सत्यं शिवमानम्

# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2018

## MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

### DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates are required to give their answers in their own words as far as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) A function $f(x)$ is defined as follows: 2

$$f(x)=|x-2|+1$$

Examine whether $f'(2)$ exists.

(b) Examine whether $f(x,y)=x^{-1/3}y^{4/3}\cos\left(\frac{y}{x}\right)$ is a homogeneous function of $x$ and $y$. If so, find its degree. 2

(c) Find the value of $\frac{d^n}{dx^n}\{\sin(ax+b)\}$ 2

(d) Is Rolle's theorem applicable to the function $|x|$ in the interval $[-1, 1]$? Justify your answer. 2

(e) Find the radius of curvature at the origin for the curve $x^3+y^3-2x^2+6y=0$. 2

(f) Find the asymptotes parallel to co-ordinate axes of the curve $(x^2+y^2)x-ay^2=0$. 2

(g) If $e^{a\sin^{-1}x}=a_0+a_1x+a_2x^2+\ldots\ldots$, then find the value of $a_2$. 2

(h) Evaluate: $\lim\limits_{x\to0}(\cos x)^{\cot x}$ 2

2. (a) If $\lim\limits_{x\to a}f(x)$ and $\lim\limits_{x\to a}g(x)$ exists finitely for two functions $f$ and $g$, then prove that $\lim\limits_{x\to a}\{f(x)+g(x)\}=\lim\limits_{x\to a}f(x)+\lim\limits_{x\to a}g(x)$ 3

(b) Using ε-s definition (Cauchy's definition) show that the function $f$ defined by, 3

$$\begin{aligned} f(x) &= x^2, \quad x \text{ is rational} \\ &= -x^2, \quad x \text{ is irrational} \end{aligned}$$

is continuous at 0.

(c) Find the co-ordinates of the points on the curve $y = x^2 - 8x + 5$ at which the tangents pass through the origin. 2

3. (a) If $f(x) = \begin{cases} x+1 \quad , & \text{when } x \le 1 \\ 3 - ax^2 \, , & \text{when } x > 1 \end{cases}$ 3

then find the value of $a$ for which $f$ is continuous at $x = 1$.

(b) Find the Taylor series expansion of $f(x) = \sin x$. 5

4. (a) If $u(x,y) = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, $x \neq y$, apply Euler's theorem to find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ and hence show that $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$ 5

(Assume $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$)

(b) If $y = \dfrac{x}{x+1}$, find $y_n$ (where $y_n$ is the $n$-th differential coefficient of $y$ w.r.t $x$) and hence find $y_7(0)$. 3

5. (a) If $f(x,y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} \quad , & (x,y) \neq (0,0) \\ 0 \quad , & (x,y) = (0,0) \end{cases}$ 4

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

(b) Find the asymptotes of the cubic $y^3 + x^2y + 2xy^2 - y + 1 = 0$ 4

6. (a) State and prove Cauchy's Mean Value Theorem. 5

(b) If $\lim\limits_{x \to 0} \dfrac{\sin 2x + a\sin x}{x^3}$ is finite, find $a$ and the value of the limit. 3

7. (a) Find the radius of curvature at any point $(r, \theta)$ for the curve $r = a(1 - \cos\theta)$. Hence show if $\rho_1$ and $\rho_2$ be the radii of curvature at the extremities of any chord of this cardioid which pass through the pole; then prove that $\rho_1^2 + \rho_2^2 = \dfrac{16a^2}{9}$ 5

(b) Show that the normal to the curve $3y = 6x - 5x^3$ drawn at the point $\left(1, \frac{1}{3}\right)$ passes through the origin. 3

8. (a) If $H = f(y-z,\ z-x,\ x-y)$, then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$ 3

(b) Verify Rolle's theorem for the function $f(x) = x^2 + \cos x$ on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$. 2

(c) If $V = x\sin^{-1}\left(\frac{y}{x}\right) + y\tan^{-1}\left(\frac{x}{y}\right)$, find the value of $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}$ at (1, 1) 3

9. (a) Show that at any point of the curve $by^2 = (x+a)^3$, the subnormal varies as the square of the subtangent. 4

(b) Prove that of all the rectangular parallelopiped of the same volume, the cube has the least surface area. 4

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