

```

% set global config options
global iters;
global tol;
iters = 2000;
tol = 10^-6;

% setup plot
f = figure();
title('2.b Convergence of methods');
ploti = 0;

for N = [31 63]
    for alpha = sqrt([0 10 1000])
        % increment plot number
        ploti = ploti + 1;

        % compute B and b from Bx=b
        n = N^2;
        h = 1/(N+1);
        c = (alpha * h)^2;
        B = delsq(numgrid('S', N + 2)) + c*speye(n);
        b = B*ones(N^2,1);

        % x is the initial guess
        x = zeros(n, 1);

        k2 = (4 - 2 * (cos(floor(N) * pi / (N+1)) + cos(floor(N) * pi / (N+1)))) + (alpha./
(N+1)).^2)./(4-2*(cos(pi./(N+1)) + cos(pi./(N+1)))) + (alpha./(N+1)).^2);
        fprintf('N = %d, n = %d, k2(B) = %d\n', N, n, k2);

        % compute the residuals and iteration counts for each method.
        [~, jr, i] = jacobi(B, x, b);
        fprintf('jacobi: %d iters\n', i)
        [~, gsr, i] = gaussSeidel(B, x, b);
        fprintf('gaussSeidel: %d iters\n', i)
        [~, sorr, i] = SOR(B, x, b);
        fprintf('SOR: %d iters\n', i)
        [~, cgr, i] = CG(B, x, b);
        fprintf('cg: %d iters\n', i)
        [~, pcgr, i] = PCG(B, x, b);
        fprintf('pcg: %d iters\n', i)

        % plot the residual vectors
        subplot(3, 2, ploti);
        semilogy(getx(jr), jr, getx(gsr), gsr, getx(sorr), sorr, getx(cgr), cgr, getx(pc
gr), pcgr);
        legend('jacobi', 'gaussSeidel', 'SOR', 'CG', 'PCG');
        xlabel('iterations');
        ylabel('relative residual');
        title(sprintf('N = %d, alpha = %d', N, alpha))

        fprintf('\n');
    end
end

% save plot
saveas(f, 'q2.png');

%% getx returns a vector 1, 2, ..., len(b).
function x = getx(b)
    x = 1:max(size(b));
end

% gaussSeidel solves Bx=b using the Gauss-Seidel method.
function [x, relres, i] = gaussSeidel(B, x, b)
    global iters;
    global tol;

    % Create matrix splitting of B
    D = diag(diag(B));
    L = -1*tril(B,-1);
    U = -1*triu(B,1);

```

```

% Compute iteration matrix
M = inv(D-L);

r0 = residual(B, x, b);
rs = [r0];
for i = 1:iters
    x = M * (b + U * x); % compute current solution
    r = residual(B, x, b);
    rs = [rs, r];
    if r/r0 < tol
        break
    end
end
relres = rs/norm(b, 2);
end

%% jacobi solves Bx=b using the Jacobi method.
function [x, relres, i] = jacobi(B, x, b)
    global iters;
    global tol;

    % Create matrix splitting of B
    D = diag(diag(B));
    L = -1*tril(B,-1);
    U = -1*triu(B,1);

    % Compute iteration matrix
    M = inv(D);

    % cache LPU for better performance
    LPU = (L+U);

    r0 = residual(B, x, b);
    rs = [r0];
    for i = 1:iters
        x = M*(LPU*x + b); % compute current solution
        r = residual(B, x, b);
        rs = [rs, r];
        if r/r0 < tol
            break
        end
    end
    relres = rs/norm(b, 2);
end

%% CG solves Bx=b using the Conjugate Gradient method.
function [x, relres, i] = CG(B, x, b)
    global iters;
    global tol;

    [x, ~, ~, i, relres] = pcg(B, b, tol, iters, [], [], x);
end

%% PCG solves Bx=b using the Conjugate Gradient method with a incomplete
%% Cholesky IC(0).
function [x, relres, i] = PCG(B, x, b)
    global iters;
    global tol;

    L = ichol(B);
    [x, ~, ~, i, relres] = pcg(B, b, tol, iters, L, L', x);
end

function [x, relres, i] = SOR(B, x, b)
    global iters;
    global tol;

    % Create matrix splitting of B
    D = diag(diag(B));
    L = -1*tril(B,-1);
    U = -1*triu(B,1);

    % Compute iteration matrix

```

```
M = inv(D-L);
jacobiM = inv(D);

% compute damping factor
spectral_radius = max(abs(eig(jacobiM)));
w = 2 / (1 + sqrt(1 - spectral_radius^2));

r0 = residual(B, x, b);
rs = [r0];
for i = 1:iters
    xnew = M * (b + U * x); % compute GS
    x = w * xnew + (1-w) * x; % damp GS according to damping factor w
    r = residual(B, x, b);
    rs = [rs, r];
    if r/r0 < tol
        break
    end
end
relres = rs/norm(b, 2);
end

%% residual computes the residual of Bx = b
function r = residual(B, x, b)
    r = norm(b - B*x, 2);
end
```