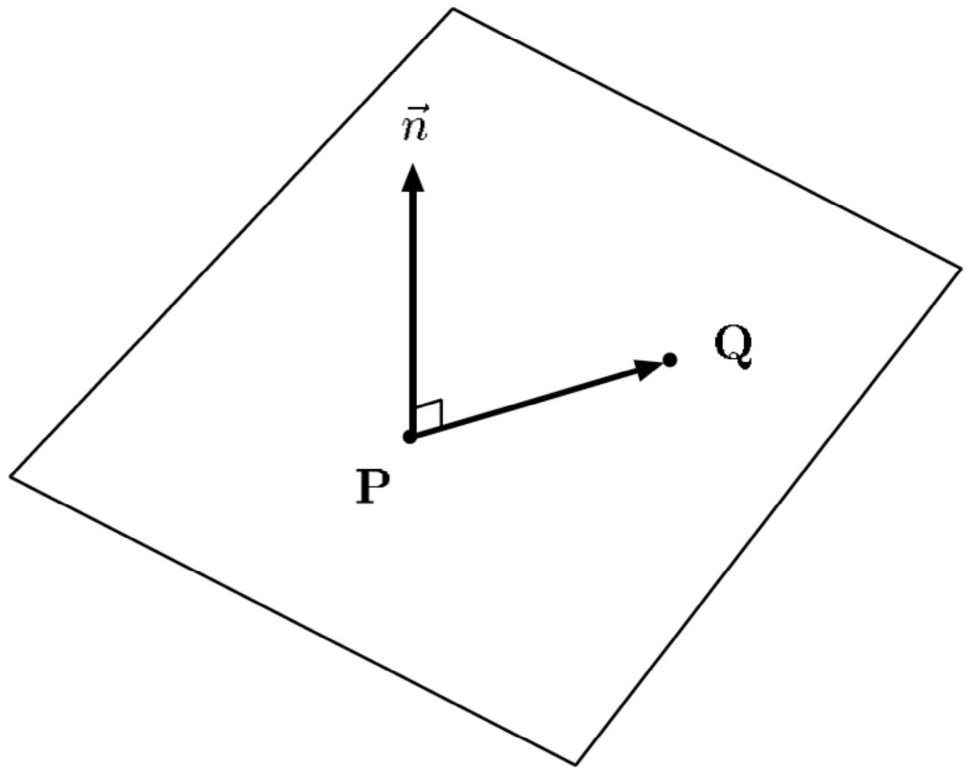
Clipping

Clipping is one of the fundamental operations of computer graphics modeling.

It is easy!

However, it is one algorithm that everyone struggles with…

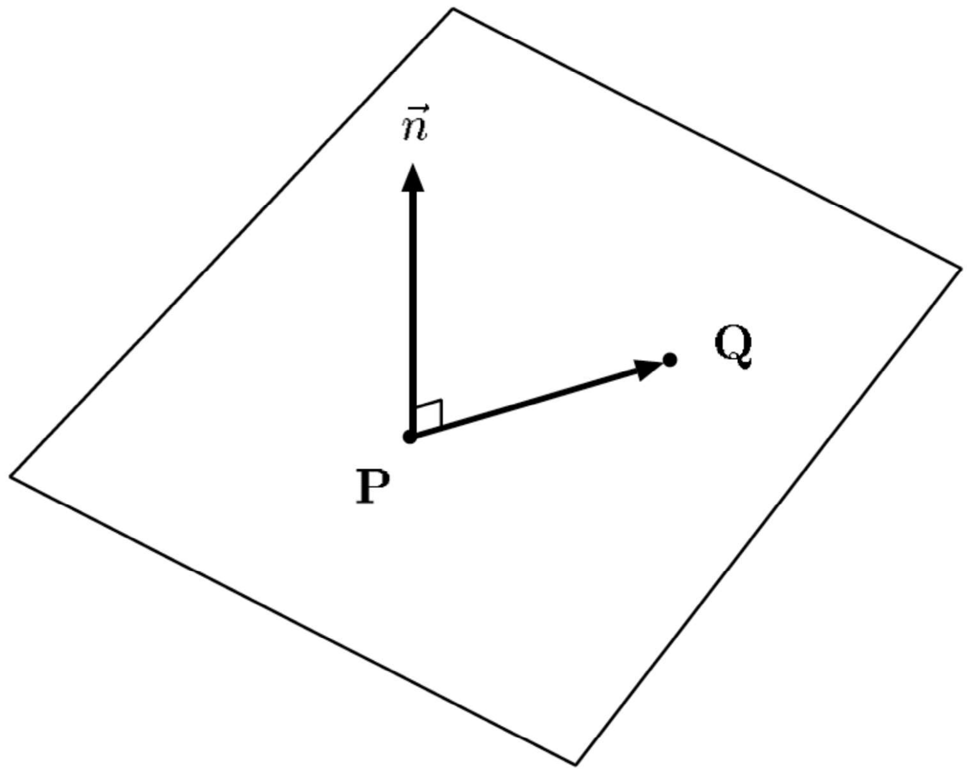
Planes and Normal Vectors



Defined by a point $\mathbf{P}$

and a normal vector $\vec{n}$

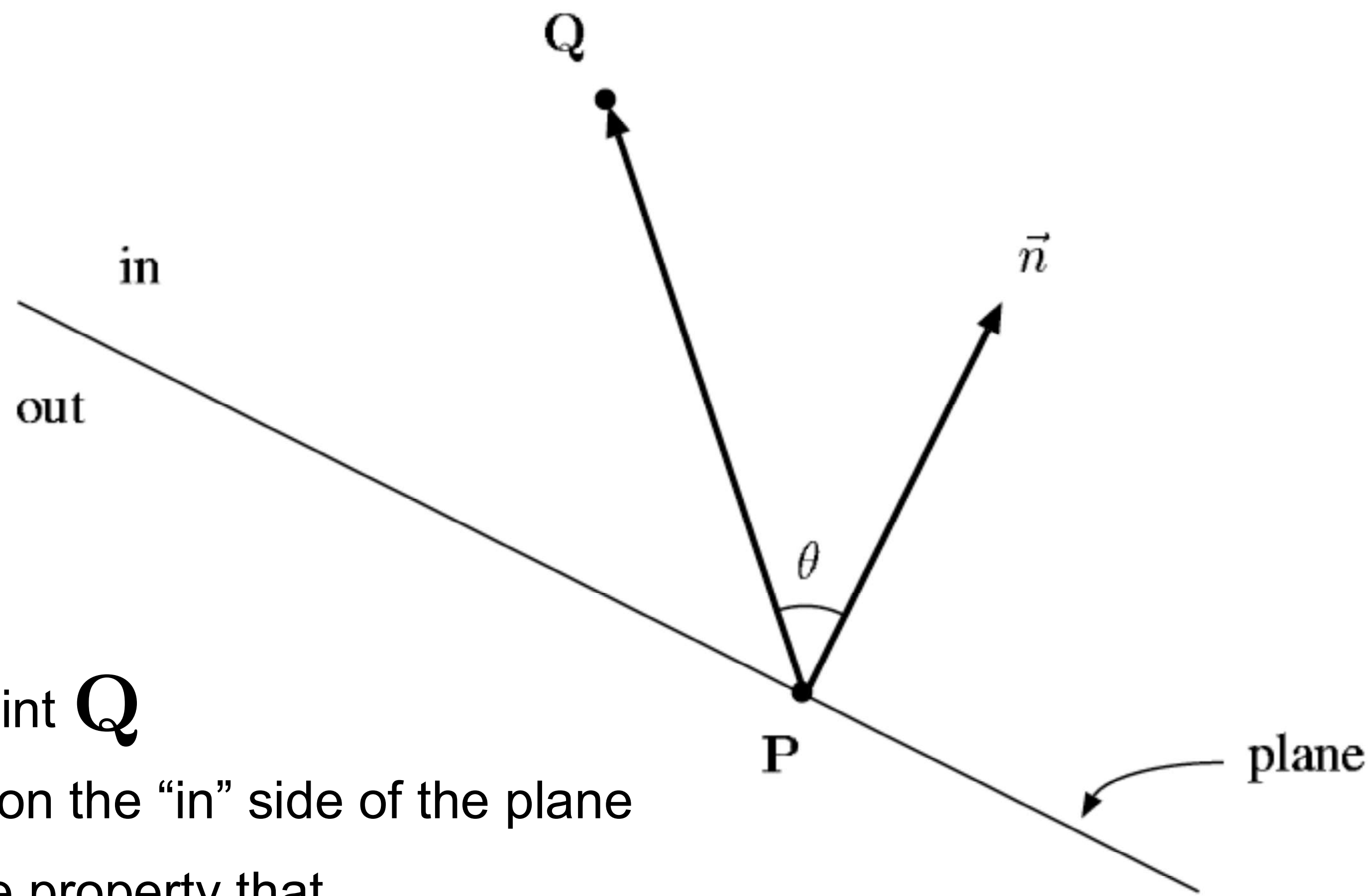
Planes and Normal Vectors



Any point $\mathbf{Q}$

has the property that $(\mathbf{Q} - \mathbf{P}) \cdot \vec{n} = 0$

Planes and Normal Vectors



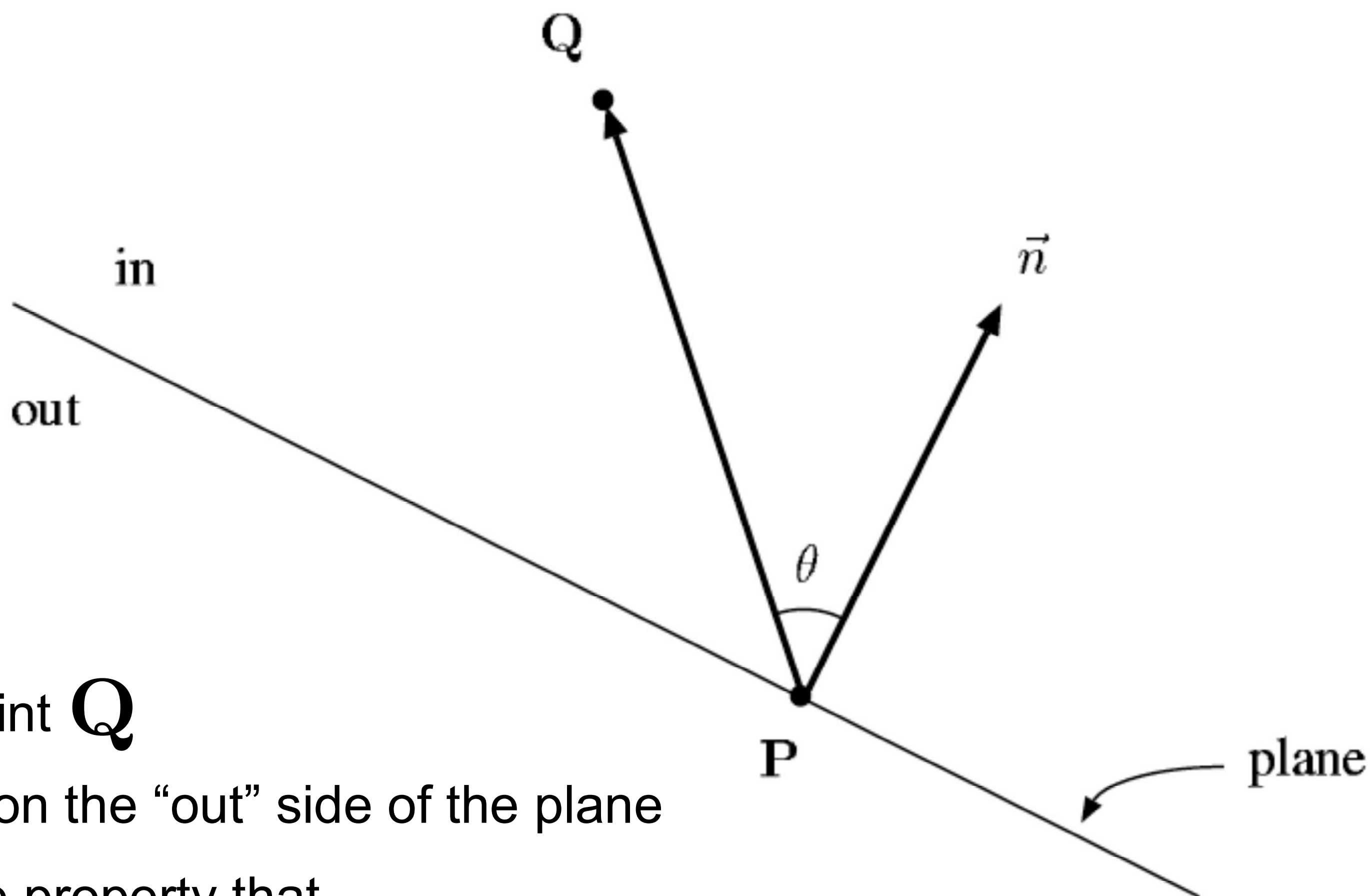
Any point $\mathbf{Q}$

that is on the “in” side of the plane

has the property that

$$(\mathbf{Q} - \mathbf{P}) \cdot \vec{n} = |\mathbf{Q} - \mathbf{P}||\vec{n}| \cos\theta > 0$$

Planes and Normal Vectors



Any point $\mathbf{Q}$

that is on the "out" side of the plane

has the property that

$$(\mathbf{Q} - \mathbf{P}) \cdot \vec{n} = |\mathbf{Q} - \mathbf{P}||\vec{n}| \cos\theta < 0$$

Planes and Normal Vectors

$\mathbf{Q}$ is on the "in" side of the plane if

$$(\mathbf{Q} - \mathbf{P}) \cdot \vec{n} = |\mathbf{Q} - \mathbf{P}||\vec{n}| \cos\theta > 0$$

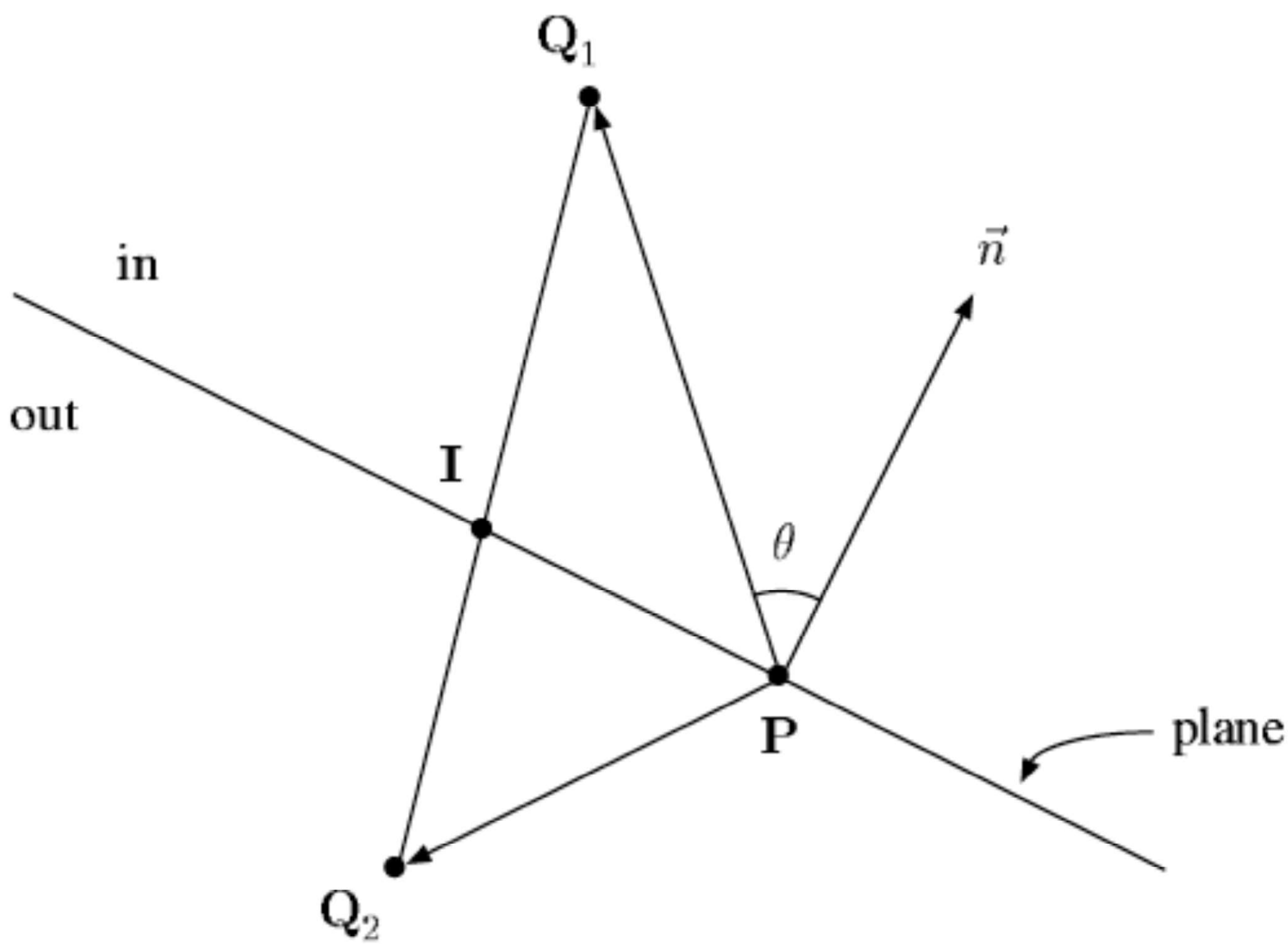
$\mathbf{Q}$ is on the "out" side of the plane if

$$(\mathbf{Q} - \mathbf{P}) \cdot \vec{n} = |\mathbf{Q} - \mathbf{P}||\vec{n}| \cos\theta < 0$$

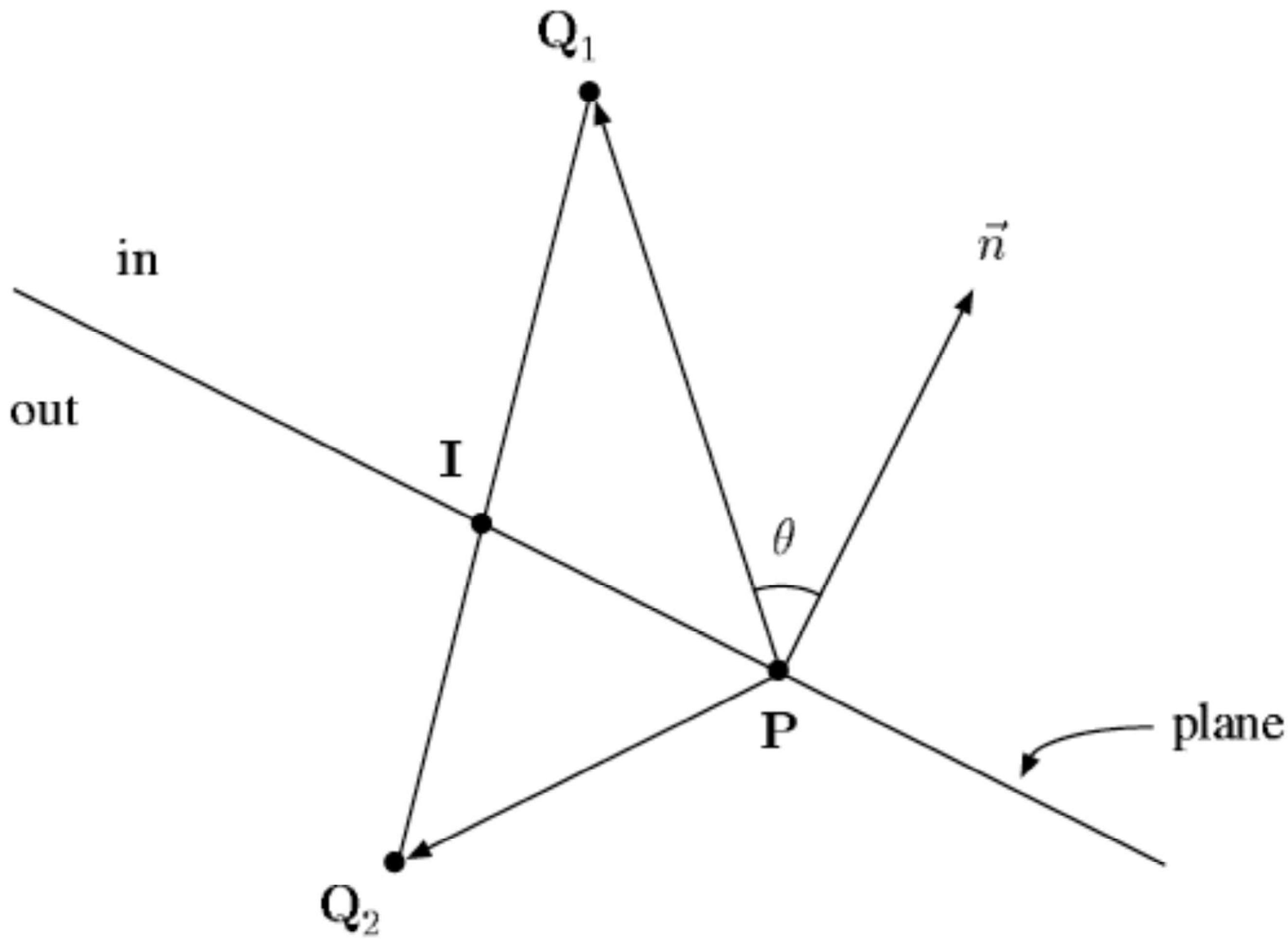
$\mathbf{Q}$ is "on" the plane if

$$(\mathbf{Q} - \mathbf{P}) \cdot \vec{n} = 0$$

Clipping Line Segments

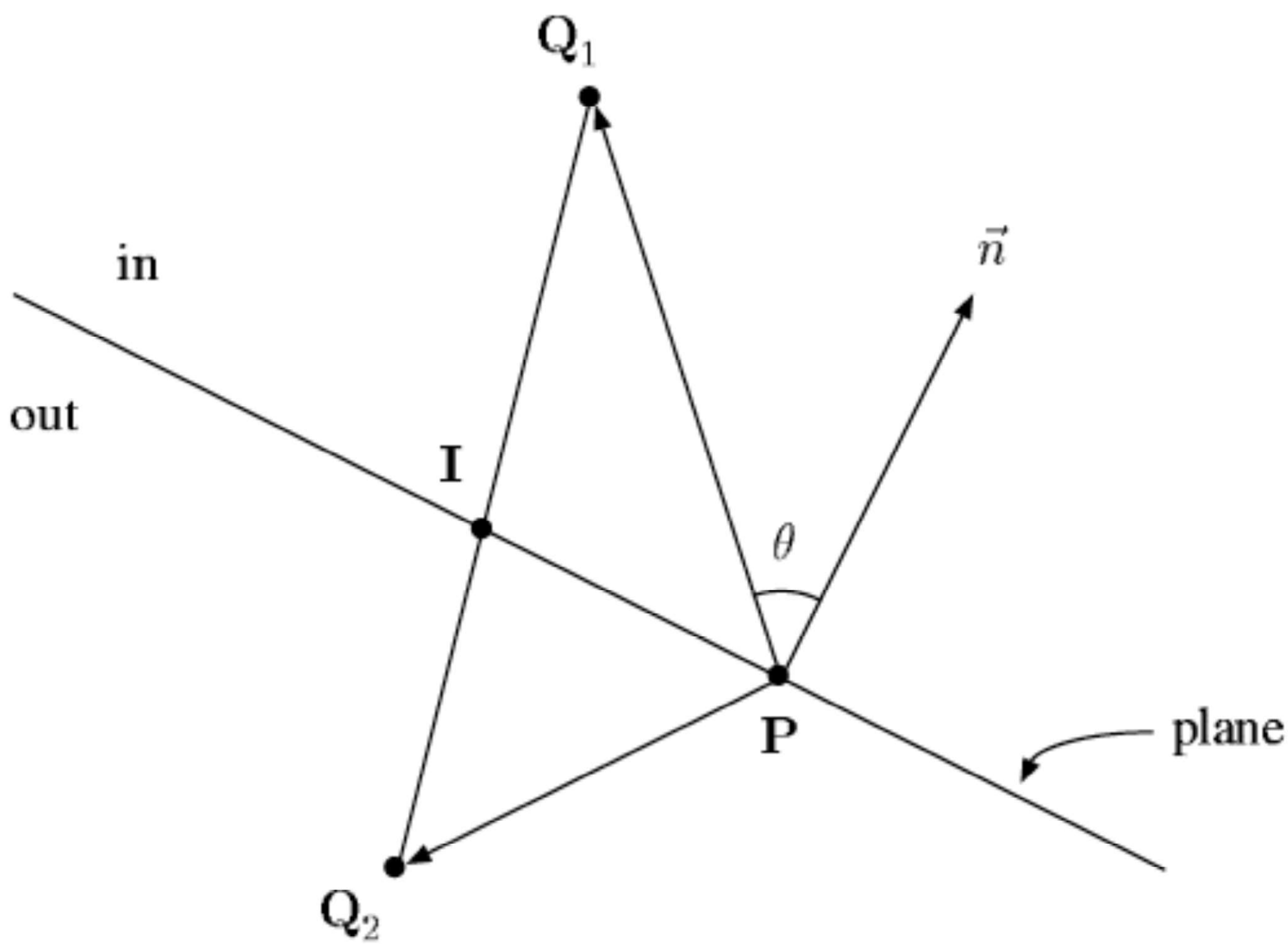


Clipping Line Segments



$$d_1 = (\mathbf{Q}_1 - \mathbf{P}) \cdot \vec{n} \qquad d_2 = (\mathbf{Q}_2 - \mathbf{P}) \cdot \vec{n}$$

Clipping Line Segments



$$\mathbf{I} = \mathbf{Q}_1 + t(\mathbf{Q}_2 - \mathbf{Q}_1)$$

Clipping Line Segments

$$\mathbf{I} = \mathbf{Q}_1 + t(\mathbf{Q}_2 - \mathbf{Q}_1)$$

Clipping Line Segments

$$\mathbf{I} = \mathbf{Q}_1 + t(\mathbf{Q}_2 - \mathbf{Q}_1)$$

Subtract $\mathbf{P} = \mathbf{P} + t(\mathbf{P} - \mathbf{P})$

$$\mathbf{I} - \mathbf{P} = (\mathbf{Q}_1 - \mathbf{P}) + t((\mathbf{Q}_2 - \mathbf{P}) - (\mathbf{Q}_1 - \mathbf{P}))$$

Clipping Line Segments

$$\mathbf{I} = \mathbf{Q}_1 + t(\mathbf{Q}_2 - \mathbf{Q}_1)$$

Subtract $\mathbf{P} = \mathbf{P} + t(\mathbf{P} - \mathbf{P})$

$$\mathbf{I} - \mathbf{P} = (\mathbf{Q}_1 - \mathbf{P}) + t((\mathbf{Q}_2 - \mathbf{P}) - (\mathbf{Q}_1 - \mathbf{P}))$$

dot with $\vec{n}$

$$\begin{aligned}(\mathbf{I} - \mathbf{P}) \cdot \vec{n} = (\mathbf{Q}_1 - \mathbf{P}) \cdot \vec{n} \\ + t((\mathbf{Q}_2 - \mathbf{P}) \cdot \vec{n} - (\mathbf{Q}_1 - \mathbf{P}) \cdot \vec{n})\end{aligned}$$

Clipping Line Segments

$$\mathbf{I} = \mathbf{Q}_1 + t(\mathbf{Q}_2 - \mathbf{Q}_1)$$

Subtract $\mathbf{P} = \mathbf{P} + t(\mathbf{P} - \mathbf{P})$

$$\mathbf{I} - \mathbf{P} = (\mathbf{Q}_1 - \mathbf{P}) + t((\mathbf{Q}_2 - \mathbf{P}) - (\mathbf{Q}_1 - \mathbf{P}))$$

dot with $\vec{n}$

$$\begin{aligned}(\mathbf{I} - \mathbf{P}) \cdot \vec{n} = (\mathbf{Q}_1 - \mathbf{P}) \cdot \vec{n} \\ + t((\mathbf{Q}_2 - \mathbf{P}) \cdot \vec{n} - (\mathbf{Q}_1 - \mathbf{P}) \cdot \vec{n})\end{aligned}$$

which is

$$(\mathbf{I} - \mathbf{P}) \cdot \vec{n} = d_1 + t(d_2 - d_1)$$

Clipping Line Segments

$$(\mathbf{I} - \mathbf{P}) \cdot \vec{n} = d_1 + t(d_2 - d_1)$$

Clipping Line Segments

$$(\mathbf{I} - \mathbf{P}) \cdot \vec{n} = d_1 + t(d_2 - d_1)$$

$$0 = d_1 + t(d_2 - d_1)$$

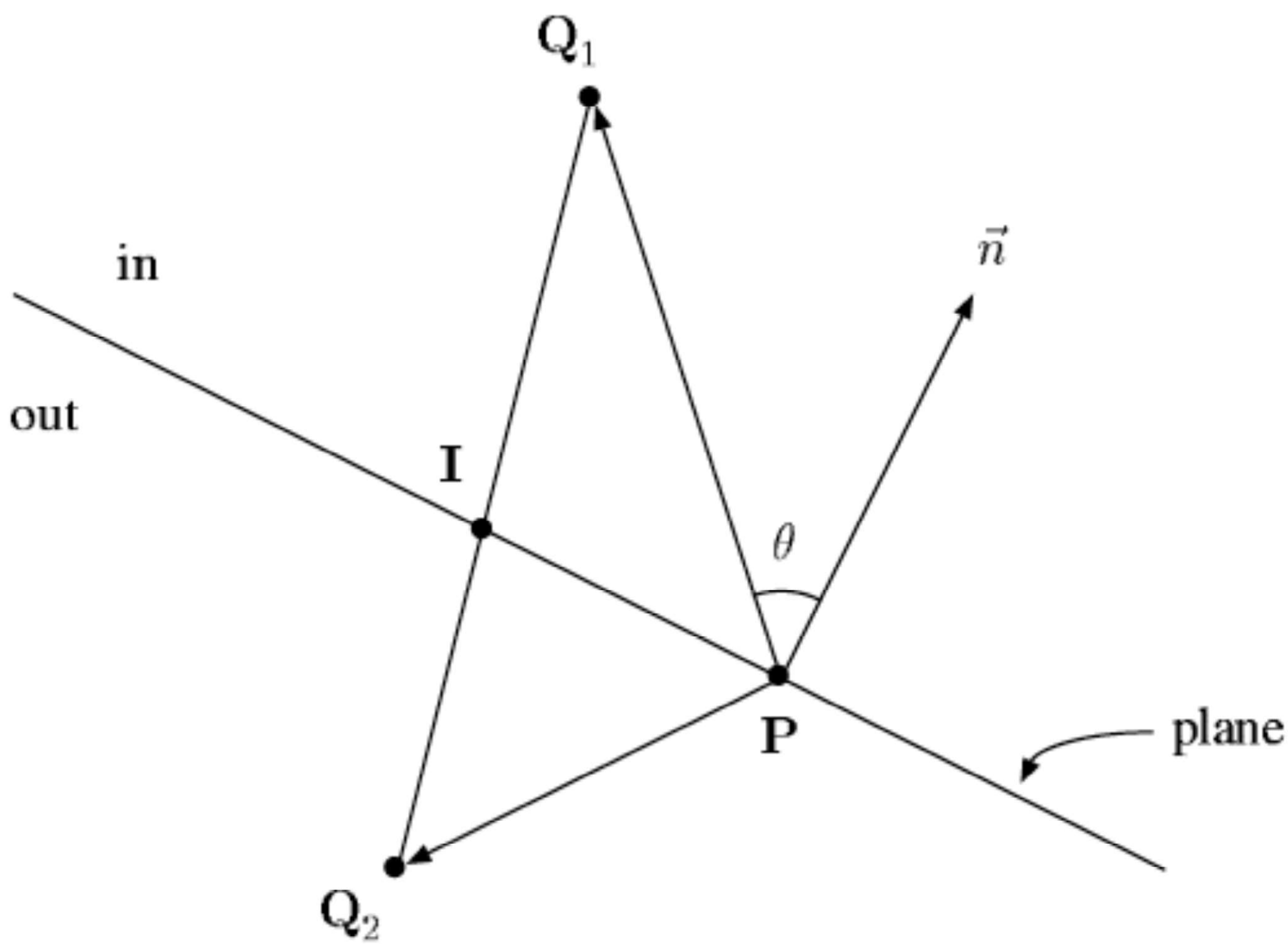
Clipping Line Segments

$$(\mathbf{I} - \mathbf{P}) \cdot \vec{n} = d_1 + t(d_2 - d_1)$$

$$0 = d_1 + t(d_2 - d_1)$$

$$t = \frac{d_1}{d_1 - d_2}$$

Clipping Line Segments



$$\mathbf{I} = \mathbf{Q}_1 + \frac{d_1}{d_1 - d_2}(\mathbf{Q}_2 + \mathbf{Q}_1)$$

Clipping a Convex Polygon

Keep two lists

The “in” list

The “out” list

a point goes on the “in” list, if it is “in” or “on”

a point goes on the “out” list, if it is “out” or “on”

If a crossing happens (dot products of two consecutive points are of different signs), calculate the intersection point and put it on both lists.

Clipping a Convex Polygon

Clipping Algorithm

Given a plane defined by $\vec{n}$ and $\mathbf{P}$

Given vertices $\mathbf{Q}_0, \mathbf{Q}_1, ..., \mathbf{Q}_{n-1}$

Let $pdot = 0$

Let $idot = \vec{n} \cdot < \mathbf{Q}_0 - \mathbf{P} >$

for each vertex i

  Let $dot = \vec{n} \cdot < \mathbf{Q}_i - \mathbf{P} >$

  if $dot * pdot < 0$ **then**

    calculate $\mathbf{I} = \mathbf{Q}_{i-1} + t(\mathbf{Q}_i - \mathbf{Q}_{i-1})$ **with** $t = \frac{pdot}{pdot - dot}$

    insert I into the "in" list

    end if

  if $dot > 0$ **then**

    insert $\mathbf{Q}_i$ into the "in" list

    end if

    Let $pdot = dot$

  end for

If $pdot * idot < 0$ **then**

  calculate $\mathbf{I} = \mathbf{Q}_{n-1} + t(\mathbf{Q}_0 - \mathbf{Q}_{n-1})$ **with** $t = \frac{pdot}{pdot - idot}$

  insert I into the "in" list

  end if