MATHEMATICS GRADE 10

UNIT 6 SUMMARY

PLANE GEOMETRY

BY: ADDIS ABABA EDUCATION BUREAU

MAY 2020

You Tube: https://www.youtube.com/watch?v=TWkBGoeYCNQ

Telegram ፡ https://t.me/wwwAddisAbabaeducationbureau

Twitter ፡ https://twitter.com/aacaebc

Face book : https://www.facebook.com/aacityeducationcommunication

Main Contents

6.1. Theorems on triangles

6.2. Special quadrilaterals

6.3. More on circles

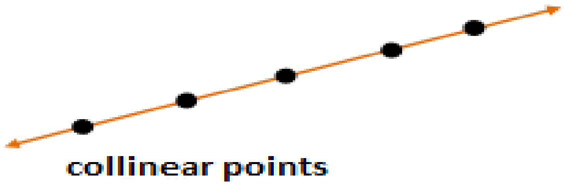
6.4. Regular polygons

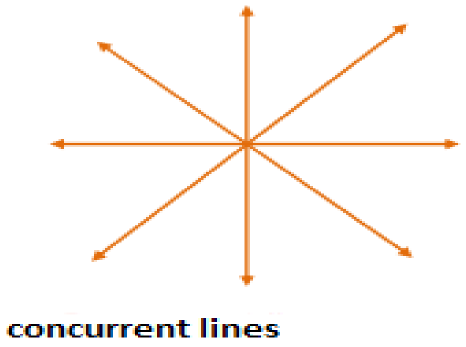
INTRODUCTION

- Plane Geometry (sometimes called Euclidean Geometry) is a branch of Geometry dealing with the properties of flat surfaces and plane figures, such as triangles, quadrilaterals or circles.

6.1 THEOREMS ON TRIANGLES

- Three or more points that lie on one line are called collinear points.
- Three or more lines that pass through one point are called concurrent lines.



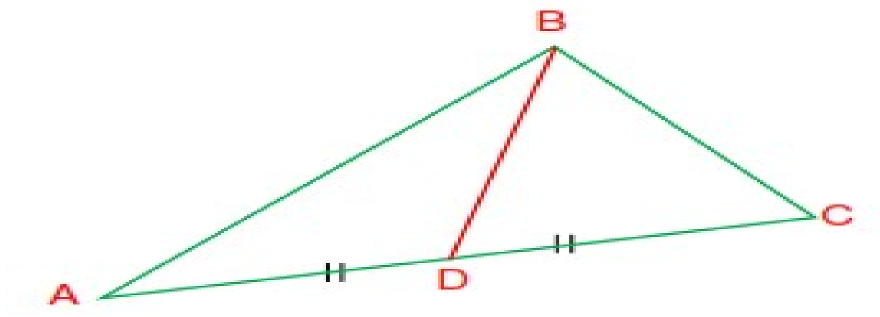


Median of a triangle

- A median of a triangle is a line segment drawn from any vertex to the mid – point of the opposite side.
- is a median of triangle ABC

- A median of a triangle is a line segment drawn from any vertex to the mid – point of the opposite side.

- $\overline{BD}$ is a median of triangle ABC



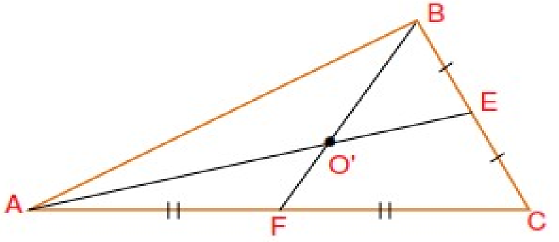
- Note: The point of intersection of the medians of a triangle is called the centroid of the triangle

- Note: The point of intersection of the medians of a

Theorem

- The medians of a triangle are concurrent at a point $\frac{2}{3}$ of the distance from each vertex to the mid –point of the opposite side

- The medians of a triangle are concurrent at a point $\frac{2}{3}$ of
the mid –poi



It follows that $AO = AO'$ and hence $O = O'$ as O and O' are on $\overline{AE}$.
Therefore, all the three medians of ΔABC are concurrent at a single point O located at $\frac{2}{3}$ of the distance from each vertex to the mid-point of the opposite side.

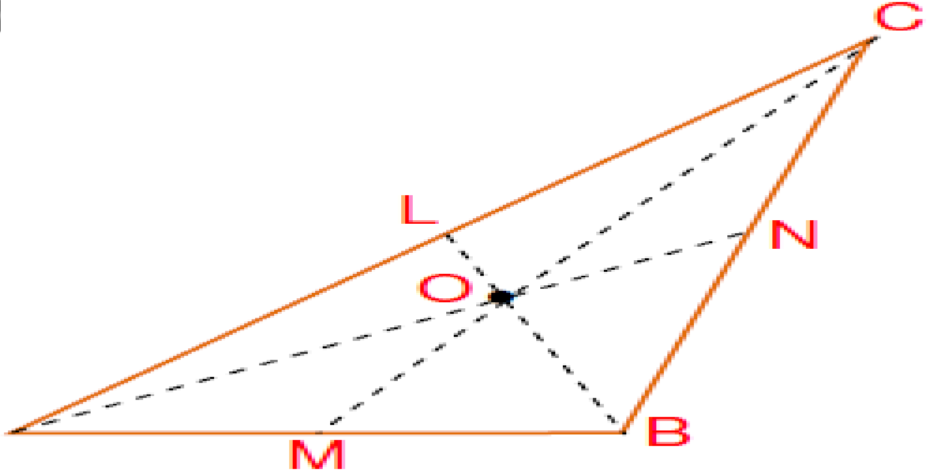
$$\overline{OE} = \frac{1}{3}\overline{AE}$$

$$\overline{OA} = \frac{2}{3}\overline{AE}$$

Example

In Figure given below, are medians of *ABC*. If , = 12cm, = 5 cm and = 6 cm, find, and .

In Figure given below, $\overline{AN}$, $\overline{CM}$ *and* $\overline{BL}$ are medians of ΔABC. If , $\overline{AN}$ = 12cm, $\overline{OM}$ = 5 cm and , $\overline{BO}$ = 6



A

solutions

- ➢ $\overline{OB} = \frac{2}{3}\overline{BL}$, but $\overline{OB}$ = 6cm

  6cm = $\frac{2}{3}\overline{BL}$

  18cm = 2 $\overline{BL}$

  $\overline{BL}$ = 9cm

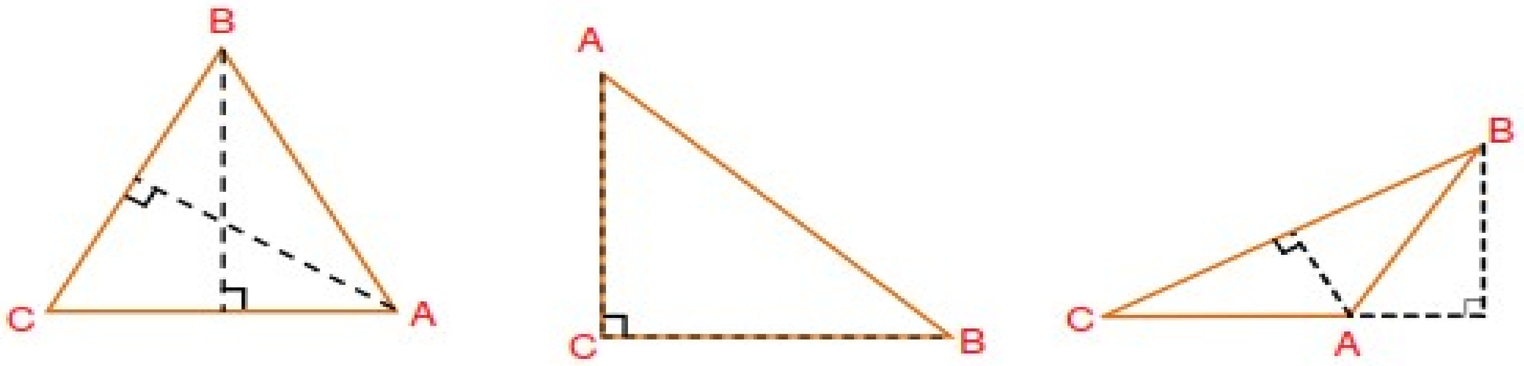
  ➢ $\overline{ON} = \frac{1}{3}\overline{AN}$

  $\overline{ON} = \frac{1}{3}$ x 12cm

  $\overline{ON} = 4cm$

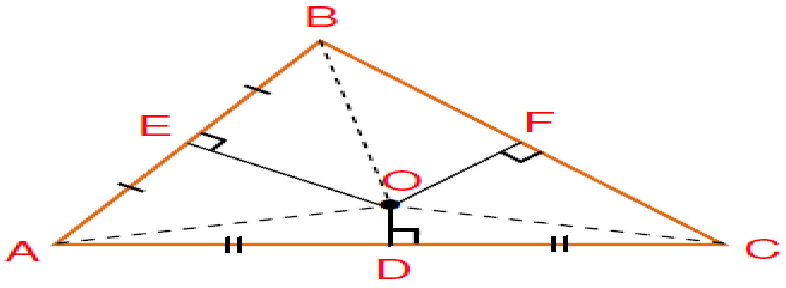
Altitude of a triangle

- **The altitude of a triangle is a line segment drawn from a vertex, perpendicular to the opposite side, or to the opposite side**



Theorem

The perpendicular bisectors of the sides of any triangle are concurrent at a point which is equidistant from the vertices of the triangle.

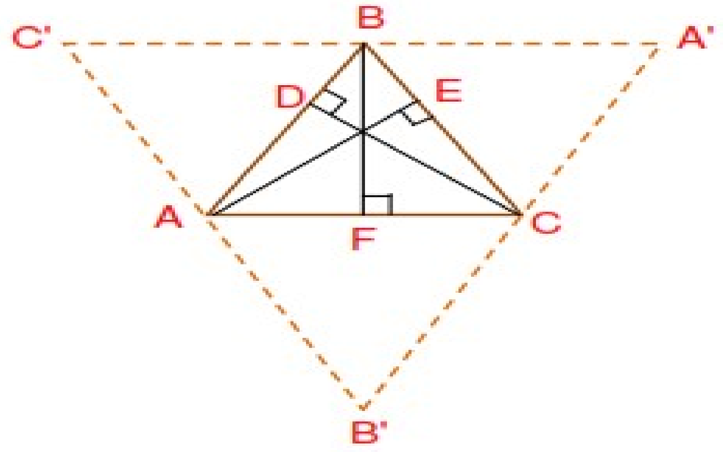


Note: the point of intersection of the perpendicular bisectors of a triangle is called circumcentre of the triangle.

Theorem

- **The altitudes of a triangle are concurrent.**

- **The altitudes of a triangle are concurrent.**



- $\overline{AE}, \overline{BF}$ *and* $\overline{CD}$ **are concurrent**

- **are concurrent**

- **Note: The point of intersection of the altitudes of a triangle is called orthocenter of the triangle.**

- **Note: The point of intersection of the altitudes of a triangle is called orthocenter of the triangle.**

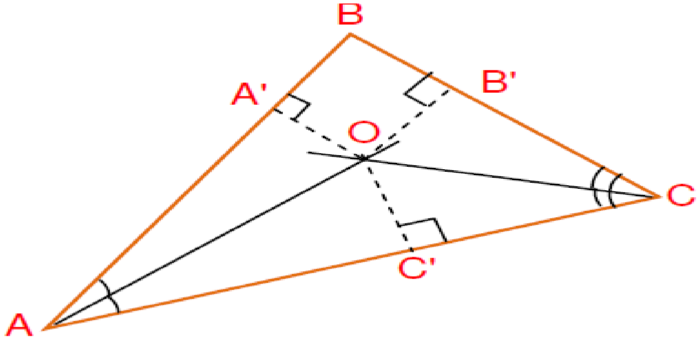
Angle bisector of a triangle

Theorem

- **The angle bisectors of any triangle are concurrent at a point which is equidistant from the sides of the triangle.**

Theorem

- **The angle bisectors of any triangle are concurrent at a point which is equidistant from the sides of the triangle.**



- **The angle bisectors of *ABC* m**

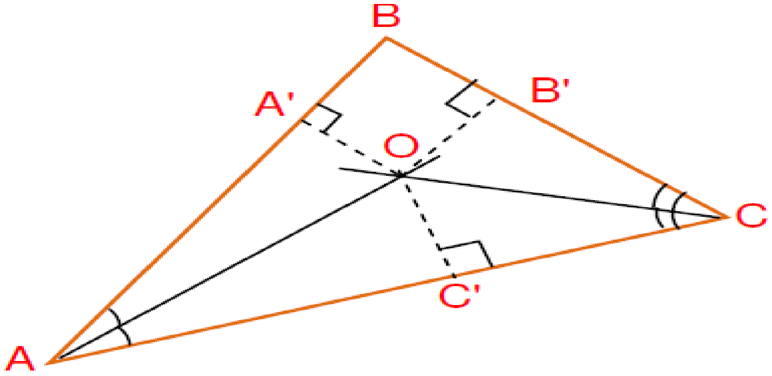
  heir point of intersection is equidistant from the three sides of *ABC*.

- $\overline{OA'} = \overline{OB'} = \overline{OC'}$

- **The angle bisectors of Δ*ABC* meet at a single point. Also their point of intersection is equidistant from the**

Note:

The point of intersection of the bisectors of the angles of a triangle is called the incentre of the triangle.



Altitude theorem

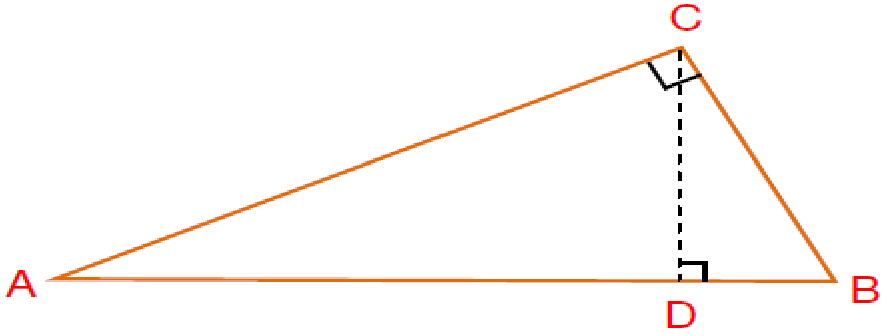
In a right angled triangle ABC with altitude to the ____ hypotenuse

In a right angled triangle ABC with altitude $\overline{CD}$ to the hypotenuse $\overline{AB}$

$$\frac{AD}{CD} = \frac{DC}{DB}$$

or

$$(CD)^2 = (\overline{AD})(\overline{DB})$$



- **The square of the length of the altitude is the product of the lengths of the segments of the hypotenuse.**

- **The square of the length of the altitude is the product of the lengths of the segments of the hypotenuse**

Example

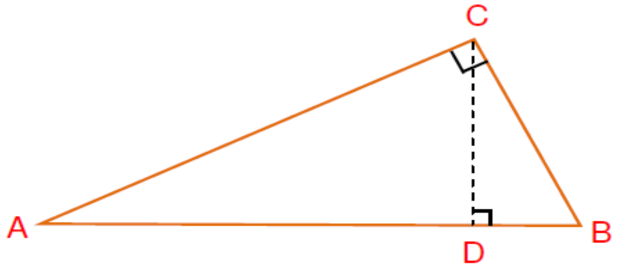
In *ABC*, is the altitude to the hypotenuse = 16 cm and = 4 cm. How long is the altitude ?

Solution

CD =

CD = 8cm

In ΔABC, $\overline{CD}$ is the altitude to the hypotenuse $\overline{AB}$. $\overline{AD}$ = 16 cm and $\overline{BD}$ = 4 cm. How long is the altitude $\overline{CD}$?



Solution

$$(CD)^2 = (\overline{AD})(\overline{DB})$$

$$(CD)^2 = (16cm)(4cm)$$

$$= 64cm^2$$

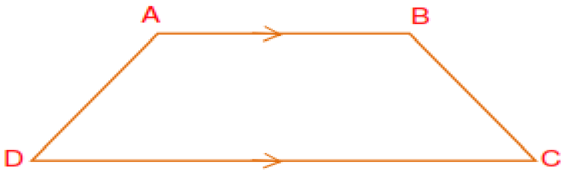
$$CD = \sqrt{64cm^2}$$

6.2. SPECIAL QUADRILATERALS

➢ We consider the following special quadrilaterals: trapezium, parallelogram, rectangle, rhombus and square.

A, Trapezium

➢ A trapezium is a quadrilatera … e sides are parallel.



➢ In the figure above the quadrilateral ABCD is a trapezium. The sides are non-parallel sides of the trapezium ABCD.

➢ We consider the following special quadrilaterals: trapezium, parallelogram, rectangle, rhombus and square.

A, Trapezium

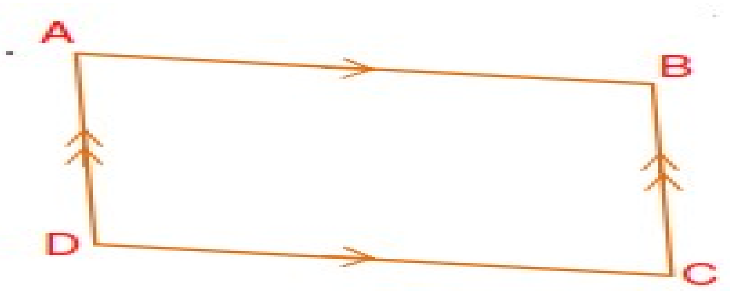
➢ A trapezium is a quadri … wo of the sides are parallel.

➢ In the figure above the quadrilateral ABCD is a trapezium. The sides $\overline{AD}$ and $\overline{BC}$ are non-

B, Parallelogram

- A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.

- A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.



- In the above figure the quadrilateral ABCD is a parallelogram. // and // .

- In the above figure the quadrilateral ABCD is a parallelogram. $\overline{AD} // \overline{BC}$ and $\overline{AB} // \overline{DC}$

Theorem

a) The opposite sides of a parallelogram are congruent.

b) The opposite angles of a parallelogram are congruent.

c) The diagonals of a parallelogram bisect each other.

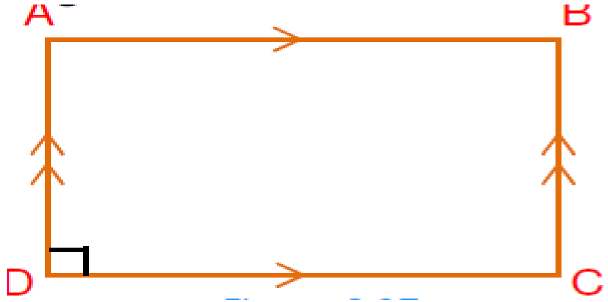
d) If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

e) If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

f) If the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

C, Rectangle

- A rectangle is a parallelogram in which one of its angles is a right angle.

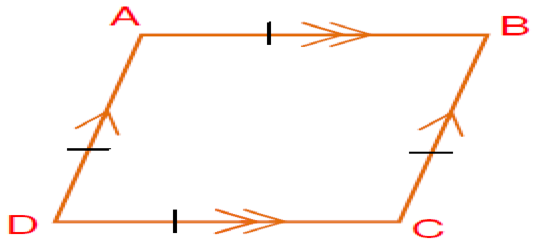


- **Some properties of a rectangle**

1. A rectangle has all properties of a parallelogram.
2. Each interior angle of a rectangle is a right angle. Figure
3. The diagonals of a rectangle are congruent.

D, Rhombus

- A rhombus is a parallelogram which has two congruent adjacent sides.

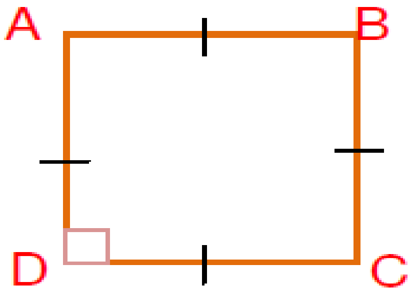


- Some properties of a rhombus

1. A rhombus has all the properties of a parallelogram.
2. A rhombus is an equilateral quadrilateral.
3. The diagonals of a rhombus are perpendicular to each other.
4. The diagonals of a rhombus bisect its angles.

Square

- A square is a rectangle which has congruent adjacent sides.



- The rectangle *ABCD* is a square.
- Some properties of a square

1. A square has the properties of a rectangle.
2. A square has all the properties of a rhombus.

Theorem

If the diagonals of a quadrilateral are congruent and are perpendicular bisectors of each other, then the quadrilateral is a square.

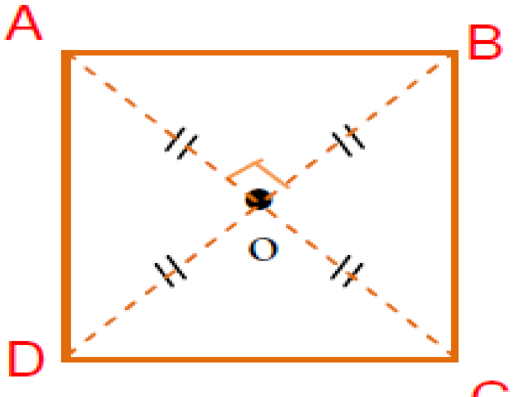
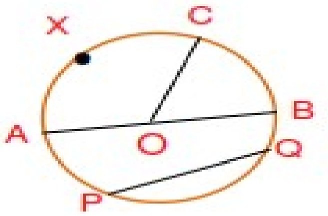


Figure 6.30

6.3 MORE ON CIRCLES

- A circle is a plane figure, all points of which are equidistant from a given point called the center of the circle.
- , are chords of the circle with Cen
- is the largest chord (diameter).
- is an arc of the circle.
- *BOC* is a central angle.

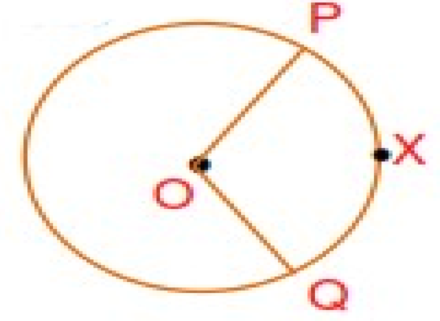
- A circle is a plane figure, all points of which are equidistant from a given point called the center of the circle.
- $\overline{PQ}$, $\overline{AB}$ are chords of the circle with Centre o.
- $\overline{AB}$ is the largest chord (diameter).
- $\widehat{AXC}$ is an arc of the circle.



Angles and Arcs Determined by Lines Intersecting Inside and On a Circle

Measure of a central angle:

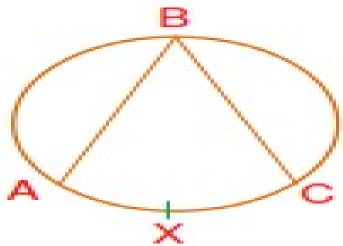
Note that : the measure of a central angle is the measure of the arc it intercepts.



So, $m(\angle POQ) = m(\overset{\frown}{PXQ})$.

Theorem

- The measure of an angle inscribed in a circle is half the measure of the arc subtending it.



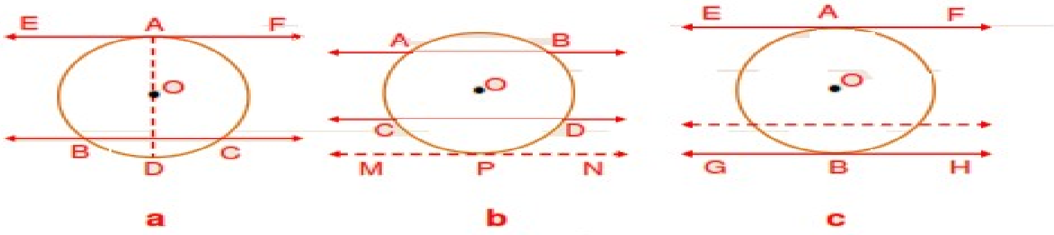
- Given: Circle O with angle B an inscribed angle intercepting arc AC.

$$\text{Therefore, } m\,(\angle ABC) = \frac{1}{2}\, m\,(\overset{\frown}{AXC})$$

- An angle inscribed in a semi-circle is a right angle.

Theorem

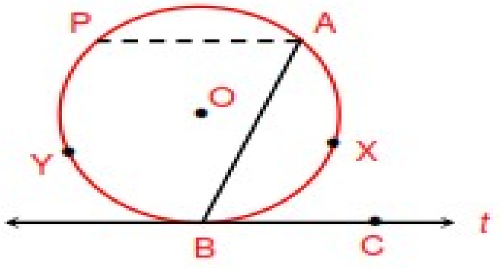
- Two parallel lines intercept congruent arcs on the same circle.



- When one of the parallel lines is a tangent line and the other is a secant line as shown in figure above a.
- When both parallel lines and are secants as shown in figure above b.
- When both parallel lines and are tangents as shown in figure above c.

Theorem

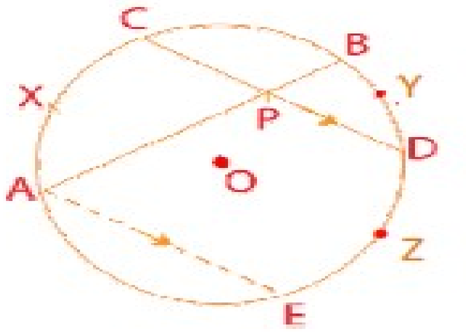
- **An angle formed by a tangent and a chord drawn from the point of tangency is measured by half the arc it intercepts.**



$$\therefore m(\angle ABC) = \frac{1}{2} m(\overset{\frown}{AXB})$$

Theorem

- **The measure of an angle formed by two chords intersecting inside a circle is half the sum of the measures of the arc subtending the angle and its vertically opposite angle.**

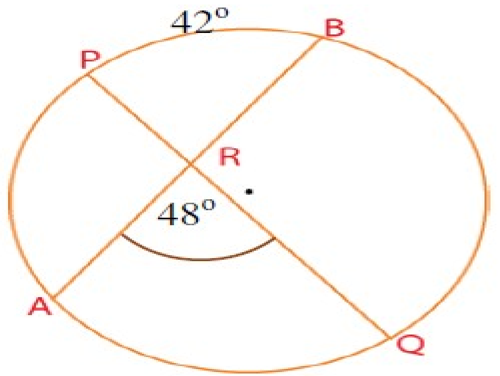


$$m(\angle BPD) = \frac{1}{2}\left[m(\widehat{AXC}) + m(\widehat{BYD})\right]$$

Example

An angle formed by two chords intersecting within a circle is 48°, andone of the intercepted arcs measures 42°. Find the measures of the otherintercepted arc.

An angle formed by two chords intersecting within a circle is 48°, and one of the intercepted arcs measures 42°. Find the measures of the other intercepted arc.



$$(\angle PRB) = \frac{1}{2}(\widehat{PB}) + \frac{1}{2}(\widehat{AQ})$$

$$48^{o} = \frac{1}{2}(42^{o}) + \frac{1}{2}(\widehat{AQ})$$

$$48^{o} = 21^{0} + \frac{1}{2}(\widehat{AQ})$$

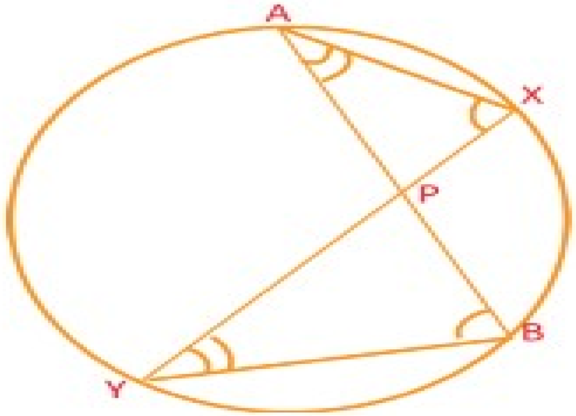
$$27^{0} x\, 2 = \widehat{AQ}$$

$$m(\widehat{AQ}) = 54^{0}$$

product or rectangle property of a circle

- If two chords intersect in a circle as shown in figure below, then

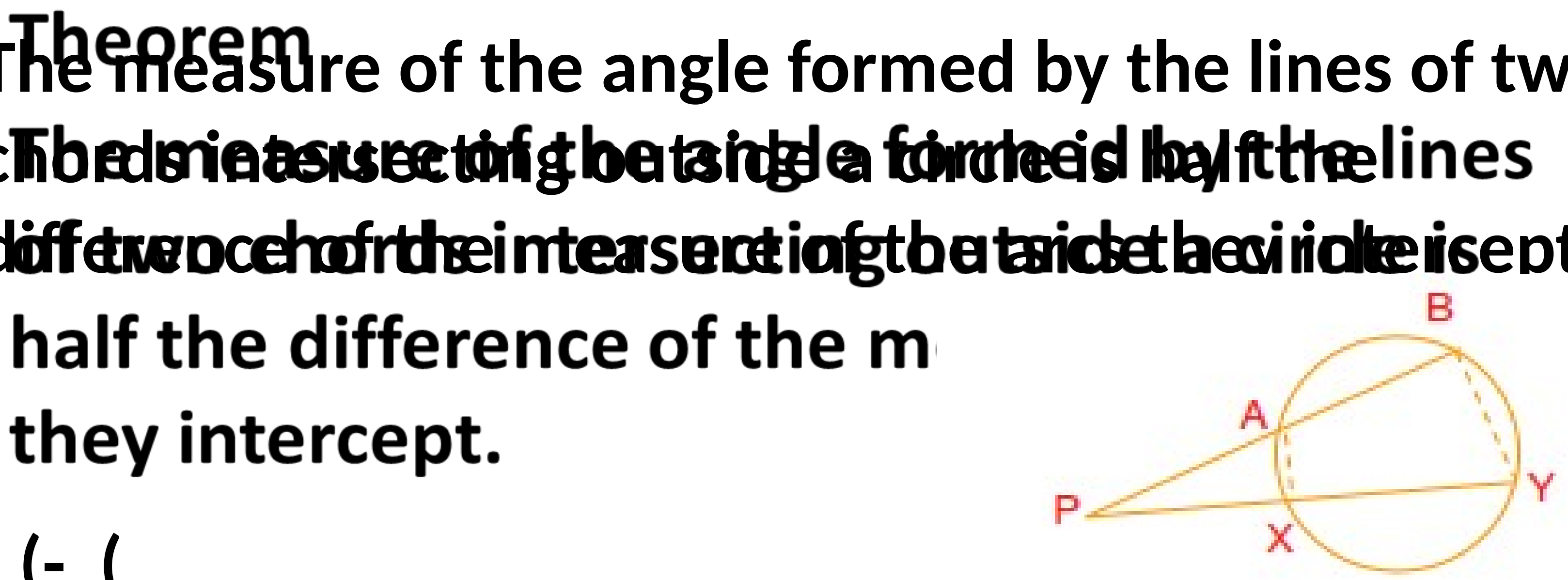
- If two chords intersect in a circle as shown in f



Angles and Arcs Determined by Lines intersecting Outside a Circle

Theorem

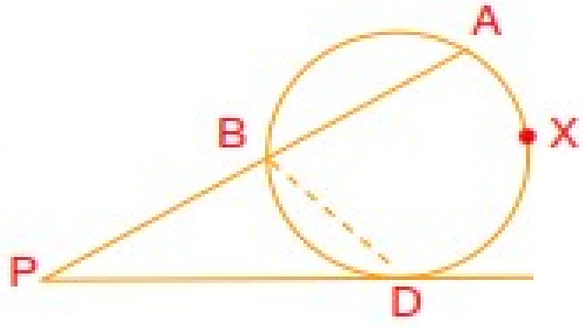
The measure of the angle formed by the lines of two chords intersecting outside a circle is half the difference of the measure of the arcs they intercept.



((- (

Theorem

The measure of an angle formed by a tangent angle and a secant drawn to a circle from a point outside the circle is equal to one-half the difference of the measures of the intercepted arcs.

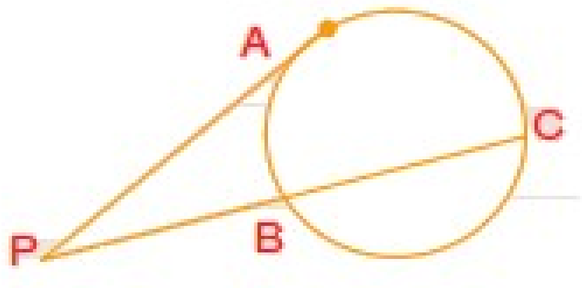


$$m(\angle P) = \frac{1}{2}[m(\overset{\frown}{AXD}) - m(\overset{\frown}{BD})]$$

Theorem

If a secant and a tangent are drawn from a point outside a circle, then the square of the length of the tangent is equal to the product of the lengths of line segments given by

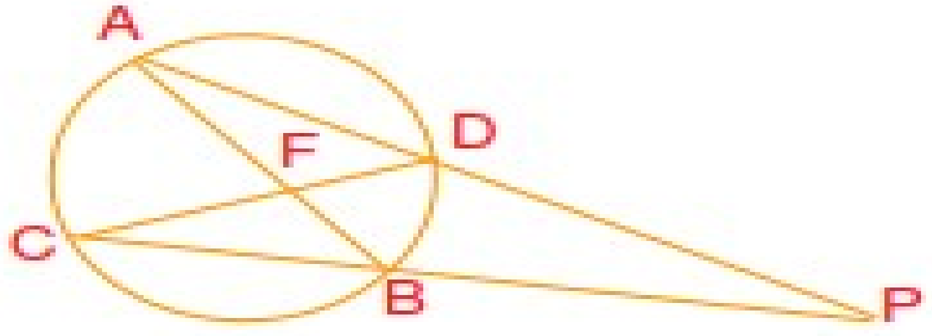
If a secant and a tangent are drawn from a point outside a circle, then the square of the length of the tangent is equal to the product of the lengths of line segments give



$$(PA)^2 = (PB)(PC)$$

Example

In Figure below, from P secants $\overline{PA}$ and $\overline{PC}$ are drawn so that $m(<APC) = 40^0$; chords $\overline{AB}$ and $\overline{CD}$ intersect at F such that $m(<AFC) = 86^0$. Find the measure of arc AC, measure of arc BD and measure of $(<ABC)$.



Solution: Let $m(\widehat{AC}) = x$ and $m(\widehat{DB}) = y$

Since $m(\angle AFC) = \frac{1}{2} m(\widehat{AC}) + \frac{1}{2} m(\widehat{BD})$

$$86^\circ = \frac{1}{2}(x + y)$$

$$x + y = 172^\circ \text{ . . . . (1)}$$

Again as $m(\angle APC) = \frac{1}{2} m(\widehat{AC}) - \frac{1}{2} m(\widehat{BD})$

$$40^\circ = \frac{1}{2}(x - y)$$

$$x - y = 80^\circ \text{ . . . . (2)}$$

Solving equation 1 and equation 2 simultaneously, we get

$$\begin{cases} x + y = 172^\circ \\ x - y = 80^\circ \end{cases}$$

$$2x = 252^\circ$$

$$x = 126^\circ$$

Substituting for x in equation 2,

$$126^\circ - y = 80^\circ$$

$$y = 46^\circ$$

Therefore, $m(\overset{\frown}{AC}) = 126^\circ$ and $m(\overset{\frown}{DB}) = 46^\circ$.

$$m(\angle ABC) = \frac{1}{2} m(\overset{\frown}{AC}) = \frac{1}{2}(126^\circ) = 63^\circ.$$

6.4 REGULAR POLYGONS

Perimeter of a Regular Polygon

Theorem

Formulae for the length of side *s*, apothem *a*, perimeter *P* and area *A* of a regular polygon with *n* sides and radius *r* are:

1. s = 2rsin $\frac{180^0}{n}$
2. a = r cos
3. P = 2nrsin $\frac{180^0}{n}$
4. A= $\frac{1}{2}a$P

Area of a Regular Polygon

Theorem

The area A of a regular polygon with n sides and radius r is

Theorem

The area A of a regular polygon with n sides and radius r is

n

$$A = \frac{1}{2} \mathrm{n} r^2 \sin \frac{360^0}{n}$$

Example

Find the area of a regular twelve-sided polygon with radius 3 units.

Solution

$n = 12$; $r = 3$units

$$A = \frac{1}{2} n\, r^2 \sin\left(\frac{360^0}{n}\right)$$

$$A = \frac{1}{2} x 12 x\ (3)^2 \sin\left(\frac{360^0}{12}\right)$$

$$A = 6 \text{ x } 9 \text{ x } \sin(30^0)$$

$$A = 6 \text{ x } 9\text{x } \frac{1}{2} = 27 \text{ square units}$$

Activity 1

<u>Write true if the statement is correct and false if the statement is incorrect</u>

1. The incentre of a triangle is equidistant from all three vertices.
2. The incentre of a triangle always lies inside the triangle.
3. The bisectors of the angles of a triangle are concurrent.
4. The perpendicular bisectors of the sides of a triangle are concurrent.
5. Four lines intersecting in one point are concurrent.
6. A rhombus is a square.
7. A square is a rectangle.
8. Every parallelogram is a square.
9. Every rhombus is a parallelogram.
10. Every rectangle is a parallelogram

<u>For each of the following questions choose the correct answer from the given alternatives</u>

11. The point of concurrency of the medians of a triangle

A orthocenter C circum centre

B in centre D centroid

12.The Point of concurrency of the perpendicular bisector of a triangle

A orthocenter C circum centre

B in centre D centroid

13. The area of a regular 12 sided polygon of radius 6units long is;

A. 98 unit^2 B. 108 unit^2 C. 48 unit^2 D. 144unit^2

workout

14. What is the length of side a regular hexagon whose area is $318\sqrt{2}cm^2$?

15. What is area A of an equilateral triangle inscribed in a circle of radius r?

16. In the figure below, $\overline{CQ}, \overline{AT}$ and $\overline{BR}$ are medians of ΔABC . If $\overline{AT}$ =18cm, $\overline{OQ}$ =3cm, then what is, $\overline{CO}$?