

CPSC 340 Assignment 5

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I'm using one late day to turn in the assignment the Monday after the original Saturday 2pm due date. I'll have 2 late days remaining after this.

1 Principal Component Analysis

1.1 PCA by Hand

x_1	x_2
-2	-1
-1	0
0	1
1	2
2	3

1. What is the first principal component?

Mean is (0, 1).

Centered:

x_1	x_2
-2	-2
-1	-1
0	0
1	1
2	2

You can fit a line through this perfectly, since it's just $x_1 = (x_2 - 1)$.

2. What is the (L2-norm) reconstruction error of the point (3,3)? (Show your work.)

We need to calculate the orthogonal L2 distance between the first principle component. We can do this with the equation:

$$distance(ax + by + c = 0, (x_0, y_0)) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$$distance(x_1 - x_2 + 1 = 0, (3, 3)) = \frac{|3 - 3 + 1|}{\sqrt{1^2 + (-1)^2}} = \frac{0}{\sqrt{2}} = 0.7071$$

3. What is the (L2-norm) reconstruction error of the point (3,4)? (Show your work.)

$$distance(x_1 - x_2 + 1 = 0, (3, 4)) = \frac{|3 - 4 + 1|}{\sqrt{1^2 + (-1)^2}} = \frac{0}{\sqrt{2}} = 0$$

1.2 Data Visualization

```
load animals.mat
```

```
k = 2;
```

```
[n,d] = size(X);
```

```
X = standardizeCols(X);
```

```

figure(1);
imagesc(X);
figure(2);
i = 1
j = 2

model = dimRedPCA(X,k);

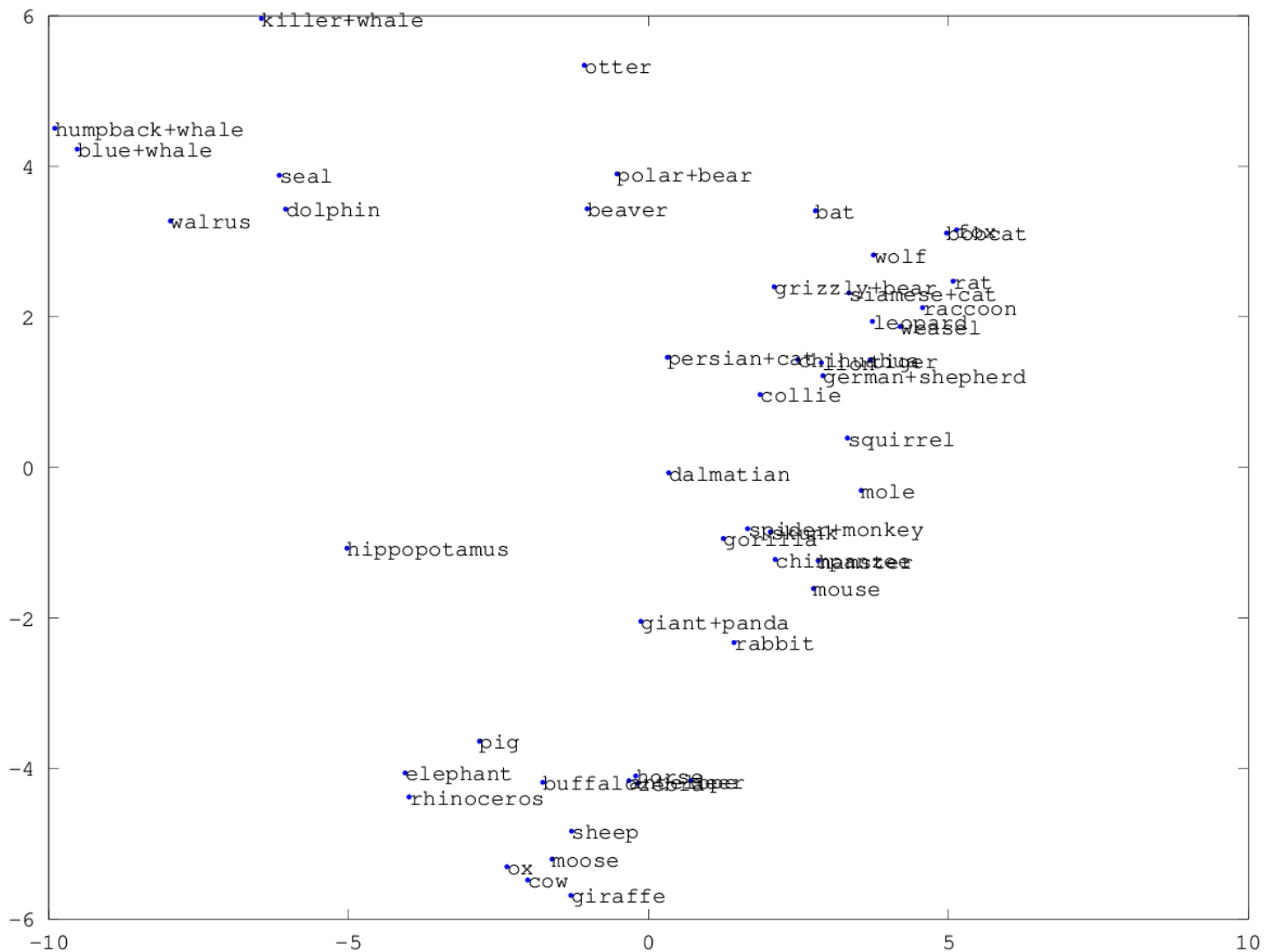
Z = model.compress(model,X);

plot(Z(:,i),Z(:,j),'.');

for k = 1:n
    text(Z(k,i),Z(k,j),animals(k,:));
end

print -dpng 1.2.png

```



1.3 Data Compression

1. How much of the variance is explained by our two-dimensional representation from the previous question?

```

Xhat = model.expand(model,Z);
variance = norm(Xhat-X, "fro")^2/(norm(X, "fro")^2)

```

```
=> variance = 0.69806
> (1 - 0.69806)*100
[1] 30.194
```

Thus, our two dimensional representation represents 30.194% of the variance.

2. How many PCs are required to explain 50% of the variance in the data?

If we set $k = 5$ we get that 50.59% of the variance is explained.

2 PCA Generalizations

2.1 Robust PCA

dimRedPCA_alternate.m

```
function [model] = dimRedPCA_alternate(X,k)
```

```
[n,d] = size(X);
```

```
% Subtract mean
```

```
mu = mean(X);
```

```
X = X - repmat(mu,[n 1]);
```

```
% Initialize W and Z
```

```
W = randn(k,d);
```

```
Z = randn(n,k);
```

```
R = Z*W-X;
```

```
f = sum(sum(R.^2));
```

```
for iter = 1:50
```

```
    fOld = f;
```

```
% Update Z
```

```
Z(:) = findMin(@funObjZ,Z(:),10,0,X,W);
```

```
% Update W
```

```
W(:) = findMin(@funObjW,W(:),10,0,X,Z);
```

```
R = Z*W-X;
```

```
f = sum(sum(R.^2));
```

```
fprintf('Iteration %d, loss = %.5e\n',iter,f);
```

```
if fOld - f < 1e-4
```

```
    break;
```

```
end
```

```
end
```

```
model.mu = mu;
```

```
model.W = W;
```

```
model.compress = @compress;
```

```
model.expand = @expand;
```

```
end
```

```
function [Z] = compress(model,X)
```

```
[t,d] = size(X);
```

```
mu = model.mu;
```

```
W = model.W;
```

```
k = size(W,1);
```

```

X = X - repmat(mu,[t 1]);
% We didn't enforce that W was orthogonal so we need to optimize to find Z
Z = zeros(t,k);
Z(:) = findMin(@funObjZ,Z(:),100,0,X,W);
end

function [X] = expand(model,Z)
[t,d] = size(Z);
mu = model.mu;
W = model.W;

X = Z*W + repmat(mu,[t 1]);
end

function [f,g] = funObjW(W,X,Z)
% Resize vector of parameters into matrix
d = size(X,2);
k = size(Z,2);
W = reshape(W,[k d]);

% Compute function value
R = Z*W-X;
epsilon = 0.0001
% f = sum(sum(R.^2));
f = sum(sum(sqrt(R.^2+epsilon)));

% Compute derivative with respect to each residual
dR = R;

% Multiply by Z' to get elements of gradient
g = Z'*dR;

% Return a vector
g = g(:);
end

function [f,g] = funObjZ(Z,X,W)
% Resize vector of parameters into matrix
n = size(X,1);
k = size(W,1);
Z = reshape(Z,[n k]);

% Compute function value
R = Z*W-X;
% f = sum(sum(R.^2));
f = sum(sum(sqrt(R.^2+epsilon)));

% Compute derivative with respect to each residual
dR = R;

% Multiply by W' to get elements of gradient
g = dR*W';

% Return a vector
g = g(:);
end

```

2.2 L1-Regularized and Binary Latent-Factor Models

1. What is the effect of λW on the sparsity of the parameters W and Z ? What is the effect of λZ on the sparsity of W and Z ?

Increasing λ_W will increase the sparsity of W and will have little effect on Z . Increasing λ_Z will increase the sparsity of Z and have little effect on W .

2. What is the effect of λ_Z on the two parts of the fundamental trade-off in machine learning? What is the effect of k on the two parts?

Since increasing λ_Z makes Z sparser, that means it is using fewer features to represent the data. This likely will increase the test error, as it can't fit the test data exactly, but make the test error more representative of the validation error as it prevents overfitting. If λ_Z is too large, it will likely increase both test error and validation error.

3. Would the answers to (2) change if $\lambda_W = 0$?

When $\lambda_W = 0$, it is equivalent to having no L1-regularizer on W . Thus, you could potentially get no benefit from having the L2-regularizer on Z since those large weights could be shifted into W instead. The net effect will be the same no matter what λ_Z is.

4. Suppose each element of the matrix X is either +1 or -1 and our goal is to build a model that makes the sign of $w^T z_i$ match the sign of x_{ij} . Write down a (continuous) objective function that would be more suitable.

If we're trying to match the signs, this becomes a classification problem. One good continuous loss function for classification is logistic loss.

Since the logistic loss is between 0 and 1 and not -1 and +1, we first have to scale the inputs.

$$f(Z, W) = \sum_{(i,j) \in R} \log(1 + \exp(-w_j^T z_i (x_{ij} + 1)/2)) + W, Z \text{ regularizers}$$

3 Multi-Dimensional Scaling

3.1 ISOMAP

```
function [Z] = visualizeISOMAP(X,k,names)

[n,d] = size(X);

% Compute all distances
D = X.^2*ones(d,n) + ones(n,d)*(X').^2 - 2*X*X';
D = sqrt(abs(D));

DE = zeros(n, n);

for i = 1:n
    [v, j] = sort(D(i, :));
    DE(i, j(2:4)) = v(2:4);
    DE(j(2:4), i) = v(2:4);
end

for i = 1:n
    for j = (i+1):n
        i
        j
        dist = dijkstra(DE,i,j);
        D(i,j) = dist;
        D(j,i) = dist;
    end
end

% Initialize low-dimensional representation with PCA
model = dimRedPCA(X,2);
Z = model.compress(model,X);

Z(:) = findMin(@stress,Z(:),500,0,D,names);

end
```

```

function [f,g] = stress(Z,D,names)

n = length(D);
k = numel(Z)/n;

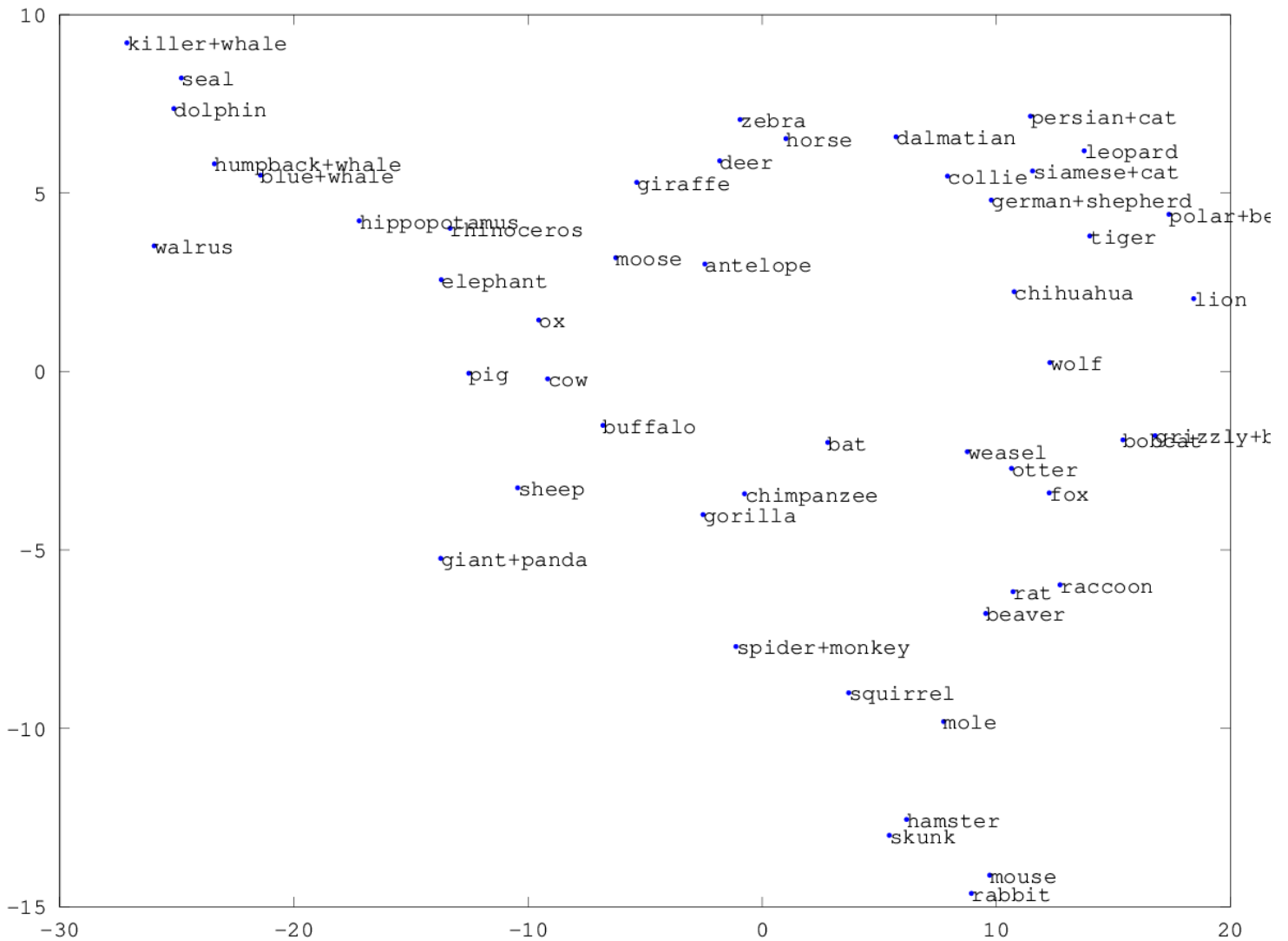
Z = reshape(Z,[n k]);

f = 0;
g = zeros(n,k);
for i = 1:n
    for j = i+1:n
        % Objective Function
        Dz = norm(Z(i,:)-Z(j,:));
        s = D(i,j) - Dz;
        f = f + (1/2)*s^2;

        % Gradient
        df = s;
        dgi = (Z(i,:)-Z(j,:))/Dz;
        dgj = (Z(j,:)-Z(i,:))/Dz;
        g(i,:) = g(i,:) - df*dgi;
        g(j,:) = g(j,:) - df*dgj;
    end
end
g = g(:);

% Make plot if using 2D representation
if k == 2
    figure(3);
    clf;
    plot(Z(:,1),Z(:,2),' .');
    if ~isempty(names)
        hold on;
        for i = 1:n
            text(Z(i,1),Z(i,2),names(i,:));
        end
    end
    pause(.01)
end
end

```



3.2 ISOMAP with Disconnected Graph

```
function [Z] = visualizeISOMAP(X,k,names)

[n,d] = size(X);

% Compute all distances
D = X.^2*ones(d,n) + ones(n,d)*(X').^2 - 2*X*X';
D = sqrt(abs(D));

DE = zeros(n, n);

for i = 1:n
    [v, j] = sort(D(i, :));
    DE(i, j(2:3)) = v(2:3);
    DE(j(2:3), i) = v(2:3);
end

for i = 1:n
    for j = (i+1):n
        i
        j
        dist = dijkstra(DE,i,j);
        D(i,j) = dist;
    end
end
```

```

        D(j,i) = dist;
    end
end

% Get rid of max infs.
D(D==inf) = max(D(D!=inf))

% Initialize low-dimensional representation with PCA
model = dimRedPCA(X,2);
Z = model.compress(model,X);

Z(:) = findMin(@stress,Z(:),500,0,D,names);

end

function [f,g] = stress(Z,D,names)

n = length(D);
k = numel(Z)/n;

Z = reshape(Z,[n k]);

f = 0;
g = zeros(n,k);
for i = 1:n
    for j = i+1:n
        % Objective Function
        Dz = norm(Z(i,:)-Z(j,:));
        s = D(i,j) - Dz;
        f = f + (1/2)*s^2;

        % Gradient
        df = s;
        dgi = (Z(i,:)-Z(j,:))/Dz;
        dgj = (Z(j,:)-Z(i,:))/Dz;
        g(i,:) = g(i,:) - df*dgi;
        g(j,:) = g(j,:) - df*dgj;
    end
end
g = g(:);

% Make plot if using 2D representation
if k == 2
    figure(3);
    clf;
    plot(Z(:,1),Z(:,2),'.');
    if ~isempty(names)
        hold on;
        for i = 1:n
            text(Z(i,1),Z(i,2),names(i,:));
        end
    end
    pause(.01)
end
end

```