CPSC 302 - Assignment 4

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1. Stationary Method

1.a

$$A = M - N$$
$$Mx_{k+1} = Nx_k + b$$

$$x_{k+1} = x_k + \alpha(b - Ax_k)$$
$$x_{k+1} = x_k + \alpha b - \alpha A x_k$$
$$x_{k+1} = (I - \alpha A)x_k + \alpha b$$
$$\frac{I}{\alpha}x_{k+1} = (\frac{I}{\alpha} - A)x_k + b$$

$$M = \frac{I}{\alpha}$$

$$T = (I - \alpha A)$$

1.b

Converges only if $\rho(T) < 1$

Thus, only converges if $\rho(I - \alpha A) < 1$.

This converges if $|1 - \alpha\lambda| < 1$ where λ is the eigenvalue that maximizes the left hand statement. That happens to be λ_n since the eigenvalues are all positive and λ_n is the smallest eigenvalue.

$-1 < 1 - \alpha\lambda_n < 1 -2 < -\alpha\lambda_n < 0\ 2 > \alpha\lambda_n > 0$

We know that λ_n won't ever be 0 since it's defined as such.

Thus, the condition for convergence is just $\alpha\lambda_n < 2$

1.c

Using the above convergence conditions, we get $\lambda_n < 2$. Thus, the smallest eigenvalue must always be smaller than 2 for the statement to hold.

A diagonal matrix is by definition strictly diagonally dominant and the eigenvalues are just the values on the diagonal.

Thus we can construct a matrix with diagonal and eigenvalues $\lambda = \{5, 4, 3\}$. Thus, the smallest eigenvalue is 3 and this statement is contradicted since our convergence condition fails.

2. Consider the two-dimensional partial differential equation …

2.a

2.b

3. Suppose we wish to solve Ax = b …

3.a

Increasing `diagonalIncrement` causes gsMorph to converge faster.

Increasing the diagonal entries of A increases the spectral radius of A. This increase causes the spectral radius of the convergence matrix to go down since the iteration matrix is $I - M^{-1}A$.

3.b

N/A

4. The smoothing factor …

4.a

4.b

4.c