\Delta t}.$$

The quantity $L = \frac{\varepsilon_L}{\Delta i/_{\Delta t}} = N\frac{\Phi}{i}$ is called the self inductance of the coil.

The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current. Its unit is henry (H). It depends on the geometry of the circuit and other physical characteristic. Inductance is opposition to changing current. An inductor is another important component of a circuit that has a circuit symbol 

Example 7.3

Inductance of a solenoid

Find the inductance of a uniformly wound solenoid having n turns and length L. Assume L is much longer than the radius of the windings and the core of the solenoid is air.

Solution;

We can assume that the interior magnetic field due to the current is uniform and given by

$$B = \mu_0 nI = \mu_0\frac{N}{L}I$$

The magnetic flux through each turn is

$$\Phi_B = BA = \mu_0\frac{NA}{L}I$$

where A is the cross-sectional area of the solenoid. Using this expression, we have

$$L = N\frac{\Phi}{I} = \mu_0\frac{N^2A}{L}$$

This result shows that L depends on geometry and is proportional
to the square of the number of turns. Because N =nL, we can also express the result in the form

$$L = \mu_0\frac{(nL)^2A}{L} = \mu_0 n^2 AL = \mu_0 n^2 V\ ,$$

where V =AL is the interior volume of the solenoid.

Example 7.4

(A) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.

(B) Calculate the self-induced emf in the solenoid if the current it carries is decreasing at the rate of 50.0 A/s.

Solution

A) Using the relation $L = N\frac{\Phi}{I} = \mu_0\frac{N^2A}{L}$ = 1.81 X10^{-4} Tm^2/A = 0.181 mH

B) Given $\frac{\Delta i}{\Delta t}$=-50A/s and applying the relation

$\varepsilon = -L\frac{\Delta i}{\Delta t}$, we get $\varepsilon = 9.05 mV$

Exercise

Calculate L of an air–core solenoid containing 300turns if the length is 25cm and A=4cm^2.

The energy stored in an inductor

Consider an inductor in a circuit as shown.

When an electric current flows through the inductor, we know
That there is an induced voltage given by

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

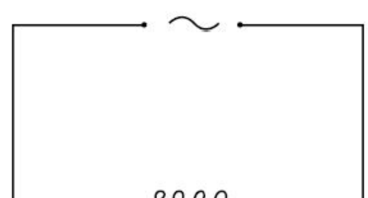


Figure 7.7 Inductive circuit

When the current is flowing through the inductor, there is energy stored in the magnetic field, which we give the symbol PE_B (it is potential energy PE stored by a magnetic field, *B*).
The instantaneous power that must be supplied to the inductor to initiate the current in the conductor, *P*, is given by

$$P = I\varepsilon = -LI\frac{\Delta I}{\Delta t}$$

We can find the energy stored when there is a final current I_f at time t by integrating the expression for power like this

$$U_B=\int_0^t Pdt=\int_0^{I_f} LIdI$$

Carrying out the integration, we obtain

$$U_B = \frac{1}{2}LI^2$$

Magnetic energy density

Magnetic energy density is defined as

$$u_B = \frac{energy}{volume}$$

Consider an inductor of length *l* and area of cross section *A* as shown in Figure 7.7
We know that the energy stored in the inductor

$$U_B = \frac{1}{2}LI^2$$

We also know that

$$L = \mu_0\frac{N^2A}{L} \text{ and } I^2 = \frac{B^2l^2}{\mu_0^2N^2}$$

Substituting these values, we get

$$U_B = \frac{AB^2l}{2\mu_0}$$

The energy density is thus, given as

$$u_B = \frac{U_B}{V} = \frac{AB^2l}{2\mu_0(Al)} = \frac{B^2}{2\mu_0}$$

Alternating Current (A.C.) Generator and Transformers

AC generator converts mechanical energy into electrical. It takes in energy by work and transfer it out by electrical transmission. To understand how they operate, let us consider the alternating current (AC) generator. In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field (Fig. 7.8). As the coil rotates the magnetic flux linking its coil is changed and results in a potential difference between its terminals. If we assume the loop has N turns the flux linkage is given as

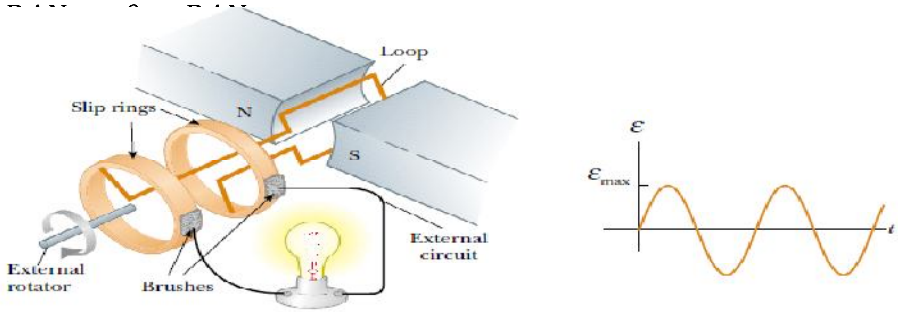


Figure 7.8 Ac generator and the voltage it produces as a function of time

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}(BANcos\omega t)$$

Taking the time derivative, we find

$$\varepsilon = BAN\omega \sin \omega t = \varepsilon_{max} \sin \omega t$$

where $\omega = 2\pi f = \frac{2\pi}{T}$ and $\varepsilon_{max} = BAN\omega$ is the maximum voltage.

Exercise

An AC generator consists of 8 turns of wire, each of area A=0.09m^2 , and the total resistance of the wire is 12 Ω. The loop rotates in a 0.50T magnetic field at a constant frequency of 60.Hz.

(A) Find the maximum induced emf.

(B) What is the maximum induced current when the output terminals are connected to a low-resistance conductor?

Transformers

Transformers are used to step-up or step-down ac voltage. It needs changing flux for its operation.

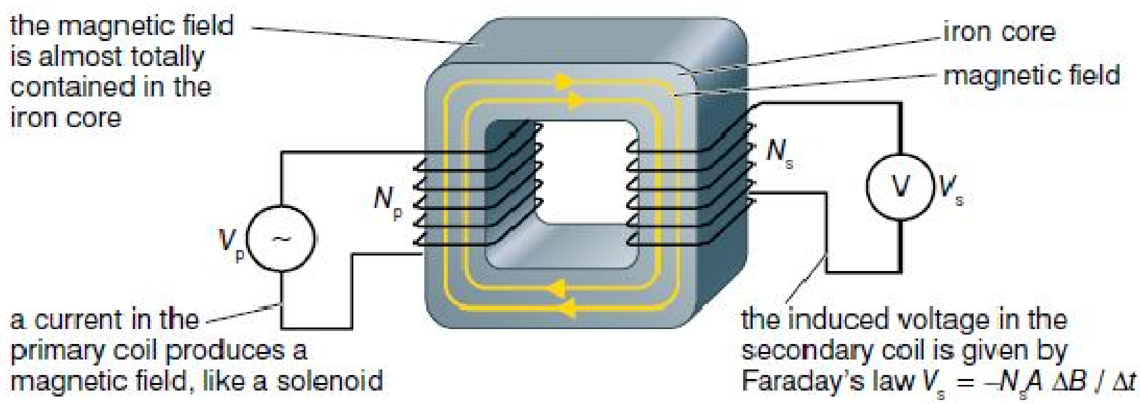


Figure 7.9 Transformer

The changing flux through the primary and secondary coils is equal. Thus, one can write

$$\frac{\Delta\Phi_p}{\Delta t} = \frac{\Delta\Phi_s}{\Delta t}$$

From Faraday's law, we have

$$V_p = -N_p \frac{\Delta\Phi_p}{\Delta t} \quad \text{and} \quad V_s = -N_s \frac{\Delta\Phi_s}{\Delta t}$$

Dividing the second equation by the first and taking into consideration the above relation,we find

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{or} \quad V_s = V_p\left(\frac{N_s}{N_p}\right)$$ This the transformer equation.

If $N_s > N_p \Rightarrow V_s > V_p$ the transform in this case is called **step-up**.

If $N_s < N_p \Rightarrow V_s < V_p$ the transformer in this case is called **step-down**.

Example 7.5

A transformer has 1000 turns on the input coil and 100 turns on the output coil. If the input voltage is 220v, what is the output voltage?

Solution:

V_p=220v N_p=1000 N_s=100

Applying the transformer equation

$V_s = \frac{100}{1000}\, 220V = 22v$

Transformers have very low resistance (their energy wastage is very small).

For ideal transformer the efficiency $(\eta) = 100\%$

$$P_{out} = P_{in}$$

$$V_o I_o = V_{in} I_{in}$$

$$\frac{I_{out}}{I_{in}} = \frac{V_{in}}{V_{out}} = \frac{N_p}{N_s}$$

7.3. Alternating Current Circuits

Resistors in an AC Circuit

Consider a simple AC circuit consisting of a resistor and an AC source, as shown in Figure 7. 10. At any instant, the sum of the voltages loop in a circuit must be zero (Kirchhoff's loop rule).

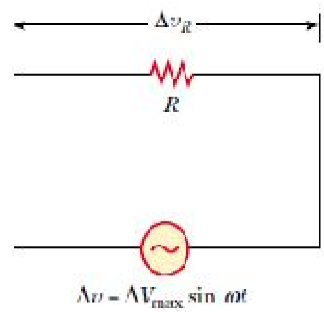


Figure 7. 10 Resistive AC circuit

$$V + V_R = 0$$

The magnitude of the source voltage equals the voltage across the resistor

$$V = V_R = V_{max} \sin\omega t$$

Thus, the instantaneous current in the resistor is

$$i_R = \frac{V_R}{R} = \frac{V_{max}}{R} \sin \omega t = I_{max} \sin\omega t ,$$

where $I_{max} = \frac{V_{max}}{R}$. The instantaneous voltage across the resistor is

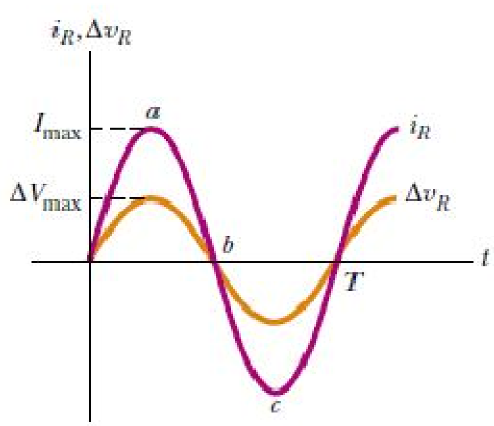


Figure 7. 11 Plots of V_R and i_R against time

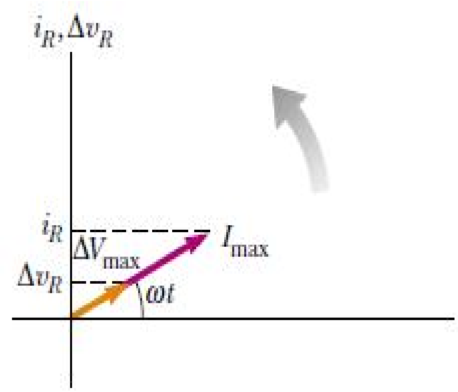


Figure 7. 12 Phasor diagram

The current and voltage are in step with each other because they vary identically with time. Because i_R and V_R both vary as sin ωt and reach their maximum values at the same time, as shown in Figure 7., they are said to be in phase,

To simplify our analysis of circuits containing two or more elements, we use graphical constructions called *phasor diagrams.* A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents (V_{max} for voltage and I_{max} for current in the present discussion) and which rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable. The projection of the phasor onto t2he 2vertical axis represents the instantaneous value of the quantity it represents.

Root mean square (rms) values in ac circuits

For the simple resistive circuit in Figure 7.10 , note that the average value of the current over one cycle is zero. The half cycle average current is

$$i_{av} = \frac{2}{\pi} I_{max} = 0.637\, I_{max}$$

The rate at which energy is delivered to a resistor is the power $P = i^2 R$, where i is the instantaneous current in R which is non-zero.

Thus, what is of important in Ac circuit is the root=mean square (rms) value; means the suare root of the mean (average) value of the square of the current.

$$I_{rms} = \sqrt{\bar{i^2}} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$$

This states that an ac current with $I_{max} = 2A$ delivers to a resistor the same power as a direct current that has a value of 0.707(2A)=1.41A. Thus the average power delivered to a resistor that carries an alternating current is

$$P_{av} = I_{rms}^2 R$$

Alternating voltage is also best discussed in terms of rms voltage

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = 0.707 V_{max}$$

Example 7.6

1. In our house if V_{rms}=220V, what is the peak voltage?
   Solution:
   The peak voltage $= \frac{V_{rms}}{0.707} = 311V$
2. The voltage output of an ac circuit is $V = (200V)\, sin\omega t$. Find the rms current when the source is connected to a 100Ω resistor.
   Solution:

$$V_{rms} = \frac{V_{max}}{\sqrt{2}} = 141V$$

$$I_{rms} = \frac{V_{rms}}{R} = 1.41V$$

Exercise

An ac supply has peak value of $I_{max} = 2.5a \ and \ V_{max} = 6v$.

a) Find I_{rms} and V_{rms}

b) The effect power supplied by the source.

Inductor in an AC circuit (Inductive circuit)

Consider an AC circuit consisting only of an inductor connected to an Ac source. The self induced instantaneous voltage across the inductor is

$$\varepsilon_L = V_L = -L\frac{di}{dt}$$

Using Kirchhoff's loop role to the circuit, we have

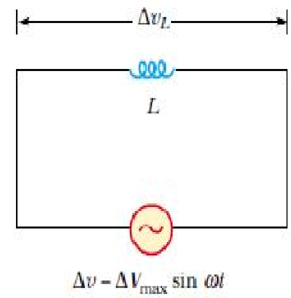


Figure 7. 13 Inductive AC circuit

$$V + V_L = 0$$

Or

$$V - L\frac{di}{dt} = 0$$

Using $V = V_{max} \sin\omega t$, we get

$$V = L\frac{di}{dt} = V_{max} \sin\omega t$$

Or

$$di = \frac{V_{max}}{L} \sin\omega t \, dt$$

Integrating this expression gives the instantaneous current i_L in the inductor

$$i_L = \frac{V_{max}}{L}\int \sin\omega t \, dt = -\frac{V_{max}}{\omega L} \cos\omega t$$

Using the identity $\cos \omega t = -\sin(\omega t - \pi/2)$, we find

$$i_L = \frac{V_{max}}{\omega L} \sin(\omega t - \pi/2)$$

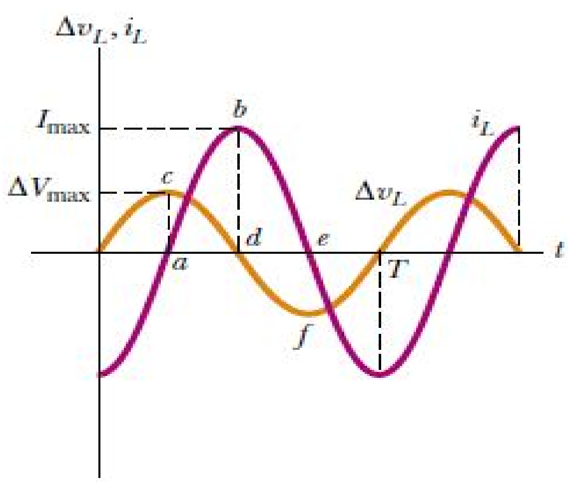


Figure 7. 15 Plots of i_L and vL across an inductor versus time.

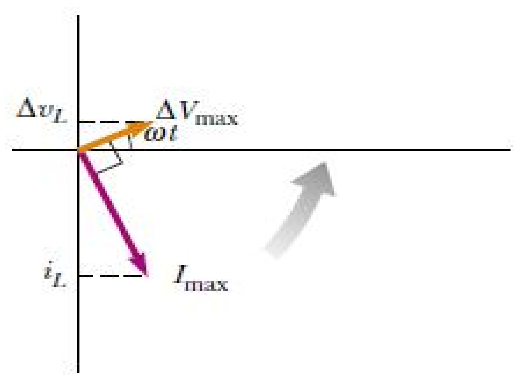


Figure7.16 Phasor diagram for inductive circuit

i_L *and* V_L in the inductor are out of phase by 90^0 ($\pi/2$ rad). i_L in an inductor always lags behind the V_L across the inductor by 90^0 (one-quarter cycle in time). The current in an inductive circuit reaches its maximum value when $\cos \omega t = -1$.

$$I_{Max} = -\frac{V_{max}}{\omega L} \cos\pi = \frac{V_{max}}{\omega L}$$

This looks similar to the relation between I, V and R in Dc circuits (I=V/R Ohm's law)

ωL behaves in a manner similar to resistance and is defined as the **inductive reactance (X_L).** Its unit is Ω and is expressible as

$$X_L = \omega L$$

And one can write

$$I_{max} = \frac{V_{max}}{X_L}$$

The instantaneous current and voltage are given by

$$i_L = -\frac{V_{max}}{X_L}cos\omega t$$

$$V_L = -L\frac{di}{dt} = -V_{max}sin\omega t = -I_{max}X_L sin\omega t$$

Example 7.7

In a purely inductive AC circuit (see Fig. 7), L=25.0 mH and the rms voltage is 150 V. Calculate the inductive reactance and rms current in the circuit if the frequency is 60.0 Hz.

Solution:

L=25.0 mH; f=60.0 Hz; V_{rms}=150 V

a) $X_L = \omega L = 2\pi f L = 9.42\Omega$

b) $I_{rms} = \frac{V_{rms}}{X_L}$=15.9V

Capacitor in Ac circuit

Consider an Ac circuit consisting only of a capacitor connected to an Ac source. Kirchhoff's loop rule applied to this circuit gives

$$V+V_c=0$$

The magnitude of the source voltage is equal to the magnitude of voltage across the capacitor.

$$V = V_c = V_{max}sin\omega t,$$

where V_c is the voltage across the capacitor. We know from the definition of capacitance that $C = \frac{q}{V_c}$; hence

$$\frac{q}{C} = V_{max}sin\omega \Rightarrow q = CV_{max}sin\omega,$$

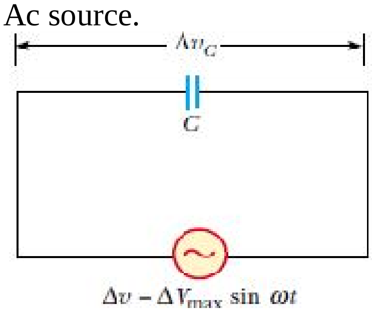


Figure 7.17 Capacitive AC circuit

where q is the instantaneous charge on the capacitor. Because $i = \frac{dq}{dt}$ differentiating the above equation with respect to time gives the instantaneous current in the circuit;

$$i_c = \frac{dq}{dt} = \omega CV_{max}\cos\omega t$$

Using the trigonometric identity $\cos\omega t = \sin(\omega t + \frac{\pi}{2})$, we have

$$i_c = \frac{dq}{dt} = \omega CV_{max}\sin(\omega t + \frac{\pi}{2}).$$

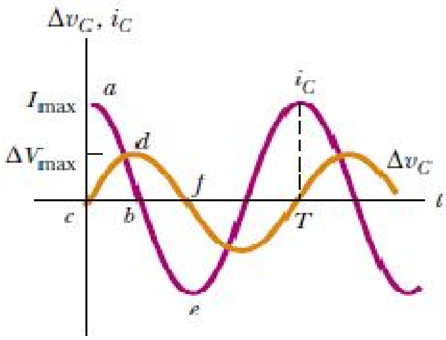


Figure 7. 18 Plots of i_L and vL across a capacitor against time.

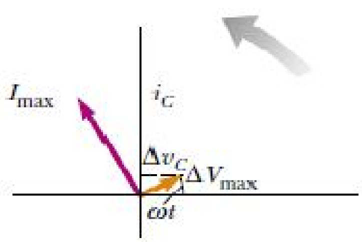


Figure 7. 19 Phasor diagram for capacitive circuit

The current is $\frac{\pi}{2}$ rad $=90^0$ out of phase with the voltage across the capacitor. The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches ite maximum. (The voltage lags or the current leads by 90^0). The current in the circuit reaches its maximum when $\cos\omega t = 1$.

$$I_{Max} = \omega C V_{max} = \frac{V_{max}}{1/\omega C} \quad \text{(This expression looks Ohm's law I=V/R).}$$

Thus, the dominator must play the role of resistance with units of ohms. We give the combination $1/\omega C$ the symbol Xc and because this function varies with frequency, we define it as capacitive reactance;

$$X_c = 1/\omega C$$

$$I_{Max} = \frac{V_{max}}{X_c}$$ We have similar expression for the rms values $I_{rms} = \frac{V_{rms}}{X_c}$.

The instantaneous voltage across the capacitor is

$$V_c = V_{max}\sin\omega t = I_{Max}X_c\sin\omega t$$

Example 7.8

An 8.00μF capacitor is connected to the terminals of a 60.0 Hz AC source whose rms voltage is 150 V. Find the capacitive reactance and the rms current in the circuit.

Solution:

C=8.00μF; f=60.0 Hz; V_{rms}=150V

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 332\Omega$

$I_{rms} = \frac{V_{rms}}{X_c} = \frac{150V}{332\Omega} = 0.452A$

What if the frequency is doubled?

f'=2f=120Hz

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f' C} = 166\Omega$

$I_{rms} = = 0.904A$

The RLC series circuit

RLC is a circuit that contains a resistor, an inductor and a capacitor connected in series across an Ac source. The voltage applied is

$$V = V_{max}\sin\omega t$$

and the current is

$$i = I_{max}\sin(\omega t - \phi),$$

where ϕ is some phase angle between iand v.

We seek to determine ϕ and $I_{max.}$ Because the elements are in series, the current everywhere in the circuit must be the same at any instant(the same phase and amplitude).

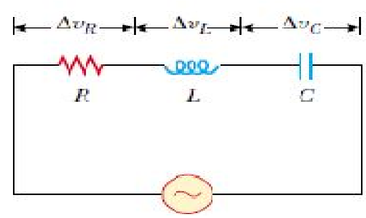


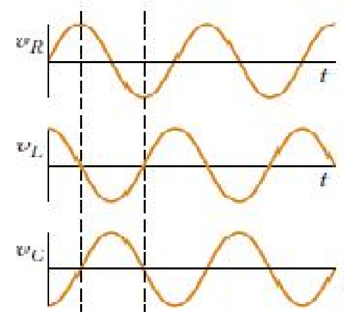
Figure 7.20 RLC circuit

The instantaneous voltages across the three circuit elements are:

$$V_R = I_{max}R\sin\omega t = V_{Rmax}\sin\omega t \Rightarrow V_{Rmax} = I_{Max}R$$

$$V_L = I_{max}X_L\sin(\omega t + \tfrac{\pi}{2}) = V_{Lmax}\cos\omega t \Rightarrow V_{Lmax} = I_{Max}X_L$$

$$V_C = I_{max}X_C\sin(\omega t - \tfrac{\pi}{2}) = V_{Cmax}\cos\omega t \Rightarrow V_{Cmax} = I_{Max}X_C$$



I_{Max}, V_{Lmax} *and* V_{Cmax} are the maximum voltages across the elements. We would proceed by noting that the instantaneous voltage V across the three elements equals the sum

$$V = V_R + V_L + V_C$$

This sum can be simply obtained by examining the phasor diagram.

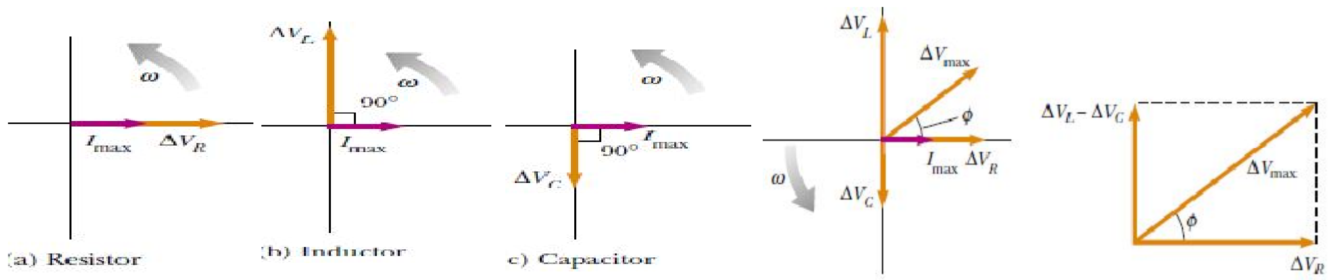


Figure 7.21 Phasor diagram for RLC circuit

Because the current at any instant is the same in all elements, we can combine the three parts vertically.

$$V_{max} = \sqrt{V_{Rmax}^2 + (V_{Lmax} - V_{Cmax})^2} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2}$$

$$V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2}$$

Therefore, we can express the maximum current as

$$I_{max} = \frac{V_{max}}{\sqrt{R^2+(X_L-X_C)^2}}$$

This has the same form as I=V/R. The denominator plays the role of resistance and is called the **Impedance** (Z) of the circuit.

$Z=\sqrt{R^2 + (X_L - X_C)^2}$ It has unit ohm.

Therefore, one can write

$V_{max}=I_{max}Z$ is the Ac equivalent of V=IR.

The impedance, therefore the current in AC circuit depends on R, L, C and ω.

From the impedance triangle, we find the phase angle between the current and voltage as

$$\phi = tan^{-1}\left(\frac{V_L-V_C}{V_R}\right) = tan^{-1}\left(\frac{X_L-X_C}{X_R}\right)$$

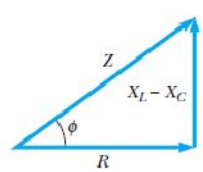


Figure 7.22 Impedance triangle

We see that cos ϕ=R/Z, when $X_L>X_C$ (at high frequency) $\phi > 0$, implies the current lags behind the voltage. The circuit is more inductive than capacitive.

When $X_L<X_C$, $\phi < 0$, indicating the current leads the voltage and the circuit is more capacitive than inductive.

When $X_L=X_C$, $\phi = 0$ and the circuit is pure resistive.

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]		
Circuit Elements	Impedance Z	Phase Angle ϕ
R	R	$0°$
C	X_L	$90°$
L	X_L	$+90°$
R C	$\sqrt{R^2 + X_C^2}$	Negative, between $-90°$ and $0°$
R L	$\sqrt{R^2 + X_L^2}$	Positive, between $0°$ and $90°$
R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Table 7.1

Example 7.9

A series RLC Ac circuit has R=425Ω, L=1.25H, C=3.5μF, $\omega = 377 s^{-1}$ and V_{max}=150V

a) Determine X_L, X_C and Z.

b) Find the maximum current in the circuit.

C) Find the phase angle

Solution:

a) $X_L = \omega L$=471 Ω, Xc=1/ωC=758 Ω Z=$\sqrt{R^2 + (X_L - X_C)^2}$=515 Ω

b) $I_{max} = \frac{V_{max}}{Z}$=0.292A

c) $\phi = tan^{-1}(\frac{X_L - X_C}{X_R})$ =-34^0

Power in AC circuit

Let us now take an energy approach to analyzing AC circuits, considering the transfer of energy from the AC source to the circuit. For *RLC* circuit the instantaneous power delivered by an Ac source to the circuit is

$$P = iV = I_{max}V_{max}\sin(\omega t - \phi)\sin\omega t.$$

We are interested in the average power over one or more cycles. By using the identity $\sin(\omega t - \phi) = sin\omega t\ cos\phi - cos\omega t\ sin\phi$, we can write

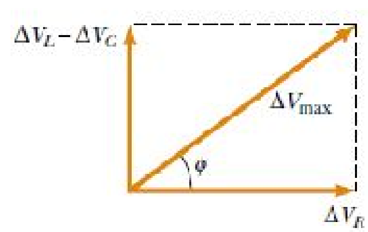
$$P = I_{max}V_{max}\sin^2\omega t\cos\phi - I_{max}V_{max}\sin\omega t\cos\omega t\sin\phi$$

We now take the average of p over one or more cycles, noting that I_{max}, V_{max}, ϕ, ω are all constants and $\langle sin^2\omega t\rangle_{av} = 1/2$ and $\langle sin\ \omega t\cos\omega t\rangle_{av} = 1/2\langle sin\ 2\omega t\rangle_{av} = 0$, the average power takes the form

$$P_{av} = 1/2\ I_{max}V_{max}\cos\phi$$

In terms of the rms values the average power can be written as

$$P_{av} = I_{rms}V_{rms}\cos\phi,$$



where the quantity cosϕ is called the **power factor.**

By inspection of the phasor diagram, we see that V_{max} across the resistor is given by

$$V_R = V_{max}\cos\phi = I_{max}R \text{ , because } \cos\phi = \frac{I_{max}R}{V_{max}}$$

Thus

$$P_{av} = I_{rms}V_{rms}\cos\phi = I_{rms}\frac{V_{max}}{\sqrt{2}}\frac{I_{max}}{V_{max}} = I_{rms}^2 R$$

In words, the average power delivered by the source is converted to internal energy in the resistor, just as in Dc circuit. When the load is purely resistive $\phi = 0$ and cos ϕ =1 and we have

$$P_{av} = I_{rms}V_{rms}$$

We find that no power losses are associated with pure capacitor and pure inductor in Ac circuit.

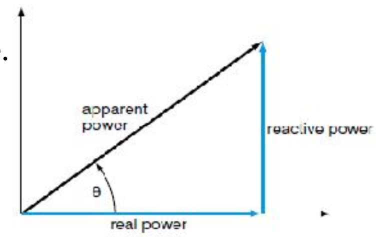
Real, apparent and ideal power in RLC circuit

The real power is the power delivered when the load is purely resistive.

Apparent power is the sum of the real and reactive power.

Apparent power= $I_{rms}^2 Z$

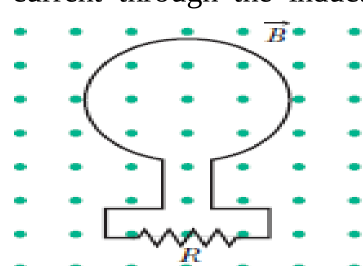
The ideal situation (ideal power) is where apparent power and real (true) power are equal. The Power factor=1



The power factor= $\frac{Real\ power}{Apparent\ power}$

Problems

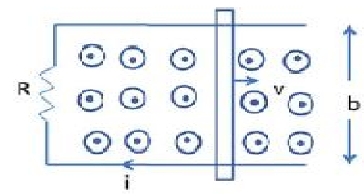
1. An AC circuit consists of only a resistor R = 100Ω and a source voltage V = 0.5 V_{max} at time t=1/360 s. Assuming that at t = 0, V = 0, find the frequency.
2. A 6 Ω resistor, a 12 Ω inductive reactance and a 20 Ω capacitive reactance are connected in series to a 250Vrms AC generator. (a) Find the impedance. (b) Estimate the power dissipated in the resistor.
3. At 600 Hz an inductor and a capacitor have equal reactances. Calculate the ratio of the capacitive reactance to the inductive reactance at 60 Hz.
4. A 3 Ω resistor is joined in series with a 10mH inductor of negligible resistance, and a potential difference (rms) of 5.0V alternating at 200/π Hz is applied across the combination.
   (a) Calculate the potential difference V_R across the resistor and V_L across the inductor.
   (b) Determine the phase difference between the applied PD and the current.
5. An inductance stores 10 J of energy when the current is 5A. Find its value.
6. An inductor in the form of a solenoid contains 420 turns, is 16.0 cm in length, and has a cross-sectional area of 3.00 cm2. What uniform rate of decrease of current through the inductor induces an emf of 175 'V?
7. In the Fig., the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when t= 2.0 s?
   (b) Is the direction of the current through R to the right or left?



8. A wire loop of area $0.2 m^2$ has a resistance of 20 Ω. A magnetic field, normal to the loop, initially has a magnitude of 0.25 T and is reduced to zero at a uniform rate in 10^{-4} s. Estimate the induced emf and the resulting current.

9. A coil is 30 turns of wire, each of area 10 cm^2, is placed with its plane at right angles to a magnetic field of 0.1 T. When the coil is suddenly withdrawn from the field, a galvanometer in series with the coil indicates that 10^{-5} C passes around the circuit. What is the combined resistance of the coil and the galvanometer?

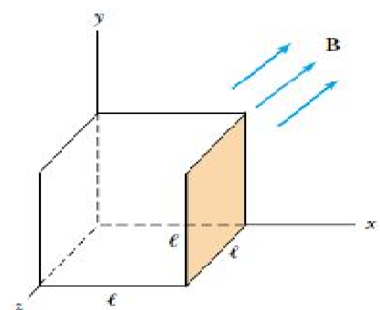
10. A bar slides on rails separated by 20 cm, Fig. If the current flowing through the resistor R = 5Ω is 0.4A and the field B = 1 T, what is the speed of the bar



11. a) A solenoid with 100 turns, length 10 cm and of radius 1 cm, carries a current of 5A. Calculate the magnetic energy stored in the solenoid.

b) The current in the solenoid of part (a) is reduced to zero at a uniform rate over 5 s. Calculate the emf induced in the coil.

12. A cube of edge length L= 2.50 cm is positioned as shown in Figure. A uniform magnetic field given by **B** = (5***i*** + 4***j*** + 3***k***)T exists throughout the region.
(a) Calculate the flux through the shaded face.
(b) What is the total flux through the six faces?



Unit 8

ATOMIC AND NUCLEAR PHYSICS

Classical physics is developed between 1600 and 1900. It includes classical mechanics, thermodynamics and electromagnetism. In classical physics one distinguishes the difference between two entities: material particle and wave. Material particle is an entity that possesses definite mass but with spatial dimension that can be ignored in all practical purposes. Electrons, protons and neutrons can considered as point particles having no spatial dimensions. Wave is an entity that spreads in space from its source. It has no definite position. Travelling wave transports energy.

In classical physics, there are some views that were taken for granted.

- Electromagnetic radiation or light is a continuous wave produced by acceleration charges.
- A material particle has definite position and linear momentum.
- Classical physics is deterministic. If the initial conditions and the net force acting on a particle are known, its position and momentum at any later time can be predicted by solving Newton’s second Law.
- The energy of a particle is a continuous variable. Hence there is no restriction on the amount of energy the particle gains or losses when it interacts with other particle or wave. When particles and waves interact, they can exchange any amount of energy.

Employing the mutually exclusive concepts of particle and wave classical physics successfully described several physical phenomena. However, in the beginning of the twentieth century, classical physics fails to explain certain physical phenomena. This failer leads to the development Quantum Physics.

In quantum physics, the classical views of pure wave and pure particle are replaced by wave-particle duality. Electromagnetic radiation has dual aspects or properties. It has the properties of a wave as well as a particle. It behaves simultaneously like a wave and like a stream of mass less particles called photons. A photon has definite energy and momentum like a material particle. It has also wavelength like a wave. Physical processes such as Blackbody Radiation, Photoelectric Effect, and Compton Effect cannot be explained unless both aspects are taken into account. Material particles also have dual properties. An electron has definite masse and charge like a particle but no definite position like a wave. Beams of electrons can interfere like waves. They can also collide with photons like particles.

One of the shocking results of quantum physics is that the classical view of energy as a continuous variable fails on a microscopic scale. It takes a certain discrete values only. It is quantized.

In the present state of scientific knowledge, quantum physics plays a fundamental role in the description and understanding of natural phenomena which occur on atomic or subatomic scale. Atomic and nuclear properties, emission and absorption spectra of atoms, chemical and nuclear reaction, physical, electrical, magnetic and some their properties of materials cannot be explained outside the framework of quantum physics.

Blackbody Radiation

When heated, a solid object glows and emits thermal radiation. As the temperature increases, the object becomes red, then yellow, then white. The thermal radiation emitted by glowing solid contains a continuous distribution of frequencies ranging from infrared to ultraviolet. The continuous pattern of the distribution is in sharp contrast to the radiation emitted by heated gases; then radiation emitted by gases has a discrete or line spectrum.

Understanding the continuous character of the radiation emitted by a glowing solid object was one of the major unsolved problems during the second half of the nineteenth century. All attempts to explain this phenomenon by means the available theories of classical physics ended up in miserable failure. This problem consisted in essence of specifying the proper theory that describes how energy is exchanged between radiation and matter.

When radiation falls on an object, some of it might be absorbed and some reflected. An idealized "blackbody" is a material object that absorbs all the radiation falling on it, and appears black under reflection when illuminated from the outside. When an object is heated, it radiates electromagnetic energy. The intensity of the radiation depends on its frequency and on the temperature of the object. The light emitted by the object ranges over the entire spectrum. An object in thermal equilibrium with its surroundings radiates as much energy as it absorbs. It thus follows that a blackbody is a perfect absorber as will as a perfect emitter of radiation.

In laboratory a blackbody can be approximated by a hollow cavity whose walls perfectly reflect electromagnetic radiation (e.g., metallic walls) and which has a very small hole leading to its interior. Radiation that enters the hole will be trapped inside the cavity and gets completely absorbed. On the other hand, when this cavity is heated to temperature T, the radiation that leaves the hole is blackbody radiation. To understand the radiation inside the cavity, one needs simply to analyze the spectral distribution of the radiation leaving the hole.

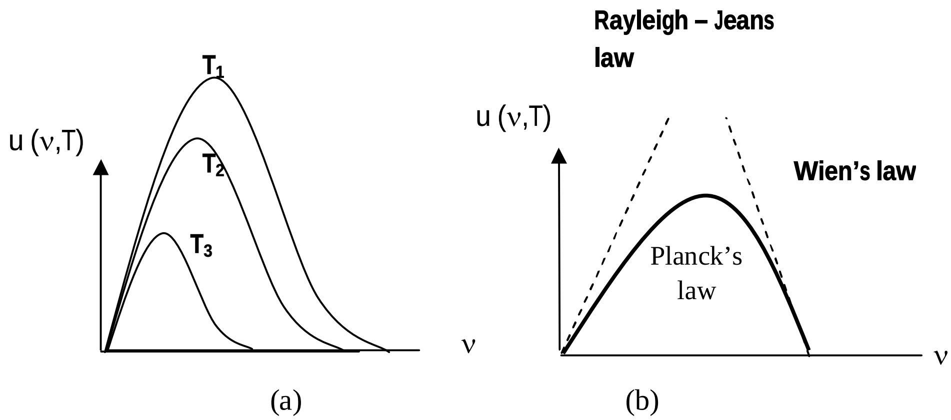


Figure 8.1 (a) experimental spectral energy density for several temperatures T_1, T_2 and T_3, with $T_1>T_2>T_3$. (b) The solid curve represents the experimental and Planck's distributions; the dashed curves correspond to Rayleigh-Jeans and Wien distributions.

Experiments showed that at equilibrium, the radiation emitted has a continuous energy distribution. To each frequency there corresponds an energy density which depends only on the temperature of the cavity's wall. The energy density shows a pronounced maximum at a given frequency, which increases with temperature.

A number of attempts aimed at explaining the origin of the continuous character of this radiation were carried out. The most important among these attempts were due to Wilhelm Wien in 1889 and by Rayleigh and Jeans in 1900. Both attempts failed. The origin of this failure is the premise that the energy exchange between radiation and matter is continuous; any amount of energy can be exchanged. The failure of the Rayleigh-Jeans formula at short wavelengths or high frequencies is known as the **ultraviolet catastrophe** and represents a serious problem for classical physics, because the theories of thermodynamics and electromagnetism on which Rayleigh-Jeans formula is based have been carefully tested in many other circumstances and found to give extremely good agreement with experiment. It was apparent in the case of blackbody radiation that the classical theories would not work, and that a new kind of physical theory was needed.

Planck's Theory of Blackbody Radiation

In 1900 Max Planck discovered an empirical formula-formula not supported by theory-for blackbody radiation that was in complete agreement with the experimental results at all wavelengths. He also introduced a new universal constant h-helping constant-know as Planck's constant. Later, Planck derived his empirical formula theoretically.

In his theory, Planck made two controversial postulates or assumptions about the nature of the oscillating atoms of the cavity walls.

- The oscillating atoms that emit radiation can have only discrete energies given by

$E = nh\nu$, where ν is the frequency of vibration of the atoms, n=0, 1, 2, 3….
and h= 6.626×10^{-34} Js is Planck's constant. The energies the oscillating atoms are said to be quantized and the allowed energy states are called quantum states.

Planck's theory was the first breakthrough in the development of quantum physics. However, Planck himself was uncomfortable with the idea of energy quantization. Years later he referred to his assumptions as an "act of desperation ". He didn't realize the significance of what he had done. Neither did anyone else except Einstein who applied Planck's idea in describing the Photoelectric Effect.

Photoelectric Effect

The photoelectric effect provides a direct confirmation for the energy quantization of light. In 1887 Hertz discovered the photoelectric effect: which is ejection of electrons from metals when irradiated with light.

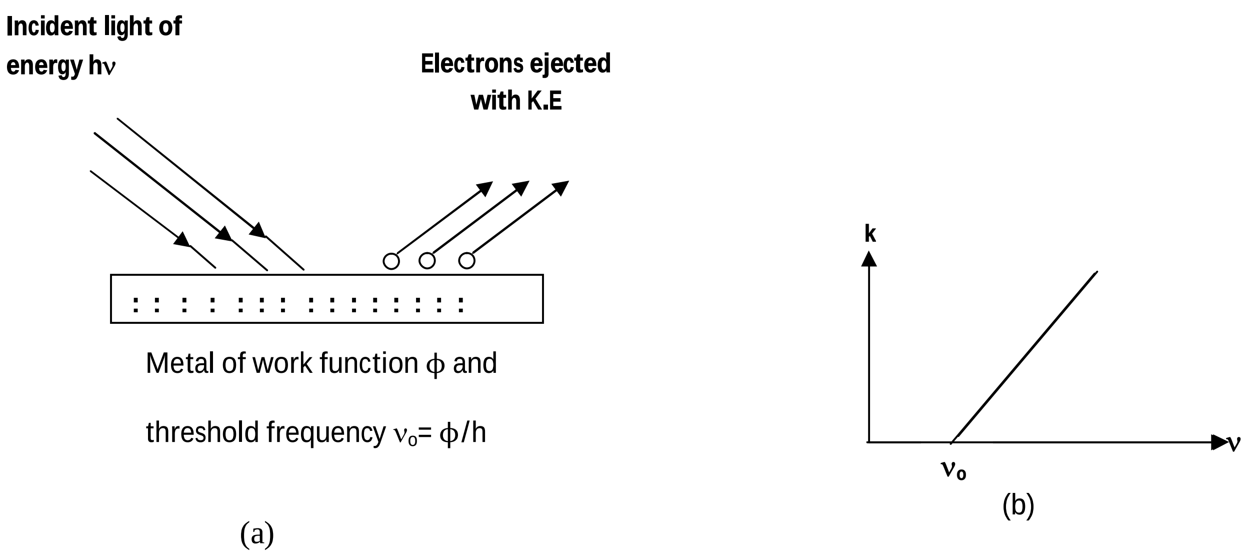


Figure 8.2 (a) Photoelectric effect: when a metal is irradiated with light, electrons may get emitted. (b) Kinetic energy K of the electron leaving the metal when irradiated with a light of frequency ν; when $\nu < \nu_0$ no electron is ejected from the metal regardless of the intensity of the radiation.

Discovered experimental laws:

- If the frequency of the incident radiation is smaller than the metal's threshold frequency, a frequency that depends on the properties of the metal, no electrons can be emitted regardless of the radiation's intensity (Philip Lenard, 1902).
- No matter how low the intensity of the incident radiation, electrons will be ejected instantly the moment the frequency of the radiation exceeds the threshold frequency.

- At a fixed frequency, the number of ejected electrons increases with the intensity of the light but does not depend on its frequency.
- The kinetic energy of the ejected electrons depends on the frequency but not on the intensity of the beam; the kinetic energy of the ejected electrons increases linearly with the incident frequency.

These experimental findings cannot be explained with in the context of a purely classical picture of radiation, notably the dependence of the effect on the threshold frequency.

Einstein succeeded in 1905 in giving a theoretical explanation. He assumed that light is made of corpuscles each carrying an energy hν, called photons. When a beam of light is incident on a metal, the photon is entirely absorbed by the electron. The electron will thus absorb energy only in quanta of energy hν, irrespective of the intensity of the incident radiation. If hν is larger than the metal's work function ϕ, the energy required to dislodging the electron from the metal (every metal has free electrons that move from one atom to another; the minimum energy required to free the electrons from the metal is called the work function of the metal), the electron will then be knocked out of the metal. Hence no electron can be emitted from the metal's surface unless hν> ϕ:

$$h\nu = \phi + K,$$

where K is the kinetic energy of the electron leaving the metal. This can also put in the form

$$K = h\nu - \phi = h(\nu - \nu o),$$

where $\nu_o = \phi/h$ is called the threshold or cut off frequency of the metal.

Example 8. 1

When two ultraviolet beams of wavelengths λ_1 = 280 nm and λ_2= 490nm fall on a lead surface, they produce photoelectrons with maximum energies 8.57ev and 6.67ev, respectively.

(a) Estimate the value of the Planck constant.
(b) Calculate the work function and the cutoff frequency of lead.

Solution:

(a) Using Einstein's formula on can write

$$k_1 = \frac{hc}{\lambda_1} - \phi \quad \text{and} \quad k_2 = \frac{hc}{\lambda_2} - \phi$$

Subtracting the second equation from the first, we get

$$k_1 - k_2 = hc\frac{(\lambda_2 - \lambda_1)}{\lambda_2\lambda_1},$$

Hence solving for h, we find

$$h = \frac{k_1 - k_2}{c} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}$$

Since 1 eν = 1.5x10^{-19}J, the numerical value of h follows:

$$h = \frac{(8.57 - 6.67)x1.6x10^{-19}\,J}{3x10^8\,{}^{m}\!/\!_{s}} = \frac{(280x10^{-9}\,m)(490x10^{-9}\,m)}{490x10^{-9}\,m - 280x10^{-9}\,m} \approx 6.62x10^{-34}\,Js$$

(b) The work function of the metal

$$\phi = \frac{hc}{\lambda_1} - k_1 = \frac{6.62x10^{-34}\,JSx3x10^8\,{}^{m}\!/\!_{s}}{280x10^{-9}\,m} - 8.57x1.6x10^{-19}\,J$$

$$\phi = -6.62x10^{-19}\,J = -4.14w$$

The cutoff frequency of the metal is

$$\nu_o = {}^{\phi}\!/\!_{\lambda} = \frac{6.62x10^{-19}\,J}{6.62x10^{-34}\,JS} = \underline{\underline{10^{15}\,Hz}}$$

Then in 1923 Arthur Compton made an important discovery that gave the most conclusive conformation for the corpuscular(particle) aspect of light. By scattering X-rays with electrons, he confirmed that the X-ray protons behave like particles with momenta bν/c, ν is the frequency of the X- rays.

These and other series of breakthroughs gave both theoretical foundations as well as the conclusive experimental confirmation for the particle aspect of waves; that is, the concept that waves exhibit particle behavior at the microscopic scale.

The De Broglie's Hypothesis

In 1923 Luis de Broglie suggested that wave-particle is not restricted to radiation, but must be universal; all material particles should also display a dual wave-particle behavior.

Starting from the momentum of a photon $P = \frac{h\upsilon}{c}$, we can generalize this relation to any material particle with non zero rest mass; each material particle of momentum **P** behave as a group of waves (matter waves) where wavelength λ and wavevector k are:

$$\lambda = \frac{\mathrm{h}}{\mathrm{P}} \qquad \mathbf{k} = \frac{\mathbf{P}}{\hbar}\ , \quad \text{where } \hbar = \frac{\mathbf{h}}{2\pi} \qquad \text{de Broglie relation.}$$

de Broglie's idea was confirmed experimentally in 1927 by Davisson and Germer and later by Thomas, who obtained interference patterns with electrons.

Example 8.2

Compute the de Broglie wavelength of an electron having momentum 5.4 $x10^{-25}$kgm/s

($h=6.63x10^{-31}$ Js).

Solution

The de Broglie wave length is given as

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\,\text{Js}}{5.4 \times 10^{-25}\,\text{kgms}^{-1}}$$

$$= 1.2 \times 10^{-9}\,\text{m} = 1.2\text{nm}$$

Exercise

Electrons are accelerated through potential difference of 4000V before striking a layer of graphite and being diffracted ($m_e=9.11x10^{-31}$ Kg and $h=6.63x10^{-31}$ Js). Calculate

a) the speed of electrons when they hit the graphite

b) the momentum of the electron

c) the wavelength of the electron

Heisenberg's Uncertainty principle

In any measurement there is always a certain amount of error or uncertain involved. Suppose one want to determine the location of electron. There should be an experiment. Since electrons behave as a wave and particle, the position determined with certain value of uncertain.

If the momentum of a particle is measured with an uncertainty $\Delta P_{X,}$ then the position cannot at the same time, be measured more accurately than $\Delta X = \frac{\hbar}{2\Delta P_x}$.

That is for all types of experiment the uncertainties ΔX and ΔP_X will always be related by

$$\Delta P_x \Delta X \geq \frac{\hbar}{2}$$

This equation is known as Heisenberg **uncertainty relation**.

Example 8.3

The position X of a 0.01g object has been carefully measured and is known with in $\pm 0.5\mu m$. According to the uncertainty principle, what are the minimum uncertainties in its momentum and velocity, consistent with our knowledge of X?

Solution:

If X is known with in $\pm 0.5\mu m$, the spread $\pm \Delta X$ in the position is certainly no larger than 0.5μm. Thus

$$\Delta X \leq 0.5\mu m$$

According to the uncertainty relation, this implies that the momentum is uncertain by an amount

$$\Delta P_x \geq \frac{\hbar}{2\Delta X} \geq \frac{10^{-34}\,J.S}{10^{-6}\,m} = 10^{-28}\,kgm/s$$

Therefore, the velocity $V = \frac{P}{m}$ is uncertainty

$$\Delta V = \frac{\Delta P}{m} \geq \frac{10^{-28}\,kgm/s}{10^{-5}\,kg}$$

$$= 10^{-23}\,m/s$$

8.2 Atoms and Nuclei

Atomic model

A number of atomic models have been proposed to describe the distribution of the positive and the negative charges in the atom. In this section you will revise the main models, like Thomson, Rutherford, Bohr atomic models and the wave mechanics model.

The Thomson atomic model

J. J. Thomson, who discovered the electron in 1897, was proposed an atomic model in 1904 before the discovery of the atomic nucleus. He proposed a model of the atom that is sometimes called the "Plum Pudding" model.

The Thomson model considered the atom as a homogeneous sphere of positive charge with sufficient electrons embedded in it so that the atom as a whole is electrically neutral (see Fig 8.3). It is like pudding with plums distributed throughout.

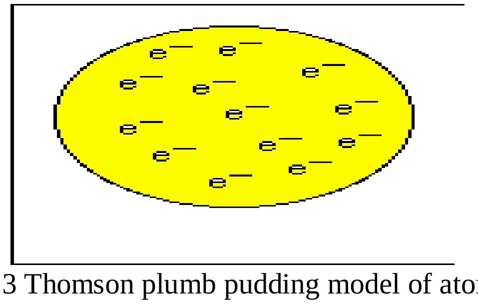


Figure 8.3 Thomson plumb pudding model of atom

This model failed to explain the observed frequencies in optical spectra and the result of alfa-scattering experiment of Geiger and Marsden

The Rutherford atomic models

In 1911 Ernest Rutherford proposed a nuclear theory for the structure of an atom. The 1904 Thomson model was disproved by the 1909 gold foil experiment. The Geiger–Marsden experiment (also called the Gold foil experiment or the Rutherford experiment) was an experiment to probe the structure of the atom performed by Hans Geiger and Ernest Marsden in 1909 under the direction of Ernest Rutherford. The unexpected results of the experiment demonstrated for the first time the existence of the atomic nucleus, leading to the downfall of the plum pudding model of the atom, and the development of the Rutherford (or planetary) model.

The Bohr model of the hydrogen atom

In 1913 Niels Bohr combining Rutherford's, Planck's and Einstein's concepts forwarded accurate account of hydrogen atom and emerges a new explanation for atomic stability.

Bohr made the following bold postulates regarding the hydrogen atom:

i. The electron revolves round the nucleus only in certain definite circular orbits with out radiating energy. The possible orbits are called the stationary states of the atom.

ii. The allowed states are those for which the orbital angular momentum of the electron mvr is equal to integral multiple of $\hbar$

$$mvr_n = n\hbar, \qquad n = 1,2,3,........$$

Where, r_n is the radius of the n^{th} orbit.

iii. Radiation of energy hυ is emitted only when the electron jumps from one stationary state of energy E_j to another state of low energy E_i. When such jump occurs, energy conservation gives

$$h\nu = E_i - E_j$$

In the figure below (Fig 8.3), the model of the atom may look familiar. This is the Bohr model. In this model, the nucleus is orbited by electrons, which are in different energy levels.

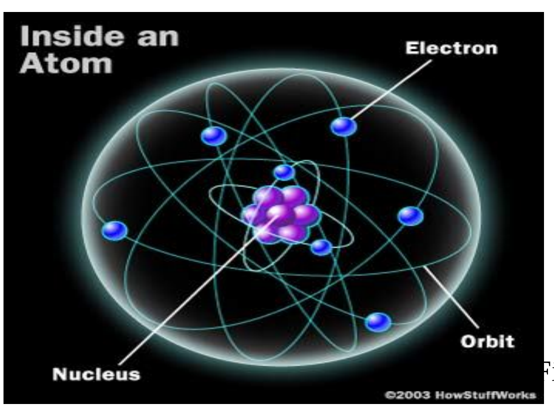


Figure 8.4. Bohr atomic model

Example 8.4

What is the wavelength of radiation emitted when a hydrogen atom is jumped from the 1st exited staet (-3.4ev) to the ground-state (-13.6 ev)

Solution:

Using the relation

$$h\nu = E_2 - E_1$$

we esily get the frequency of the ration to be

$$h = \frac{E_2 - E_1}{h} = \frac{-3.4eV + 13.6eV}{6.63 \times 10^{-34}Js} =$$

and the wavelength is

$$\lambda = \frac{c}{\nu}$$

The atomic orbital model or wave mechanics model of the atom used now was proposed by Heisenberg in 1920. The “probability clouds” represent what are referd to as electron “orbitals”.

Atomic Nucleus and Radioactivity

Here we wish to study about nucleus, nucleons and isotopes, and then we are going to study about binding energy, radioactive process, about fusion and fission reaction. Finally we will study how ionizing radiations are harmful and what precaution has to be taken in the area of radioactive element.

The atomic nucleus

From experiments done in the late 19th and early 20th century it was deduced that atoms are made up of three fundamental or sub-atomic particles called **protons, neutrons and electrons.**

The **electrons** which are moving around the nucleus are responsible for the optical and chemical properties of the element.

The **nucleus** is made up of neutrons and protons, collectively called nucleons (N). Neutrons have no charge. Neutrons and protons have about the same mass, with the neutrons being slightly heavier. The **atomic number** Z of an element is the number of protons in each of its atomic nuclei, which is the same as the number of electrons in a neutral atom of the element.

The **atomic mass number** is A is given by

$$A = Z + N$$

We name elements by

$$^{A}_{Z}X$$

Where, X = the chemical symbol.

Z = atomic number of the element

= number of proton in the nucleus

A = mass number of the nuclide

= number of nucleons in the nucleus

An ordinary hydrogen atom has a single proton, whose charge is +e and whose mass is 1836 times that of the electron. All other elements have nuclei that contain neutrons as well as protons.

The element is determined by the number of protons. Elements with different number of neutrons but the same number of protons are called **Isotopes.**

For example, Hydrogen has three isotopes. Namely ordinary hydrogen (Z=1, A=1), deuterium (Z = 1, A = 2), and tritium (Z = 1, A = 3). In symbol,

$$^{1}_{1}H \qquad ^{2}_{1}H \qquad ^{3}_{1}H$$

The strong nuclear force

It held protons and neutrons together. It is the strongest of the four forces but has the shortest range (about diameter of proton/neutron). It is created between nucleons by the exchange of particles called **mesons** (extremely fast, temp. extremely high and under immense pressure).

Neutrons help to reduce the repulsion between protons in the nucleus.

The nuclei are roughly spherical and appear to have a constant density: the Fermi Model

$$R=1.2x10^{-15}xA^{1/3}m,$$

where r is the radius of the nucleus and A is the mass number.

Example 8.5
The mass number of oxygen is 16. Calculate the radius of the oxygen nucleus.
Solution;
$r=1.2x10^{-15}x16^{1/3}m=3.024x10^{-15}m= 3.024x10^{-5}4$, where $A^0 is\ Angstrom$: $1A^0 = 10^{-10}m$.

Nuclear Properties

Nuclei of atoms can be ordered according to Z and N.

- For lighter nuclei, if we look at the most common isotops, N$\cong Z$
- As we get to heavier nuclei, past Z=20, N>Z, this becomes more apparent.
- Bismuth is the heaviest stable nucleus. Heavier nuclei exist but they are unstable (radioactive). Nuclei from Z=84(polonium) to 92 (uranium) are found in nature (on earth) and all their isotopes are radioactive
- Nuclei heavier than uranium exist but they are all artificial. The heaviest known nucleus has Z=118

The Atomic Mass and Binding energy

Einstein's theory

Mass and energy are equivalent. Thus, mass and energy can interconvert. The energy equivalence of mass is given by

$$E = mc^2$$

This relation helps to calculate binding energy.

Mass are often given in atomic mass units. One atomic mass unit is equal to 1/12 the average weight of a carbon nucleus and is approximately equal to the mass of a proton.

$$1u = 1.6605 \times 10^{-27} kg$$

Its energy equivalent is found, using Einstein's relation, to be

$$E = 931.5 MeV$$

The total mass of a stable nucleus is always less than the sum of the masses of its constituents by an amount Δm, called **mass deficiency**. The energy equivalent of this mass difference is referred to as the **binding energy** BE of the nucleus. That is

$$BE = \left[Nm_n + Zm\left({}_1^1H\right) - m\left({}_Z^AX\right)\right]c^2$$

where N = A – Z is the neutron number,

m_n is the mass of a neutron,

$m\,{}_1^1H$ is the mass of a hydrogen atom, and

$m\,{}_Z^AX$ is the mass of the isotope which includes the mass of Z electrons. Thus for deuterium,

$$BE = (1.008665u + 1.007825u - 2.014102u)931.5\frac{MeV}{u}$$
$$= 2.224MeV$$

The **binding energy** is responsible for holding the nucleons together in the nucleus. It is defined as the energy required breaking up a given nucleus into its constituent parts of N neutrons and Z protons.

The **average binding energy** per nucleon is defined as the binding energy of a nucleus divided by the number of nucleons it contains.

Example 8.6

Calculate the binding energy and average binding energy per nucleon of ${}^{12}_{6}C$.

Solution:

${}^{12}_{6}C$ has 6 proton and neutrons.

Mass of 6 neutrons = 6 X 1.008665 u = 6.05199 u

Mass of 6 hydrogen atoms = 6 x 1.007825 u = 6.04695 u (this includes the mass of electrons as well)

Total mass of constituents = 12.09894 u

Mass of ${}^{12}_{6}C$ =12.0 u

Mass deficiency = 0.09894 u

Binding energy = (0.09894 u)(931.5MeV/u) = 92.16 MeV

Average binding energy per nucleon = $\frac{92.16MeV}{12} = 7.68MeV$

Nuclear stability and Radioactivity

Nuclear stability

Isotopes are collectively known as nuclides. About 256 (76%) of the nuclides are “stable isotopes”. For 80 of the elements, there is at least one stable isotope. The average number of stable isotopes per element among those that have stable isotopes is 3:1. Twenty seven elements have only a single stable isotope, while the largest number of stable isotopes observed for an element is 10, for the element tin. Elements 43, 61 and all elements numbered 83 or higher have no stable isotopes.

Nuclear stability is affected by the ratio of proton to neutron.

As Z increases, the number of stable Isotopes diverges from the line Z=N

Nuclear stability is also determined by binding energy per nucleon.

Is the energy required to disassemble a nucleus into the same number.

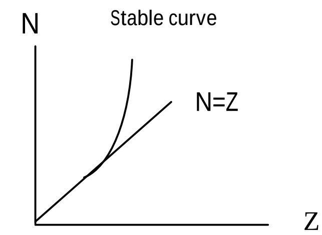


Figure. 8.5 Nucleus stability curve

of free un bounden protons and neutrons. (E_b=Nuclear force-Coulomb force: stability increases with $\frac{E_b}{A}$). As nucei get heavier than helium, their net binding energy per nucleon grows more and more slowly and reaches its peak at iron.

Radioactive decay

In a nuclear reaction, the rearrangement of the protons and neutrons in nuclei results in the formation of new nuclei. The unstable isotopes are radioactive, that is their spontaneous nuclear reactions are accompanied by the emission of **alpha particle**, **beta particle** or **gamma rays**. The process is called **radioactive decay** and thus is three types:

Alpha decay($\boldsymbol{\alpha - decay}$) an atomic nucleus emit an alpha particle (helium nucleus) and decays into an atom with A that is reduced by four and Z that is reduced by two.

$$ {}_{z}^{A}X \rightarrow {}_{z-2}^{A-4}Y + {}_{2}^{4}\alpha $$

For example U-238 decays to form thorium -234

$$ {}_{82}^{238}U \rightarrow {}_{80}^{234}Th + {}_{2}^{4}\alpha $$

- Have K.E. 5Mev and low speed $1.45\text{x}10^5$m/s to $2.2\text{x}10^5$m/s.
- Have larger mass, q=+2e and are very likely to interact with other atoms
- Have list penetrating power
- Have high ionization power

Beta decay ($\boldsymbol{\beta - decay}$) - is a decay in which beta ray (fast energetic electron or positron) and neutrino are emitted from an atomic nucleus.

Example

$Neutron \rightarrow Proton + electron$ $\quad n \rightarrow P + \bar{e} + \bar{v}_e,$ $where\ \bar{v}_e\ is\ electron\ antineutrion$

$Proton \rightarrow Neutron + positron(e^+)$ $\quad p \rightarrow n + e^+ + v_e,$ $where\ v_e\ is\ electron\ neutrion$

- They are fast moving electrons: speed ranging fro 1.6x108m/s to $2.8x10^8$m/s
- Their masses vary with velocity
- They are emitted by an atom with different velocity
- They are easily scattered by atoms when collide
- They have penetration power hindereds times larger than $\alpha -$ particles.
- Much less ionization power

Gamma decay($\boldsymbol{\gamma - decay}$) gamma rays are produced during gamma decay, which normally occur after other forms of decay occur such as alpha or beta decay.

Example $\quad {}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni \rightarrow \bar{e} + \bar{v}_e + \gamma + 1.7Mev$

- They have no charge and no mass
- They have high energy and high frequency
- They rarely interact with particles
- They have very high penetration power
- They have least ionizing power

Radioactive isotopes are of two types. Those **naturally occurring** and those **artificially produced**, and they are called natural radioactivity and artificial radioactivity respectively.

Radioactive isotopes often decay to other isotopes that are also radioactive. The initial isotope is called **the parent** and its decay product is called **daughter**. The daughter may also be radioactive; its decay product is then called the granddaughter, and so on.

Decay law

Radioactivity is a statistical process. Nuclei decay one by one over a period of time. Assuming that each nucleus has the same probability of decaying in one second, we can determine how many nuclei in a sample will decay over a given period.

The rate at which particular radioactive materials decay is a constant. That is it is almost independent of all physical and chemical condition.

If the sample contains N undecayed nuclei at time t and dN decay at time dt,

$$\frac{dN}{dt} = -N\lambda$$

Where, λ is a constant of proportionality called **disintegration constant** or decay constant which is constant in time too. The negative sign implies N decreases with increasing time.

Assuming that there are No nuclei at time t = 0, from the above equation, we have

$$\int_{N_o}^{N} \frac{dN}{N} = -\int_0^t \lambda dt$$

Integrating both sides gives

$$\ln N - \ln N_o = -\lambda t$$

Taking exponential on both side gives

$$N = N_O e^{-\lambda t}$$

This is called the **Radioactive Decay law.**

From the above equation λ can be expressed as

$$\lambda = -\frac{dN/dt}{N}$$

λ Represent the probability per unit time.

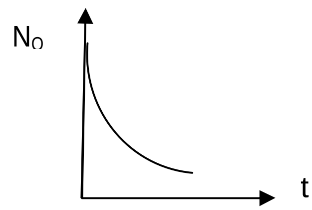


Figure 8.6 Exponential of Radioactive elements with time

Activity

The activity of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms decay. If N is the number of nuclei present in the sample at a certain time, its activity R is given by

$$R = -\frac{dN}{dt}$$

The minus sign is used to make R a positive quantity since dN/dt is, of course, intrinsically negative.

The SI unit of activity is named after Becquerel.

1 Becquerel = 1Bq = 1decay/sec

The other traditional units is curie (Ci),

1Ci = 3.7 x 1010 decay/sec

The above equation can be written as

$$R = \lambda N^{o} e^{-\lambda t} = \lambda N$$

At t = 0, $R_o = \lambda N_O$, then

$$R = R_O e^{-\lambda t}$$

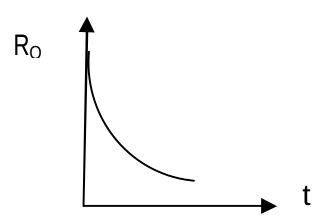


Figure.8.7 The activity of a

radioactive sample as a function of time

The figure shows activity as a function of time.

Example 8.7

From the relation $R = R_0 e^{-\lambda t}$, find expression for the time

Solution

one can be written as

$$\ln \frac{R}{R_o} = \ln e^{-\lambda t}$$
$$= -\lambda t$$

$$\Rightarrow t = \frac{-1}{\lambda} \ln\left(\frac{R}{R_o}\right)$$

Half life

Radioactivity can be characterized by different parameters like, Half-life and activity. Half-life is the time needed for an initial activity to drop by half. Half-life is different for different element.

After a half-life has elapsed, that is when $t=T_{1/2}$, the activity drops to $R_o/2$, one can write

$$\frac{1}{2} R_o = R_o e^{-\lambda t}$$

$$\Rightarrow e^{\lambda T_{1/2}} = 2$$
$$\Rightarrow \ln 2 = \lambda T_{1/2}$$

Simplifying this yields

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Example 8.8

The half life of a nucleus is 2.1min. What is its decay constant?

Solution:

Using the equation for the half life,

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\Rightarrow \lambda = \frac{0.693}{T_{1/2}}$$
$$= \frac{0.693}{2.1x60} = 0.0055\,s^{-1}$$

Example 8.9

Find the activity of 1.0mg of ^{198}AU whose decay constant is 2.97 x $10^{-6}s^{-1}$, if the sample at some time contains 1.0mg.

Solution

The number of nuclei in the sample,

$$N = \frac{mass \times Avogadronumber}{molar\ mass}$$

Or

$$N = \frac{1.o \times 10^{-3}\,g)(6.02 \times 10^{23}\,atoms / mol)}{198g / mole}$$
$$= 3.04 \times 10^{12}\,atoms$$

Hence, Activity is

$$A = \lambda N$$
$$= (2.97 \times 10^{-6}\,s^{-1})(3.04 \times 10^{12})atoms$$
$$= 9.03 \times 106di\,sin\,tegration\,per\,sec\,ond$$
$$2.44 \times 10^{-4}\,Ci$$

Nuclear reactions

In addition to natural radioactivity, it is possible to induce nuclear transformations by bombarding the nucleus with another particle or nucleus. Such collisions are called **nuclear reactions**.

Nuclear fusion reactions

Nuclear fusion occurs when atoms are joined together to make larger ones.

What are the conditions under which fusion can occur?

In order for atoms to fuse together, they must not have any electrons around them, and they must be moving very fast.

Fusion reactions occur in stars like the sun, in supernovas, and in particle accelerators

Example of fusion reaction**:**

$$^{1}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + \gamma(5.49\ MeV)$$

Nuclear Fission reaction

Nuclear fission occurs when large, unstable atoms break apart into separate atoms. The energy released may be used as for destruction (a nuclear bomb) or for generating electrical power (a nuclear power plant). The difference is that in a bomb, the energy is released in one large blast, while in a nuclear reactor the reaction is controlled so that it is released at a steady pace. Below is an example of a fission reaction.

Examples of fission reaction:

$$^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + energy$$

This is a fission reaction – note that the products are much smaller than the original atom, showing that it has split into pieces. Here is a schematic of what is going on.

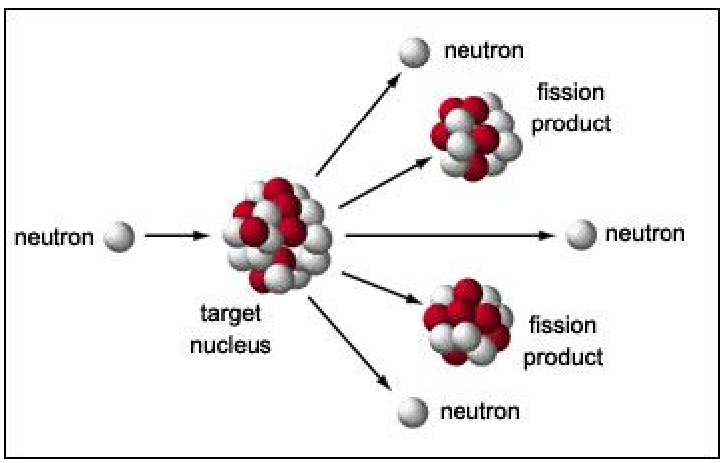


Figure 8.8 Schematic representation of fission reaction

Only a few unstable atoms undergo fission reactions – most of them undergo radioactive decay instead. The most common isotopes that undergo fission are uranium-236 and plutonium-240. These are the ones that are used in nuclear explosions.

Effect of radioactive emission

All radioactive emissions are **extremely dangerous** to living organisms. When alpha, beta or gamma radioactive emissions hit living cells they cause **ionization** (ioni**z**ation) **effects**, they can kill cells directly or cause genetic damage for example to the DNA molecules. High radiation doses cause burn effects and can kill cells. However, low doses don't kill the cells, but if they are genetically damaged and can still replicate, these **mutations** can lead to the formation of **cancerous cells and tumor development later**. When alpha, beta and gamma radiation **collide with neutral atoms or molecules** they knock off electrons and convert them into **charged or ionized particles (ions)**. Positive ions are formed on electron loss and negative ions are formed by electron gain. The positive ions maybe unstable and very reactive and cause other chemical changes in the cell molecules.

Examples of precautions that can be taken include:

In research laboratories, experiments are conducted in sealed fume cupboards at the laboratory side and technicians work through sealed whole arm gloves through a thick lead glass front. You can also reduce the pressure in the fume cupboard so there is no chance of pressure leakage out into the laboratory area.

- The figure below is a sign used as a warning to protect people from being exposed to radioactivity. This sign is posted where radioactive materials are handled, or where radiation-producing equipment is used.

Figure 8.8 Symbols for warning peoples from being exposed to radioactivity

Problems

1. Nickel has a work function of 5.0 eV.
   a. What is the maximum kinetic energy of photoelectrons knocked out of a nickel surface by a 1mW ultraviolet source at 2000 $\dot{A}$?
   b. What is the maximum kinetic energy of photoelectrons knocked out by a 1.5w argon laser source at a wavelength of 4658 $\dot{A}$?
2. In a photoelectric experiment in which monochromatic light and a sodium cathode are used, we find a stopping potential of 1.85V for λ =300 Å and 0.82 V for λ = 4000 A Form these data determine the
   a. value of Planck 's constant,
   b. work function of sodium light in eV and
   c. threshold wavelength for sodium.
3. Stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength λ=4910 Å is 0.71V. When the incident wavelength is changed the stopping potential is found to be 1.43V. What is the new wavelength?
4. Find the de Broglie wavelength of electron with a velocity of 10^5m/s
5. Determine the energy of the electron of a hydrogen atom at n=2.
6. Calculate the binding energy per nucleons, in Mev, for ${}_2^3He$ with a mass of 3.016029u.

7. If the number of radioactive nuclei present is cut in half, how does the activity change?

8. The half-life of ^{24}Na is 15.0hr. How long does it take for 80 percent of a sample of this nuclide to decay?

9. 1g of ^{226}Ra has an activity of nearly 1 Cl. Determine the half-life of ^{226}Ra.

10. The half –life of ${}^{238}_{92}U$ is 4.5 x 109 year. What is the decay constant? Find the activity of 1g of ^{238}U.

11. What fraction of sample is left after (i) 3 half-lives (ii) 5 half – lives?

12. The half-life of ^{14}C isotope is 5730 years. If a sample of 14C contains 10^{22} nuclei, what is the activity of the sample?