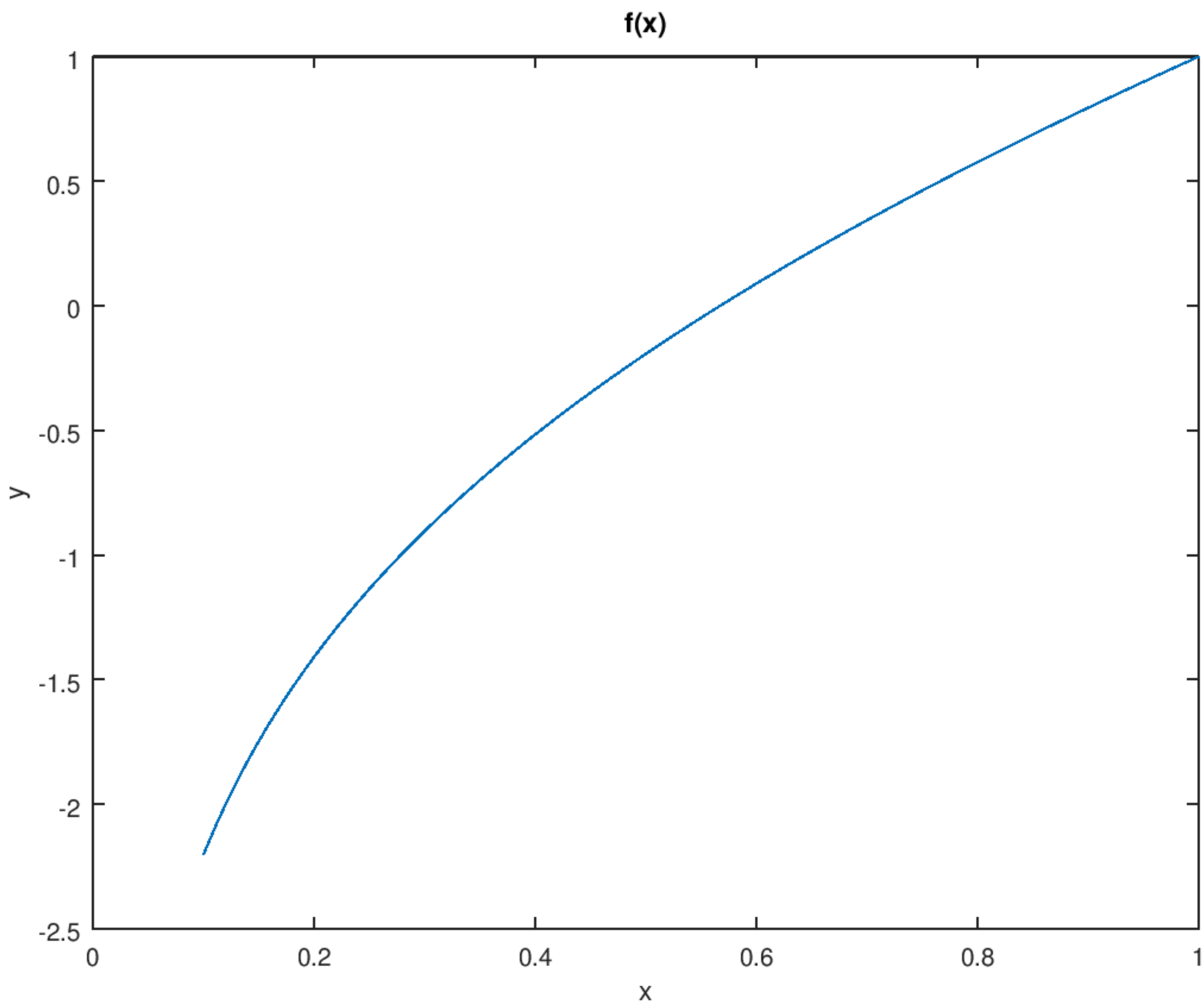
CPSC 302 - Assignment 2

Tristan Rice, q7w9a, 25886145

1

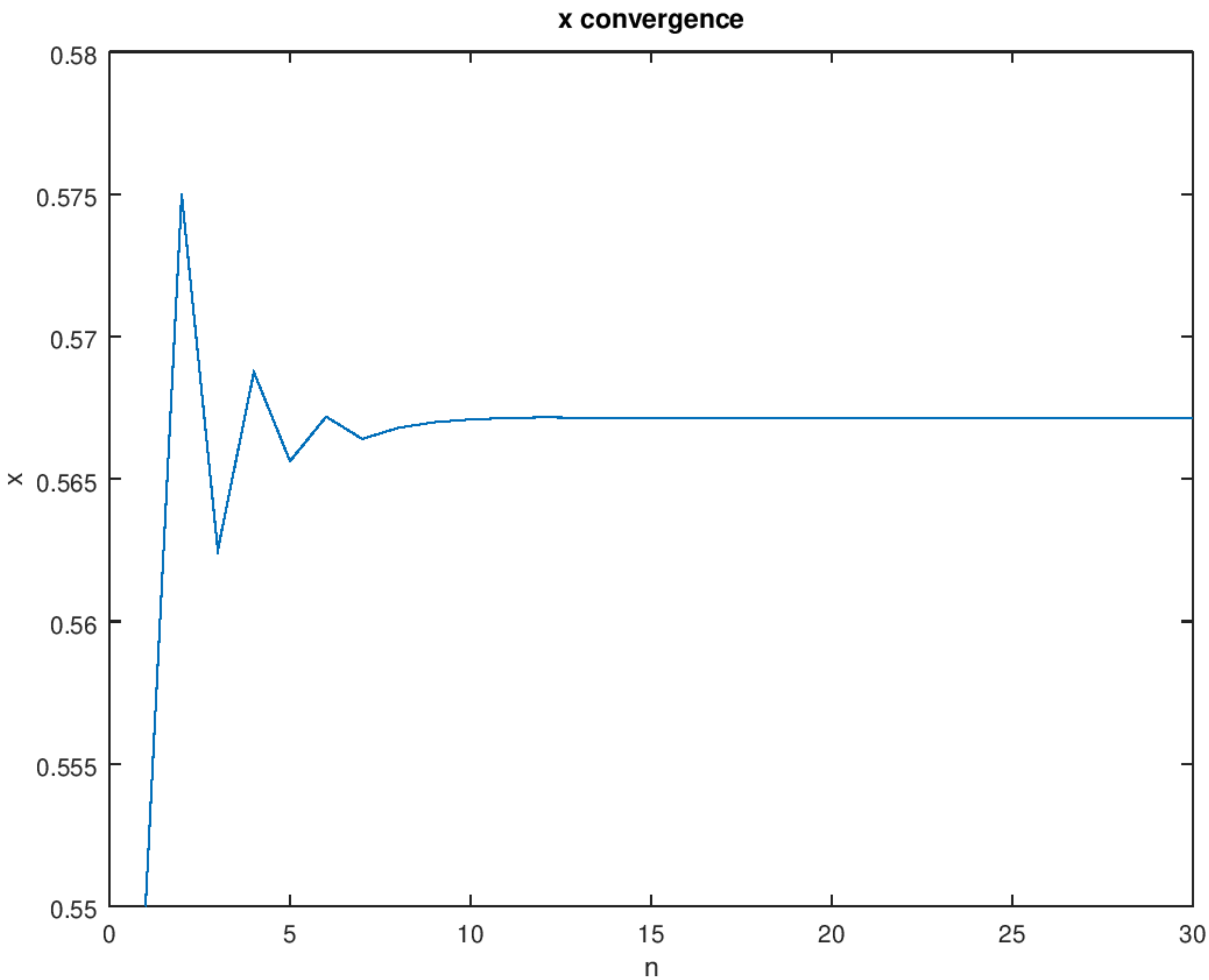
1.a



1.b

1.b.i

This is valid since the function is continuous on the interval, and one side is less than 0 and one side greater. Thus, bisection works just fine.



Bisection took 30 iteration to find the value within the specified tolerance. This very closely matches the theoretical value since the search space is halved each iteration. $\log_2(\frac{0.6-0.5}{10^{-10}}) = 29.89735$.

1.b.ii

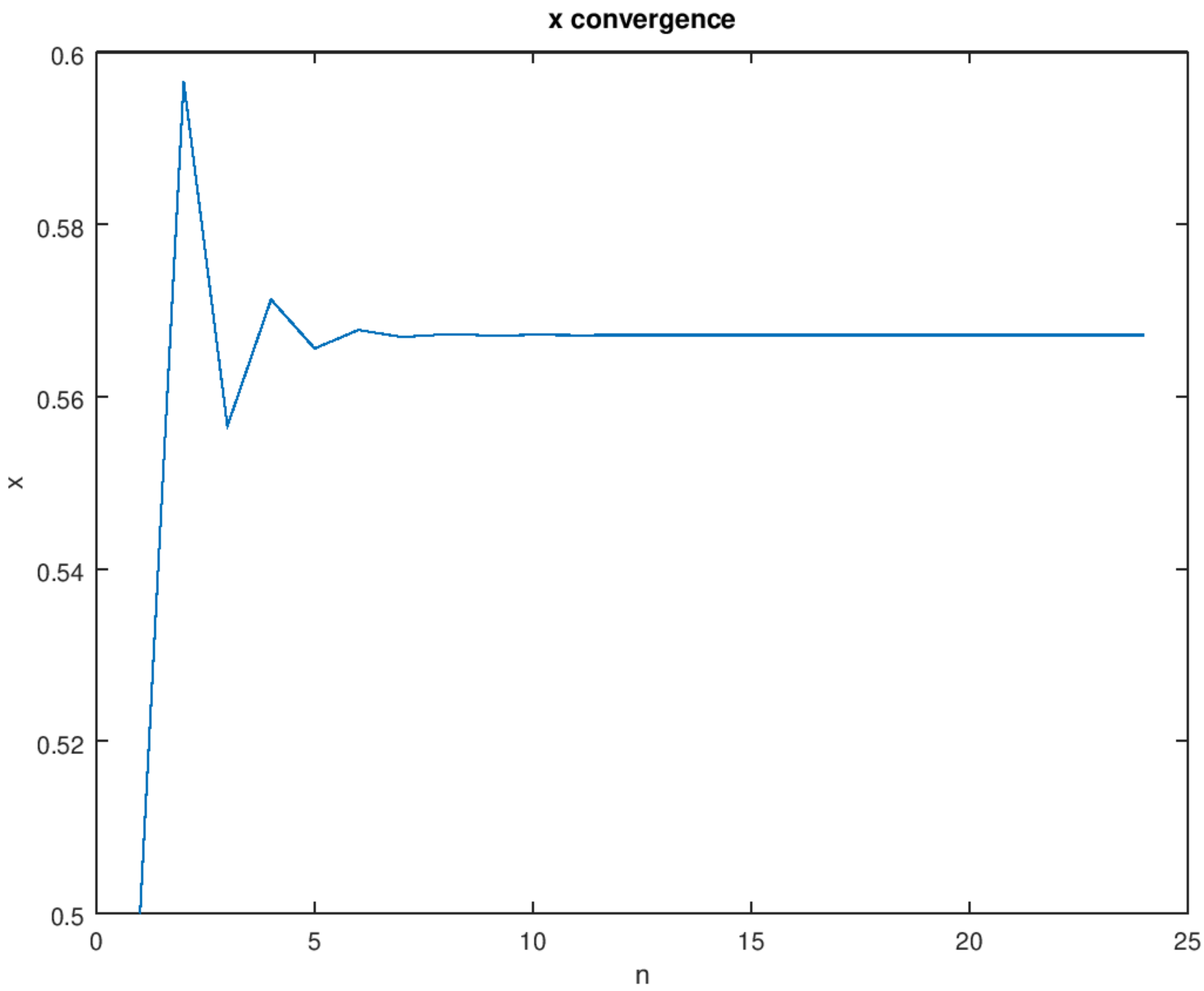
$$g(x) = x - \frac{f(x)}{2}$$

Existence:

- $g(0.5) = 0.596574 \geq 0.5$
- $g(0.6) = 0.555413 \leq 0.6$

Uniqueness: The derivative $g'(x) = \frac{x-1}{2x}$ exists and there is a constant $\rho < 1$ between $0.5, 0.6$.

Thus, the fixed point theorem holds and there is only one fixed point in the range $[0.5, 0.6]$.



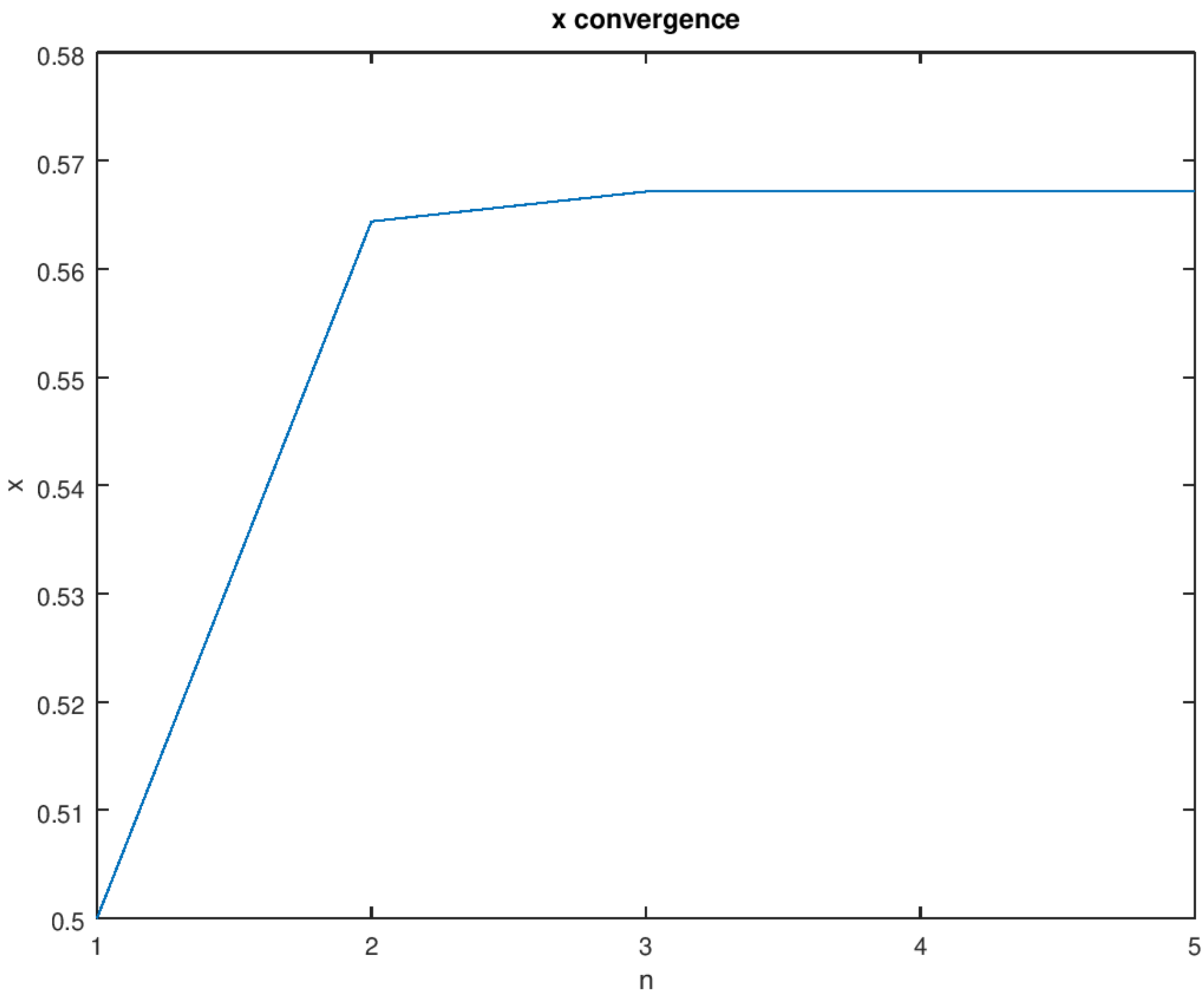
The maximum $\rho = \frac{1}{2}$ and minimum $\rho = \frac{1}{3}$ over the range $[0.5, 0.6]$.

If we use those to extrapolate the number of iterations it will take we get:

```
> log((0.6-0.5)/(1e-10), base=1/(1/3))
[1] 18.86313
> log((0.6-0.5)/(1e-10), base=1/(1/2))
[1] 29.89735
```

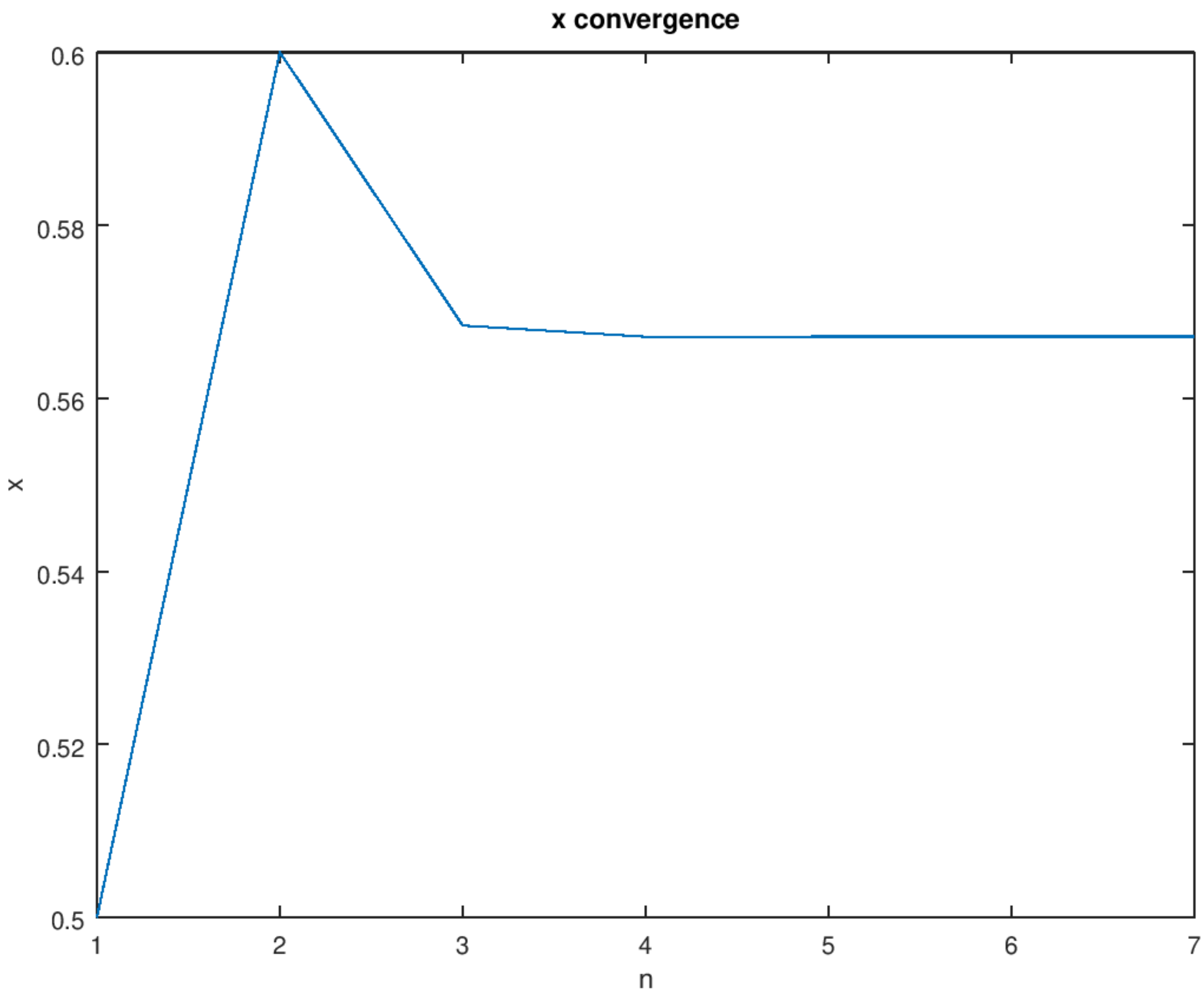
Since the number of iterations was actually 24, that fits the theoretical value since that's the average of the endpoint estimates.

1.b.iii



Newton's method takes 5 iterations. We expect Newton's method to converge quadratically. Our empirical results match the theoretical since it takes roughly the square root of how long our linearly convergent fixed point function.

1.b.iv



The secant method takes 7 iterations. We know that the order of convergence of the secant method is roughly 1.618, so it's not quite quadratic, but still superlinear. Since 7 iterations is a lot fewer than the 24 the linearly convergent method took and 7 iterations is slightly more than the 5 the quadratically convergent function took, this fits our expectations quite well.

2

2.a

$$x^2 + \frac{3}{16} = x$$
$$x^2 - x + \frac{3}{16} = 0$$
$$x = \{\frac{1}{4}, \frac{3}{4}\}$$

2.b

$$f'(x) = 2x$$

You can be sure that $\frac{1}{4}$ will converge using fixed point iteration since the magnitude of the slope is less than 1. $f'(\frac{3}{4}) = \frac{3}{2}$.

2.c

Need to find ρ.

$$\rho^n = \frac{1}{100}$$

.

$$n = \frac{\log \frac{1}{100}}{\log \rho}$$

$$|x_k^2 + \frac{3}{16} - (x^*)^2 - \frac{3}{16}| \leq \rho|x_k - x^*|$$

$$|x_k^2 - (x^*)^2| \leq \rho|x_k - x^*|$$

$$|x_k^2 - (x^*)^2| \leq \rho|x_k - x^*|$$

$$\frac{|x_k^2 - (x^*)^2|}{|x_k - x^*|} \leq \rho$$

$$\rho \geq x_k + x^*$$

$$n = \frac{\log \frac{1}{100}}{\log(x_k + x^*)}$$

If we start at $x = \frac{1}{8}, n = 5$, if $x = 1/2, n = 16$.

3

3.a

$$f(x) = (x-1)^2 e^x$$

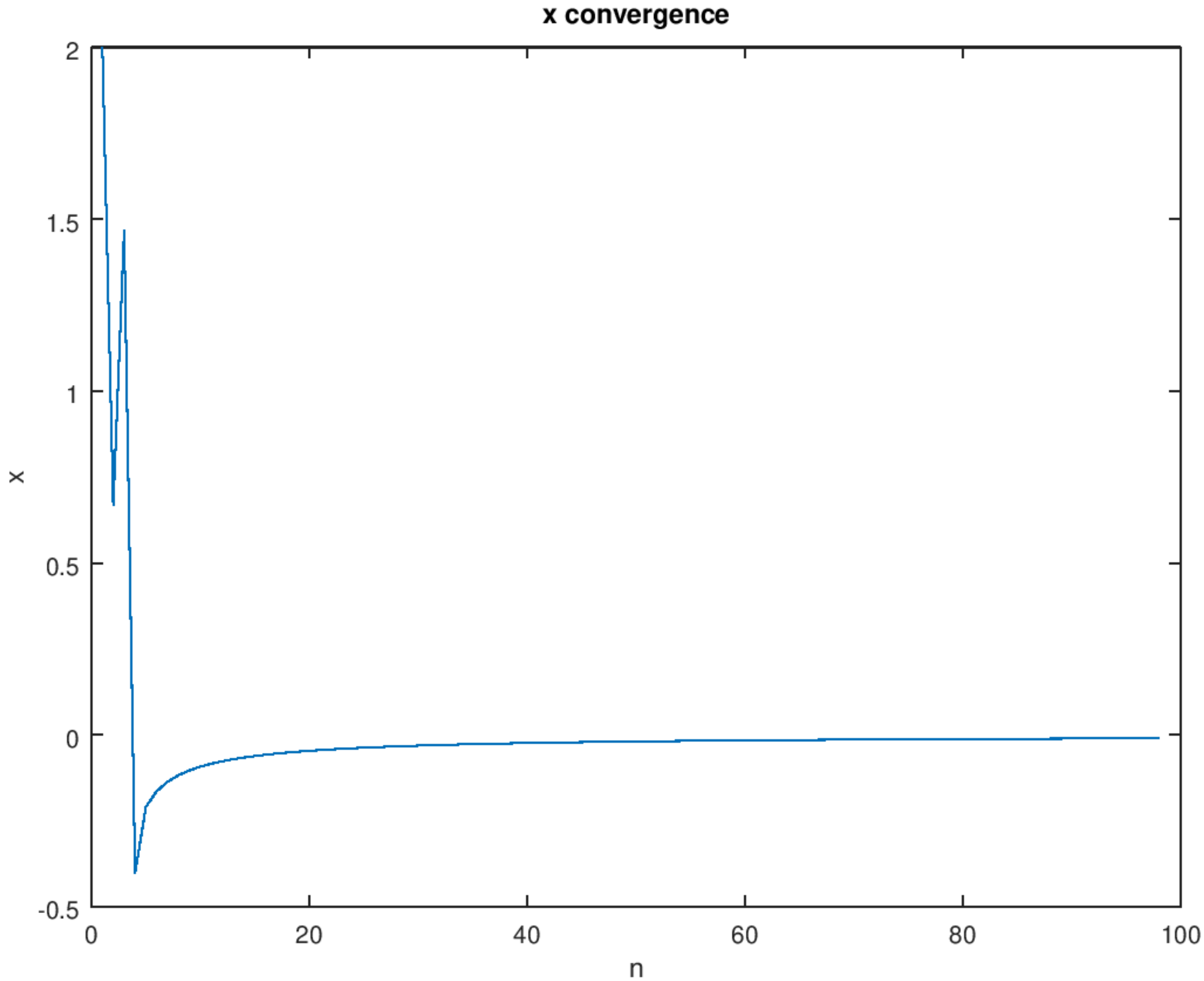
$$f'(x) = e^x(x^2 - 1)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n)^2 e^x}{e^x(x_n^2 - 1)}$$

$$x_{n+1} = x_n - \frac{x_n^2}{x_n^2 - 1}$$

3.b



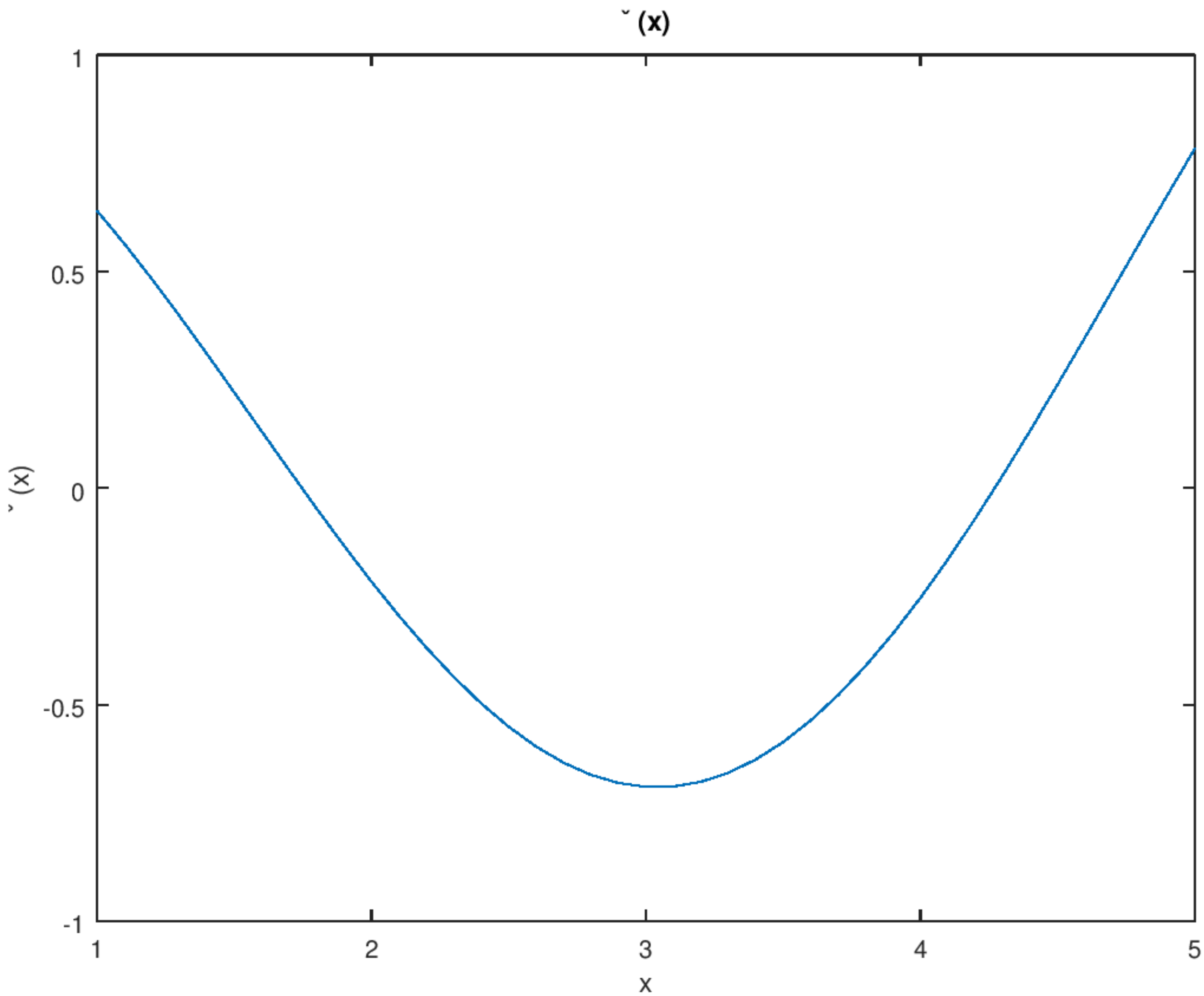
3.c

It'd be hard to implement the bisection method since the root doesn't cross the x-axis. The intermediate value theorem doesn't help you since both sides of the root are positive and thus bisection doesn't work.

However, you can apply a transformation to the function and use that. Since the derivative at the root is zero, you should be able to take the derivative of the function and then apply bisection.

4

4.a

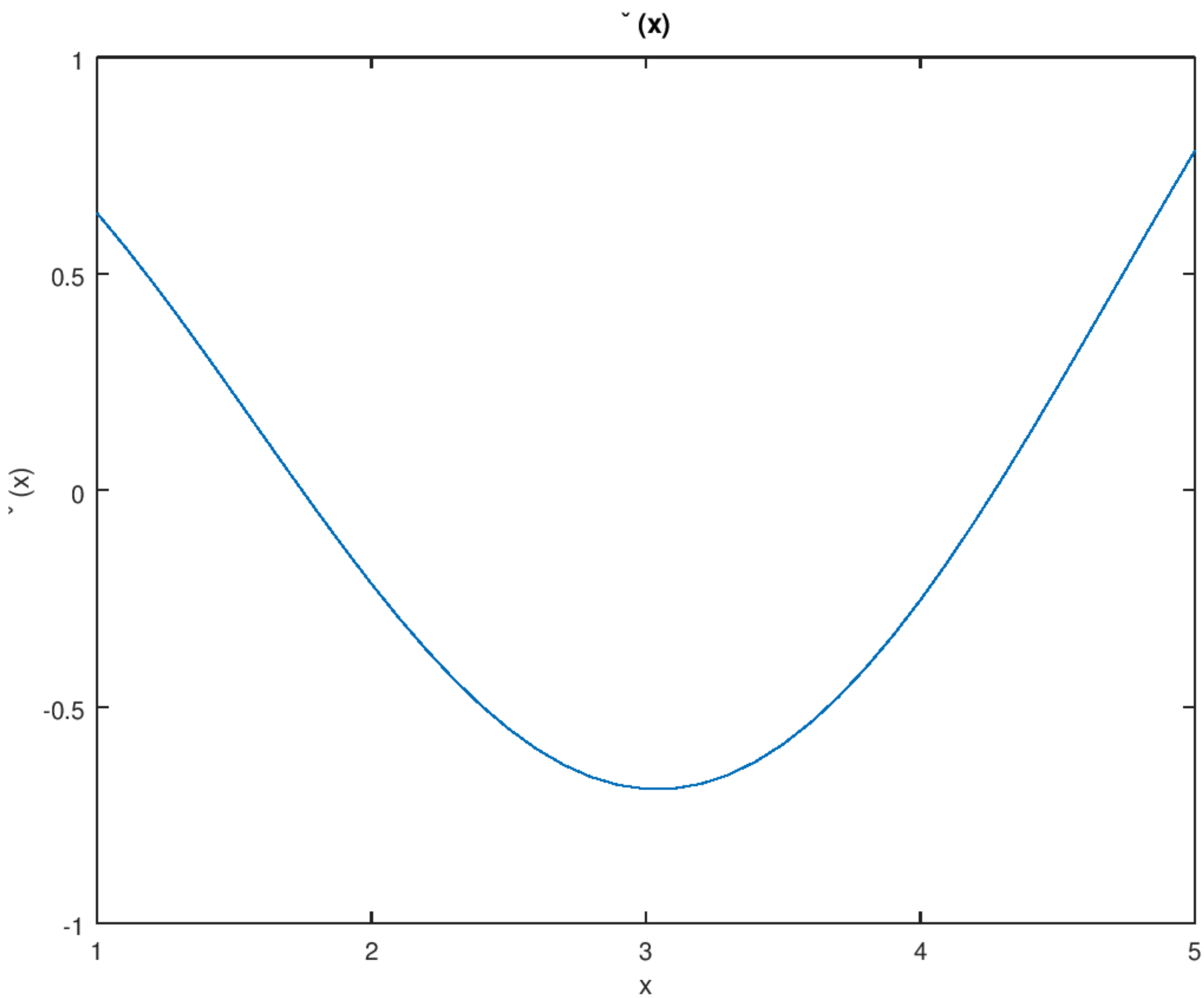


I expect it to converge to the global minimum since there's only one local minimum in the range [1,5] and it's also the global.

The computed minimum took 42 iterations, and found $x = 3.0414, \phi(x) = -0.69084.$

That matches the minimum in the plot quite well.

4.b



I don't expect it to necessarily converge to the global minimum since there's multiple local minimums, and the algorithm can't know which way to go to find the global minimum.

It takes 42 iterations once again. The computed minimum does not match the analytical, global minimum. The global minimum is at $x* = -2$, where the found minimum is at $x = 2.4$. It found a local minimum but not the global minimum.

```
1;

function y = f (x)
  y = x + log(x);
end

function y = fp(x)
  y = 1/x + 1;
end

% 1.a
xs = 0.1:0.01:1;
ys = f(xs);

plot(xs, ys);
xlabel('x');
ylabel('y');
title('f(x)');
print('1.a.png', '-dpng');

% 1.b.i

y = 0;
left = 0.5;
right = 0.6;
xs = []

while abs(left-right)>1e-10
  mid = (left+right)/2;
  xs = [xs, mid];
  y = f(mid);
  if y > 0
    right = mid;
  else
    left = mid;
  end
end

left
right
y

plot(1:length(xs), xs);
xlabel('n');
ylabel('x');
title('x convergence');
print('1.b.i.png', '-dpng');

% 1.b.ii

function y = g(x)
  y = x - f(x)/2;
end

x = 0.5;
last_x = -100000000;

xs = [x];

while abs(x-last_x)>1e-10
  last_x = x;
  x = g(x);
  xs = [xs, x];
end
x

plot(1:length(xs), xs);
xlabel('n');
ylabel('x');
title('x convergence');
print('1.b.ii.png', '-dpng');

% 1.b.iii
```

```
x = 0.5;
last_x = -100000000;

xs = [x];

while abs(x-last_x)>1e-10
  last_x = x;
  x -= f(x)/fp(x);
  xs = [xs, x];
end
x

plot(1:length(xs), xs);
xlabel('n');
ylabel('x');
title('x convergence');
print('1.b.iii.png', '-dpng');

% 1.b.iv

x = 0.6;
last_x = 0.5;

xs = [last_x, x];

while abs(x-last_x)>1e-10
  tmp_x = x;
  x -= f(x)*(x-last_x)/(f(x)-f(last_x));
  last_x = tmp_x;
  xs = [xs, x];
end
x

plot(1:length(xs), xs);
xlabel('n');
ylabel('x');
title('x convergence');
print('1.b.iv.png', '-dpng');
```

```
x = 2;
last_x = -1000;
xs = [x];

while abs(x-last_x)>1e-4
  last_x = x;
  x -= x^2/(x^2-1)
  xs = [xs, x];
end

plot(1:length(xs), xs);
xlabel('n');
ylabel('x');
title('x convergence');
print('3.b.png', '-dpng');
```

```
1;

function y = phi(x)
  y = cos(x) + x/10;
end

% 4.a

x = 1:0.1:5;
y = phi(x);

plot(x,y);
xlabel('x');
ylabel('¿(x)');
title('¿(x)');
print('4.a.png', '-dpng');

a = 1;
b = 5;
tol = 1e-8;

tau = (sqrt(5) - 1)/2;
x1 = a + (1-tau)*(b-a);
f1 = phi(x1);
x2 = a + tau *(b-a);
f2 = phi(x2);

iters = 0

while ((b-a) > tol)
  iters += 1;

  if (f1 > f2)
    a = x1;
    x1 = x2;
    f1 = f2;
    x2 = a + tau * (b-a);
    f2 = phi(x2);
  else
    b = x2;
    x2 = x1;
    f2 = f1;
    x1 = a + (1-tau)*(b-a);
    f1 = phi(x1);
  end
end

x1
x2
a
b
f1
f2
iters



% 4.b
x = -2:0.1:2.4;
y = phi(x);

plot(x,y);
xlabel('x');
ylabel('¿(x)');
title('¿(x)');
print('4.b.png', '-dpng');

a = -2;
b = 2.4;
tol = 1e-8;

tau = (sqrt(5) - 1)/2;
x1 = a + (1-tau)*(b-a);
```

```
f1 = phi(x1);
x2 = a + tau *(b-a);
f2 = phi(x2);

iters = 0

while ((b-a) > tol)
  iters += 1;

  if (f1 > f2)
    a = x1;
    x1 = x2;
    f1 = f2;
    x2 = a + tau * (b-a);
    f2 = phi(x2);
  else
    b = x2;
    x2 = x1;
    f2 = f1;
    x1 = a + (1-tau)*(b-a);
    f1 = phi(x1);
  end
end

x1
x2
a
b
f1
f2
iters
```