CpSc 418 **Final Exam** December 5 2013

Solution set

1. Reduce and Scan (**24 points**) One of the problems below can be solved using reduce, the other can be solved using scan. Identify which is which. For the problem that can be solved using reduce, describe the `Leaf` and `Combine`, and `Root` functions – the `Root` function may (or may not) be the identity function. For the problem that can be solved using scan, describe the `Leaf1` and `Leaf2`, and `Combine` functions.

   (a) (**12 points**) Given a list of numbers that is distributed across the worker processes, compute a new list whose i^{th} element is the maximum element up to and including the current one minus the minimum element up to and including this one. I'll call this function `gap` and here are a few examples:

```
gap([5,2,6,18,-3,4,8,-1,20]) -> [0,3,4,16,21,21,21,21,23];
gap([]) -> [];
gap([42]) -> [0];
```

   Solution The problem is an instance of scan. In my solution, the `Leaf1` function reads the list and returns a tuple of the form `{Min, Max}` where `Min` is the minimum element of the list and `Max` is the maximum element. If the list is empty, `Leaf1` returns the atom `'empty'`. The `Combine` function takes two such tuples (or `'empty'` atoms) and returns the maximum and minimum for the combined subtrees (or `'empty'` if both are empty). The `Leaf2` function takes an `AccIn` value that is the tuple with the maximum and minimum for everything to the left of this leaf. It computes the gap for each element and stores is with with the given key. I wrote a helper function, `gap(List, Accin)` that returns a tuple of the form `{GapList, {Min, Max}}`, where `GapList` is the gap for each element of `List`, and the `{Min, Max}` tuple is the tuple produced by `Leaf1` or `Combine` (or the atom `'empty'`).

```
gap(L, AccIn) -> mapfoldl(
   fun
      (E, empty) -> {0, {E, E}};
      (E, {Min, Max}) ->
         X = min(Min,E),
         Y = max(Max, E),
         {Y-X, {X, Y}}
   end, AccIn, L).

gap(W, Src, Dst) ->
   wtree:scan(W,
      fun(S) -> % Leaf1
         element(2, gap(workers:get(S, Src), empty))
      end,
      fun(S, AccIn) -> % Leaf2
         workers:put(S, element(1, gap(workers:get(S, Src), AccIn)))
      end,
      fun % Combine
         (empty, Right) -> Right;
         (Left, empty) -> Left;
         ({Lmin, Lmax}, {Rmin, Rmax}) ->
            {min(Lmin, Rmin), max(Lmax, Rmax)}
      end
   ).
```

   Note: I haven't tried compiling and running this code yet – you didn't have that opportunity on the exam. When I do, there will almost certainly be some minor error. I'll fix them and might add

that version. You can think of the difference between the code written here and the final version as indicating the range of what I will consider to be an acceptable solution. Of course, there are many other valid implementations other than the one shown here. They will get full credit as well.

(b) (**12 points**) Given a list of numbers, report the number of consecutive Pythagorean triples. If `x`, `y`, `z` are consecutive elements of the list, then they are a Pythagorean triple iff $x^2 + y^2 = z^2$. Here are few examples:

```
nPythag3([1,2,4,6,8,10,11,3,4,5,12,13,14]) -> 3; % [6,8,10], [3,4,5], [5,12,13]
nPythag3([1,-2,3,-4,5]) -> 1; nPythag3([0,1,0,1,0,1,0,1]) -> 3;
```

Solution The problem is an instance of reduce. In my solution, each leaf worker process produces a tuple of the form `{First2, N_triple, Last2}` where `First2` is a list of the first two elements of the worker's piece of the lists; `Last2` is a list of the last two elements of the worker's piece of the lists; and `N_triple` is the number of Pythagorean triples in the worker's piece of the list. If the worker's piece of the list has fewer than two elements, then `Leaf` just returns the list. The `Combine` function combines these in the "obvious" way. Have a different pattern for leaves with short lists makes handling the special cases fairly straightforward. The `Root` function extracts the count from the triple, or if the top-value is a list, returns 0. Finally, I wrote `nPythag3(List)`, a sequential implementation which is used by `Leaf` and `Combine`.

```
nPythag3(L=[A, B | _]) ->
{N, Last2} = nPythag3(L, 0),
{[A,B], N, Last2};
nPythag3(L) -> L.
nPythag3(W, Src) ->
   wtree:reduce(W,
      fun(S) -> nPythag3(workers:get(S, Src)) end, % Leaf
      fun % Combine
         ({LL, LN, LR}, RL, RN, RR) ->
            {LL, LN + element(2, nPythag3(LR++RL)) + RN, RR};
         (L, {RL, RN, RR}) ->
            {First2, MidN, RL} = nPythag3(L++RL),
            {First2, MidN+RN, RR};
         ({LL, LN, LR}, R) ->
            {LR, MidN, Last2} = nPythag3(LR++R),
            {LL, LN+MidN, Last2};
         (L,R) -> nPythag3(L++R)
      end,
      fun % Root
         ({_, N, _}) -> N;
         (L) -> 0
      end
   ).
```

2. **Filter Locks (16 points)** I've mentioned several times that Peterson's mutual exclusion algorithm can be extended to apply to any number of clients. The algorithm is called a "filter lock". Here's the code:

```
00: int level[N], victim[N]; // initially all 0

01: lock(i) {
02:   for(int j = 1; j < N; j++) {
03:     level[i] = j;
04:     victim[j] = i;
```

```
05:     while((victim[j] == i) && ∃ 0 ≤ k < N. (k ≠ i) && level[k] ≥ j);
06:   }
07: }

08: unlock(i) {
09:   level[i] = 0;
10: }
```

The key idea in the lock is that each thread must pass through $N-1$ "levels" of locking (the `for(j...)` loop) before entering its critical section. To advance from level `j` to level `j+1`, there must be no other threads at level `j` or higher, **or** there must be some other thread that is the "victim" at level `j`. A consequence is that if there is another thread at this level or higher, then there must be at least one thread "left behind" at each of the lower levels. In particular, if thread x is in the critical section, and thread y is at level $N-1$, there are threads at every level from 1 to $N-1$; thread y is the victim at level $N-1$; and thread y can't advance to the critical section.

(a) (**8 points**) Show that the filter lock is **not** starvation free. For example, consider a filter lock with three clients: 0, 1, and 2. Show that one of the clients can spin forever while the other two alternately acquire and release the lock.

Solution: Proof that filter locks are starvation free. If a thread is stuck spinning at line `05`, then this thread must be the victim, and there must be another thread at or above this level. If there is another thread at this level, then that thread can advance because it is not the victim. This shows that the filter lock is deadlock free. Once the threads that are ahead of this one make it to the critical section and back to idle, this one must will be able to advance.

The concern is that some other threads may come reach this level in the meantime – for example, threads that were above this one could reach their critical section, come back around and then pass this one again. The key observation (and this is where I blew it) is that if thread i_1 is at level j, and thread i_2 enters level k then thread i_2 sets the `victim[j]` variable to i_2. It will not get set back to i_2 (until after i_1 has reached its critical section and is trying again for the lock), because only thread i_1 can set a victim variable to i_1. Thus, once some other thread enters level j, thread i_1 can progress at least to level $j+1$. Eventually, thread i_1 gets the lock.

(b) (**8 points**) Show that if the statements at lines `03` and `04` are exchanged (i.e. `victim[j] = i;` is performed *before* `level[i] = j;`, then mutual exclusion can be violated.

Solution: The following execution shows a violation:

0. All threads are at level 0.
1. Thread 1 sets `victim[1] = 1;`
2. Thread 2 sets `victim[1] = 2;`
3. Thread 2 sets `level[2] = 1;`
4. Thread 2 checks the condition at line `05`. It is the victim, but there is no other thread with `level` at or above 1.
5. Thread 2 enters it's critical section.
6. Thread 1 sets `level[1] = 1;`
7. Thread 1 checks the condition at line `05`. It is the not victim.
8. Thread 1 enters it's critical section.

3. **Sorting (25 points)**

The question finishes showing that the compare-and-swap elements of sorting networks can be replaced by "merge-and-split" operations. Homework 4, question 2, also looked at this. This can be used to obtain practical parallel sorting algorithms for common parallel architectures.

Let M be a positive integer. We will consider merge-and-split operations on lists of length M. In particular,

```
merge_and_split({In0, In1}) -> {Out0, Out1}
```
where:

`In0` and `In1` are lists of M elements in ascending order.

`Out0` is the first M elements of `merge(In0, In1)`, sorted into ascending order.

`Out1` is the last M elements of `merge(In0, In1)`, sorted into ascending order.

For example:

```
merge_and_split({[1, 4, 9, 16, 25, 36], [3, 5, 7, 9, 11, 13]}) ->
   {[1, 3, 4, 5, 7, 9], [9, 11, 13, 16, 25, 36]}.
```

(a) (**5 points**) What is the result of

```
merge_and_split({[0, 0, 0, 0, 1, 1], [0, 0, 0, 1, 1, 1]})
```

?

Solution: `{[0, 0, 0, 0, 0, 0], [0, 1, 1, 1, 1, 1]}`.

(b) (**5 points**) Show that if `In0` and `In1` are lists of 0s and 1s in ascending order, and

```
{Out0, Out1} = merge_and_split({In0, In1})
```

then at least one of `Out0` or `Out1` must be clean (i.e. all 0s or all 1s).

Solution: There are a total of $2M$ elements in `In0` and `In1`. If at least M of these are 0s, then `Out0` is all 0s and therefore clean. Otherwise, at least $M + 1$ of the input elements are 1s, and `Out1` is all 1s and therefore clean.

(c) (**10 points**) Let S be a sorting network with N inputs and N outputs. Let S' be the network obtained by replacing every compare-and-swap in S with a merge-and-split operation. Now, let A_0, A_1, ..., A_{N-1} be any N sorted lists of 0s and 1s of length M. Apply these lists as the inputs to S', and let X_0, X_1, ..., X_{N-1} be the resulting outputs.

Show that there is an input to S', B_0, B_1, ..., B_{N-1} such that B is a permutation of X and when B is applied as the input to S', X is the output.

Solution: The main idea is that we can derive B by working backwards through the sorting network from X. We use the forward computation that started with A to see where the clean lists are in the network. If A produces a clean list at the output of some merge-and-split operation, we'll keep that list in our backwards pass for deriving B. We note that if A produces a dirty list at the output of some merge-and-split operation, we can change the number of 1s or 0s in that list, and still be able to find a valid input to that merge-and-split that produces the outputs that we are using in our backwards sweep. This lets us keep the lists as a permutation of the lists from X, and we never have to break a clean lists from X into the dirty lists of A that went into it. Quite a bit of partial credit will be given for making observations like those above.

The rest of my solution gives the details for the proof. I give lots more details than I require in a full-credit solution. I'm trying to write a solution that will be clear to someone who has already thought about the problem but may not have solved it.

We can describe a sorting network as a list of tuples. The list `[{I,J} | Tl]` describes a network that first does a merge-and-split of lists `I` and `J`, and applies the resulting lists to the sorting network described by `Tl`. In particular, Let

```
derive_b(A, []) -> A;
derive_b(A, S1 = [{I, J} | Tl]) ->
   {AI, AJ} = {nth(I, A), nth(J, A)},
   {RI, RJ} = merge_and_split({AI, AJ}),
   R = replace(I, RI, replace(J, RJ, A)),
   Y = derive_b(R, Tl),
```

```
{YI, YJ} = {nth(I, Y), nth(J, Y)}
{BI, BJ} = if
   clean(AI) and (YJ == AI) -> YJ, YI;
   clean(AJ) and (YI == AJ) -> YJ, YI;
   true -> YI, YJ
end,
B = replace(I, BI, replace(J, BJ, Y)).
```

Where `nth(I, List)` is the I^{th} element of `List`; `replace(I, X, List)` is the list obtained by replacing the I^{th} element of `List` with `E`; and `clean(List)` is true if every element of `List` is 0 or every element of `List` is 1. If `S` is a sorting network (using merge-and-split operations) and `LL` is a list of lists, then I'll write `S(LL)` to denote the list of lists obtained by applying sorting network `S` with input `LL`.

To understand the code, note that `{I,J}` are the indices of the lists for the next merge-and-split. Thus `AI` and `AJ` are the inputs to that merge-and-split, and `RI` and `RJ` are the outputs. The goal now is to derive `B` so that `S1(A) = S1(B)` and the lists of `B` are a permutation of the lists of `S1(A)`. The code does this recursively. It constructs `Y` so that `Tl(Y) = S1(A)` and the lists of `Y` are a permutation of the lists of `S1(A)`. Then, it derives `B` from `A` and `Y`.

Let `B = derive_b(A, S')`. I'll show that `B` has the properties described above. My proof is by induction on `S1`, and my induction hypothesis is

`S1(B) == S1(A)`,

and The lists of `B` are the same (but possibly permuted) as the lists of `S1(A)`,

and If the I^{th} list of `A` is clean, then the I^{th} list of `B` is the same as the I^{th} list of `A`.

The base case is when `S1 = []`, then `B = A = X`, and the claims are immediate. Otherwise, let `S1 = [{I,J} | Tl]`. From the induction hypothesis, we have that `Tl(R) = S1(A)`. There are two cases to consider:

`clean(AI) and clean(AJ)`: then `RI` and `RJ` are both clean. From the induction hypothesis, `YI=RI` and `YJ=RJ` are both clean. From the declarations for `BI` and `BJ`, `BI=AI` and `BJ=AJ`, therefore `B=A` and `S1(B) = S1(A)`, and all clean lists of `A` are equal to their counterparts in `B`. From the declarations for `BI` and `BJ`, `B` is a permutation of `Y`. From the induction hypothesis, `Y` is a permutation of `S1(A)`. Therefore, `B` is a permutation of `S1(S)`.

Otherwise: On of `AI` or `AJ` (or both) are dirty. From question 3b, one of `RI` or `RJ` is clean. If `AI` is clean and all 0s, then `RI` will be clean and all 0s (def. `merge_and_split`); `YI` will be clean and all 0s (induction hypothesis); and `BI` will be clean and all 0s (declaration of BI). Similar arguments apply in the other three cases where `AI` or `AJ` are clean. If `AI` and `AJ` are both dirty, then one of `RI` or `RJ` will be clean (question 3b); and the corresponding `YI` or `YJ` will be clean. The other one may be either clean or dirty. In this case `BI=YI` and `BJ=YJ`. By construction,

`{YI, YJ} = merge_and_split({BI, BJ})`

Therefore, `S1(B) = T1(Y) = S1(A)`. Furthermore, the lists of `B` are a permutation of the lists of `S1(A)` by the same argument as used in the previous case.

This completes the induction proof. Therefore, `B = derive_b(A, S')` satisfies the induction hypothesis, which means that it is a permutation of `X = S'(A)` and `S'(B) = X` as required.

(d) (**5 points**) The induction argument from question 3c shows that if the lists $B_1, \ldots, B_{N-1}$ are applied as the inputs to S', then for each `merge_and_split` operation in S', either `{Out0, Out1} = {In0, In1}`, or `{Out0, Out1} = {In1, In0}`. For $0 \leq i < N$, let C_i be the number of 1s in list B_i. Show that if S' sorts the B lists incorrectly, S must sort C incorrectly as well.

Solution: If `B` is applied as an input to `S'` then each merge-and-split operation has at least one clean input – otherwise, the number of 1s in the lists wouldn't be preserved by the merge-and-split operations. If one input to a merge-and-split is clean, then it must have a clean output, and the other output must match the other input. In other words, the merge and split either copies `{In0, In1}` to `{Out0,`

`Out1`} or it swaps them. In the problem statement, I told you you could assume this; so, a solution can get full credit without the argument of this paragraph.
The lists consist of 0s and 1s and are sorted into ascending order; so, there are only $M+1$ possible lists, depending on how many 1s are in the list. Thus, each input to a merge-and-split can be described using integers in $0, \ldots, M$, and the outputs of the merge-and-split are the same of the inputs if `In0` has fewer 1s than `In1` and swapped if `In0` has more 1s than `In1`. Thus, a compare-and-swap of the integers is equivalent to merge-and-split of the lists. This can be applied to the entire sorting network. Therefore, if the merge-and-split sorting network produces an unsorted result with the B list-of-lists, then the compare-and-swap network will produce an unsorted result with the C list-of-integers.

Hint: you can do this even if you didn't solve question 3c. In particular, you are allowed to assume what I said about the `merge_and_split` operations.

4. **Matrix Multiplication** (20 points)
Let's say you get a job writing code at a local company that needs to do some moderately intense number crunching. A typical problem is computing products of $N \times N$ matrices, where N ranges from 1000 to 20000.

(a) (**4 points**) Let's say you've got a laptop with a 2.5GHz, quad-core processor. From mini-assignment 4, we know that multiplying a $N_1 \times N_2$ by a $N_2 \times N_3$ matrix on a machine with a clock frequency of f takes time of roughly $3N_1N_2N_3/f$. What is the time to compute a matrix products for $N = 1000$, $N = 5000$, and $N = 20000$?

Solution: The time is $3N^3/f$. For $f = 2.5$ GHz, this is $(1.2 \text{ seconds})(N/1000)^3$.
$N = 1000$: $T_{seq} = 1.2$ seconds.
$N = 5000$: $T_{seq} = 1.2 * (5^3)$ seconds $= 1.2 * 125$ seconds $= 150$ seconds.
$N = 20000$: $T_{seq} = 1.2 * (20^3)$ seconds $= 1.2 * 8000$ seconds $= 9600$ seconds.

(b) (**4 points**) You try to convince your manager to buy a cluster of linux machines for your computation. Your proposal is for 32, 8-core processors running at 2.5GHz. The total number of processors is 256. For simplicity, we'll pretend that there isn't any multi-threading. Assume that the time to transfer a block m double-precision values between processors (over the network, it's a cluster) is:

$$T_{network} \quad = \quad 0.1\text{ms} + m * 50\text{ns}$$

where $1\text{ms} = 10^{-3}$seconds, and $1\text{ns} = 10^{-9}$seconds.
Using the algorithm we derived in class (Sept. 24), each processor performs P matrix products where it multiplies a $(N/P) \times (N/P)$ matrix by a $(N/P) \times N$ matrix. Furthermore, each processor sends (and receives) $P-1$ messages of size N^2/P double precision numbers. For simplicity, assume that the time for sending messages cannot be overlapped with computation time. What is the speed-up when computing matrix products for $N = 1000$, $N = 5000$, and $N = 20000$?

Solution: Each processor computes P multiplications of a $(N/P) \times (N/P)$ matrix by a $(N/P) \times N$ matrix. Each matrix multiplication takes $3N^3/(P^2 f)$ time. The total compute time for each processor is $P\frac{3N^3}{P^2 f} = 3N^3/(Pf)$ which is just T_{seq}/P. The communication time is $(P-1) * (0.1\,\text{ms} + 50(N^2/P)\,\text{ns})$. That gives us the following table:

N	T_{seq}	$T_{par,1}$	$T_{par,2}$	$T_{par,3}$	T_{par}	*speed_up*
1000	1.2s	4.6875ms	1.5ms	49.8ms	55.9875ms	21.4
5000	150s	586ms	1.5ms	1.245s	1.8326s	81.9
20000	9600s	37.5s	1.5ms	19.92s	57.42s	167.2

where $T_{par,1} = 3N^3/(Pf)$ is the compute time for the parallel version; $T_{par,2} = (P-1) * 0.1\,\text{ms}$ is the part of the communication time that is independent of matrix size; and

$$T_{par,3} \quad = \quad (P-1)50\tfrac{N^2}{P}\,\text{ns} \quad = \quad 50\tfrac{P-1}{P}\left(\tfrac{N}{1000}\right)^2\,\text{ms}$$

is the part of the communication time that depends on the matrix size. Of course, to save time, you didn't need to copy numbers from your solution to question 4a. I put them all into the table to make it easier to read.

Note: for all parts of this problem, I did most of the calculations in my head, and used a calculator for the multi-digit divisions. My answers may be slightly off by one or two in the least significant digits. That's indicative of the error-tolerance that I'll include in grading.

(c) (**4 points**) Your manager isn't convinced, but you really want to her to buy that linux cluster. You read up on algorithms for matrix multiplication and find an algorithm where each processor performs $\sqrt{P}$ matrix products where it multiplies a $(N/\sqrt{P}) \times (N/\sqrt{P})$ matrix by another $(N/\sqrt{P}) \times (N/\sqrt{P})$. Furthermore, each processors sends (and receive) $\sqrt{P}$ messages of size N^2/P double precision numbers. With the new algorithm, what is the speed-up when computing matrix products for $N = 1000$, $N = 5000$, and $N = 20000$?

Solution: The computation time stays the same. There are now $\sqrt{P}$ messages sent per processor instead of $P - 1$. I'll just scale the communication times ($T_{par,2} + T_{par,3}$) from the previous problem by $\sqrt{P}/(P-1) = 0.62745$. The new table is:

N	T_{seq}	$T_{par,1}$	$T_{par,2+3}$	T_{par}	$speed\_up$
1000	1.2s	4.6875ms	3.22ms	7.91ms	151.7
5000	150s	586ms	78.2ms	664.2ms	225.8
20000	9600s	37.5s	1.25s	38.75s	247.7

Reducing the number of messages makes a big difference!

(d) (**8 points**) A coworker installs an optimized BLAS (numerical library) which is faster than the simple code I showed in class. You find that the time to multiply a $N_1 \times N_2$ matrix by a $N_2 \times N_3$ matrix on a machine with a clock frequency of f now takes time of roughly $1.2N_1N_2N_3/f$. What is the sequential time (as in question 4a and parallel time (as in question 4c when the BLAS library is used?

i. What is the sequential time (as in question 4a and parallel time (as in question 4c when the BLAS library is used?

Solution: The sequential times get divided by $3/1.2 = 2.5$. For the parallel version, the compute time is divided by 2.5, but the communication time stays the same.

N	T_{seq}	$T_{par,1}$	$T_{par,2+3}$	T_{par}	$speed\_up$
1000	0.48s	1.875ms	3.22ms	4.1ms	117.5
5000	60s	234.4ms	78.2ms	312.6ms	191.9
20000	3840s	15s	1.25s	16.25s	236.3

ii. Write one sentence to summarize the impact of the faster sequential algorithm on the execution time.

Solution: Execution times dropped for both the sequential and parallel versions.

iii. Write one sentence to summarize the impact of the faster sequential algorithm on the speed-up.

Solution: Speed-ups decreased as well.

iv. Write one sentence to explain the relationship you observe between execution time and speed-up?

Solution: A faster sequential implementation results in lower speed-ups because the overheads (communication) become a larger percentage of the total execution time for the parallel program.

5. **Other Stuff (16 points)**

Answer each question below with 1-3 sentences. Points may be taken off for long answers.

(a) (**4 points**) What is instruction level parallelism?

Solution: Executing multiple instructions in parallel by identifying their dependencies and executing instructions when their dependencies are resolved. Super-scalar architectures are one way to exploit instruction level parallelism.

(b) (**4 points**) What is "single-instruction, multiple-thread" parallelism?

Solution: Fetching and decoding a single instruction stream and sending it to multiple execution pipelines. Each pipeline can resolve branch conditions independently. Instead of taking or not taking a branch, instructions are executed conditionally based on the branch outcome. This is called "predicated execution." GPUs are an example of an architecture that provides single-instruction multiple-thread parallelism.

(c) (**4 points**) How does map-reduce handle machine and network failures?

Solution: The map and reduce processes can be (re-)executed on a different machine if the original machine fails to respond (e.g. to a ping). The entire job can be re-executed with a different master process if the master fails to respond.

(d) (**4 points**) What is a memory fence?

Solution: An operation that forces pending memory operations to complete, thus ensuring that all cache coherence operations have finished as well.

Note, my answer for single-instruction, multiple-thread was five sentences. A three sentence answer is fine as long as it clearly gets two or three key points. The same applies for the other problems as well.

CpSc 418 **Midterm** March 1, 2017

Graded out of **86 points**

This exam has 10 pages (plus the cover sheet). Please check that you have a complete exam.

Answer question 0, and any **four** out of questions 1–5. If you write solutions for all questions, please indicate which you want to have graded; otherwise the graders will select which one to discard. Note that questions 1–3 are worth 20 points each and questions 4–5 are worth 24 points each. Depending on which questions you choose to answer, your maximum score will be either 86 or 90; however, no matter which questions you choose to answer the exam will be graded on a scale of 86 points.

0. (2 points)

   Sign below to confirm that you have read and understood the exam instructions.

   I, Mark Greenstreet, confirm that I have read and understood the exam instructions and that I only need to attempt **four** questions out of questions 1–5 below. If I attempt all questions, I understand that I should clearly indicate which ones to grade. If I attempt all questions and do not indicate which ones to grade, then I accept that the graders will choose at their own convenience which to grade.

1. **Erlang** (20 points)

   The final two pages of the exam contain Erlang definitions of the functions `last`, `lastv2` and `biggest_product`, along with associated functions that are used to define these functions. You may find it convenient to tear off the last sheet of the exam so you can refer to these functions while answering this question.

   For each of these functions, state its asymptotic worst-case run time (using big-O notation) and briefly justify your claim (you do not need to provide a formal proof or recurrence).

   For your run time analysis you may use the variable N to refer to the length of the list, the variable P to refer to the size of the worker pool `W`, and variable λ to refer to the ratio between the time taken to complete a global action (such as communicating a message between workers) and the time taken to complete a local action (such as a basic arithmetic operation). You may assume that the following operations are all unit time local actions:

   - `hd`, `tl`, pattern matching
   - `+`, `-`, `*`, `/`, `abs`, `div`, `max`, `rem`
   - `<`, `=<`, `=:=`, `==`, `/=`, `=/=`, `>=`, `>`
   - `and`, `andalso`, `or`, `orelse`, `not`.

   You may also assume that `lists:foldl(Fun, Acc0, List)` takes time linear in the length of `List` plus the time for the (linear number of) calls to `Fun`.

   The possible run-time complexities are: $\mathcal{O}(\log N)$, $\mathcal{O}(N)$, $\mathcal{O}(N \log N)$, $\mathcal{O}(N^2)$, $\mathcal{O}(N + P)$, $\mathcal{O}(N + \lambda)$, $\mathcal{O}(NP)$, $\mathcal{O}(\lambda N)$, $\mathcal{O}((N/P) + \log P)$, $\mathcal{O}((N/P) + \lambda \log P)$, $\mathcal{O}((N/P) + \lambda)$, or $\mathcal{O}((N/P) + \lambda P)$. Not all of these complexities occur in this question.

   (a) (5 points) `last`: return the last element of a list. Asymptotic complexity and brief justification:

   $\mathcal{O}(N)$. Each recursive call to `last` takes $\mathcal{O}(1)$ time. There are N such recursive calls.
   Note: from the review problems.

   (b) (5 points) `lastv2`: another implementation of `last`. Asymptotic complexity and brief justification:

   $\mathcal{O}(N^2)$. Each recursive call to `lastv2` takes $\mathcal{O}($`length(List)`$)$ to check the "`when length(List) > 1`" part of the guard. This is done for `length(List)` $= N, N-1,$ `...,` 1. The total time is

   $$\sum_{i=1}^{N} i \;=\; \frac{N^2+N}{2} \;\in\; \mathcal{O}(N^2)$$

   Note: from the review problems.

(c) (10 points) `greatest_product`: return the greatest product of pairs of adjacent elements in a list (parallel implementation). Asymptotic complexity and brief justification:

$O\left(\frac{N}{P} + \lambda \log P\right)$.
Each worker calls `bp_leaf` with a list of $\frac{N}{P}$ elements. `bp_leaf` does $\mathcal{O}(1)$ work plus the work of `gp(List)`. `gp(List)` calls a tail recursive version of itself. The tail-recursive `gp(List, Acc)` does $\mathcal{O}(1)$ work per recursive call – note that `my_max` takes $\mathcal{O}(1)$ time. Thus `gp(List, Acc)` and therefore `gp(List)` and `bp_leaf(List)` take $\mathcal{O}(\texttt{length(List)}) = \mathcal{O}(N/P)$ time.
Each level of the combine tree involves one send-and-receive pair at a cost of λ and $\mathcal{O}(1)$ sequential computation. There are $\lceil \log_2 P \rceil$ levels to the tree. The total cost for the combine is $\mathcal{O}(\lambda \log P)$.
`pb_root` extracts the second element from a three element tuple. This takes $\mathcal{O}(1)$ time.
Combining the results for `bp_leaf`, `bp_combine`, and `bp_root` yields the total time of $O\left(\frac{N}{P} + \lambda \log P\right)$.
Note: We've covered this in class many times. Just writing the answer with a reference to the lecture slides is a sufficient justification. For example, "See Jan. 13 lecture slides, slide 3.".

2. `largest_gap(W, Key)` (20 points)

`largest_gap` should return the largest gap between adjacent numbers of the list of numbers stored on the worker processes of `W` and associated with `Key`. Here is the sequential version:

```
largest_gap([]) -> undef;
largest_gap([H | Tail]) ->
   {Gap, _} = lists:foldl(fun(X, {G, PrevX}) -> {my_max(abs(X-PrevX), G), X} end,
                          {undef, H}, Tail),
   Gap.
```

where `my_max` is the function defined in question 1. For example,

```
largest_gap([]) -> undef.
largest_gap([1,4,9,16,20]) -> 7.   % |16 − 9| = 7
largest_gap([1,4,9,16,2]) -> 14.   % |2 − 16| = 14
```

The parallel version of `largest_gap` is a typical reduce pattern, and you will write the functions `lg_leaf`, `lg_combine` and `lg_root` in the following implementation:

```
largest_gap(W, Key) ->
   wtree:reduce(W,
      fun(ProcState) -> lg_leaf(wtree:get(ProcState, Key)) end,
      fun(Left, Right) -> lg_combine(Left, Right) end,
      fun(Top) -> lg_root(Top) end
   ).
```

Assuming that there are N elements in the list spread evenly across P workers in the worker pool `W` (where $P \ll N$), your implementation should achieve a speedup close to P. Your implementation should fail if there is no value associated with `Key` or if the value associated with `Key` is not a list of numbers.

You may call any of the functions defined in question 1, the sequential `largest_gap(List)` function defined above, any Erlang built-in functions, and any functions from the `lists`, `misc`, `workers`, or `wtree` modules.
Hint: Some functions from question 1 may prove inspirational even if they cannot be called directly.

(a) (8 points) Your code for `lg_combine(Left, Right)`:

```
% All three parts of this problem can be solved by copying the code
% for biggest_product from Q1 with a few small changes.
lg_combine(empty, Right) -> Right; % adapted from bp_combine
lg_combine(Left, Empy) -> Left;
lg_combine({LL, LBig, LR}, {RL, RBig, RR}) ->
   {LL, my_max([LBig, abs(LR-RL), RBig]), RR}.   % my_max from Q1.
```

(b) (8 points) Your code for `lg_leaf(Data)`:

```
% adapted from bp_leaf.
% largest_gap(List) is from the problem statement.
lg_leaf([]) -> empty;
lg_leaf(List) -> {hd(List), largest_gap(List), tl(List)}.
```

(c) (4 points) Your code for `lg_root(Top)`:

```
lg_root(V) -> bp_root(V).
```

3. **Message Passing Network Topologies** (20 points)

We are building a message passing machine with 64 nodes, and we need to decide how to connect its network. For each of the following topologies, specify the bisection bandwidth, diameter and maximum number of network ports needed by a node.

(a) (3 points) Ring.

bisection bandwidth: 2

diameter: 32

network ports: 2

(b) (3 points) 2D mesh.

bisection bandwidth: 8
Note: $\sqrt{P}$ assuming a simple mesh, not toroidal because the problem didn't use the word "toroidal". Draw a line that separates the left and right halves of the mesh.

diameter: 14
Note: 7 hops in each dimension.

network ports: 4
Note: connections to north, south, east, and west neighbours.

(c) (3 points) 3D mesh.

bisection bandwidth: 16
Note: $\sqrt[3]{P}$ assuming a simple mesh, not toroidal because the problem didn't use the word "toroidal". Draw a plan that separates the top half of the mesh from the bottom half.

diameter: 9
Note: 3 hops in each dimension.

network ports: 6
Note: same connections as above, plus up and down.

(d) (3 points) 6D hypercube.

bisection bandwidth: 32

diameter: 6
Note: example, node 0 to node 63.

network ports: 6
Note: one for each bit of the node index.

(e) (4 points) State **two** advantages of the 6D hypercube over the 2D mesh? Answers which give more than two will receive zero.

- Small diameter → low routing latency.
- High bisection → fewer communication bottlenecks.
- Note: for the solution set, I'll include another response that gets full credit:
  - No special cases at edges.

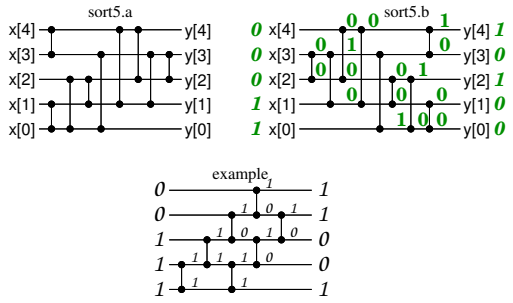


Figure 1: Sorting Networks

(f) (4 points) State **two** advantages of the 2D mesh over the 6D hypercube? Answers which give more than two will receive zero.

- Easily mapped to 2D technologies such as chips or circuit boards.
- Bounded degree nodes make the design more modular.
- Note: for the solution set, I'll include another response that gets full credit:
  - Doesn't become "all wire" for networks with a large number of processors.

4. **Sorting Networks** (24 points)

Figure 1 shows two sorting networks with five inputs: sort5.a, and sort5.b. One of these networks sorts correctly and one does not.

(a) (10 points) Identify which of the networks **does not** sort correctly. Label the inputs with values of **0** or **1** that the network does not sort correctly. Label the output of each compare-and-swap with its value for this input, and label the outputs of the sorting network with their values. The "example" network provides an example of such a labeling (and also an incorrect attempt at sorting).

Which network do you claim is incorrect (circle one): sort5.a (sort5.b)

Note: The counter-example has 1s for `x[0]` and `x[1]` and 0s for `x[2]`, `x[3]` and `x[4]`. The compare-and-swap (1,3) moves the 1 for `x[1]` into row 3. Note that the only path for the 1 for `x[0]` to rows 3 or 4 is through the compare-and-swap (0,3). However, it is blocked by the 1 from `x[1]` that has already moved to row 3. This shows that the network cannot sort this input correctly.

Remember to label that network with a counter-example in the figure above.

(b) (14 points) For the other network, explain specifically how it sorts any combination of three **0**s and two **1**s correctly.

We observe that the three compare-and-swap modules in the bottom left, i.e. (0,1), (0,2), and (1,2) sort rows 0, 1, and 2 into ascending order. Likewise, the compare-and-swap module in the top-left, i.e. (3,4), sorts rows 3 and 4 into ascending order. Call these four compare-and-swap operations "the initial compare and swaps"
Now, consider the number of 1s in $\{$`x[3]`, `x[4]`$\}$.

zero 1s: In this case, the values of rows 0 to 4 after the initial compare and swaps must be (0,1,1,0,0). The 1 in row 2 moves to row 4 with the compare-and-swap (2,4), and the 1 in row 1 moves to row 3 with the compare-and-swap (1, 3). Thus, the two ones make it to the top two rows.

one 1: In this case, the values of rows 0 to 4 after the initial compare and swaps must be (0,0,1,0,1). The 1 in row 2 moves to row 3 with the final compare-and-swap (2,3).

two 1s: In this case, the values of rows 0 to 4 after the initial compare and swaps must be (0,0,0,1,1). The data are already sorted, and all of the compare-and-swaps preserve this order.

Compare-and-swap operations preserve the total number of 1s in the network, and the total number of 0s. Showing that the two 1s arrive at `y[3]` and `y[4]` implies that `y[0]` = `y[1]` = `y[2]` = 0.

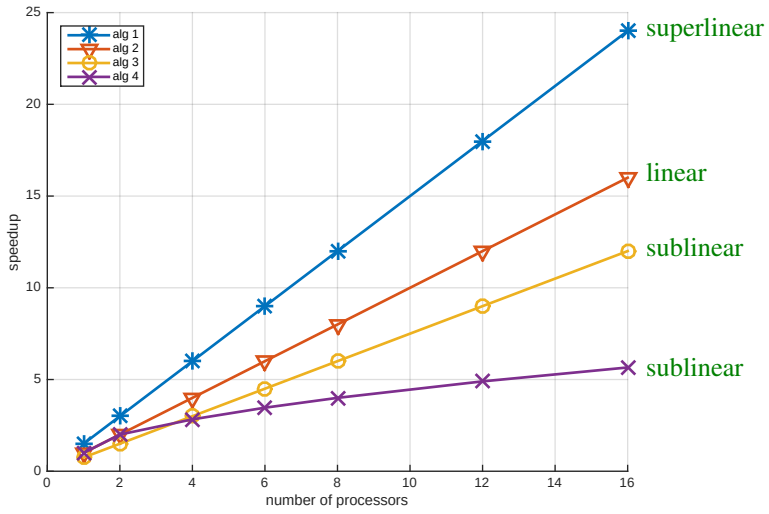


Figure 2: Speedup for some parallel algorithms.

5. **Short Answer** (24 points)

(a) (7 points) Figure 2 shows the speedup measured for four algorithms: alg 1, alg 2, alg 3 and alg 4. In the space to the right of the plot, label each curve as either *sublinear*, *linear* or *superlinear* speedup. Not all speedups may be present in the plot.

Give **two** general reasons why parallel programs sometimes display sublinear speedup *other than computational or communication overhead*. Answers which give more than two reasons will receive zero.

- Non-parallelizable code.
- Extra computation.
- Note: for the solution set, I'll include other responses that get full credit:
  - memory overhead
  - synchronization overhead
  - resource contention
  - idle processors

Give **one** general reason why parallel programs sometimes display superlinear speedup. Answers which give more than one reason will receive zero.

- more fast memory (in total)
- Note: the "fast memory" answer is the right explanation 99% of the time. occasionally, a problem is so naturally parallel that the sequential algorithm has control overhead to serialize the operations that the parallel code avoids. A common explanation is "invalid comparison" because there's something wrong with the way the times were measured, but I don't think that's an intended answer.

(b) (6 points) For sequential computing there is the successful and near ubiquitous Von Neumann architecture; in the early days of computing there were other sequential architectures, but they have essentially disappeared from use. In contrast, we have discussed three very different approaches to achieving parallel computation that are still widely used, often in combination with one another: superscalar, shared memory and message passing. For **each** of the three, give **one** reason why it has **not** become dominant over the others; in other words, one significant disadvantage. Note that your reasons need not be unique. Answers which give more than one reason for any approach will have marks deducted.

superscalar: Limited instruction level parallelism.

shared memory: Poor scaling of coherence models to large numbers of processors.

message passing: Network delays.

(c) (6 points) You should remember from your algorithms course that sequential merge sort requires $\mathcal{O}(N \log N)$ time and $\mathcal{O}(N \log N)$ comparisons for N elements. We saw that under reasonable assumptions a bitonic sorting network requires $\mathcal{O}((\frac{N}{P})(\log N + \log^2 P))$ time and $\mathcal{O}(N \log N \log^2 P)$ comparisons for N elements with P processors. Assume now that we have M (where $M \gg P$) independent sorting problems each of size N. In this question we consider two methods of solving these M problems: either connecting the P processors together into a single bitonic sorting network and running the problems through the network one at a time, or running up to P independent sequential merge sorts at a time. We will assume in each case that the data is already distributed in a manner which is optimal for that case. Is the *latency* of a bitonic sorting network better, worse or about the same as independent sequential merge sorts? Is the *throughput* of a bitonic sorting network better, worse or about the same as independent sequential merge sorts? Briefly explain your answers.

Bitonic latency (circle one and briefly explain): (BETTER) WORSE SAME

The latency for parallel bitonic sort is $\mathcal{O}(\frac{N}{P} \log N \log^2 P)$ which is less than the latency for sequential merge sort, $\mathcal{O}(N \log N)$ when $N \gg P$.

Note: I found this one a little confusing. The problem statement says

> "Assume now that we have M (where $M \gg P$) independent sorting problems each of size N."

Is latency the time to solve one of these problems are all M of them? I can suggest rephrasings of the problem that would make this unambiguous, but it's a bit late for that.

Bitonic throughput (circle one and briefly explain): BETTER (WORSE) SAME

With $M \gg P$, we can keep P processors busy working on independent merge sort problems. Thus, the bottleneck is the execution time of the processors. Sequential merge sort requires fewer total processor cycles than bitonic sort *and* it avoids the communication costs of the parallel algorithm. Thus, sequential merge sort will achieve higher throughput.

(d) (5 points) We developed a tree-based reduce operation in Erlang—a language which assumes a message passing architecture, although it can be run on either message passing or shared memory hardware—to speed up parallel computations that produce a small amount of summary information at a single process from a large amount of data spread across many processes. If we were to use a language and hardware which supported shared memory, is a tree-based reduce still useful to speed up parallel computation? Briefly explain your answer. (If it is relevant to your answer, you may assume an idealized memory where access to any item costs the same amount for any process.)

The tree based algorithm is still faster. We need some kind of synchronization between the threads, and that comes at a cost of λ per synchronization. A tree has a cost of $\lambda \log P$ for synchronization. If a single master processor synchronized with each worker, the synchronization cost would be λP.

Note: I'm a bit concerned about "idealized memory". If I have atomic reads and writes that take $\mathcal{O}(1)$ time, then I can implement a mutual-exclusion algorithm (such as Peterson's) that provides lock acquistion or transfer in $\mathcal{O}(1)$ time. This removes the λs from the analysis. In this case, a

tree-algorithm takes time $\mathcal{O}(\log P)$ and a singel master process takes tiem $\mathcal{O}(P)$. Again, the tree is faster, but it is likely that P is much less than the time for the worker processes to do their tasks before meeting at a reduce.

Functions for Q1a and Q1b

Do not write your answer on this page—it will not be graded. You may find it convenient to tear this page off when answering questions 1 and 2. If you tear it off, you need not submit it with the rest of your exam.

```
%%%%%%%%%   Q1.a
% last(List) -> LastElement.
%   Return the last element of List. If List is not
%   a list or if List is empty, then an error will be thrown.
%   Example:
%     last([1, 4, 9, 16]) -> 16.
%     For run-time analysis, N = length(List).
last([E]) -> E;
last([_ | T]) -> last(T).
```

```
%%%%%%%%%   Q1.b
% lastv2(List) -> LastElement.
%   Another implementation of last(List).
%     For run-time analysis, N = length(List).
lastv2(List) when length(List) > 1 -> lastv2(tl(List));
lastv2([X]) -> X.
```

Repeated from the question 1 problem statement for your convenience:

For your run time analysis you may use the variable N to refer to the length of the list, the variable P to refer to the size of the worker pool `W`, and variable λ to refer to the ratio between the time taken to complete a global action (such as communicating a message between workers) and the time taken to complete a local action (such as a basic arithmetic operation). You may assume that the following operations are all unit time local actions:

- `hd`, `tl`, pattern matching
- `+`, `-`, `*`, `/`, `abs`, `div`, `max`, `rem`
- `<`, `=<`, `=:=`, `==`, `/=`, `=/=`, `>=`, `>`
- `and`, `andalso`, `or`, `orelse`, `not`.

You may also assume that `lists:foldl(Fun, Acc0, List)` takes time linear in the length of `List` plus the time for the (linear number of) calls to `Fun`.

The possible run-time complexities are: $O(\log N)$, $O(N)$, $O(N \log N)$, $O(N^2)$, $O(N+P)$, $O(N+\lambda)$, $O(NP)$, $O(\lambda N)$, $O((N/P) + \log P)$, $O((N/P) + \lambda \log P)$, $O((N/P) + \lambda)$, or $O((N/P) + \lambda P)$. Not all of these complexities occur in this question.

Function for Q1c

Do not write your answer on this page—it will not be graded. You may find it convenient to tear this page off when answering questions 1 and 2. If you tear it off, you need not submit it with the rest of your exam.

```
%%%%%%%%%    Q1.d
% greatest_product(W, Key) -> Number.
%   W is a worker-tree. Key is the key for a list of numbers distributed
%   across the workers We return the largest product of adjacent numbers.
%   If the list is empty or a singleton, then greatest_product returns 'undef'.
%   If there is no value associated with Key, or if the value associated with
%   Key is not list of numbers , then greatest_product fails.
%   Example:
%     If Key corresponds to the list [1, 2, 7, 3, 5, 4, 2, -8, 6, 3],
%     Then the products of adjacent numbers are
%       [2, 14, 21, 15, 20, 8, -16, -48, 18],
%     and the largest product of adjacent numbers is 21.
%   For run-time analysis: let N denote the total length of the list
%   corresponding ot Key; let P denote the number of processes.
%   Assume that the list is divided into equal sized segments.
%   Assume that there are enough processors so that all processes
%   can run at the same time.
greatest_product(W, Key) ->
   wtree:reduce(W,
      fun(ProcState) -> bp_leaf(wtree:get(ProcState, Key)) end,
      fun(Left, Right) -> bp_combine(Left, Right) end,
      fun(Top) -> bp_root(Top) end
   ).

bp_leaf([]) -> empty;
bp_leaf(List) -> {hd(List), gp(List), last(List)}.

bp_combine(empty, Right) -> Right;
bp_combine(Left, empty) -> Left;
bp_combine({LL, LBig, LR}, {RL, RBig, RR}) ->
   {LL, my_max([LBig, LR*RL, RBig]), RR}.

bp_root({_, Big, _}) when is_number(Big) -> Big;
bp_root(_) -> undef.

% gp(List) -> Number.
%   a sequential version of greatest_product.
gp(List) -> gp(List, undef).
gp([], BigProd) -> BigProd;
gp([_], BigProd) -> BigProd;
gp([A | Tail = [B | _]], BigProd) ->
   gp(Tail, my_max(A*B, BigProd)).

% my_max: calculate max where 'undef' is less thn any number
my_max(undef, Y) -> Y;
my_max(X, undef) -> X;
my_max(X, Y) -> max(X, Y).

% my_max for lists.
my_max(List) -> lists:foldl(fun(X, Acc) -> my_max(X, Acc) end, undef, List).
```

1. The zero-one principle (**20 points** + **5 points Extra Credit**)
   Let $X[0..N-1]$ be a *bitonic* array of N elements, where N is even, and every element of X is either 0 or 1.

   (a) (**5 points**) Explain what it means that an array is bitonic. Use at most three sentences.

   ## Solution

   Any of the solutions below (or similar) are acceptable:

   - A bitonic sequence is either a monotonically increasing sequence followed by a decreasing sequence or the other way around.
   - (Because I said X is all 0s and 1s,) A bitonic sequence is of the form $0^*1^*0^*$ or $1^*0^*1^*$.
   - If a solution just says "increasing followed by decreasing" or " a sequence of 0s followed by a sequence of 1s followed by a sequence of 0s", that will get full credit, even though the more general definition is needed for the proof in question 1c.

   (b) (**10 points**) Let Y be an array with N elements such that:

$$\begin{aligned} Y[i] &= \min(X[i], X[i+(N/2)]), && \text{if } 0 \le i < N/2; \\ &= \max(X[i-(N/2)], X[i]), && \text{if } N/2 \le i < N. \end{aligned}$$

   Show that either

   - $Y[i] = 0$ for $0 \le i < N/2$, or
   - $Y[i] = 1$ for $N/2 \le i < N$.

   Both may hold for particular choices of X, but you just need to show that at least one of these conditions holds.

   ## Solution

   - If X is all 0s, then Y is all 0s, and $Y[i] = 0$ for $0 \le i < N/2$ which satisfies the claim.
   - Otherwise, let i_{lo} and i_{hi} be defined as suggested in the hint for the problem. Note, that this assumes that X is increasing-then-decreasing bitonic. The argument if X is decreasing-then-increasing is equivalent. Consider the three cases below:
     - case $N/2 \le i_{\text{lo}}$. In this case, $X[i] = 0$ for $0 \le i < N/2 \le i_{\text{lo}}$; $Y[i] = 0$ for $0 \le i < N/2 \le i_{\text{lo}}$; and the claim is satisfied.
     - case $i_{\text{hi}} < N/2$, this is analogous to the previous case. In particular, $X[i] = 1$ for $i_{\text{hi}} < N/2 \le i < N \le i_{\text{lo}}$; $Y[i] = 1$ for $i_{\text{hi}} < N/2 \le i < N \le i_{\text{lo}}$; and the claim is satisfied.
     - case $i_{\text{lo}} < N/2 \le i_{\text{hi}}$. In this case, $Y[0..(N/2)-1]$ is a sequence of 0s followed by a sequence of 1s, and $Y[(N/2)..(N-1)]$ is a sequence of 1s followed by a sequence of 0s. This produces two cases depending on how their "breaks" line up:
       * If $i_{\text{hi}} \le i_{\text{lo}} + (N/2)$, then for all $0 \le i < (N/2)$ either $X[i] = 0$ (i.e. for $0 \le i < i_{\text{lo}}$) or $X[i+(N/2)] = 0$ (i.e. for $i_{\text{hi}} < i+(N/2) < N$). Therefore $Y[i] = 0$ for $0 \le i < N/2$, and the claim is satisfied.
       * Otherwise $i_{\text{lo}} + (N/2) < N$, and a similar argument shows that $Y[i] = 1$ for $N/2 \le i < N$. In a bit more detail, for all $N/2 \le i < N$, either either $X[i] = 1$ (i.e. for $N/2 \le i \le i_{\text{hi}}$) or $X[i-(N/2)] = 1$ (i.e. for $i_{\text{lo}} \le i-(N/2) < N/2$). Therefore $Y[i] = 1$ for $N/2 \le i < N$, and the claim is satisfied.

   (c) (**5 points**) Let Y be defined as in question 1b. Show that $Y[0..(N/2)-1]$ is bitonic and that $Y[(N/2)..(N-1)]$ is bitonic.

   ## Solution

- If $(N/2) \leq i_{\text{lo}}$ then $X[0..(N/2)-1]$ is all 0s; $Y[0..(N/2)-1]$ is all 0s; and the claim holds.
- If $i_{\text{hi}} < (N/2)$ then $X[(N/2)..(N-1)]$ is all 0s; $Y[0..(N/2)-1]$ is all 0s; and the claim holds.
- The final case is $0 \leq i_{\text{lo}} < (N/2) \leq i_{\text{hi}} < N$. If $i_{\text{lo}} + (N/2) \leq i_{\text{hi}}$, then the 1s in the lower half of X overlap the 1s in the upper half of X, and $Y[0..(N/2)-1]$ is $0^*1^*0^*$ bitonic. Otherwise, $i_{\text{lo}} + (N/2) > i_{\text{hi}}$; the 0s in the lower half of X overlap the 0s in the upper half of X, $Y[0..(N/2)-1]$ is all 0s (and the top half of Y is $1^*0^*1^*$ bitonic).

(d) (**5 points, Extra Credit**) Let A be a $N \times M$ array, where N is even, and $A[i, j]$ denotes the element of A in row i and column j. Let $A[i, 0..M-1]$ denote a row of A. Assume that every element of A is either 0 or 1, and that for $0 \leq i < N$, row $A[i, 0..M-1]$ is sorted into ascending order of i is even and into descending order if i is odd.

Let B be the $N \times M$ array where for $0 \leq j < M$, column $B[0..N-1, j]$ is sort$(A[0..N-1, j])$, where sort sorts the elements of the column into ascending order.
Prove that at least $N/2$ rows of B are either all 0s are all 1s. For example, if $N = 16$, and B has 6 rows that are all 0s and three rows that are all 1s, then the claim is satisfied.

Solution

Note that we could sort the columns of A by first doing a compare and swap of $A[2i, j]$ and $A[2i+1, j]$ for $0 \leq i < (N/2)$, and then sorting the result. Doing this for $0 \leq j < M$ is the same as doing a compare-and-swap of rows $2i$ and $2i+1$. From questions 1b and 1c this produces (at least) one row that is all 0s or all 1s. The sort will take that row to the bottom of B if the row is all 0s or to the top of B if it is all 1s. Thus, each pair of rows of A produces at least one row that is all 0s or all 1s in B. Because A has N rows, B must have at least $N/2$ rows that are all 0s or all 1s.

2. CUDA (**20 points**) Let's say that I have an array of

$$n = 2^{27} = 2^{17} \cdot 2^{10} = 131,072 \cdot 1,024 = 134,217,728$$

`float`s, and I want to compute their sum using my blazing-fast GPU.

(a) (**10 points**) I've written three approaches to compute the sum below. Each is either incorrect or inefficient. Identify the problem with each, giving a short (at most three sentences) explanation for each one describing why it is wrong or why it is slow.

i. Create one block with 134,217,728 threads executing the kernel:

```
0  __global__ void sum1(float *x, int n) {
1     // assume n is a power of 2
2     int myId = blockDim.x * blockIdx.x + threadIdx.x;
3     if(myId < n) {
4        for(int stride = 2; stride < n; stride *= 2) {
5           __syncthreads();
6           if((myId % stride) == 0)
7              x[myId] = x[myId] + x[myId + stride]
8        }
9     }
10    // the result is in x[0]
11 }
```

The kernel launch is:

```
sum1<<<1, 134217728>>>(x, 134217728)
```

ii. The same kernel as for version [i], but this time the kernel launch is:

```
sum1<<<131072, 1024>>>(x, 134217728)
```

iii. Create one block with 1024 threads executing the kernel where each thread computes the sum of `n/1024` elements before the threads perform a reduce:

```
0    __global__ void sum3(float *x, int n, int m) {
1       // assume n and m are powers of 2
2       int myId = blockDim.x * blockIdx.x + threadIdx.x;
3       if(myId < n) {
4          // compute the sum for my part of the array
5          float sum = 0.0;
6          for(int i = 0; i < m; i++)
7             sum += x[myId + i];

8          // reduce
9          for(int stride = 2; stride < blockDim.x; stride *= 2) {
10            __syncthreads();
11            if((myId % stride) == 0)
12               x[myId] = x[myId] + x[myId + stride]
13         }
14      }
15      // the result is in x[0]
16   }
```

The kernel launch is:

```
sum3<<<1, 1024>>>(x, 134217728, 131072)
```

Solution

- Version (i) is incorrect because a block can have at most 1024 threads.
- Version (ii) is incorrect `__syncthreads()` only acts as a barrier for threads in the same block. It does not provide synchronization across blocks.
- Version (iii) is incorrect because
  - the threads compute their initial sums over overlapping parts of the array and miss most of the array.
  - the initial sum is not copied back to memory; so, it's not available for the reduce.

  Note, these two errors were unintentional – oops. But, the problem lets you point out any error or efficiency issue; so, this just made the problem a little bit easier. No bug-bounty for these.

  Version (iii) is also inefficient for many reasons (any one is fine):
  - Thread divergence. In the reduce loop, half of the threads in a warp are active in the first iteration, one fourth in the second, and so on.
  - Global memory accesses in the reduce loop. The global accesses in the initial sums are unavoidable, but we could use local memory here.
  - Lots of synchronization: we could use "warp synchronous" execution once all of the active threads are part of the same warp.

(b) (**10 points**) There are many ways to improve the performance of the slow kernel. Pick one such as coalescing global memory references, or using shared memory, or avoiding bank conflicts, or reducing thread divergence, or ....

- (**2 points**) State what optimization you are performing.
- (**4 points**) Write a short (no more than three sentence) description of what the optimization does.

- (**2 points**) Write the revised kernel. You can just write the lines that you change compared with the kernels I wrote above, and write
  lines 05-08 of sum2
  or similar to refer to lines from the kernels above.

Solution

I'll fix each of the problems listed above, but a real solution only needs to address one.

Fix sum3:

Change line 7 to:

```
sum += x[m*myId + i];
```

Just before the reduce (i.e. between lines 7 and 8) add

```
x[myId] = sum;
```

But now it will be slow due to uncoalesced memory references.

Global Memory Coalesscing:

Change line 6 to:

```
int stride = gridDim.x * blockDim.x;
for(int i = myId; i < n; i += stride) {
```

Global memory accesses in the reduce:

Declare a shared array, for example between lines 1 and 2 add

```
__shared__ float y[1024];
```

Then, change the line I added between lines 7 and 8 to:

```
y[myId] = sum;
```

and change line 12 to

```
y[myId] = y[myId] + y[myId+stride];
```

For an even faster solution, each thread would continue to use `sum` until the last iteration before it becomes idle. It would copy `sum` to `y[myId]` in that last iteration.

Reducing thread divergence:

As described in lecture and the text, start stride at `blockDim.x/2` and divide it by two until it is 0. This changes line 9-12 to:

```
 9       for(int stride = blockDim.x/2; stride > 0; stride /= 2) {
10          __syncthreads();
11          if((myId < stride) == 0)
12             y[myId] = y[myId] + y[myId + stride]
```

Using warp-synchronous execution:

Change the revised line 9 to:

```
for(int stride = blockDim.x/2; stride >= warpsize; stride /= 2) {
```

Then add another copy of the loop where `stride` goes from `warpsize/2` down to 0. Omit the `__syncthreads()` in this second loop.

(c) (**10 points**) Give two reasons that a GPU needs thousands of active threads to fully utilize the processors. For each, you can give its name (e.g. the nVidia™ jargon word or phrase) and a short description (at most four or five sentences for each of your two reasons).

Solution

Here are more than two, but a real solution should just have two.

- **Mitigating data hazards:** Using lots of threads allows the SPs to have deep pipelines (easier to implement, lower power, higher clock speeds) without the complications of bypasses.

- **Mitigating control hazards:** Even though a branch instruction takes 10s of cycles (or more) on a GPU, the SM can fetch from other warps until the branch is resolved.
- **Hiding global memory latency:** Accesses to global memory are slow. Other warps can execute while a warp is waiting for a load to finish.
- **Warps have lots of threads:** This amortizes the hardware, and especially the power consumption for instruction fetch, decode, and other pipeline control issues across many SPs.
- **A GPU has lots of SMs:** This is how it gets lots of parallelism. But many SMs times many warps per SM times many threads per warp results in needing thousands of threads to get high performance from GPU.

3. GPUs (**20 points**)
Earlier this month, nVidia™ announced the GP100 GPU. It has 3584 single-precision floating point cores with a peak 10.6 teraflops of single-precision floating point computation (1 teraflop = 1000 gigaflops = 10^{12} floating point operations per second). The interface to off-chip, global memory has a bandwidth of 720 GBytes/sec.

(a) (**10 points**) How many floating point operations must the GP100 perform on each `float` loaded from global memory if it is to achieve its peak floating point operation rate (and not by limited by memory bandwidth)?

Solution

A `float` is 4 bytes. Thus, the GP100 can load

$$\begin{aligned} & 720 \frac{\text{Gbytes}}{\text{sec}} \cdot \frac{1\text{float}}{4\text{bytes}} \\ = \; & 180 \frac{\text{Gfloats}}{\text{sec}} \end{aligned}$$

As stated in the problem, The GP100 can perform 10.6 teraflops. The number of floating point operations we need to do per global memory access to allow for peak floating-point rate is:

$$\begin{aligned} & 10.6 \cdot 10^{12} \frac{\text{float}-\text{ops}}{\text{sec}} \cdot \left(180 \cdot 10^{9} \frac{\text{floats}-\text{from}-\text{gmem}}{\text{sec}} \right)^{-1} \\ = \; & 58.\overline{8} \frac{\text{float}-\text{ops}}{\text{float}-\text{from}-\text{gmem}} \end{aligned}$$

This is more than twice the CGMA needed to fully utilize the SPs on the GTX 550 Ti on the `linXX` boxes that we used.

(b) (**10 points**) Give two reasons that a GPU needs thousands of active threads to fully utilize the processors. For each, you can give its name (e.g. the nVidia™ jargon word or phrase) and a short description (at most four or five sentences for each of your two reasons).

Solution

Here are more than two, but a real solution should just have two.

- **Mitigating data hazards:** Using lots of threads allows the SPs to have deep pipelines (easier to implement, lower power, higher clock speeds) without the complications of bypasses.
- **Mitigating control hazards:** Even though a branch instruction takes 10s of cycles (or more) on a GPU, the SM can fetch from other warps until the branch is resolved.
- **Hiding global memory latency:** Accesses to global memory are slow. Other warps can execute while a warp is waiting for a load to finish.
- **Warps have lots of threads:** This amortizes the hardware, and especially the power consumption for instruction fetch, decode, and other pipeline control issues across many SPs.
- **A GPU has lots of SMs:** This is how it gets lots of parallelism. But many SMs times many warps per SM times many threads per warp results in needing thousands of threads to get high performance from GPU.

4. **Speed-up** Let's say that I spend all day running four programs in sequence on my x86 linux machine:

   - Program-A runs for 1 minute on the x86.
   - Program-B runs for 1 minute on the x86.
   - Program-C runs for 1 minute on the x86.
   - Program-D runs for 1 minute on the x86.

   Each program depends on the results of the one that came before it; so, I can't run them in parallel ☹. After Program-D finishes, I go back and start the sequence with Program-A again.

   (a) (**5 points**) Assuming that I work 8 hours per day and get paid $1 each time I complete all four tasks, how much do I earn per day?

   ## Solution

   Completing all four tasks takes 4 minutes. That means 15 tasks in an hour, and 120 tasks in an eight-hour workday. I get paid $120/day.

   (b) (**5 points**) An nVidia™ sales person drops by my office and tells me that with a new, GP100 GPU, I can get the following speedups:

   - Program-A: speed-up = 100.0;
   - Program-B: speed-up = 10.0;
   - Program-C: speed-up = 1.0;
   - Program-D: speed-up = 0.1.

   If I sell my linux machine and move all four of my computing tasks to the GPU, long will it take me to complete all four tasks? What is the overall speed-up?

   ## Solution

   The total time for all four tasks is

   $$\begin{array}{rl} & \left(\frac{1}{100} + \frac{1}{10} + \frac{1}{1} + \frac{1}{0.1}\right) \text{ minutes} \\ = & 11.11 \text{ minutes} \end{array}$$

   Running on the x86, the time for all four is 4 minutes. The speedup is

   $$\begin{array}{rcl} SpeedUp & = & \frac{4\text{min}}{11.11\text{min}} \\ & = & 0.36 \end{array}$$

   Yikes! I've been swindled, and now I'll only get $43/day. Ouch!

   (c) (**5 points**) Maybe selling my linux box wasn't such a good idea. Let's say I run Program-D on my linux machine, and I run the other three programs on the GPU. For simplicity, we'll ignore the time for communication when switching between running one task on the GPU and the next on the CPU (or the other way around). Because the tasks have sequential dependencies, I can't use the CPU and the GPU at the same time. With this combined CPU + GPU approach what is the speed-up compared with running everything on the CPU?

   ## Solution

   By calculation like the previous one, the total time is now for all four tasks is 2.11 minutes, and the speed-up is slightly less than 1.9.

   On a happier note, I'll now earn $211/day. Will the extra $91/day cover the cost of the GP100? It should.

(d) (**5 points**) What is Amdahl's law? Write the mathematical formula, and write two or three sentences to explain the intuition it gives for the speed-up results in the previous two sections.

Solution

The formula is:

$$SpeedUp \quad = \quad \frac{1}{f/s+(1-f)}$$

where $0 \leq f \leq 1$ is the fraction of an application that can be improved, and s is the speed-up for that fraction. In the limit that $s \to \infty$, the speed-up goes to $1/(1-f)$. This is what we are seeing in the example above. Half of the tasks (programs C and D) don't benefit from the speed up. If the other two had really large speed-ups (and 100 isn't bad), then we would still only get a speed-up of 2. The speed-up of 1.896 is getting close to the value of 2. Note that if the "improvement" results in a slow-down for some portion (Program-D in the part a), then that can result in a very large performance degradation.

Simple version: it's tempting to just try to average the speed-ups. For the problem above the average is 27.775. That sounds great. In reality, we need to account for the slowest part. If we can adjust our workload to have most of it get the large speed-up, then we can get a huge pay-off. In practice, this means:

- Parallel computing doesn't solve all of your problems.
- We adjust our uses of computers to line up with the problems that do get large speed-ups, such as graphics, animation, simulations, data analytics, and machine learning. Applications that don't benefit from parallel computing may still be important, but they will evolve much more slowly.

5. **Other stuff (20 points)**
Answer each question below with 1-3 sentences. Points may be taken off for long answers.

(a) (**4 points**) What is a pipeline bypass?

Solution

A mechanism that allows a result that is in the pipeline from an earlier issued instruction to be made available to a later issued instruction, even if the first instruction is still in the pipeline.

(b) (**4 points**) What is a fused multiply-add?

Solution

A common feature in hardware for floating-point arithmetic where a multiply followed by an add, e.g. `a*x + y`, can be computed as single operation. This can be very useful for common numerical algorithms (e.g. matrix multiplication), and it lets the marketing people claim higher GFlop or TFlop numbers by counting the fused multiply-add as two floating point operations.

(c) (**4 points**) What is the cross-section bandwidth of a message-passing multiprocessor?

Solution

Consider all ways to partition the N processors of the multiprocessor into two groups of size $N/2$. For each such partition, let B be the bandwidth of all the links between the two parts of the partition. The cross-section bandwidth is the smallest value of B for all possible partitions.

(d) (**4 points**) What is a "straggler" in a map-reduce computation?

Solution

A straggler is a task that takes much longer than the others.

(e) (**4 points**) How do "back-up tasks" mitigate the problems caused by stragglers?

Solution

We can use "work stealing". Near the end of a map-reduce (or other parallel) computation, assign incomplete tasks to idle processors. Often, the stragglers are due to processor being slow (other jobs running on it, configuration problems, etc.), failed, or network problems. By replicating the task, the computation can finish when either the original task or the back-up task completes.

Useful stuff

Amdahl's Law $speedup = \frac{1}{\frac{f}{s}+(1-f)}$

Erlang Cheat Sheet

lists:foldl

```
fsum(L) -> lists:foldl(fun(X,S) -> S+X end, 0, L).
```

reduce

```
largest_gap_seq([]) -> {};
largest_gap_seq([V]) -> {V, V};
largest_gap_seq([V1, V2 | Tail]) ->
    F = fun(XNew, {CurGap, XOld}) -> {gap_max(CurGap, {XOld, XNew}), XNew} end,
    {Gap, _XLast} = lists:foldl(F, {{V1, V2}, V2}, Tail),
    Gap.

largest_gap_par(WTree, DataKey) ->
  Leaf = fun(ProcState) -> leaf(workers:get(ProcState, DataKey)) end,
  Combine = fun(Left, Right) -> combine(Left, Right) end,
  Root = fun(Value) -> root(Value) end,
  wtree:reduce(WTree, Leaf, Combine, Root).

leaf([]) -> {};
leaf([V]) -> {V, {V,V}, V};
leaf(List) -> {hd(List), largest_gap_seq(List), lists:last(List)}.

combine({}, Right) -> Right;
combine(Left, {}) -> Left;
combine({L1, {L2, L3}, L4}, {R1, {R2, R3}, R4}) ->
  {L1, gap_max(gap_max({L2, L3}, {L4, R1}), {R2, R3}), R4}.

root({_MinV, {V1, V2}, _MaxV}) -> {V1, V2}.

gap_max({A1, A2}, {B1, B2}) when (A2 - A1) >= (B2 - B1) -> {A1, A2};
gap_max(_, B) -> B.
```

scan

```
bank_statement(W, SrcKey, DstKey, InitialBalance) ->
  Leaf1 = fun(ProcState) ->
    process_transactions(wtree:get(ProcState, SrcKey), {1, 0})
  end,
  Leaf2 = fun(ProcState, AccIn) ->
    {_S, C} = AccIn,
    Src = wtree:get(ProcState, SrcKey),
    Result = process_transactions_cum(Src, C),    % compute the cummulative sum
    wtree:put(ProcState, DstKey, Result) % save the result -- must be the last expression
  end,                                   %   in the Leaf2 function
  Combine = fun({C1, S1}, {C2, S2}) -> {C1*C2, S1*C2 + S2} end,
  wtree:scan(W, Leaf1, Leaf2, Combine, {1, InitialBalance}).
```

cuda

```
__shared__ float v[1024];
__device__ float f(float x) {
  return ((5/2)*(x*x*x - x));
}

__device__ void compute and reduce(uint n, uint m, float *x) {
  uint myId = threadIdx.x;
  if(myId < n) {
    float y = x[myId];
    for(uint i = 0; i < m; i++) {
      y = f(y); % y = f^(i + 1) (x[myId])
    }
    v[myId] = y; % shared memory is much faster than global memory
    for(uint m = n >> 1; m > 0; m = n >> 1) { % reduce
      n -= m;
      __syncthreads();
      if(myId < m) {
    v[myId] += v[myId+n];
      }
    }
    x[myId] = v[myId]; % move result to global memory
  }
}
```

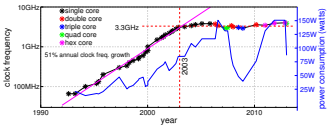
Parallel Computation

Mark Greenstreet

CpSc 418 – Jan. 4, 2017

Outline:

Why Parallel Computation Matters



Clock Speed and Power of Intel Processors vs. Year Released
[Wikipedia CPU-Power, 2011]

- In the good-old days, processor performance doubled roughly every 1.5 years.
- Single thread performance has seen small gains in the past 14 years.
  - Too bad. If it had, we would have 1000GHz CPUs today. ☺
- Need other ways to increase performance.

Why Sequential Performance Can't Improve (much)

Power

- CPUs with faster clocks use more energy per operation than slower ones.
- For mobile devices: high power limits battery life.
- For desktop computers and gaming consoles: cooling high-power chips requires expensive hardware.
- For large servers and clouds, the power bill is a large part of the operating cost.

More Barriers to Sequential Performance

- The memory bottleneck.
  - Accessing main memory (i.e. DRAM) takes hundreds of clock cycles.
  - But, we can get high bandwidth.
- Limited instruction-level-parallelism.
  - CPUs already execute instructions in parallel.
  - But, the amount of this "free" parallelism is limited.
- Design complexity.
  - Designing a chip with 100 simple processors is **way** easier than designing a chip with one big processor.
- Reliability.
  - If a chip has 100 processors and one fails, there are still 99 good ones.
  - If a chip has 1 processor and it fails, then the chip is useless.
- See [Asanovic et al., 2006].

Parallel Computers

- Mobile devices:
  - multi-core to get good performance on apps and reasonable battery life.
  - many dedicated "accelerators" for graphics, WiFi, networking, video, audio, . . .
- Desktop computers
  - multi-core for performance
  - separate GPU for graphics
- Commercial servers
  - multiple, multi-core processors with shared memory.
  - large clusters of machines connected by dedicated networks.

Outline

Topics

Parallel Architectures

- **There isn't one, standard, parallel architecture for everything.** We have:
  - Multi-core CPUs with a shared-memory programming model. Used for mobile device application processors, laptops, desktops, and many large data-base servers.
  - Networked clusters, typically running linux. Used for web-servers and data-mining. Scientific supercomputers are typically huge clusters with dedicated, high-performance networks.
  - Domain specific processors
    - GPUs, video codecs, WiFi interfaces, image and sound processing, crypto engines, network packet filtering, and so on.
- As a consequence, **there isn't one, standard, parallel programming paradigm**.

Parallel Performance

The incentive for parallel computing is to do things that wouldn't be practical on a single processor.

- Performance matters.
- We need good models:
  - Counting operations can be very misleading – "adding is free."
  - Communication and coordination are often the dominant costs.
- We need to measure actual execution times of real programs.
  - There isn't a unified framework for parallel program performance analysis that works well in practice.
  - It's important to measure actual execution time and identify where the bottlenecks are.
- Key concepts with performance:
  - Amdahl's law, linear speed up, overheads.

Parallel Algorithms

- We'll explore some old friends in a parallel context:
  - Sum of the elements of an array
  - matrix multiplication
  - dynamic programming.
- And we'll explore some uniquely parallel algorithms:
  - Bitonic sort
  - mutual exclusion
  - producer consumer

Parallel Programming Frameworks

- Erlang: functional, message passing parallelism
  - Avoids many of the common parallel programming errors: races and side-effects.
    You can write Erlang programs with such bugs, but it takes extra effort (esp. for the examples we consider).
  - Allows a simple presentation of many ideas.
  - But it's slow, for many applications, when compared with C or C++.
  - OTOH, it finds real use in large-scale distributed systems.
- CUDA: your graphics card is a super-computer
  - Excellent performance on the "right" kind of problem.
  - The data-parallel model is simple, and useful.

Syllabus

- January: Erlang
  - **Jan. 4– 9:** Course overview, intro. to Erlang programming.
  - **Jan. 11–18:** Parallel programming in Erlang, reduce and scan.
  - **Jan. 20–27:** Parallel architectures
  - **Jan. 29–Feb. 6:** Performance analysis
- February: Erlang, Midterm
  - **Feb. 8–17:** Sorting
  - **Feb. 20–19:** Midterm break.
  - **Feb. 27:** Midterm Review
  - **Mar. 1:** Midterm
- March: CUDA and other topics
  - **Mar. 3–10:** Introduction to SIMD and CUDA.
  - **Mar. 13–24:** More algorithms in CUDA (and a bit of Erlang)
  - **Mar. 27–Apr. 6:** Map-Reduce, Mutual Exclusion, & More Fun.
- Note: I'll make adjustments to this schedule as we go.

Administrative Stuff – Who

- The instructors
  - **Mark Greenstreet,** mrg@cs.ubc.ca
    - ICCS 323, (604) 822-3065
    - Office hours: Tuesdays, 1pm – 2:30pm, ICCS 323
  - **Ian Mitchell,** mitchell@cs.ubc.ca
    - ICCS 217, (604) 822-2317
    - Office hours: Fridays, 12noon – 1pm, ICCS 217
- The TAs

Devon Graham,	drgraham@cs.ubc.ca
Chenxi Liu,	chenxil@cs.ubc.ca
Carolyn Shen,	shen.carolyn@gmail.com
Brenda Xiong,	krx.sky@gmail.com

- Course webpage: http://www.ugrad.cs.ubc.ca/~cs418.
- Online discussion group: on piazza.

Textbook(s)

- For Erlang: *Learn You Some Erlang For Great Good*, Fred Hébert,
  - Free! On-line at http://learnyousomeerlang.com.
  - You can buy the dead-tree edition at the same web-site if you like.
- For CUDA: *Programming Massively Parallel Processors: A Hands-on Approach* (2nd **or** 3rd ed.), D.B. Kirk and W-M.W. Hwu.
  - Please get a copy by late February – I'll assign readings starting after the midterm. It's available at amazon.ca and many other places.
- I'll hand-out copies of some book chapters:
  - *Principles of Parallel Programming* (chap. 5), C. Lin & L. Snyder – for the reduce and scan algorithms.
  - *An Introduction to Parallel Programming* (chap. 2), P.S. Pacheco – for a survey of parallel architectures.
  - Probably a few journal, magazine, or conference papers.

Why so many texts?

- There isn't one, dominant parallel architecture or programming paradigm.
- The Lin & Snyder book is a great, paradigm independent introduction,
- But, I've found that descriptions of real programming frameworks lack the details that help you write real code.
- So, I'm using several texts, but
  - You only have to buy one! ☺

Grades

Homework:	35%	roughly one assignment every two weeks
Midterm:	25%	March 1, in class
Final:	40%	
Mini-Assignments:	see description on slide 19	
Bug Bounties:	see description on slide 20	

Homework

- Collaboration policy
  - You are welcome and encouraged to discuss the homework problems with other students in the class, with the TAs and me, and find relevant material in the text books, other book, on the web, etc.
  - You are expected to work out your own solutions and write your own code. Discussions as described above are to help understand the material. Your solutions must be your own.
  - You must properly cite your collaborators and any outside sources that you used. You don't need to cite material from class, the textbooks, or meeting with the TAs or instructor. See slide 22 for more on the plagiarism policy.
- Late policy
  - Each assignment has an "early bird" date before the main date. Turn in you assignment by the early-bird date to get a 5% bonus.
  - **No late homework accepted.**

Exams

- Midterm, in class, on March 1.
- Final exam will be scheduled by the registrar.
- Both exams are open book, open notes, open homework and solutions – open anything printed on paper.
  - You can bring a calculator.
  - No communication devices: laptops, tablets, cell-phones, etc.

Mini-Assignments

- Mini-assignments
  - Worth 20% of points missed from HW and exams.
    - If your raw grade is 90%, you can get at most 2% from the minis. Missing one or two isn't a big deal.
    - If your raw grade is 70%, you can get 6% from the minis. This can move your letter grade up a notch (e.g. C+ to B−).
    - If your raw grade is 45%, you can get up to 11% from the minis. Do the mini-assignments – I hate turning in failing grades.
  - The first is at http://www.ugrad.cs.ubc.ca/~cs418/2016-2/mini1/mini1.pdf, and due Jan. 9.
  - **If you are on the course waitlist,** we will select from the students who submit acceptable solutions to **Mini Assignment 1** to fill any slots that open up.

Bug Bounties

- If I make a mistake when stating a homework problem, then the first person to report the error gets extra credit.
  - If the error would have prevented solving the problem, then the extra credit is the same as the value of the problem.
  - Smaller errors get extra credit in proportion to their severity.
- Likewise, bug bounties are awarded (as homework extra credit) for finding errors in mini-assignments, lecture slides, the course web-pages, code I provide, etc.
- The midterm and final have bug bounties awarded in midterm and final exam points respectively.
- If you find an error, report it.
  - Suspected errors in homework, lecture notes, and other course materials should be posted to piazza.
  - The first person to post a bug gets the bounty.
  - Bug-bounties reward you for looking at the HW when it first comes out, and not waiting until the day before it is due.

Grades: the big picture

$$
\begin{aligned}
\mathit{RawGrade} &= 0.35 * \mathit{HW} + 0.25 * \mathit{MidTerm} + 0.40 * \mathit{Final} \\
\mathit{MiniBonus} &= 0.20 * (1 - \min(\mathit{RawGrade}, 1)) * \mathit{Mini} \\
\mathit{BB} &= 0.35 * \mathit{BB}_{\mathit{HW}} + 0.25 * \mathit{BB}_{\mathit{MT}} + 0.40 * \mathit{BB}_{\mathit{FX}} \\
\mathit{CourseGrade} &= \min(\mathit{RawGrade} + \mathit{MiniBonus} + \mathit{BB}, 1) \times 100\%
\end{aligned}
$$

Plagiarism

- I have a very simple criterion for plagiarism:
  Submitting the work of another person, whether that be another student, something from a book, or something off the web and representing it as your own is plagiarism and constitutes academic misconduct.
- If the source is clearly cited, then it is not academic misconduct.
  If you tell me "This is copied word for word from Jane Foo's solution" that is not academic misconduct. It will be graded as one solution for two people and each will get half credit. I guess that you could try telling me how much credit each of you should get, but I've never had anyone try this before.
- I encourage you to discuss the homework problems with each other.
  If you're brainstorming with some friends and the key idea for a solution comes up, that's OK. In this case, add a note to your solution that lists who you collaborated with.
- More details at:
  - http://www.ugrad.cs.ubc.ca/~cs418/plagiarism.html
  - http://learningcommons.ubc.ca/guide-to-academic-integrity/

Learning Objectives (1/2)

- Parallel Algorithms
  - Familiar with parallel patterns such as reduce, scan, and tiling and can apply them to common parallel programming problems.
  - Can describe parallel algorithms for matrix operations, sorting, dynamic programming, and process coordination.
- Parallel Architectures
  - Can describe shared-memory, message-passing, and SIMD architectures.
  - Can describe a simple cache-coherence protocol.
  - Can identify how communication latency and bandwidth are limited by physical constraints in these architectures.
  - Can describe the difference between bandwidth and inverse latency, and how these impact parallel architectures.

Learning Objectives (2/2)

- Parallel Performance
  - Understands the concept of "speed-up": can calculate it from simple execution models or measured execution times.
  - Can identify key bottlenecks for parallel program performance including communication latency and bandwidth, synchronization overhead, and intrinsically sequential code.
- Parallel Programming Frameworks
  - Can implement simple parallel programs in Erlang and CUDA.
  - Can describe the differences between these paradigms.
  - Can identify when one of these paradigms is particularly well-suited (or badly suited) for a particular application.

Lecture Outline

Erlang Intro – very abbreviated!

- Erlang is a functional language:
  - Variables are given values when declared, and the value never changes.
  - The main data structures are lists, `[Head | Tail]`, and tuples (covered later).
  - Extensive use of pattern matching.
- The source code for the examples in this lecture is available at:
  http://www.ugrad.cs.ubc.ca/~cs418/2016-2/lecture/01-06/code.html

Lists

- `[1, 4, 9, 16, 25, 36, 49, 64, 81, 100]` is a list of 10 elements.
- If `L1` is a list, then `[0 | L1]` is the list obtained by prepending the element `0` to the list `L1`. In more detail:

```
1> L1 = [1, 4, 9, 16, 25, 36, 49, 64, 81, 100].
[1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
2> L2 = [0 | L1].
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100]
3> L3 = [0 , L1].
[0, [1, 4, 9, 16, 25, 36, 49, 64, 81, 100]]
```

- Of course, we traverse a list by using recursive functions:

Lists traversal example: sum

```
sum(List) ->
   if (length(List) == 0) -> 0;
      (length(List) > 0)  -> hd(List) + sum(tl(List))
   end.
```

- `length(L)` returns the number of elements in list `L`.
- `hd(L)` returns the first element of list `L` (the head), and throws an exception if `L` is the empty list.
  `hd([1, 2, 3]) = 1`. `hd([1]) = 1` as well.
- `tl(L)` returns the list of all elements after the first (the tail).
  `tl([1, 2, 3]) = [2, 3]`. `tl([1]) = []`.
- See `sum_wo_pm` ("sum without pattern matching") in simple.erl

Pattern Matching – first example

We can use Erlang's pattern matching instead of the `if` expression:

```
sum([]) -> 0;
sum([Head | Tail]) -> Head + sum(Tail).
```

- `sum([Head | Tail])` matches any non-empty list with `Head` being bound to the value of the first element of the list, and `Tail` begin bound to the list of all the other elements.
- More generally, we can use patterns to identify the different cases for a function.
- This can lead to very simple code where function definitions follow the structure of their arguments.
- See `sum` in simple.erl

Count 3's: a simple example

Given an array (or list) with `N` items, return the number of those elements that have the value `3`.

```
count3s([]) -> 0;
count3s([3 | Tail]) -> 1 + count3s(Tail);
count3s([_Other | Tail]) -> count3s(Tail).
```

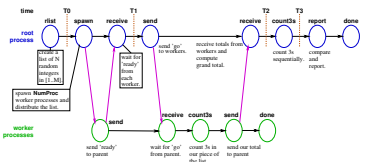
- We'll need to put the code in an erlang module. See `count3s` in count3s.erl for the details.
- To generate a list of random integers, count3s.erl uses the function `rlist(N, M)` from course Erlang library that returns a list of `N` integers randomly chosen from `1..M`.

Running Erlang

```
bash-3.2$ erl
Erlang/OTP 18 [erts-7.0] [source] ...
Eshell V7.0 (abort with ^G)

1> c(count3s).
{ok,count3s}
2> L20 = count3s:rlist(20,5).
[3,4,5,3,2,3,5,4,3,3,1,2,4,1,3,2,3,3,1,3]
3> count3s:count3s(L20).
9
4> count3s:count3s(count3s:rlist(1000000,10)).
99961
5> q().
ok
6> bash-3.2$
```


A Parallel Version



The code is in
http://www.ugrad.cs.ubc.ca/~cs418/2016-2/lecture/01-04/src/count3s.erl

Preview of the next month

January 6: Introduction to Erlang Programming
Reading: *Learn You Some Erlang*, the first eight sections – Introduction through Recursion. Feel free to skip the stuff on bit syntax and binary comprehensions.

January 9: Processes and Messages
Reading: *Learn You Some Erlang*, Higher Order Functions and The Hitchhiker's Guide..., through More on Multiprocessing
Homework: **Homework 1 goes out (due Jan. 18)** – Erlang programming
Mini-Assignment: Mini-Assignment 1 due **10:00am**
Mini-Assignment 2 goes out (due Jan. 13)

January 11: Reduce
Reading: *Learn You Some Erlang*, Errors and Exceptions through A Short Visit to Common Data Structures

January 13: Scan
Reading: Lin & Snyder, chapter 5, pp. 112–125
Mini-Assignment: Mini-Assignment 2 due **10:00am**

January 16: Generalized Reduce and Scan
Homework: Homework 1 deadline for early-bird bonus (11:59pm)
Homework 2 goes out (due Feb. 1) – Reduce and Scan

January 18: Reduce and Scan Examples
Homework: Homework 1 due **11:59pm**

January 20–27: Parallel Architecture

January 29–February 6: Parallel Performance

Review Questions

- Name one, or a few, key reasons that parallel programming is moving into mainstream applications.
- How does the impact of your mini assignment total on your final grade depend on how you did on the other parts of the class?
- What are bug-bounties?
- What is the count 3's problem?
- How did we measure running times to compute speed up?
  - Why did one approach show a speed-up greater than the number of cores used?
  - Why did the other approach show that the parallel version was slower than the sequential one?

Supplementary Material

- Erlang Resources
- Bibliography
- Table of Contents – at the end!!!

Erlang Resources

- Learn You Some Erlang
  http://learnyousomeerlang.com
  An on-line book that gives a very good introduction to Erlang. It has great answers to the "Why is Erlang this way?" kinds of questions, and it gives realistic assessments of both the strengths and limitations of Erlang.
- Erlang Examples:
  http://www.ugrad.cs.ubc.ca/~cs418/2012-1/lecture/09-08.pdf
  My lecture notes that walk through the main features of Erlang with examples for each. Try it with an Erlang interpreter running in another window so you can try the examples and make up your own as you go. This will cover everything you'll need to make it through all (or most) of what we'll do in class, but it doesn't explain how to think in Erlang as well as "Learn You Some Erlang" or Armstrong's Erlang book (next slide).

More Erlang Resources

- The erlang.org tutorial
  http://www.erlang.org/doc/getting_started/users_guide.html
  Somewhere between my "Erlang Examples" and "Learn You Some Erlang."
- Erlang Language Manual
  http://www.erlang.org/doc/reference_manual/users_guide.html
  My go-to place when looking up details of Erlang operators, etc.
- On-line API documentation:
  http://www.erlang.org/erldoc.
- The book: *Programming Erlang: Software for a Concurrent World*, Joe Armstrong, 2007,
  http://pragprog.com/book/jaerlang/programming-erlang
  Very well written, with lots of great examples. More than you'll need for this class, but great if you find yourself using Erlang for a big project.
- More resources listed at http://www.erlang.org/doc.html.

Getting Erlang

- You can run Erlang by giving the command `erl` on any departmental machine. For example:
  - Linux: bowen, thetis, lin01, ..., lin25, ..., all machines above are .ugrad.cs.ubc.ca, e.g. bowen.ugrad.cs.ubc.ca, etc.
- You can install Erlang on your computer
  - Erlang solutions provides packages for Windows, OSX, and the most common linux distros
    https://www.erlang-solutions.com/resources/download.html
  - Note: some linux distros come with Erlang pre-installed, but it might be an old version. You should probably install from the link above.

Starting Erlang

- Start the Erlang interpreter.

```
theis % erl
Erlang/OTP 18 [erts-7.0] [source] ...

Eshell V7.0 (abort with ^G)

1> 2+3.
5
2>
```

- The Erlang interpreter evaluates expressions that you type.
- Expressions end with a "`.`" (period).

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Table Of Contents (1/2)

Table Of Contents (2/2)

Introduction to Erlang

Mark Greenstreet

CpSc 418 – January 6, 2016

Outline:

Objectives

- Learn/review key concepts of functional programming:
  - Referential transparency.
  - Structuring code with functions.
- Introduction to Erlang
  - Basic data types and operations.
  - Program design by structural decomposition.
  - Writing and compiling an Erlang module.

Erlang Basics

- Numbers:
  - Numerical Constants: `1`, `8#31`, `1.5`, `1.5e3`, but not: `1.` or `.5`
  - Arithmetic: `+`, `-`, `*`, `/`, `div`, `band`, `bor`, `bnot`, `bsl`, `bsr`, `bxor`
- Booleans:
  - Comparisons: `=:=`, `=/=`, `==`, `/=`, `<`, `=<`, `>`, `>=`
  - Boolean operations (strict): `and`, `or`, `not`, `xor`
  - Boolean operations (short-circuit): `andalso`, `orelse`
- Atoms:
  - Constants: `x`, `'big DOG-2'`
  - Operations: tests for equality and inequality. Therefore pattern matching.

Lists and Tuples

- Lists:
  - Construction: `[1, 2, 3]`, `[Element1, Element2, ..., Element_N | Tail]`
  - Operations: `hd`, `tl`, `length`, `++`, `--`
  - Erlang's list library, http://erlang.org/doc/man/lists.html: `all`, `any`, `filter`, `foldl`, `foldr`, `map`, `nth`, `nthtail`, `seq`, `sort`, `split`, `zipwith`, and **many** more.
- tuples:
  - Construction: `{1, dog, "called Rover"}`
  - Operations: `element`, `setelement`, `tuple_size`.
  - Lists vs. Tuples:
    - **Lists** are typically used for an **arbitrary** number of elements of the same "type" – like arrays in C, Java, ...
    - **Tuples** are typically used for an **fixed** number of elements of the varying "types" – likes a `struct` in C or an object in Java.

Strings

What happened to strings?!

- Well, they're lists of integers.
- This can be annoying. For example,

```
1> [102, 111, 111, 32, 98, 97, 114].
"foo bar"
2>
```

- By default, Erlang prints lists of integers as strings if every integer in the list is the ASCII code for a "printable" character.
- *Learn You Some Erlang* discusses strings in the "Don't drink too much Kool-Aid" box for lists.

Functional Programming

- Imperative programming (C, Java, Python, ...) is a programming model that corresponds to the von Neumann computer:
  - A program is a sequence of statements.
    In other words, a program is a recipe that gives a step-by-step description of what to do to produce the desired result.
  - Typically, the operations of imperative languages correspond to common machine instructions.
  - Control-flow (`if`, `for`, `while`, function calls, etc.) Each control-flow construct can be implemented using branch, jump, and call instructions.
  - This correspondence between program operations and machine instructions simplifies implementing a good compiler.
- Functional programming (Erlang, lisp, scheme, Haskell, ML, ...) is a programming model that corresponds to mathematical definitions.
  - A program is a collection of definitions.
  - These include definitions of expressions.
  - Expressions can be evaluated to produce results.
- See also: the *LYSE* explanation.

Erlang Makes Parallel Programming Easier

- Erlang is functional
  - Each variable gets its value when it's declared – it **never** changes.
  - Erlang eliminates many kinds of races – another process **can't** change the value of a variable while you're using it, because the values of variables never change.
- Erlang uses message passing
  - Interactions between processes are under explicit control of the programmer.
  - Fewer races, synchronization errors, etc.
- Erlang has simple mechanisms for process creation and communication
  - The structure of the program is not buried in a large number of calls to a complicated API.

Big picture: Erlang makes the issues of parallelism in parallel programs more apparent and makes it easier to avoid many common pitfalls in parallel programming.

Referential Transparency

- This notion that a variable gets a value when it is declared and that the value of the variable never changes is called **referential transparency**.
  - You'll here me use the term many times in class – I thought it would be a good idea to let you know what it means. ☺
- We say that the value of the variable is **bound** to the variable.
- Variables in functional programming are much like those in mathematical formulas:
  - If a variable appears multiple places in a mathematical formula, we assume that it has the same value everywhere.
  - This is the same in a functional program.
  - This is not the case in an imperative program. We can declare `x` on line 17; assign it a value on line 20; and assign it another value on line 42.
  - The value of `x` when executing line 21 is different than when executing line 43.

Loops violate referential transparency

```
// vector dot-product
sum = 0.0;
for(i = 0; i < a.length; i++)
   sum += a[i] * b[i];
```

```
// merge, as in merge-sort
while(a != null && b != null) {
   if(a.key <= b.key) {
      last->next = a;
      last = a;
      a = a->next;
      last->next = null;
   } else {
      ...
   }
}
```

- Loops rely on changing the values of variables.
- Functional programs use recursion instead.
- See also the *LYSE* explanation.

Life without loops

Use recursive functions instead of loops.

```
dotProd([], []) -> 0;
dotProd([A | At1], [B | Bt1]) -> A*B + dotProd(At1, Bt1).
```

- Functional programs use recursion instead of iteration:

```
dotProd([], []) -> 0;
dotProd([A | At1], [B | Bt1]) -> A*B + dotProd(At1, Bt1).
```

- Anything you can do with iteration can be done with recursion.
  - But the converse is not true (without dynamically allocating data structures).
  - Example: tree traversal.

Example: Sorting a List

- The simple cases:
  - Sorting an empty list: `sort([]) ->` _____
  - Sorting a singleton list: `sort([A]) ->` _____
- How about a list with more than two elements?
  - Merge sort?
  - Quick sort?
  - Bubble sort (NO WAY! Bubble sort is DISGUSTING!!!).
- Let's figure it out.

Merge sort: Erlang code

- If a list has more than one element:
  - Divide the elements of the list into two lists of roughly equal length.
  - Sort each of the lists.
  - Merge the sorted list.
- In Erlang:

```
sort([]) -> [];
sort([A]) -> [A];
sort([A | Tail]) ->
   {L1, L2} = split([A | Tail]),
   L1_sorted = sort(L1),
   L2_sorted = sort(L2),
   merge(L1_sorted, L2_sorted).
```

- Now, we just need to write `split`, and `merge`.

`split(L)`

Identify the cases and their return values according to the shape of `L`:

```
% If L is empty (recall that split returns a tuple of two lists):
split([]) -> {          ,          }

% If L

split(          ) ->

% If L
```


Finishing merge sort

- An exercise for the reader – see slide 29.
- Sketch:
  - Write `merge(List1, List2) -> List12` – see slide 30
  - Write an Erlang module with the `sort`, `split`, and `merge` functions – see slide 31
  - Run the code – see slide 33

Fun with functions

- Programming with patterns
  - often, the code just matches the shape of the data
  - like CPSC 110, but pattern matching makes it obvious
  - see slide 16
- Fun expressions
  - in-line function definitions
  - see slide 17
- Higher-order functions
  - encode common control-flow patterns
  - see slide 18
- List comprehensions
  - common operations on lists
  - see slide 19
- Tail call elimination
  - makes recursion as fast as iteration (in simple cases)
  - see slide 20

Programming with Patterns

```
% leafCount: count the number of leaves of a tree represented by a nested list
leafCount([]) -> 0; % base case – an empty list/tree has no leaves

leafCount([Head | Tail]) -> % recursive case
   leafCount(Head) + leafCount(Tail);
leafCount(_Leaf) -> 1; % the other base case – _Leaf is not a list
```

- Let's try it

```
2> examples:leafCount([1, 2, [3, 4, []], [5, [6, banana]]]).
7
```

- Notice how we used **patterns** to show how the recursive structure of `leafCount` follows the shape of the tree.
- See Pattern Matching in *Learn You Some Erlang* for more explanation and examples.
- Style guideline: if you're writing code with lots of `if`'s `hd`'s, and `tl`'s, you should think about it and see if using patterns will make your code simpler and clearer.

Anonymous Functions

```
3> fun(X, Y) -> X+X + Y+Y end.  % fun ... end creates an "anonymous function"
#Fun<erl_eval.12.52032458> % ok, I guess, but what can I do with it?!
4> F = fun(X, Y) -> X+X + Y+Y end.
#Fun<erl_eval.12.52032458>
5> F(3, 4).
25
6> Factorial = % We can even write recursive fun expressions!
     fun Fact(0) -> 1;
         Fact(N) when is_integer(N), N > 0 -> N*Fact(N-1)
     end.
7> Factorial(3).
6
8> Fact(3).
* 1: variable 'Fact' is unbound
9> Factorial(-2).
** exception error: no function clause matching
       erl_eval:'-inside-an-interpreted-fun-'(-2)
10> Factorial(frog).
** exception error: no function clause matching
       erl_eval:'-inside-an-interpreted-fun-'(frog)
```

See Anonymous Functions in *Learn You Some Erlang* for more explanation and examples.

Higher-Order Functions

- `lists:map(Fun, List)` apply `Fun` to each element of `List` and return the resulting list.

```
1l> lists:map(fun(X) -> 2*X+1 end, [1, 2, 3]).
[3, 5, 7]
```

- `lists:fold(Fun, Acc0, List)` use `Fun` to combine all of the elements of `List` in left-to-right order, starting with `Acc0`.

```
12> lists:foldl(fun(X, Y) -> X+Y end, 100, [1, 2, 3]).
106
```

- For more explanation and examples:
  - See Higher Order Functions in *Learn You Some Erlang*.
  - See the lists module in the Erlang standard library. Examples include
    - `all(Pred, List)`: true iff `Pred` evaluates to true for **every** element of `List`.
    - `any(Pred, List)`: true iff `Pred` evaluates to true for **any** element of `List`.
    - `foldr(Fun, Acc0, List)`: like `foldl` but combines elements in right-to-left order.

List Comprehensions

- Map and filter are such common operations, that Erlang has a simple syntax for such operations.
- It's called a **List Comprehension**:
  - *[Expr || Var <- List, Cond, ...]*.
  - *Expr* is evaluated with *Var* set to each element of *List* that satisfies *Cond*.
  - Example:

```
13>R = count3s:rlist(5, 1000).
[444,724,946,502,312].
14>[X*X || X <- R, X rem 3 == 0].
[197136,97344].
```

- See also List Comprehensions in *LYSE*.

Head vs. Tail Recursion

- I wrote two versions of computing the sum of the first `N` natural numbers:

```
sum_h(0) -> 0; % "head recursive"
sum_h(N) -> N + sum_h(N-1).

sum_t(N) -> sum_t(N, 0).
sum_t(0, Acc) -> Acc; % "tail recursive"
sum_t(N, Acc) -> sum_t(N-1, N+Acc).
```

- Here are some run times that I measured:

N	t_{head}	t_{tail}	N	t_{head}	t_{tail}
1K	21μs	13μs	1M	21ms	11ms
10K	178μs	114μs	10M	1.7s	115ms
100K	1.7ms	1.1ms	100M	28s	1.16s
			1G	> 8 min	11.6s

Head vs. Tail Recursion – Comparison

- Both grow linearly for $N \leq 10^6$.
  - The tail recursive version has runtimes about 2/3 of the head-recursive version.
- For $N > 10^6$,
  - The tail recursive version continues to have run-time linear in `N`.
  - The head recursive version becomes much slower than the tail recursive version.
- The Erlang compiler optimizes tail calls
  - When the last operation of a function is to call another function, the compiler just revises the current stack frame and jumps to the entry point of the callee.
  - The compiler has turned the recursive function into a while-loop.
  - Conclusion: **When people tell you that recursion is slower than iteration – don't believe them.**
- The head recursive version creates a new stack frame for each recursive call.
  - I was hoping to run my laptop out of memory and crash the Erlang runtime – makes a fun, in-class demo.
  - But, OSX does memory compression. All of those repeated stack frames are very compressible. The code doesn't crash, but it's very slow.

Tail Call Elimination – a few more notes

- I doubt we'll have time for this in lecture. I've included it here for completeness.
- Can you count on your compiler doing tail call elimination:
  - In Erlang, the compiler is **required** to perform tail-call elimination. We'll see why on Monday.
  - In Java, the compiler is forbidden from performing tail-call elimination. This is because the Java security model involves looking back up the call stack.
  - `gcc` performs tail-call elimination when the `-o` flag is used.
- Is it OK to write head recursive functions?
  - Yes! Often, the head-recursive version is much simpler and easier to read. If you are confident that it won't have to recurse for millions of calls, then write the clearer code.
  - Yes! Not all recursive functions can be converted to tail-recursion.
    - Example: tree traversal.
    - Computations that can be written as "loops" in other languages have tail-recursive equivalents.
    - But, recursion is more expressive than iteration.

Summary

- Why Erlang?
  - Functional – avoid complications of side-effects when dealing with concurrency.
  - But, we can't use imperative control flow constructions (e.g. loops).
    - Design by declaration: look at the structure of the data.
    - More techniques coming in upcoming lectures.
- Sequential Erlang
  - Lists, tuple, atoms, expressions
  - Using structural design to write functions: example sorting.
  - Functions: patterns, higher-order functions, head vs. tail recursion.

Preview

January 9: Processes and Messages
Reading: *Learn You Some Erlang*, Higher Order Functions and The Hitchhiker's Guide... through More on Multiprocessing
Homework: **Homework 1 goes out (due Jan. 18)** – Erlang programming
Mini-Assignment: Mini-Assignment 1 due **10:00am**
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January 11: Reduce
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January 16: Generalized Reduce and Scan
Homework: Homework 1 deadline for early-bird bonus (11:59pm)
Homework 2 goes out (due Feb. 1) – Reduce and Scan

January 18: Reduce and Scan Examples
Homework: Homework 1 due **11:59pm**

January 20–27: Parallel Architecture
January 29–February 6: Parallel Performance
February 8–17: Parallel Sorting

Review Questions

- What is the difference between `==` and `=:=` ?
- What is an atom?
- Which of the following are valid Erlang variables, atoms, both, or neither?
  `Foo`, `foo`, 25, `'25'`, `'Foo foo'`, `"4 score and 7 years ago"`, X2, `'4 score and 7 years ago'`.
- Draw the tree corresponding to the nested list `[X, [[Y, Z], 2, [A, B+C, [], 23]], 14, [[[8]]]]`.
- What is referential transparency?
- Why don't functional languages have loops?
- Use an anonymous function and `lists:filter` to implement the body of `GetEven` below:

```
% GetEven(List) -> Evens, where Evens is a list consisting of all
%     elements of List that are integers and divisible by two.
%     Example: GetEven([1, 2, frog, 1000]) -> [2, 1000]
GetEven(List) ->
   you write this part.
```


A Few More Review Questions

- Use a list comprehension to implement to body of `Double` below:

```
% Double(List) -> List2, where List is a list of numbers, and
%     List2 is the list where each of these are doubled.
%     Example: Double([1, 2, 3.14159, 1000]) ->
%         [2, 4, 6.28318, 2000]
Double(List) ->
   you write this part.
```

- Use a list comprehension to write the body of `Evens` as described on the previous slide.
- What is a tail-recursive function?
- In general, which is more efficient, a head-recursive or a tail-recursive implementation of a function? Why?

Supplementary Material

The remaining material is included in the web-version of these slides:
http://www.ugrad.cs.ubc.ca/~cs418/2016-2/lecture/01-06/slides.pdf
I'm omitting it from the printed handout to save a few trees.

- Erlang resources.
- Finishing the merge sort example.
- Common mistakes with lists and how to avoid them.
- A few remarks about atoms.
- Suppressing verbose output when using the Erlang shell.
- Forgetting variable bindings (only in the Erlang shell).
- Table of Contents.

Erlang Resources

- *LYSE* – you should be reading this already!
- Install Erlang on your computer
  - Erlang solutions provides packages for Windows, OSX, and the most common linux distros
    http://www.erlang-solutions.com/resources/download.html
  - Note: some linux distros come with Erlang pre-installed, but it might be an old version. You should probably install from the link above.
- http://www.erlang.org
  - Searchable documentation
    http://erlang.org/doc/search/
  - Language reference
    http://erlang.org/doc/reference_manual/users_guide.html
  - Documentation for the standard Erlang library
    http://erlang.org/doc/man_index.html
- The CPSC 418 Erlang Library
  - Documentation
    http://www.ugrad.cs.ubc.ca/~cs418/resources/erl/doc/index.html
  - `.tgz` (source, and pre-compiled `.beam`)
    http://www.ugrad.cs.ubc.ca/~cs418/resources/erl/erl.tgz

Finishing the merge sort example

- Write `merge(List1, List2) -> List12` – see slide 30
- Write an Erlang modle with the `sort`, `split`, and `merge` functions – see slide 31
- Run the code – see slide 33

`merge(L1, L2)`

- Precondition: We assume `L1` and `L2` are each in non-decreasing order.
- Return value: a list that consists of the elements of `L1` and `L2` and the elements of the return-list are in non-decreasing order.
- Identify the cases and their return values.
  - What if `L1` is empty?
  - What if `L2` is empty?
  - What if both are empty?
  - What if neither are empty?
  - Are there other cases?
    Do any of these cases need to be broken down further?
    Are any of these case redundant?
- Now, try writing the code (an exercise for the reader).

Modules

- To compile our code, we need to put it into a module.
- A module is a file (with the extension `.erl`) that contains
  - Attributes: declarations of the module itself and the functions it exports.
    - The module declaration is a line of the form:
      `-module(moduleName).`
      where `moduleName` is the name of the module.
    - Function exports are written as:
      `-export([functionName1/arity1, functionName2/arity2, ...]).`
      The list of functions may span multiple lines and there may be more than one `-export` attribute.
      `arity` is the number of arguments that the function has. For example, if we define
      `foo(A, B) -> A+A + B.`
      Then we could export `foo` with
      `-export([..., foo/2, ...]).`
    - There are many other attributes that a module can have. We'll skip the details. If you really want to know, it's all described here.
  - Function declarations (and other stuff) – see the next slide

A module for `sort`

```
-module(sort).
-export([sort/1]).
% The next -export is for debugging. We'll comment it out later
-export([split/1, merge/2]).
sort([]) -> [];
...
```


Let's try it!

```
1> c(sort).
{ok,sort}
2> R20 = count3s:rlist(20, 100).  % test case: a random list
[45,73,95,51,32,60,92,67,48,60,15,21,70,16,56,22,46,43,1,57]
3> S20 = sort:sort(R20).  % sort it
[1,15,16,21,22,32,43,45,46,48,51,56,57,60,60,67,70,73,92,95]
4> R20 -- S20.  % empty if each element in R20 is in S20
[]
5> S20 -- R20.  % empty if each element in S20 is in R20
[]
```

- Yay – it works!!! (for one test case)
- The code is available at
  http://www.ugrad.cs.ubc.ca/~cs418/2016-2/lecture/01-06/src/sort.erl

Remarks about Constructing Lists

It's easy to confuse `[A, B]` and `[A | B]`.

- This often shows up as code ends up with crazy, nested lists; or code that crashes; or code that crashes due to crazy, nested lists; ....
- Example: let's say I want to write a function `divisible_drop(N, L)` that removes all elements from list `L` that are divisible by `N`:

```
divisible_drop(_N, []) -> []; % the usual base case
divisible_drop(N, [A | Tail]) ->
   if A rem N == 0 -> divisible_filter(N, Tail);
      A rem N /= 0 -> [A | divisible_filter(N, Tail)]
   end.
```

It works. For example, I included the code above in a module called `examples`.

```
6> examples:divisible_drop(3, [0, 1, 4, 17, 42, 100]).
[1,4,17,100]
```


Misconstructing Lists

Working with `divisible_drop` from the previous slide...

- Now, change the second alternative in the `if` to
  `A rem N /= 0 -> [A, divisible_filter(N, Tail)]`
  Trying the previous test case:

```
7> examples:divisible_drop(3, [0, 1, 4, 17, 42, 100]).
[1,[4,[17,[100,[]]]]]
```

  Moral: If you see a list that is nesting way too much, check to see if you wrote a comma where you should have used a `|`.
- Restore the code and then change the second alternative for `divisible_drop` to `divisible_drop(N, [A, Tail])` -> Trying our previous test:

```
8> examples:divisible_drop(3, [0, 1, 4, 17, 42, 100]).
** exception error: no function clause matching...
```


Punctuation

- Erlang has lots of punctuation: commas, semicolons, periods, and `end`.
- It's easy to get syntax errors or non-working code by using the wrong punctuation somewhere.
- Rules of Erlang punctuation:
  - Erlang declarations end with a period: `.`
  - A declaration can consist of several alternatives.
    - Alternatives are separated by a semicolon: `;`
    - Note that many Erlang constructions such as `case`, `fun`, `if`, and `receive` can have multiple alternatives as well.
  - A declaration or alternative can be a block expression
    - Expressions in a block are separated by a comma: `,`
    - The value of a block expression is the last expression of the block.
  - Expressions that begin with a keyword end with `end`
    - `case` *Alternatives* `end`
    - `fun` *Alternatives* `end`
    - `if` *Alternatives* `end`
    - `receive` *Alternatives* `end`

Remarks about Atoms

- An atom is a special constant.
  - Atoms can be compared for equality.
  - Actually, any two Erlang can be compared for equality, and any two terms are ordered.
  - Each atom is unique.
- Syntax of atoms
  - Anything that looks like an identifier and starts with a lower-case letter, e.g. `x`.
  - Anything that is enclosed between a pair of single quotes, e.g. `'47 BIG apples'`.
  - Some languages (e.g. Matlab or Python) use single quotes to enclose string constants, some (e.g. C or Java) use single quotes to enclose character constants.
    - But not Erlang.
    - The atom `'47 big apples'` is not a string or a list, or a character constant.
    - It's just its own, unique value.
  - **Atom constants can be written with single quotes, but they are not strings.**

Avoiding Verbose Output

- Sometimes, when using Erlang interactively, we want to declare a variable where Erlang would spew enormous amounts of "uninteresting" output were it to print the variable's value.
  - We can use a comma (i.e. a block expression) to suppress such verbose output.
  - Example

```
9> L1_to_5 = lists:seq(1, 5).
[1, 2, 3, 4, 5].
10> L1_to_5M = lists:seq(1, 5000000), ok.
ok
11> length(L1_to_5M).
5000000
12>
```


Forgetting Bindings

- Referential transparency means that bindings are forever.
  - This can be nuisance when using the Erlang shell.
  - Sometimes we assign a value to a variable for debugging purposes.
  - We'd like to overwrite that value later so we don't have to keep coming up with more name.s
- In the Erlang shell, `f(Variable).` makes the shell "forget" the binding for the variable.

```
12> X = 2+3.
5.
13> X = 2+3.
** exception error: no match of right hand side value 6.
14> f(X).
ok
15> X = 2+3.
6
16>
```


Table of Contents

Processes and Messages

Mark Greenstreet

CpSc 418 – Jan. 9, 2017

Outline:

Objectives

- Introduce Erlang's features for concurrency and parallelism
  - Spawning processes.
  - Sending and receiving messages.
- Describe timing measurements for these operations and the implications for writing efficient parallel programs.
  - **Communication often dominates the runtime of parallel programs.**
- The source code for the examples in this lecture is available here: procs.erl.

Processes – Overview

- The built-in function spawn creates a new process.
- Each process has a process-id, pid.
  - The built-in function self() returns the pid of the calling process.
  - `spawn` returns the pid of the process that it creates.
  - The simplest form is `spawn(Fun)`.
    - A new process is created – "the child".
    - The pid of the new process is returned to the caller of `spawn`.
    - The function *Fun* is invoked with no arguments in that process.
    - The parent process and the child process are both running.
    - When *Fun* returns, the child process terminates.

Processes – a friendly example

```
hello(N)->
    [     spawn(fun() -> io:format(
            "hello world from process ~b~n", [I])
          end)
      || I <- lists:seq(1,N)
    ].
```

Running the code:

```
1> c(procs).
{ok,procs}
2> procs:hello(3).
hello world from process 1
hello world from process 2
hello world from process 3
[<0.40.0>,<0.41.0>,<0.42.0>]
```


Messages

- To solve tasks in parallel, the processes need to communicate.
- Sending a message: `Pid ! Expr`.
  - `Expr` is evaluated, and the result is sent to process `Pid`.
  - We can send **any** Erlang term: integers, atoms, lists, tuples, ...
- Receiving a message:

```
receive
    Pattern1 -> Expr1;
    Pattern2 -> Expr2;
    ...
    PatternN -> ExprN
end
```

If there is a pending message for this process that matches one of the patterns,

- The message is delivered, and the value of the `receive` expression is the value of the corresponding *Expr*.
- Otherwise, the process blocks until such a message is received.

- Message passing is asynchronous: the sending process can continue its execution before the receiver gets the message.

Adding two numbers using processes and messages

- The plan:
  - We'll spawn a process in the shell for adding two numbers.
  - This child process receives two numbers, computes the sum, and sends the result back to the parent.

```
add_proc(PPid) ->
    receive
        A -> receive
                 B ->
                     PPid ! A+B
             end
    end.

adder() ->
    MyPid = self(),
    spawn(fun() ->
            add_proc(MyPid)
    end).
```

```
3> Apid = procs:adder().
<0.44.0>
4> Apid ! 2.
2
5> Apid ! 3.
3
6> receive Sum -> Sum end.
5
```


Reactive Processes and Tail Recursion

- Often, we want processes that do more than add two numbers together.
- We want processes that wait, receive a message, process the message, and then wait for the next message.
- In Erlang, we do this with recursive functions for the child process:

```
acc_proc(Tally) ->
    receive
      N when is_integer(N) ->
        acc_proc(Tally+N);
      {Pid, total} ->
        Pid ! Tally,
        acc_proc(Tally)
    end.

accumulator() ->
    spawn(fun() ->
      acc_proc(0)
    end).
```

```
7> BPid = procs:accumulator().
<0.53.0>
8> BPid ! 1.
1
9> BPid ! 2.
2
10> BPid ! 3.
3
11> BPid ! {self(), total}.
{<0.33.0>, total}
12> receive T1 -> T1 end.
6
```


Reactive Processes and Tail Recursion

- Often, we want processes that do more than add two numbers together.
- We want processes that wait, receive a message, process the message, and then wait for the next message.
- In Erlang, we do this with recursive functions for the child process:

```
acc_proc(Tally) ->
    receive
      N when is_integer(N) ->
        acc_proc(Tally+N);
      {Pid, total} ->
        Pid ! Tally,
        acc_proc(Tally)
    end.

accumulator() ->
    spawn(fun() ->
      acc_proc(0)
    end).
```

```
13> BPid ! 4.
4
14> BPid ! {self(), total}.
{<0.33.0>, total}
15> BPid ! 5.
5
16> BPid ! 6.
6
17> BPid ! {self(), total}.
{<0.33.0>, total}
18> receive T2 -> T2 end.
10
19> receive T3 -> T3 end.
21
```


Message Ordering

- Given two processes, *Proc1* and *Proc2*, messages sent from *Proc1* to *Proc2* are received at *Proc2* in the order in which they were sent.
- Message delivery is reliable: if a process doesn't terminate, any message sent to it will eventually be delivered.
- Other than that, Erlang makes no ordering guarantees.
  - In particular, the triangle inequality is not guaranteed.
  - For example, process *Proc1* can send message *M1* to process *Proc2* and after that send message *M2* to *Proc3*.
  - Process *Proc3* can receive the message *M2*, and then send message *M3* to process *Proc2*.
  - Process *Proc2* can receive messages *M1* and *M3* in either order.
  - Draw a picture to see why this is violates the spirit of the triangle inequality.

Tagging Messages

- It's a very good idea to include "tags" with messages.
- This prevents your process from receiving an unintended message:

  "Oh, I forgot that another process was going to send me that. I thought it would happen later."

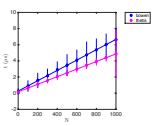
- For example, my accumulator might be better if instead of just receiving an integer, it received

  `{2, add}`

Timing Measurements

- We write parallel code to solve problems that would take too long on a single CPU.
- To understand performance trade-offs, I'll measure the time for some common operations in Erlang programs:
  - The time to make *N* recursive tail calls.
  - The time to spawn an Erlang process.
  - The time to send and receive messages:
    - Short messages.
    - Messages consisting of lists of varying lengths.

Tail Call Time



`bowen.ugrad.cs.ubc.ca`:

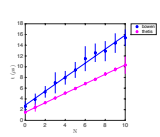
$t = (6.4N + 269)$ns,	line of best fit
$t = 64.3\mu s$,	$N = 10K$
$t = 640\mu s$,	$N = 100K$

`thetis.ugrad.cs.ubc.ca`:

$t = (4.7N + 170)$ns,	line of best fit
$t = 46.9\mu s$,	$N = 10K$
$t = 466\mu s$,	$N = 100K$

- Measurement: start the timing measurement, make *N* tail calls, end the timing measurement.
- The measurements on this slide and throughput the lecture were made using the `time_it:t` function from the course Erlang library.
  - `time_it:t(Fun)` repeatedly calls *Fun* until about one second has elapsed. It then reports the average time and standard deviation.
  - `time_it:t` has lots of options.

Process Spawning Time



`bowen.ugrad.cs.ubc.ca`:

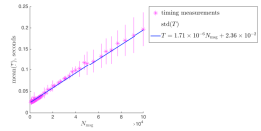
$t = (1.30N + 2.8)\mu s$,	line of best fit
$t = 127\mu s$,	$N = 100$
$t = 1.2$ms,	$N = 1000$

`thetis.ugrad.cs.ubc.ca`:

$t = (0.88N + 1.5)\mu s$,	line of best fit
$t = 89.4\mu s$,	$N = 100$
$t = 887\mu s$,	$N = 1000$

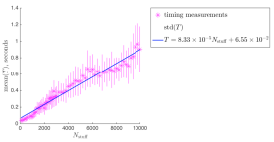
- Measurement: root spawns *Proc1*; *Proc1* spawns *Proc2*, and then *Proc1* exits; *Proc2* spawns *Proc3*, and then *Proc2* exits; ...; *ProcN* sends a message to the root process, and then *ProcN* exits. The root process measures the time from just before spawning *Proc1* until receiving the message from *ProcN*.

Send+Receive Time



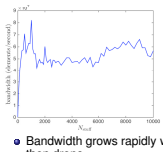
- Set-up: Two processes do a fixed amount of "work" while exchanging short messages with non-blocking receives.
- N_{msg} is the number of messages sent and received by each process.
- The slope of the line is the time per message:
  - $\sim 1.7\mu s$/message on `thetis.ugrad.cs.ubc.ca`, `erts` 18.2.
  - My laptop is about three-times faster. I'm running `erts` 19.2.

Message Time vs. Message Size



- Set-up: as on the previous slide. This time each message consists of a list of N_{stuff} small integers.
- Each process sends and receives 5000 messages per run.
- The slope of the line divided by 5000 is the time per element:
  - ~ 17ns/message on `thetis.ugrad.cs.ubc.ca`, `erts` 18.2.

Bandwidth vs. Message Size



Subtract the "non-message" time from the run-time and calculate:

$$\frac{N_{\text{msg}} \times N_{\text{stuff}}}{T}$$

To get elements per second.

- Bandwidth grows rapidly with message length for $N_{\text{stuff}} < 1000$, then drops.
  - Short messages have low bandwidth due to fixed overheads with each message.
  - I'm guessing that bandwidth drops some for messages with more than 1000 elements because the Erlang runtime is somehow optimized for short messages.

Summarizing the numbers

- Interprocess operations such as `spawn`, `send`, and `receive` are **much** slower than operations within a single process such as `+` or a function call.
- An Erlang tail call is about 4.7ns, roughly 10 machine instructions.
- An Erlang tail call and add is about 4.7ns, roughly 10 machine instructions.
- Spawning a process is about 200× the cost of a tail call.
- For short messages, send and receive are about 350× the cost of a tail call.
  - The send/receive overhead can be amortized by sending longer message.
  - Each additional list element is about 3× the cost of a tail call.
  - Beware of any model that just counts the overhead and ignores the length, or just considers bandwidth and ignores the overhead.
- We will often refer to the ratio of the relationship between the time for interprocess operations and local operations as **big**.
  - In practice, **big** is 100 to 10000 for shared-memory computers.
  - **Big** can be even bigger for other architectures.

How to Write Efficient Parallel Code

- Think about **communication costs**
  - Message passing is good – it makes communication explicit.
  - Pay attention to both the number of messages and their size.
  - Combining small messages into larger ones often helps.
- Think globally, but **compute locally**
  - Move the computation to the data, not the other way around.
  - Keep the data distributed across the parallel processes.
- Think about **big–*O***
  - If *N* is the problem size, you want the computation time to grow faster with *N* than the communication costs.
  - Then, your solution becomes more efficient for larger values of *N*.

Summary

- Processes are easy to create in Erlang.
  - The `spawn` mechanism can be used to start other processors on the same CPU or on machines spread around the internet.
- Processes communicate through messages
  - Message passing is asynchronous.
  - The receiver can use patterns to select a desired message.
- Reactive processes are implemented with tail-recursive functions.
- Interprocess operations are much slower than local ones
  - This is a key consideration in designing parallel programs.
  - We'll learn **why** when we look at parallel architectures later this month.

Preview

January 11: Reduce	
Reading:	*Learn You Some Erlang*, Errors and Exceptions through A Short Visit to Common Data Structures
January 13: Scan	
Reading:	Lin & Snyder, chapter 5, pp. 112–125
Mini-Assignment:	Mini-Assignment 2 due **10:00am**
January 16: Generalized Reduce and Scan	
Homework:	Homework 1 deadline for early-bird bonus (11:59pm)
	Homework 2 goes out (due Feb. 1) – Reduce and Scan
January 18: Reduce and Scan Examples	
Homework:	Homework 1 due **11:59pm**
January 20–27: Parallel Architecture	
January 29–February 6: Parallel Performance	
February 8–17: Parallel Sorting	

Review Questions

- How do you spawn a new process in Erlang?
- What guarantees does Erlang provide (or not) for message ordering?
- Give an example of using patterns to select messages.
- Why is it important to use a tail-recursive function for a reactive process?
  - In other words, why is it a bad idea to use a head-recursive function for a reactive process.
  - The answer isn't explicitly on the slides, but you should be able to figure it out from what we've covered.
- Modify one of the examples in this lecture to use a time-out with one or more `receive` operations. Try it and show that it works.
- Implement the message flushing described in *LYSE* to show pending messages on a time-out. Demonstrate how it works.

Supplementary material

- Debugging concurrent Erlang Code.
- Table of contents.

Tracing Processes

When you implement a reactive process, it can be handy to trace the execution. Here's a simple approach:

- Add an `io:format` call when entering the function and after matching each receive pattern.
- Example:

```
acc_proc(Tally) ->
    io:format("~p:  acc_proc(~b)~n", [self(), Tally]),
    receive
      N when is_integer(N) ->
        io:format("~p:  received ~b~n", [self(), N]),
        acc_proc(Tally+N);
      Msg = {Pid, total}
        io:format("~p:  received ~p~n", [self(), Msg]),
        Pid ! Tally,
        acc_proc(Tally)
    end.
```

- Try it (e.g. with the example from slide 7.
- Don't forget to delete (or comment out) such debugging output before releasing your code.

Time Outs

- If your process is waiting for a message that never arrives, e.g. because
  - You misspelled a tag for a message, or
  - The receive pattern is slightly different than the message that was sent, or
  - Something went wrong in the sending process, and it died before sending the message, or
  - You got the message ordering slightly wrong, and there's a cycle of processes waiting for each other to send something, or
  - ...
- Then your process can wait forever, your Erlang shell can hang, and it's a very unhappy time in life.
- Time-outs can handle these problems more gracefully.
  - See Time Out in *LYSE*.
  - Note: time-outs are great for debugging. They should be used with great caution elsewhere because they are sensitive to changes in hardware, changes in the scale of the system, and so on.

Table of Contents

Reduce

Mark Greenstreet

CpSc 418 – Jan. 11, 2017

Outline:

Objectives

- Understand why using a tree-structure for communication improves efficiency.
- Learn how to implement reduce using Erlang processes and messages.
- Learn how to use the `reduce` function in the course Erlang library.

Problem Statement

Given a list, *L* of *N* values, how can we use *P* processors to efficiently compute the sum of the values of the elements?

Possible in-class exercise:

- Divide class into groups of five or six.
- Hand each group a sheet of numbers. We could arrange the numbers is blocks, or perhaps hand each group a stack of five or six sheets, each of which has around 10 small integers (one to three digits) to add.
- Give them the task that the team that computes the sum of the numbers first wins (perhaps have a bag of M&M's or similar as a prize.
- See what they do.

Now, go back to the observations we made from the previous lecture.

Summarizing the numbers

- Interprocess operations such as `spawn`, `send`, and `receive` are **much** slower than operations within a single process such as `+` or a function call.
- Let's use a tail-call as the cost of an operation within a process.
- Spawning a process is about 200× the cost of a tail call.
- For short messages, send and receive are about 350× the cost of a tail call.
- For longer messages, the time grows with message length. Sending 100 numbers takes about twice as long as sending 1.

My guess is that many of the groups had a "team captain" who handed out the sheets of paper. Each team member would compute their local sum and report it to the team captain. The team captain computed the final sum and reported it (FTW).

We can do a bit of front-of-class theatre, you, Devon, me (and if we could get one or two others that would be great). Act it out with **slow** communication actions, e.g. "tai chi" style. Two problems should become apparent: starting where the team captain has all the data and distributes it is a bottleneck. Having everyone communicate send their result directly to the captain is a bottleneck.

How to Write Efficient Parallel Code

This is a review from the previous lecture.

- Think about **communication costs**
  - Message passing is good – it makes communication explicit.
  - Pay attention to both the number of messages and their size.
  - Combining small messages into larger ones often helps.
- Think globally, but **compute locally**
  - Move the computation to the data, not the other way around.
  - Keep the data distributed across the parallel processes.
- Think about **big–*O***
  - If *N* is the problem size, you want the computation time to grow faster with *N* than the communication costs.
  - Then, your solution becomes more efficient for larger values of *N*.

Interactive Exercise

- Design an efficient way to add N numbers using P processes.
- Should plan to start with each process having $\sim N/P$ values – this is the "Keep the data distributed across the parallel processes concept.
- Should "discover" the tree structure for communication
  - Point to bring out: we are not using a tree to get more parallelism in the final $P-1$ additions. These adds don't take long enough to matter. The $P-1$ communication actions **do** matter.
  - Reducing the depth of the communication actions from $P-1$ (when the team captain handles all of them) to $\log P$ is what matters.

Now, translate it into code

- Let $N = 2^K$.
- We can have one process that creates a binary tree with N leaves.
- Each leaf process:
  - waits to receive a task with a tag.
  - does the task.
  - sends the result to its parent (with a parent provided tag).
- Each intermediate node:
  - Waits to receive a two functions and a tag.
  - Call the two functions *LeafTask* and *Combine*.
  - The node sends the two functions with a `left` or `right` tag to its children.
  - The node receives results from its children, combines them with *Combine* and sends the result with the parent provided tag to its parent.
  - We now discover that the leave probably receives {*LeafTask*, *Combine*, `LeftRightTag, PPid`} just like the intermediate nodes. The leaves just ignore the *Combine*.

Finish the code sketch

- The function for the process is pretty much like the intermediate nodes except
  - There's one function to create the tree.
  - Another function takes a process tree and the *LeafTask* and *CombineTask* functions and sends the result home.

It's not quite that simple

- We want the leaf nodes to generate their arrays as one task, and compute the sums as another task.
- This means that the children need to "remember" state between the two tasks.
- There are a several possible solutions:
  - The *LeafTask* function could take an argument of `ProcState` and return a tuple of `{ValueForCombine, NewProcState}`.
  - The leaf process makes its recursive call with `NewProcState`.
  - Or, we could use the Erlang process dictionary with `erlang:put` and `erlang:get`, but that's not very functional. I prefer the `ProcState` approach.
- And, we need to deal with end-of-life issues.
  - Use the atom `exit` instead of the {*LeafTask*, *Combine*} tuple.

We can do better

- **Note:** I'm not sure how far we can make it through this material with the various in-class activities. If we make it through the simple implementation of reduce on the previous slide, I'm happy. The rest of this is optional – equivalently, it's material we could move into the Jan. 13 or Jan. 16 lecture.
- With the design above, half of the processes sit around idle, while waiting for the leaves to do their work.
- We can make a tree where each process forwards messages to its right subtree(s) and then does its own *LeafTask*.
- I'm sure I've got a figure from some previous year, I'll find it.

But we don't need to code the better version in class

- The better version is implemented in the course Erlang library.
- We now show how to do the reduce example with `wtree:reduce`.

Summary

Preview

After we make it through reduce, we'll cover

- scan – I want to have one lecture on scan by the end of the Jan. 16 lecture to have the students at a place that they can start on HW2.
- generalized scan and reduce.
- Then, we transition to $\sim$4 lectures on parallel architectures.

Review Questions

Scan

Mark Greenstreet

CpSc 418 – Jan. 13, 2017

Outline:

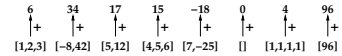
- Reduce Redux
  - The basic algorithm.
  - Performance model.
  - Implementation considerations.
- Scan
  - Understand how reduce generalizes to a method that produces all N values for a "cumulative" operation in $O(\log N)$ time.
- A few implementation notes

Reduce Redux

Problem statement:
Given P processes that each hold part of an array of numbers, compute the sum of all the numbers in the combined array.

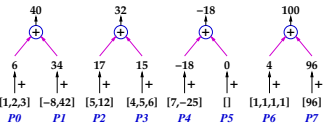
Reduce Redux

Accumulate step:
Each process computes the total of the elments in its local part of the array.



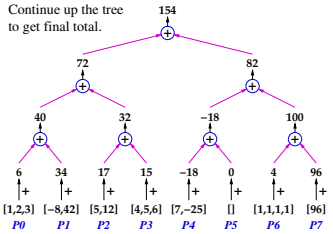
Reduce Redux

Combine step:
Each process sends its result to a coombiner process. The combiners compute the sums of the values from adjacent pairs of processes.



Reduce Redux

Continue up the tree to get final total.



Reduce Notes

- For simplicity, I drew the tree as if we used separate processes for accumulating the local arrays and doing the combining.
  - In practice, we use the same processes for both accumulating and combining.
  - Note that ½ of the processes are active in the first level of combine; ¼ of the processes are active in the second level; and so on.
- Simple time model:

$$T \in O\left(\frac{N}{P} + \lambda \log P\right)$$

where λ is **big** – i.e. the communication time.

Scan Problem Statement

- Given an array, A, with N elements.
  - Let $B = \text{scan}_+(A)$:

$$B_i = \sum_{k=0}^{i} A_k$$

  - Example:

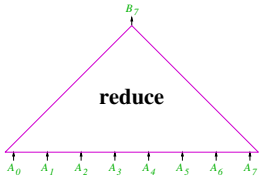
$$A = [1, 2, 3, -8, 42, 5, 12, 4, 5, 6, 7, -25, 1, 1, 1, 1, 96]$$
$$B = [1, 3, 6, -2, 40, 45, 57, 61, 66, 72, 79, 54, 55, 56, 57, 58, 154]$$

- Is there an efficient parallel algorithm for computing $\text{scan}_+(A)$?
  - I wrote scan_+ because our solution works for any associative operator.

Scan Example: Monthly Bank Statement

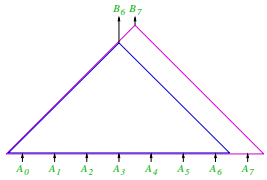
- Assumptions:
  - You make **lots** of transactions; so, the bank needs to use a parallel algorithm just for your account.
  - Months have 32 days – the power-of-two version of the algorithm is simpler. It generalizes to any number of processors.
  - Each processes has the transaction data for one day.
- Using parallel scan:
  - Each process computes the total of the transactions for its day.
  - Using parallel scan, we determine the balance at the beginning of each day for each process.
  - The process can use its start-of-day balance, and compute the balance after each transaction for that day.

Brute force Scan



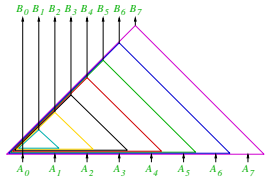
- Use a reduce tree to compute B_7.

Brute force Scan



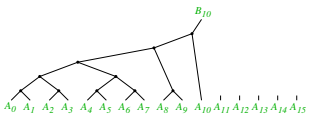
- Use a reduce tree to compute B_7.
- Use another reduce tree to compute B_6.

Brute force Scan



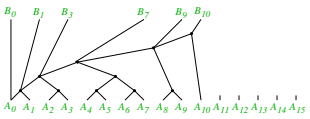
- Use a reduce tree to compute B_7.
- Use another reduce tree to compute B_6.
- Use 6 more reduce trees to compute $B_{5\ldots0}$
- It works. It's $O(\log P)$ time! But it's not very efficient.

Reuse trees



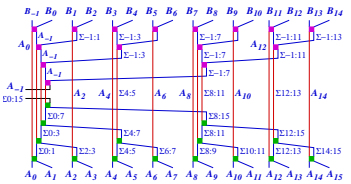
- Key idea: we don't need the trees to be balanced.
- We just want them to be $O(\log P)$ in height.
- If we need a tree for 2^K nodes, we'll make a balanced tree.
- Otherwise:
  - Make the largest balanced tree we can on the left.
  - Repeat this process for what's left on the right.

Reuse trees



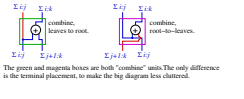
- Key idea: we don't need the trees to be balanced.
- We just want them to be $O(\log P)$ in height.
- If we need a tree for 2^K nodes, we'll make a balanced tree.
- Otherwise:
  - Make the largest balanced tree we can on the left.
  - Repeat this process for what's left on the right.
- Notice that while computing B_{10}, we produced many other of the Bs as intermediate results.

Scan



- See the next slide for an explanation of the notation, etc.

Scan: explained



- Notation
  - A_{-1} is initializer for the sum.
  - $A_0, A_1, \ldots A_{15}$ is the initial array.
  - $B_{-1}, B_1, \ldots B_{15}$ is the result of the scan.

$$B_i = \sum_{k=-1}^{i} A_k, \text{ Include the initializer } A_{-1}$$

  - $\Sigma i : j$ is shorthand for $\sum_{k=i}^{j} A_k$
- Each process needs to compute its local part of the scan at the end, starting from the value it receives from the tree.

A few implementation notes

- On slide 3 I pointed out that for efficiency, it is better to use the same processes for the leaves and the combine.

```
% reduce:
   treeLevels = ceil(log2(NProcs0);
   tally = localAccumulate(...);
   for(k = 0; k < treeLevels; k++) {
      if((myPid & (1 << k)) != 0) {
         send(myPid - (1 << k), myPid, tally);
         break;
      } else
         tally += receive(myPid + (1<< k));
   } // Process 0 now has the grand total.
   // We can use another loop to broadcast the result.
```

- I'll provide an Erlang version on Monday.

Reduce & Scan

Scan is very similar to reduce. We just change the downward tree.

- For reduce, each process just forwards the grand total to its descendants.
- For scan:
  - Each process records the tallies from its left subtree(s) during the upward sweep.
  - During the downward sweep, each process receives the tally for **everything to the left of the subtree for this process**.
    - The process adds the tally from its own left subtree to the value from its parent, and sends this to its own right subtree.
    - The process continues the downward sweep for its own left subtree.
    - When we reach a leaf, the process does the final accumulate.

Preview

January 16: Generalized Reduce and Scan
Homework: Homework 1 deadline for early-bird bonus (11:59pm)
Homework 2 goes out (due Feb. 1) – Reduce and Scan
January 18: Reduce and Scan Examples
Homework: Homework 1 due **11:59pm**
January 20: Architecture Review
Reading: Pacheco, Chapter 2, through section 2.2
January 23: Shared Memory Architectures
Reading: Pacheco, Chapter 2, through section 2.3
Homework: Homework 2 deadline for early-bird bonus (11:59pm)
Homework 3 goes out (due Feb. 17)
January 25: Message Passing Architectures
Homework: Homework 2 due **11:59pm**
January 27–February 6: Parallel Performance
February 8–17: Parallel Sorting

Review Questions

- What is the cumulative sum of `[1,7,-5,12,73,19,0,12]`?
  - For the same list as above, what is the cumulative product?
  - For the same list as above, what is the cumulative maximum?
- Draw a tree showing how the sum (simple, not cumulative) of the values in the list above can be computed using reduce. Assume that there are eight processes, and each starts with one element of the list.
- Draw a graph like the one on slide 8 for a scan of eight values.
- Label each edge of your graph with the value that will be sent along that edge when computing the cumulative sum of the values in the list above. Assume that there are eight processes, and each starts with one element of the list.
- Add a second label to each edge indicating whether the value is local to that process or if the edge requires inter-process communication. Write 'L' for local, and 'G' for global (i.e. inter-process communication).

Generalize Reduce and Scan

Mark Greenstreet

CpSc 418 – Jan. 16, 2017

Outline:

- Reduce in Erlang
- Scan in Erlang

Objectives

- Understand relationship between reduce and scan
  - Both are tree walks.
  - The initial combination of values from leaves is identical.
  - Reduce propagates the grand total down the tree.
  - Scan propagates the total "everything to the left" down the tree.
- Generalized Reduce and Scan
  - Understand the role of the *Leaf*, `Combine`, and `Root` functions.
  - Understand the use use of higher-order functions to implement reduce and scan.
- The CS418 class library
  - Able to create a tree of processes.
  - Able to distribute data and tasks to those processes.
  - Able to use the `reduce` and `scan` functions from the library.
  - Know where to find more information.

Reduce in Erlang

- Build a tree.
- Each process creates a lists of random digits.
- The processes meet at a barrier so we can measure the time to count the 3s.
- Each process counts its threes.
- The processes use reduce to compute the grand total.
- Each process reports the grand total and its own tally.
- The root process reports the time for the local tallies and the reduce.
- Get the code at

http://www.ugrad.cs.ubc.ca/~cs418/2016-2/lecture/01-16/code/reduce.erl

The Reduce Pattern

- It's a parallel version of *fold*, e.g. `lists:foldl`.
- Reduce is described by three functions:
  - *Leaf()*: What to do at the leaves, e.g.
    `fun() -> count3s(Data) end.`
  - *Combine()*: What to do at the root, e.g.
    `fun(Left, Right) -> Left+Right end.`
  - *Root()*: What to do with the final result. For count 3s, this is just the identity function.

The `wtree` module

- Part of the course Erlang library.
- Operations on worker trees"
  - `wtree:create(NProcs) -> [pid()].`
    Create a list of `NProcs` processes, organized as a tree.
  - `wtree:broadcast(W, Task, Arg) -> ok.`
    Execute the function `Task` on each process in `W`. Note: `W` means "worker pool".
  - `wtree:reduce(P, Leaf, Combine, Root) -> term().`
    A generalized reduce.
  - `wtree:reduce(P, Leaf, Combine) -> term().`
    A generalized reduce where `Root` defaults to the identity function.

Store Locally

- Communication is expensive – each process should store its own data whenever possible.
- How do we store data in a functional language?
  - Our processes are implemented as Erlang functions that receive messages, process the message, and make a tail-call to be ready to receive the next message.
  - We add a parameter to these functions, `State`, that is a mapping from *Keys* to *Values*.
- What this means when we write code:
  Functions such as *Leaf* for `wtree:reduce` or *Task* for `wtree:broadcast` have a parameter for `State`.
  - `worker:put(State, Key, Value) -> NewState.`
    Create a new version of `State` that associates `Value` with `Key`.
  - `worker:get(State, Key, Default) -> Value.`
    Return the value associated with `Key` in `State`. If no such value is found, `Default` is returned. Note: `Default` can be a function in which case it is called to determine a default value – see the documentation.

Count3s using `wtree`

```
count3s_par(N, P) ->
    W = wtree:create(P),
    wtree:rlist(W, N, 10, 'Data'),
    wtree:reduce(W,
        fun(ProcState) ->
            count3s(workers:get(ProcState, 'Data'))
        end,
        fun(Left, Right) -> Left+Right end
    ).
```


Reduce and Scan

- The root node:
  - Reduce: `count3s_reduce(none, [], Total3s) -> Total3s;`
  - Scan: `count3s_scan(none, [], Total3s) -> 0;`
- Internal nodes:

```
% Reduce:
count3s_reduce(Parent, [Child | MoreKids], ThreesInLeftSubtree) ->
    ThreesInRightSubtree = count3s_wait(Child),
    ThreesInMyTree = ThreesInLeftSubtree + ThreesInRightSubtree,
    Total3s = count3s_reduce(Parent, MoreKids, ThreesInMyTree),
    count3s_notify(Child, Total3s).

% Scan:
count3s_scan(Parent, [Child | MoreKids], ThreesInLeftSubtree) ->
    ThreesInRightSubtree = count3s_wait(Child),
    ThreesInMyTree = ThreesInLeftSubtree + ThreesInRightSubtree,
    ThreesToMyLeft = count3s_scan(Parent, MoreKids, ThreesInMyTree),
    count3s_notify(Child, ThreesToMyLeft + ThreesInLeftSubtree),
    ThreesToMyLeft.
```


Scan in Erlang

- Remarkably like reduce.
- Reduce has
  - an upward pass to compute the grand total
  - a downward pass to broadcast the grand total.
- Scan has
  - an upward pass where the grand total – **just like reduce**
  - On the downward pass, we compute the total of all elements to the left of each subtree.
- Get the code at

http://www.ugrad.cs.ubc.ca/~cs418/2016-2/lecture/01-16/code/scan.erl

The Scan Pattern

- It's a parallel version of *mapfold*, e.g. `lists:mapfoldl` and `lists:mapfoldr`.
- `wtree:scan(`*Leaf1, Leaf2, Combine, Acc0*`)`
  - *Leaf1*`(`*ProcState*`) -> `*Value*
    Each worker process computes its *Value* based on its *ProcState*.
  - *Combine*`(`*Left*`, `*Right*`) -> `*Value*
    Combine values from sub-trees.
  - *Leaf2*`(`*ProcState*`, `*AccIn*`) -> `*ProcState*
    Each worker updates its state using the *AccIn* value – i.e. the accumulated value of everything to the worker's "left".
  - *Acc0*: The value to use for *AccIn* for the leftmost nodes in the tree.

Scan example: prefix sum

```
prefix_sum_par(W, Key1, Key2) ->
    wtree:scan(W,
        fun(ProcState) -> % Leaf1
            lists:sum(wtree:get(ProcState, Key1)) end,
        fun(ProcState, AccIn) -> % Leaf2
            wtree:put(ProcState, Key2,
                prefix_sum(wtree:get(ProcState, Key1), AccIn)
            ) end,
        fun(Left, Right) -> % Combine
            Left + Right end,
        0 % Acc0
    ).

prefix_sum(L, Acc0) ->
    element(1,
        lists:mapfoldl(fun(X, Y) -> Sum = X+Y, {Sum,Sum} end,
                       Acc0, L).
```


More Examples of scan

- Account balance with interest:
  - Input: a list of transactions, where each transaction can be a deposit (add an amount to the balance), a withdrawal (subtract an amount from the balance), or interest (multiply the balance by an amount). For example:
    `[{deposit, 100.00}, {withdraw, 5.43}, {withdraw, 27.75}]`
  - Output: the account balance after each transaction. For example, if we assume a starting balance of $1000.00 in the previous example, we get
    `[1100.00, 1094.57, 1066.82, 1067.40, ...]`
- Delete 3s
  - Given a list that is distributed across *NProc* processes, delete all 3s, and rebalance the list so each process has roughly the same length sublist.
  - Solution (sketch):
    - Using scan, each process determines how many 3s preceed its segment, the total list length preceeding it, and the total list length after deleting 3s.
    - Each process deletes its 3s and send portions of its lists

Preview

January 18: Reduce and Scan Examples
Homework: Homework 1 due **11:59pm**
January 20: Finish Reduce and Scan
Mini-assignments: Mini assignment 3 goes out.
January 23: Architecture Review
Reading: Pacheco, Chapter 2, through section 2.2
January 27: Shared Memory Architectures
Reading: Pacheco, Chapter 2, through section 2.3
Mini-assignments: Mini assignment 3 due, 10am.
January 27: Message Passing Architectures
January 27–February 6: Parallel Performance
January 30: HW 2 Earlybird due (11:59pm), HW 3 goes out.
February 1: HW 2 due (11:59pm).
February 8–17: Parallel Sorting
February 15: HW 3 Earlybird (11:59pm).
February 17: HW 3 due (11:59pm).
February 27: TBD
March 1: Midterm

Scan

Mark Greenstreet

CpSc 418 – Jan. 20, 2016

Objectives

- Prefix sum
  - Spawning processes.
  - Sending and receiving messages.
- The source code for the examples in this lecture is available here: procs.erl.

Prefix Sum

- Scan is similar to reduce, but every process calculates its cumulative total.
- Example:

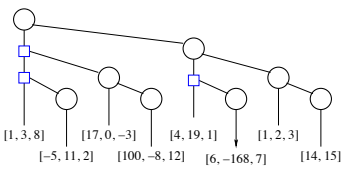
```
% prefix_sum: compute prefix sum.
prefix_sum(L) when is_list(L) -> prefix_sum_tr(L, 0).
prefix_sum_tr([], Acc) -> [];
prefix_sum_tr([H | T], Acc) ->
    MySum = H+Acc,
    [MySum | prefix_sum_tr(T, MySum)].
```

- Let's try it:

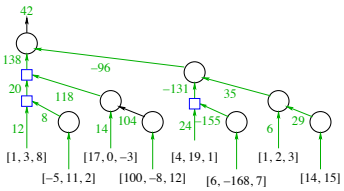
```
1> examples:prefix_sum([1, 13, 2, -5, 17, 0, 33]).
[1,14,16,11,28,28,61]
```

- How can we do this in parallel?

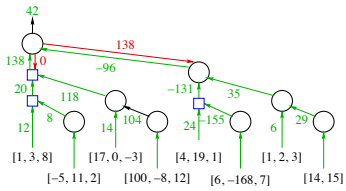
Parallel Prefix Sum



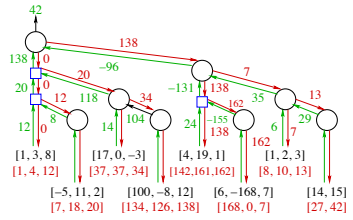
Parallel Prefix Sum



Parallel Prefix Sum



Parallel Prefix Sum



The Scan Pattern

- It's a parallel version of *mapfold*, e.g. `lists:mapfoldl` and `lists:mapfoldr`.
- `wtree:scan(`*Leaf1, Leaf2, Combine, Acc0*`)`
  - *Leaf1*`(`*ProcState*`) -> `*Value*
    Each worker process computes its *Value* based on its *ProcState*.
  - *Combine*`(`*Left*`, `*Right*`) -> `*Value*
    Combine values from sub-trees.
  - *Leaf2*`(`*ProcState*`, `*AccIn*`) -> `*ProcState*
    Each worker updates its state using the *AccIn* value – i.e. the accumulated value of everything to the worker's "left".
  - *Acc0*: The value to use for *AccIn* for the leftmost nodes in the tree.

Scan example: prefix sum

```
prefix_sum_par(W, Key1, Key2) ->
    wtree:scan(W,
        fun(ProcState) -> % Leaf1
            lists:sum(wtree:get(ProcState, Key1)) end,
        fun(ProcState, AccIn) -> % Leaf2
            wtree:put(ProcState, Key2,
                prefix_sum(wtree:get(ProcState, Key1), AccIn)
            ) end,
        fun(Left, Right) -> % Combine
            Left + Right end,
        0 % Acc0
    ).

prefix_sum(L, Acc0) ->
    element(1,
        lists:mapfoldl(fun(X, Y) -> Sum = X+Y, {Sum,Sum} end,
                       Acc0, L)).
```


Prefix Sum Using Scan, example (part 1 of 4)

- Consider the example from slide 4.
  - We'll assume that the original lists for each processes are associated with the key `raw_data`.
  - We'll store the cummulative sum using the key `cooked_data`.
- `Leaf1`: each worker computes the sum of the elements in its list:
  - Worker 0:
    ```
    Leaf1(ProcState) ->
        lists:sum(wtree:get(ProcState, raw_data)) ->
        lists:sum([1,3,8]) ->
    12.
    ```
  - Worker 1:
    `Leaf1(ProcState) -> lists:sum([-5,11,2]) -> 8.`
  - Worker 2:
    `Leaf1(ProcState) -> lists:sum([17,0,-3]) -> 14.`
  - Workers 3–6: . . .
  - Worker 7:
    `Leaf1(ProcState) -> lists:sum([14,15]) -> 29.`

Prefix Sum Using Scan, example (part 2 of 4)

- `Combine` (upward, first round):
  - Worker 0: `Combine(12, 8) -> 20.`
  - Worker 2: `Combine(14, 104) -> 118.`
  - Worker 4: `Combine(24, -155) -> -131.`
  - Worker 6: `Combine(6, 29) -> 35.`
- `Combine` (upward, second round):
  - Worker 0: `Combine(20, 118) -> 138.`
  - Worker 4: `Combine(-131, 35) -> -96.`
- `Combine` (upward, final round):
  - Worker 0: `Combine(138, -96) -> 42.`
  - This value is returned to the caller of `wtree:scan`.

Prefix Sum Using Scan, example (part 3 of 4)

- `Combine` (downward)
- The root sends `AccIn`, `0` to the left subtree.
- Each worker that did a combine remembers the arguments from the upward combines, and uses them in the downward sweep. In the code, each upward step is a recursive function call, and each downward step is a return.
- `Combine` (downward, first round)
  - Worker 0: `Combine(0, 138) -> 138.`
  - The `0` is `AccIn` from the root.
  - The `138` is the stored value from the left subtree.
  - Worker 0 sends this result to its right subtree, worker 4.
- `Combine` (downward, second round)
  - Worker 0: `Combine(0, 20) -> 20`. Send to worker 2.
  - Worker 4: `Combine(138, -131) -> 7`. Send to worker 6.
- `Combine` (downward, third round)
  - Worker 0: `Combine(0, 12) -> 12`. Send to worker 1.
  - Worker 2: `Combine(20, 14) -> 34`. Send to worker 3.
  - Worker 4: `Combine(138, 24) -> 162`. Send to worker 5.
  - Worker 6: `Combine(7, 6) -> 13`. Send to worker 7.

Prefix Sum Using Scan, example (part 4 of 4)

- `Leaf2` (update worker state)
  - Worker 0:
    ```
    Leaf2(ProcState, 0) ->
        wtree:put(ProcState, Key2,
            prefix_sum(wtree:get(ProcState, Key1), 0)) ->
        wtree:put(ProcState, Key2,
            prefix_sum([1, 3, 8], 0)) ->
        wtree:put(ProcState, Key2, [1, 4, 12]).
    ```
  - Worker 1:
    ```
    Leaf2(ProcState, 0) ->
        wtree:put(ProcState, Key2,
            prefix_sum(wtree:get(ProcState, Key1), 0)) ->
        wtree:put(ProcState, Key2,
            prefix_sum([-5, 11, 2], 12)) ->
        wtree:put(ProcState, Key2, [7, 18, 20]).
    ```
  - Workers 2–7: . . .

Let's Try It

```
2> W = wtree:create(8).
[<0.65.0>,<0.66.0>,<0.67.0>,<0.68.0>
 <0.69.0>,<0.70.0>,<0.71.0>,<0.72.0>]
3> workers:update(W, raw_data,
  [ [1,3,8], [-5,11,2], [17,0,-3], [100,-8,12],
    [4,19,1], [6,-168,7], [1,2,3],  [14,15]]).
ok
4> examples:prefix_sum_par(W, raw_data, cooked_data).  42
5> workers:retrieve(W, cooked_data).
[ [1,4,12], [7,18,20], "%%\"", [134,126,138],
  [142,161,162], [168,0,7], "\b\n\r", "\e*"] 6> $37
```

- Likewise, `$" == 34`, `$== 8`, `$\n == 10`, `$\r == 13`, `$\e == 27`, and `$* == 42`.
- All is well.

More Examples of scan

- Account balance with interest:
  - Input: a list of transactions, where each transaction can be a deposit (add an amount to the balance), a withdrawal (subtract an amount from the balance), or interest (multiply the balance by an amount). For example:
    `[{deposit, 100.00}, {withdraw, 5.43}, {withdraw, 27.75}]`
  - Output: the account balance after each transaction. For example, if we assume a starting balance of $1000.00 in the previous example, we get
    `[1100.00, 1094.57, 1066.82, 1067.40, ...]`
- Delete 3s
  - Given a list that is distributed across *NProc* processes, delete all 3s, and rebalance the list so each process has roughly the same length sublist.
  - Solution (sketch):
    - ⋆ Using scan, each process determines how many 3s preceed its segment, the total list length preceeding it, and the total list length after deleting 3s.
    - ⋆ Each process deletes its 3s and send portions of its lists and/or receives list portions to rebalance.

More[2] Examples of scan

- Carry-Lookahead Addition:
  - Given two large integers as a list of bits (or machine words), compute their sum.
    - Note that the "pencil-and-paper" approach works from the least significant bit (or digit, or machine word) and works sequentially to the most-significant bit. This takes $O(N)$ time where N is the number of bits in the work.
  - Carries can be computed using scan.
    - This allows a parallel implementation that adds two integers in $O(\log N)$ time.
    - This is how the hardware in your CPU does addition – the adder takes $O(\log N)$ gate delays to add two, machine words, where N is the number of bits in a word.
- See *Principles of Parallel Programming*, pp. 119f.
- See homework 2 (later today, I hope).

Preview

January 23: Architecture Review	
Reading:	Pacheco, Chapter 2, Sections 2.1 and 2.2.
January 25: Shared-Memory Machines	
Reading:	Pacheco, Chapter 2, Section 2.3
January 27: Distributed-Memory Machines	
Reading:	Pacheco, Chapter 2, Sections 2.4 and 2.5.
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March 1: Midterm	

Review Questions

- What is scan? Give an example.
- Compare scan with `lists:mapfoldl`?
- What property must an operator have to be amenable use with scan?
- What are the components of a generalized scan?
  As an example, what functions do you need to define to use `wtree:scan`?
- Consider the following variations on the bank account problem:
  - Add a transaction `{reset, Balance}`, where `Balance` is a number. The account balance is set to this amount. For example, this can be used to open an account with an initial balance. We'll also assume that a `reset` can be done at any point in a sequence of transactions.
  - Change interest computations so that the bank charges a daily interest of X% for negative balances, neither charges nor pays interest for positive balances less than $1000, and pays a daily interest of Y% for positive balances greater than $1000.
  - For each of these:
    - Can the account balance still be computed using scan?
    - If yes, explain how to do. If no, explain why it's not possible.

Computer Architecture Review

Mark Greenstreet

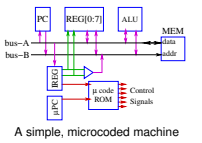
CpSc 418 – Jan. 23, 2017

- A microcoded machine
- A pipelined machine: RISC
- Let's write some code
- Superscalars and the memory bottleneck

Objectives

- Review classical, sequential architectures
  - a simple microcoded, machine
  - a pipelined, one-instruction per clock cycle machine
- Pipelining **is** parallel execution
  - the machine is supposed to appear (nearly) sequential
  - introduce the ideas of hazards and dependencies.

Microcoded machines



A simple, microcoded machine

- The microcode (μcode) ROM specifies the sequence of operations necessary to carry out an instruction.
- For simplicity, I'm assuming that the op-code bits of the instruction form the most significant bits of the μcode ROM address, and that the value of the micro-PC (μPC) form the lower half of the address.

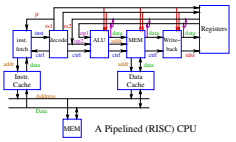
Microcode: summary

- Separates hardware from instruction set.
  - Different hardware can run the same software.
  - Enabled IBM to sell machines with a wide range of performance that were all compatible
    - I.e. IBM built an empire and made a fortune on the IBM 360 and its successors.
    - Intel has done the same with the x86.
- But, as implemented on slide 3, it's very sequential.

```
while(true) {
  fetch an instruction;
  perform the instruction
}
```

- Instruction fetch is "overhead"
  - Motivates coming up with complicated instructions that perform lots of operations per instruction fetch.
  - But these are hard for compilers to use.
  - Can we do better?

Pipelined instruction execution



A Pipelined (RISC) CPU

- Successive instructions in each stage
- When instruction `i` in `ifetch`, instruction `i-1` in decode, ...
- Allows throughput of one instruction per cycle.
- Favors simple instructions that execute on a single pass through the pipeline.
  - This is known as RISC: "Reduced Instruction Set Computer"
  - A modern x86 is CISC on the outside, but RISC on the inside.

What about Dependencies?

- Multiple-instructions are in the pipeline at the same time.
- An instruction starts before all of its predecessors have completed.
- Data hazards occur if
  - an instruction can read a different value than would have been read with a sequential execution of instructions,
  - or if a register or memory location is left holding a different value than it would have had in a sequential execution.
- Control hazards occurs if
  - an instruction is executed that would not have been executed in a sequential execution.
  - This is because the instruction "depends" on a jump or branch that hasn't finished in time.

Handling Hazards

- Bypass: If an instruction has a result that a later instruction needs, the earlier instruction can provide that result directly without waiting to go through the register file.
- Move common operations early:
  - Decide branches in decode stage
  - ALU operations in the stage after decode
  - Memory reads take longer, but they happen less often.
- Let the compiler deal with it
- If nothing else helps, stall.

Break for Live Coding

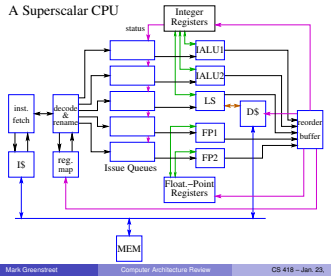
Back to Architecture

- the microcoded machine takes 5+ clock-cycles per instruction.
- the RISC machine takes 1 clock-cycle per instruction – in the best case:
  - There can be stalls due to cache misses,
  - unfilled delay slots, or
  - multi-cycle operations.
- Can we break the one-cycle-per instruction barrier?

The Memory Bottleneck

- A CPU core can execute roughly one instruction per clock-cycle.
  - With a 3GHz clock, that's roughly 0.3ns per instruction.
- Main memory accesses take 60-200ns (or longer)
  - That's 200-600 instructions per main memory access.
- Why?
  - CPUs designed for speed.
  - Memory designed for capacity:
    - fast memories are small
    - large memories are slow

Superscalar Processors



Superscalar Execution

- Fetch several, W, instructions each cycle.
- Decode them in parallel, and send them to issue queues for the appropriate functional unit.
- But what about dependencies?
  - We need to make sure that data and control dependencies are properly observed.
  - Code should execute on a superscalar as if it were executing on sequential, one-instruction-at-a-time machine.
  - Data dependencies can be handled by "**register renaming**" – this uses register indices to dynamically create the dependency graph as the program runs.
  - Control dependencies can be handled by "**branch speculation**" – guess the branch outcome, and rollback if wrong.
- The opportunity to execute instructions in parallel is called **Instruction Level Parallelism**, ILP.

What superscalars are good at

- Scientific computing:
  - often successive loop iterations are independent
  - the superscalar **pipelines** the loop
  - Perform memory reads for loop i, while doing multiplications for loop i-2, while doing additions for loop i-4, while storing the results for loop i-5.
- Commercial computing (databases, webservers, ...)
  - often have large data sets and high cache miss rates.
  - the superscalar can find executable instructions after a cache miss.
  - if it encounters more misses, the CPU benefits from **pipelined** memory accesses.
- Burning lots of power
  - many operations in a superscalar require hardware that grows quadratically with W.
  - basically, all instructions in a batch of W have to compare there register indices with all of the other ones.

Superscalar Reality

- Most general purpose CPUs (x86, Arm, Power, SPARC) are superscalar.
- Register renaming works **very** well:
- Branch prediction is also very good, often $> 90\%$ accuracy.
  - But, data dependent branches can cause very poor performance.
- Superscalar designs make multi-threading possible
  - The features for executing multiple instruction in parallel work well for mixing instructions from several threads or processes – this is called "multithreading" (or "hyperthreading", if you're from Intel).
  - In practice, superscalars are often *better at multithreading* than they are at extracting ILP from a sequential program.

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Review

- How does a pipelined architecture execute instruction in parallel?
- What are hazards?
- What are dependencies?
- What is multithreading.
- For further reading on RISC:
  "Instruction Sets and Beyond: Computers, Complexity, and Controversy"
  R.P. Colwell, *et al.*, *IEEE Computer*, vol. 18, no. 3,
  - You can download the paper for free if your machine is on the UBC network.
  - If you are off-campus, you can use the library's proxy.

Shared Memory Multiprocessors

Mark Greenstreet

CpSc 418 – Jan. 25, 2017

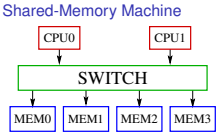
Outline:

- Shared-Memory Architectures
- Memory Consistency
- Coding Break
- Weak Consistency

Objectives

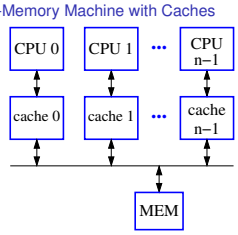
- Understand how processors can communicate by sharing memory.
- Able to explain the term "sequential consistency"
  - Describe a simple cache-coherence protocol, MESI
  - Describe how the protocol can be implemented by snooping.
  - Describe "sequential consistency".
  - Be aware that real machines make guarantees that are weaker than sequential consistency.

An Ancient Shared-Memory Machine



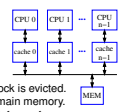
- Multiple CPU's (typically two) shared a memory
- If both attempted a memory read or write at the same time
  - One is chosen to go first.
  - Then the other does its operation.
  - That's the role of the switch in the figure.
- By using multiple memory units (partitioned by address), and a switching network, the memory could keep up with the processors.
- But, now that processors are 100's of times faster than memory, this isn't practical.

A Shared-Memory Machine with Caches



- Caches reduce the number of main memory reads and writes.
- But, what happens when a processor does a write?

Cache Inconsistency

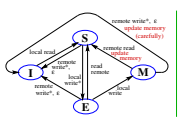


- Assume caches are write-back:
  - **write-back**: writes only update the cache. Main memory updated when the cache block is evicted.
  - **write-through**: writes update cache and main memory.
  - Modern processors have to use write-back for performance: Main memory is way too slow for write-through.
- Step 0: `CPU 0` and `CPU 1` have both read memory location `addr0` and `addr1` and have copies in their cache.
- Step 1: `CPU 0` writes to `addr0` and `CPU 1` writes to `addr1`.
- Step 2: `CPU 0` reads from `addr1` and `CPU 1` reads to from `addr0`.
  - Both CPUs see the **old** value.
  - The writes only updated the writer's cache.
  - The readers got the old values.

Cache Coherence Protocols

- **Big idea:** caches communicate with each other so that:
  - Multiple CPUs can have read-only copies for the same memory location.
  - If a cache has a dirty block, then no other cache has a copy of that block.

The MESI protocol

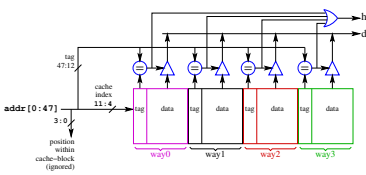


- Caches can share read-only copies of a cache block.
- When a processor writes a cache block, the first write goes to main memory.
  - The other caches are notified and invalidate their copies.
  - This ensures that writeable blocks are exclusive.

How caches work

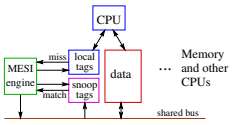
- Caching rhymes with hashing and the two ideas are similar.
  - Caches store data in "blocks" – the block size is a small power-of-two times the machine word size.
  - A cache has one or more "ways" – each way holds a power-of-two number number of blocks.
  - A hash-value is computed from the address.
    - `blockAddr = addr / blockSize; % right shift`
    - `blockIndex = blockAddr % (BlocksPerWay-1); % bit masking`
- Read:
  - The `blockIndex` is used to look up one entry in each "way".
  - Each block has a tag that includes the full-address for the data stored in that block.
  - The tags from each way are compared with the tag of the address:
    - If any tag matches, that way provides the data.
    - If no tags match, then a cache miss occurs.
    - Some current block is evicted from the cache to make room for the incoming block.
- Writes are similar to reads.

A typical cache



- Only the read-path is shown. Writing is similar.
- This is a 16K-byte, 4-way set-associative cache, with 16 byte cache blocks.

Implementing MESI: Snooping



- Caches read and write main memory over a shared memory bus.
- Each cache has two copies of the tags: one for the CPU, the other for the bus.
- If the cache sees another CPU reading or writing a block that is in this cache, it takes the action specified by the MESI protocol.

Implementing MESI: Directories

- Main memory keeps a copy of the data and
  - a bit-vector that records which processors have copies, and
  - a bit to indicate that one processor has a copy and it may be modified.
- A processor accesses main memory as required by the MESI protocol.
  - The memory unit sends messages to the other CPUs to direct them to take actions as needed by the protocol.
  - The ordering of these messages ensures that memory stays consistent.
- Comparison:
  - Snooping is simple for machines with a small number of processors.
  - Directory methods scale better to large numbers of processors.

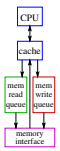
Sequential Consistency

Memory is said to be sequentially consistent if

- All memory reads and writes from all processors can be arranged into a single, sequential order, such that:
  - The operations for each processor occur in the global ordering in the same order as they did on the processor.
  - Every read gets the value of the preceding write to the same address.
- Sequential consistency corresponds to what programmers think "ought" to happen.
  - Very similar to "serializability" for database transactions.
- MESI guarantees sequential consistency

Coding Break

Weak Consistency



- CPUs typically have "write-buffers" because memory writes often come in bursts.
- Typically, reads can move ahead of writes to maximize program performance.
- Why?
  - Because there may be instructions waiting for the data from a load.
  - A transition from "shared" to "modified" requires notifying all processors – this can take a long time.
  - Memory writes don't happen until the instruction commits.
- This means that real computers don't guarantee sequential consistency.
  - Warning: classical algorithms for locks and shared buffers fail when run on a real machines!

Programming Shared Memory Machines

- Shared memory make parallel programming "easier" because:
  - One thread can pass an entire data structure to another thread just by giving a pointer.
  - No need to pack-up trees, graphs, or other data structures as messages and unpack them at the receiving end.
- Shared memory make parallel programming harder because:
  - It's easy to overlook synchronization (control to shared data structures). Then, we get data races, corrupted data structures, and other hard-to-track-down bugs.
  - A defensive reaction is to wrap *every* shared reference with a lock. But locks are slow (that λ factor for communication), and this often results is slow code, or even deadlock.
- In practice, shared memory code that works often has a message-passing structure.
- Finally, beware of weak consistency
  - Use a thread library.
  - There are elegant algorithms that avoid locking overhead, even with weak consistency, but they are beyond the scope of this class.

Shared Memory and Performance

- Shared memory can offer better performance than message passing because
  - High bandwidth: the buses that connect the caches can be very wide, especially if the caches are on a single chip.
  - Low latency: the hardware handles moving the data – no operating system calls and context-switch overheads.
- But, shared memory doesn't scale as well as message passing
  - For large machines, the latency of directory accesses can severely degrade performance.
    - In a message passing machine, each CPU has its own memory, nearby and fast.
    - For shared memory, each CPU has part of the shared main memory – accessing a directory may require accessing the memory of a distant CPU.
  - Shared memory moves the data after the cache miss
    - this stalls a thread
    - message passing can send data in advance and avoid these stalls

Summary

- Shared-Memory Architectures
  - Use cache-coherence protocols to allow each processor to have its own cache while maintaining (almost) the appearance of having one shared memory for all processors.
    - A typical protocol: MESI
    - The protocol can be implemented by snooping or directories.
  - Using cache-memory interconnect for interprocessor communication provides:
    - High-bandwidth
    - Low-latency, but watch out for fences, etc.
    - High cost for large scale machines.
- Shared-Memory Programming
  - Need to avoid interference between threads.
    - Assertional reasoning (e.g. invariants) are crucial, much more so than in sequential programming.
    - There are too many possible interleavings to handle intuitively.
    - In practice, we don't formally prove complete programs, but we use the ideas of formal reasoning.
  - Real computers don't provide sequential consistency.
    - Use a thread library.

Preview

January 27: Distributed-Memory Machines	
Reading:	Pacheco, Chapter 2, Sections 2.4 and 2.5.
Mini Assignments	Mini 4 goes out.
January 30: Parallel Performance: Speed-up	
Reading:	Pacheco, Chapter 2, Section 2.6.
Homework:	HW 2 earlybird (11:59pm). HW 3 goes out.
February 1: Parallel Performance: Overheads	
Homework:	HW 2 due (11:59pm).
February 3: Parallel Performance: Models	
Mini Assignments	Mini 4 due (10am)
February 6: Parallel Performance: Wrap Up	
January 8–February 15: Parallel Sorting	
Homework (Feb. 15):	HW 3 earlybird (11:59pm), HW 4 goes out.
February 17: Map-Reduce	
Homework:	HW 3 due (11:59pm).
February 27: TBD	
March 1: Midterm	

Review

- What is sequential consistency?
- Using the MESI protocol, can multiple processors simultaneously have entries in their caches for the same memory address?
- Using the MESI protocol, can multiple processors simultaneously modify entries in their caches for the same memory address?
- How can a cache-coherence protocol be implemented by snooping?
- How can a cache-coherence protocol be implemented using directories?
- What is false sharing (in the reading, but not covered in these slides)?
- Do real machines provide sequential consistency?
- How do these issues influence good software design practice?

Classifying Cache Misses

- **Compulsory:** The first reference to a cache block will cause a miss.
  - Note that the first access should be a write – otherwise the location is uninitialized.
  - A cache can avoid stalling the processor by using "allocate on write".
  - If a miss is a write, assign a block for the line, start the main memory read, track which bytes have been written, and merge with the data from memory when it arrives.
- **Capacity:** The cache is not big enough to hold all of the data used by the program.
- **Conflict:** Many active memory locations map to the same cache index.
  - If there are more such references than the associativity of the cache, these will cause conflict misses.
- **Coherence:** A cache block was evicted because another CPU was writing to it.
  - A subsequent read incurs a cache miss.

Cache Design Trade-Offs (1 of 2)

- **Capacity:** Larger caches have lower miss rates, but longer access times. This motivates using multiple levels of caches.
  - L1: closest to the CPU, smallest capacity (16-64Kbytes), fastest access (1-3 clock cycles).
  - L2: typically 128Kbytes to 1Mbyte, 5-10 cycle access time.
  - L3: becoming common, several Mbytes of capacity.
- **Block Size:**
  - Larger blocks can lower miss rate by exploiting spatial locality.
  - Larger blocks can raise miss rate due to conflict and coherence misses.
  - Larger blocks increase miss penalty by requiring more time to transfer all that data.
  - Typical block sizes are 16 to 256 bytes – sometimes block size changes with cache level.

Cache Design Trade-Offs (2 of 2)

- **Associativity:**
  - Increasing associativity generally reduces the number of conflict misses.
  - Increasing associativity makes the cache hardware more complicated.
  - Typical caches are direct mapped to four- or eight-way associative.
  - Associativity doesn't need to be a power of two!
- **Other stuff**
  - cache inclusion: is everything in the L1 also in the L2?
  - interaction with virtual memory: are cache addresses virtual or physical?
  - coherence protocol details:
    Example, Intel uses MESIF, the "F" stands for "forwarding". If a processor has a read miss, and another cache has a copy, one of the caches with a copy will be the "forwarding cache". The forwarding cache provides the data because it's much faster than main memory.
  - error detection and creation – caches + cosmic rays = flipped bits.
  - and all kinds of other optimizations that are beyond the scope of this class.

False Sharing

- False sharing occurs when two CPUs are actively writing different words in the same cache block.
  - Each write forces the other CPU to invalidate its cache block.
  - Each read forces the other CPU to change its cache block from `modified` or `exclusive` to `shared`.
- Example: count 3s
  - Here's an implementation with awful performance.
  - We create a global array of `int`s to hold the accumulators for each process.
  - Each time a process finds a `3`, it writes to its element in the array.
  - This forces the other CPUs whose accumulators are in the same block to invalidate their cache entry.
  - This turns accumulator accesses into main memory accesses.
  - And these accesses are serialized: one CPU at a time.

Message Passing Computers

Mark Greenstreet

CpSc 418 – Jan. 27, 2017

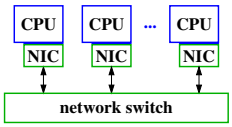
Outline:

- Network Topologies
- Performance Considerations
- Examples

Objectives

- Familiar with typical network topologies:
  rings, meshes, crossbars, tori, hypercubes, trees, fat-trees.
- Understand implications for programming
  - bandwidth bottlenecks
  - latency considerations
  - location matters
  - heterogeneous computers.

Message Passing Computers



- Multiple CPU's
- Communication through a network:
  - Commodity networks for small clusters.
  - Special high-performance networks for super-computers
- Programming model:
  - Explicit message passing between processes (like Erlang)
  - No shared memory or variables.

Some simple message-passing clusters

- 25 linux workstations (e.g. lin01 . . . lin25.ugrad.cs.ubc.ca) and standard network routers.
  - A good platform for learning to use a message-passing cluster.
  - But, we'll figure out that network bandwidth and latency are key bottlenecks.
- A "blade" based cluster, for example:
  - 16 "blades" each with 4 6-core CPU chips, and 32G of DRAM.
  - An "infiniband" or similar router for about 10-100 times the bandwidth of typical ethernet.
  - The price tag is ∼$300K.
    - Great if you need the compute power.
    - But, we won't be using one in this class.

The Sunway TaihuLight

- The world's fastest (Linpack) super-computer (as of June 2016)
- 40,960 multicore CPUs
  - 256 cores per CPU chip.
  - 1.45GHz clock frequency, 8 flops/core/cycle.
- Total of 10,485,760 cores
- LINPACK performance: 93 PFlops
- Power consumption 15MW (computer) + cooling (unspecified)
- Tree-like
  - Five levels of hierarchy.
  - Each level has a high-bandwidth switch.
  - Some levels (all?) are fully-connected for that level.
- Programming model: A version linux with MPI tuned for this machine.
- For more information, see
  Report on the Sunway TaihuLight System, J. Dongarra, June 2016.

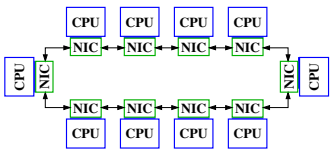
The Westgrid Clusters

- Clusters at various Western Canadian Universities (including UBC).
- Up to 9600 cores.
- Available for research use.

Network Topologies

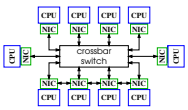
- Network topologies are to the message-passing community what cache-coherence protocols are to the shared-memory people:
  - **Lots** of papers have been published.
  - Machine designers are always looking for better networks.
  - Network topology has a strong impact on performance, the programming model, and the cost of building the machine.
- A message-passing machine may have multiple networks:
  - A general purpose network for sending messages between machines.
  - Dedicated networks for reduce, scan, and synchronization:
    - The reduce and scan networks can include ALUs (integer and/or floating point) to perform common operations such as sums, max, product, all, any, etc. in the networking hardware.
    - A synchronization network only needs to carry a few bits and can be designed to minimize latency.

Ring-Networks



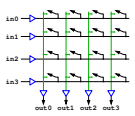
- Advantages: simple.
- Disadvantages:
  - Worst-case latency grows as $O(P)$ where P is the number of processors.
  - Easily congested – limited bandwidth.

Star Networks

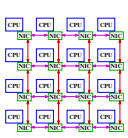


- Advantages:
  - Low-latency – single hop between any two nodes
  - High-bandwidth – no contention for connections with different sources and destinations.
- Disadvantages:
  - Amount of routing hardware grows as $O(P^2)$.
  - Requires lots of wires, to and from switch –
    Imagine trying to build a switch that connects to 1000 nodes!
- Summary
  - Surprisingly practical for 10-50 ports.
  - Hierarchies of cross-bars are often used for larger networks.

A crossbar switch

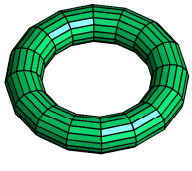


Meshes



- Advantages:
  - Easy to implement: chips and circuit boards are effectively two-dimensional.
  - Cross-section bandwidth grow with number of processors – more specifically, bandwidth grows as $\sqrt{P}$.
- Disadvantages:
  - Worst-case latency grows as $\sqrt{P}$.
  - Edges of mesh are "special cases."

Tori



- Advantages:
  - Has the good features of a mesh, and
  - No special cases at the edges.
- Disadvantages:
  - Worst-case latency grows as $\sqrt{P}$.

Hypercubes

A 0–dimensional (1 node), radix–2 hypercube

Hypercubes

A 1–dimensional (2 node), radix–2 hypercube

Hypercubes

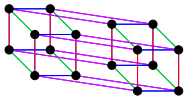
A 2–dimensional (4 node), radix–2 hypercube

Hypercubes

A 3–dimensional (8 node), radix–2 hypercube

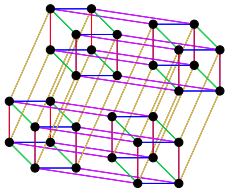
Hypercubes

A 4–dimensional (16 node), radix–2 hypercube



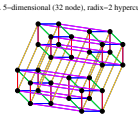
Hypercubes

A 5–dimensional (32 node), radix–2 hypercube



Hypercubes

A 5–dimensional (32 node), radix–2 hypercube

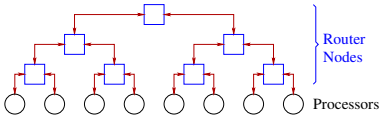


- Advantages
  - Small diameter (log N)
  - Lots of bandwidth
  - Easy to partition.
  - Simple model for algorithm design.
- Disadvantages
  - Needs to be squeezed into a three-dimensional universe.
  - Lots of long wires to connect nodes.
  - Design of a node depends on the size of the machine.

Dimension Routing

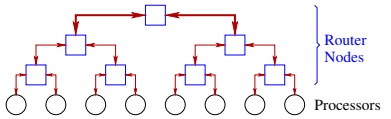
```
% Send a message, msg, from node src to node dst
for i = 1:d                     % d is dimension of the hypercube
  if(bit(i, src) != bit(i, dst))  % if different for dimension i
    send(msg, link[i]);           % then send msg to our i-neighbour
```


Trees



- Simple network: number of routing nodes = number of processors − 1.
- Wiring: $O(\log N)$ extra height ($O(N \log N)$) extra area.
  - Wiring: $O(\sqrt{N} \log N)$ extra area for H-tree.
- Low-latency: $O(\log N)$ + wire delay.
- Low-bandwidth: bottleneck at root.

Fat-Trees



- Use M^{α} parallel links to connect subtrees with M leaves.
- $0 \le \alpha \le 1$
  - $\alpha = 0$: simple tree
  - $\alpha = 1$: strange crossbar
- Fat-trees are "universal"
  - For $\frac{2}{3} < \alpha < 1$ a fat-tree interconnect with volume V can simulate any interconnect that occupies the same volume with a time overhead that is poly-log factor of N.

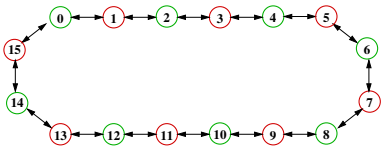
Performance Considerations

- Bandwidth
  - How many bytes per-second can we send between two processors?
    - May depend on which two processors: neighbours may have faster links than spanning the whole machine.
  - Bisection bandwidth: find the worst way to divide the processors into to sets of $P/2$ processors each.
    - How many bytes per-second can we send between the two partitions?
    - If we divide this by the number of processors, we typically get a much smaller value that the peak between two processors.
- Latency
  - How long does it take to send a message from one processor to another?
    - Typically matters the most for short messages.
    - Round-trip time is often a good way to measure latency.
- Cost
  - How expensive is the interconnect – it may dominate the total machine cost.
    - Cost of the network interface hardware.
    - Cost of the cables.

Real-life networks

- InfiniBand is becoming increasingly prevalent
- Peak bandwidths ≥ 6GBytes/sec.
  - achieved bandwidths of 2–3GB/s.
- Support for RDMA and "one-sided" communication
  - CPU A can read or write a block of memory residing with CPU B.
- Often, networks include trees for synchronization (e.g. barriers), and common reduce and scan operations.
- The MPI (message-passing interface) evolves to track the capabilities of the hardware.

Bandwidth Matters



- Assume each link has a bandwidth in each direction of 1Gbyte/sec.
- Each node, i, sends an 8Kbyte message to node $(i + 1)$ mod P, where P is the number of processors?
- How long does this take?
- What if each node, i, sends an 8Kbyte message to node $(i + P/2)$ mod P?

What this means for programmers

- Location matters.
  - The meaning of location depends on the machine.
  - Getting a good programming model is hard.
  - Challenges of heterogeneous machines.
- What it means for different kinds of computers
  - Supercomputers
  - Clouds
  - PCs of the future(?)

Summary

- Message passing machines have an architecture that corresponds to the message-passing programming paradigm.
- Message passing machines can range from
  - Clusters of PC's with a commodity switch.
  - Clouds: lots of computers with a general purpose network.
  - Super-computers: lots of compute nodes tightly connected with high-performance interconnect.
- Many network topologies have been proposed:
  - Performance and cost are often dominated by network bandwidth and latency.
  - The network can be more expensive than the CPUs.
  - Peta-flops or other instruction counting measures are an indirect measure of performance.
- Implications for programmers
  - Location matters
  - Communication costs of algorithms is very important
  - Heterogeneous computing is likely in your future.

Preview

January 30: Parallel Performance: Speed-up	
Reading:	Pacheco, Chapter 2, Section 2.6.
Homework:	HW 2 earlybird (11:59pm). HW 3 goes out.
February 1: Parallel Performance: Overheads	
Homework:	HW 2 due (11:59pm).
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February 5: Parallel Performance: Wrap Up	
January 8–February 15: Parallel Sorting	
Homework (Feb. 15):	HW 3 earlybird (11:59pm), HW 4 goes out.
February 17: Map-Reduce	
Homework:	HW 3 due (11:59pm).
February 27: TBD	
March 1: Midterm	

- There will be readings assigned from *Programming Massively Parallel Processors* starting after the midterm.
- Make sure you have a copy. Note: this year, we'll make sure the course works with either the 2nd or 3rd edition.

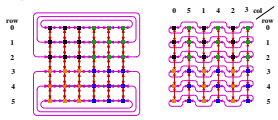
Review

- Consider a machine with 4096 processors.
- What is the maximum latency for sending a message between two processors (measured in network hops) if the network is
  - A ring?
  - A crossbar?
  - A 2-D mesh?
  - A 3-D mesh?
  - A hypercube?
  - A binary tree?
  - A radix-4 tree?

Supplementary Material

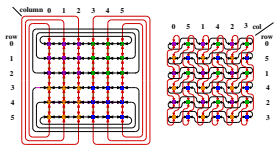
- Message-passing origami: how to fold a mesh into a torus.
- How big is a hypercube: it's all about the wires.

From a mesh to a torus (1/2)



- Fold left-to-right, and make connections where the left and right edges meet.
- Now, we've got a cylinder.
- Note that there are no "long" horizontal wires: the longest wires jump across one processor.

From a mesh to a torus (2/2)



- Fold top-to-bottom, and make connections where the top and bottom edges meet.
- Now, we've got a torus.
- Again there are no "long" wires.

How big is a hypercube?

- Consider a hypercube with $N = 2^d$ nodes.
- Assume each link can transfer one message in each direction in one time unit. The analysis here easily generalizes for links of higher or lower bandwidths.
- Let each node send a message to each of the other nodes.
- Using dimension routing,
  - Each node will send $N/2$ messages for each of the d dimensions.
  - This takes time $N/2$.
  - As soon as one batch of messages finishes the dimension-0 route, that batch can continue with the dimension-1 route, and the next batch can start the dimension 0 route.
  - So, we can route with a throughput of $\binom{N}{2}$ messages per $N/2$ time.

How big is a hypercube?

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- Assume each link can transfer one message in each direction in one time unit. The analysis here easily generalizes for links of higher or lower bandwidths.
- Let each node send a message to each of the other nodes.
- Using dimension routing,

  we can route with a throughput of $\binom{N}{2}$ messages per $N/2$ time.
- Consider any plane such that $N/2$ nodes are on each side of the plane.
  - The plane has area $O(N)$.
- Because the argument applies for *any* plane, we conclude that the hypercube has diameter $O(\sqrt{N})$ and thus volume $O(N^{\frac{3}{2}})$.
- Asymptotically, the hypercube is all wire.

Speed-Up

Mark Greenstreet

CpSc 418 – Jan. 30, 2017

Outline:

But first, USRA

Summer Undergraduate Research Opportunities

- Natural Sciences and Engineering Research Council (NSERC) Undergraduate Student Research Awards (USRAs)
  - Same process to apply for Science Undergraduate Research Experience (SURE) and Work Learn International Undergraduate Research Awards
- See what academic research really looks like
- Many research areas: …
  - Google "ubc cs usra" for full list of projects seeking students
- I have several project proposals:
  - Collaborative control of smart wheelchairs for older adults
  - Numerical software for demonstrating correctness of robots and cyber-physical systems
- 16 weeks, flexible schedule
- You get paid!
- Email potential sponsor ASAP (full applications due by Feb 10)

January 2017 Ian M. Mitchell — UBC Computer Science 1

Objectives

- Understand key measures of performance
  - Time: latency vs. throughput
  - Time: wall-clock vs. operation count
  - Speed-up: slide 5
- Understand common observations about parallel performance
  - Amdahl's law: limitations on parallel performance (and how to evade them)
  - The law of modest returns: high complexity problems are bad, and worse on a parallel machine.
  - Superlinear speed-up: more CPUs ⇒ more, fast memory – and sometimes you win.
  - Embarrassingly parallel problems: sometimes you win, without even trying.

Measuring Performance

- The main motivation for parallel programming is performance
  - Time: make a program run faster.
  - Space: allow a program to run with more memory.
- To make a program run faster, we need to know how fast it is running.
- There are many possible measures:
  - Latency: time from starting a task until it completes.
  - Throughput: the rate at which tasks are completed.
  - Key observation:

$$throughput = \frac{1}{latency}, \quad \text{sequential programming}$$

$$throughput \ge \frac{1}{latency}, \quad \text{parallel programming}$$

Speed-Up

- Simple definition:

$$speed\_up = \frac{\text{time(sequential\_execution)}}{\text{time(parallel\_execution)}}$$

- We can also describe speed-up as how many percent faster:

$$\%faster = (speed\_up - 1) * 100\%$$

- But beware of the spin:
  - Is "time" latency or throughput?
  - How big is the problem?
  - What is the sequential version:
    - The parallel code run on one processor?
    - The fastest possible sequential implementation?
    - Something else?
- More practically, how do we measure time?

Speed-Up – Example

- Let's say that count 3s of a million items takes 10ms on a single processor.
- If I run count 3s with four processes on a four CPU machine, and it takes 3.2ms, what is the speed-up?
- If I run count 3s with 16 processes on a four CPU machine, and it takes 1.8ms, what is the speed-up?
- If I run count 3s with 128 processes on a 32 CPU machine, and it takes 0.28ms, what is the speed-up?

Time complexity

- What is the time complexity of sorting?
  - What are you counting?
  - Why do you care?
- What is the time complexity of matrix multiplication?
  - What are you counting?
  - Why do you care?

Big-O and Wall-Clock Time

- In our algorithms classes, we count "operations" because we have some belief that they have something to do with how long the actual program will take to execute.
  - Or maybe not. Some would argue that we count "operations" because it allows us to use nifty techniques from discrete math.
  - I'll take the position that the discrete math is nifty because it tells us something useful about what our software will do.
- In our architecture classes, we got the formula:

$$\text{time} = \frac{(\#\text{inst. executed}) * (\text{cycles/instruction})}{\text{clock frequency}}$$

- The approach in algorithms class of counting comparisons or multiplications, etc., is based on the idea that everything else is done in proportion to these operations.
- BUT, in parallel programming, we can find that a communication between processes can take 1000 times longer than a comparison or multiplication.
  - This may not matter if you're willing to ignore "constant factors."
  - In practice, factors of 1000 are too big to ignore.

Amdahl's Law

- Given a sequential program where
  - fraction s of the execution time is inherently sequential.
  - fraction $1 - s$ of the execution time benefits perfectly from speed-up.
- The run-time on P processors is:

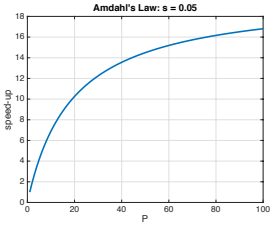
$$T_{parallel} = T_{sequential} * (s + \frac{1-s}{P})$$

- Consequences:
  - Define

$$speed\_up = \frac{T_{sequential}}{T_{parallel}}$$

  - Speed-up on P processors is at most $\frac{1}{s}$.
  - Gene Amdahl argued in 1967 that this limit means that parallel computers are only useful for a few special applications where s is very small.

Amdahl's Law



Amdahl's Law, 49 years later

Amdahl's law is not a physical law.

- Amdahl's law is mathematical theorem:
  - If $T_{parallel}$ is $(s + \frac{1-s}{P})\ T_{sequential}$
  - and $speed\_up = T_{sequential} / T_{parallel}$,
  - then for $0 < s \leq 1$, $speed\_up \leq \frac{1}{s}$.
- Amdahl's law is also an economic law:
  - Amdahl's law was formulated when CPUs were expensive.
  - Today, CPUs are cheap
    - The cost of fabricating eight cores on a die is very little more that the cost of fabricating one.
    - Computer cost is dominated by the rest of the system: memory, disk, network, monitor, ...
- Amdahl's law assumes a fixed problem size.

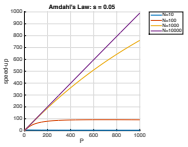
Amdahl's Law, 49 years later

- Amdahl's law is an economic law, not a physical law.
  - Amdahl's law was formulated when CPUs were expensive.
  - Today, CPUs are cheap (see previous slide)
- Amdahl's law assumes a fixed problem size
  - Many computations have s (sequential fraction) that decreases as N (problem size) increases.
  - Having lots of cheap CPUs available will
    - Change our ideas of what computations are easy and which are hard.
    - Determine what the "killer-apps" will be in the next ten years.
      - Ten years from now, people will just take it for granted that most new computer applications will be parallel.
  - Examples: see next slide

Amdahl's Law, 49 years later

- Amdahl's law is an economic law, not a physical law.
- Amdahl's law assumes a fixed problem size
  - Ten years from now, people will just take it for granted that most new computer applications will be parallel.
  - Examples:
    - Managing/searching/mining massive data sets.
    - Scientific computation.
      - Note that most of the computation for animation and rendering resembles scientific computation. Computer games benefit tremendously from parallelism.
      - Likewise for multimedia computing.

Amdahl's Law, one more try



- We can have problems where the parallel work grows faster than the sequential part.
- Example: parallel work grows as $N^{3/2}$ and the sequential part grows as log P.

The Law of Modest Returns

More bad news. ☹

- Let's say we have an algorithm with a sequential run-time $T = (12ns)N^4$.
  - If we're willing to wait for one hour for it to run, what's the largest value of N we can use?
  - If we have 10000 machines, and perfect speed-up (i.e. $speed\_up = 10000$), now what is the largest value of N we can use?
  - What if the run-time is $(5ns)1.2^N$?
- The law of modest returns
  - Parallelism offers modest returns, unless the problem is of fairly low complexity.
  - Sometimes, modest returns are good enough: weather forecasting, climate models.
  - Sometimes, problems have huge N and low complexity: data mining, graphics, machine learning.

Super-Linear Speed-up

Sometimes, $speed\_up > P$. ☺

- How does this happen?
  - Impossibility "proof": just simulate the P parallel processors with one processor, time-sharing P ways.
- Memory: a common explanation
  - P machines have more main memory (DRAM)
  - and more cache memory and registers (total)
  - and more I/O bandwidth, ...
- Multi-threading: another common explanation
  - The sequential algorithm underutilizes the parallel capabilities of the CPU.
  - A parallel algorithm can make better use.
- Algorithmic advantages: once in a while, you win!
  - Simulation as described above has overhead.
  - If the problem is naturally parallel, the parallel version can be more efficient.
- BUT: be very skeptical of super-linear claims, especially if $speed\_up \gg P$.

Embarrassingly Parallel Problems

Problems that can be solved by a large number of processors with very little communication or coordination.

- Rendering images for computer-animation: each frame is independent of all the others.
- Brute-force searches for cryptography.
- Analyzing large collections of images: astronomy surveys, facial recognition.
- Monte-Carlo simulations: same model, run with different random values.
- **Don't be ashamed if your code is embarrassingly parallel:**
  - Embarrassingly parallel problems are great: you can get excellent performance without heroic efforts.
  - The only thing to be embarrassed about is if you don't take advantage of easy parallelism when it's available.

Lecture Summary

Parallel Performance

- Speed-up: slide 5
- Limits
  - Amdahl's Law, slide 9.
  - Modest gains, slide 15.
- Sometimes, we win
  - Super-linear speedup, slide 16.
  - Embarrassingly Parallel Problems, slide 17.

Preview

February 1: Parallel Performance: Overheads	
Homework:	HW 2 due (11:59pm).
February 3: Parallel Performance: Models	
Mini Assignments	Mini 4 due (10am)
February 6: Parallel Performance: Wrap Up	
February 8: Parallel Sorting – The Zero-One Principle	
Homework (Feb. 15):	HW 3 earlybird (11:59pm), HW 4 goes out.
February 10: Bitonic Sorting (part 1)	
February 15: Family Day – no class	
February 13: Bitonic Sorting (part 2)	
Homework (Feb. 15):	HW 3 earlybird (11:59pm), HW 4 goes out.
February 17: Map-Reduce	
Homework:	HW 3 due (11:59pm).
February 27: TBD	
March 1: Midterm	

- Reading from "Programming Massively Parallel Computers" (D.B. Kirk & W.-M. Hwu) start right after the midterm. Make sure you have a copy.
- You can use either the 2nd or 3rd edition.

Review Questions

- What is speed-up? Give an intuitive, English answer and a mathematical formula.
- Why can it be difficult to determine the sequential time for a program when measuring speed-up?
- What is Amdahl's law? Give a mathematical formula. Why is Amdahl's law a concern when developing parallel applications? Why in many cases is it not a show-stopper?
- Is parallelism an effective solution to problems with high big-O complexity? Why or why not?
- What is super-linear speed-up? Describe two causes.
- What is an embarrassingly parallel problem. Give an example.

Performance-Loss

Mark Greenstreet

CpSc 418 – Feb. 1, 2017

Outline:

- Overhead: work that the parallel code has to do that the sequential version avoids.
  - Communication and Synchronization
  - Extra computation, extra memory
- Limited parallelism
  - Code that is inherently sequential or has limited parallelism
  - Idle processors
  - Resource contention

Objectives

- Learn about main causes of performance loss:
  - Overhead
  - Non-parallelizable code
  - Idle processors
  - Resource contention
- See how these arise in message-passing, and shared-memory code.

Causes of Performance Loss

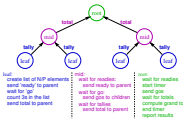
- Ideally, we would like a parallel program to run P times faster than the sequential version when run on P processors.
- In practice, this rarely happens because of:
  - Overhead: work that the parallel program has to do that isn't needed in the sequential program.
  - Non-parallelizable code: something that has to be done sequentially.
  - Idle processors: There's work to do, but some processor are waiting for something before they can work on it.
  - Resource contention: Too many processors overloading a limited resource.

Overhead

Overhead: work that the parallel program has to do that isn't needed in the sequential program.

- Communication:
  - The processes (or threads) of a parallel program need to communicate.
  - A sequential program has no interprocess communication.
- Synchronization.
  - The processes (or threads) of a parallel program need to coordinate.
  - This can be to avoid interference, or to ensure that a result is ready before it's used, etc.
  - Sequential programs have a completely specified order of execution: no synchronization needed.
- Computation.
  - Recomputing a result is often cheaper than sending it.
- Memory Overhead.
  - Each process may have its own copy of a data structure.

Communication Overhead



- In a parallel program, data must be sent between processors.
- This isn't a part of the sequential program.
- The time to send and receive data is overhead.
- Communication overhead occurs with both shared-memory and message passing machines and programs.
- Example: Reduce (e.g. Count 3s):
  - Communication between processes adds time to execution.
  - The sequential program doesn't have this overhead.

Communication with shared-memory

- In a shared memory architecture:
  - Each core has it's own cache.
  - The caches communicate to make sure that all references from different cores to the same address look like there is one, common memory.
  - It takes longer to access data from a remote cache than from the local cache. This creates overhead.
- False sharing can create communication overhead even when there is no logical sharing of data.
  - This occurs if two processors repeatedly modify different locations on the same cache line.

Communication overhead: example

- The *Principles of Parallel Programming* book considered an example of Count 3s (in C, with threads), where there was a global array, `int count[P]` where `P` is the number of threads.
  - Each thread (e.g. thread i) initially sets its count, `count[i]` to 0.
  - Each time a thread encounters a `3`, it increments its element in the array.
- The parallel version ran much slower than the sequential one.
  - Cache lines are much bigger than a single `int`. Thus, many entries for the `count` array are on the same cache line.
  - A processor has to get exclusive access to update the count for its thread.
  - This invalidates the copies held by the other processors.
  - This produces lots of cache misses and a slow execution.
- A better solution:
  - Each thread has a local variable for its count.
  - Each thread counts its threes using this local variable and copies its final total to the entry in the global array.

Communication overhead with message passing

- The time to transmit the message through the network.
- There is also a CPU overhead: the time set up the transmission and the time to receive the message.
- The context switches between the parallel application and the operating system adds even more time.
- Note that many of these overheads can be reduced if the sender and receiver are different threads of the same process running on the same CPU.
  - This has led to SMP implementations of Erlang, MPI, and other message passing parallel programming frameworks.
  - The overheads for message passing on an SMP can be very close to those of a program that explicitly uses shared memory.
  - This allows the programmer to have one parallel programming model for both threads on a multi-core processor and for multiple processes on different machines in a cluster.

Synchronization Overhead

- Parallel processes must coordinate their operations.
  - Example: access to shared data structures.
  - Example: writing to a file.
- For shared-memory programs (e.g. `pthreads` or `Java threads`, there are explicit locks or other synchronization mechanisms.
- For message passing (e.g. `Erlang` or `MPI`), synchronization is accomplished by communication.

Computation Overhead

A parallel program may perform computation that is not done by the sequential program.

- Redundant computation: it's faster to recompute the same thing on each processor than to broadcast.
- Algorithm: sometimes the fastest parallel algorithm is fundamentally different than the fastest sequential one, and the parallel one performs more operations.

Sieve of Eratosthenes

To find all primes $\leq$ `N`:

```
1.   Let MightBePrime = [2, 3, ..., N].
2.   Let KnownPrimes = [].
3.   while(MightBePrime ≠ []) do
         % Loop invariant: KnownPrimes contains all primes less than the
         % smallest element of MightBePrime, and MightBePrime
         % is in ascending order. This ensure that the first element of
         % MightBePrime is prime.
3.1.     Let P = first element of MightBePrime.
3.2.     Append P to KnownPrimes.
3.3.     Delete all multiples of P from MightBePrime.
4.   end
```

See http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes

Prime-Sieve in Erlang

```
% primes(N): return a list of all primes ≤ N.
primes(N) when is_integer(N) and (N < 2) -> [];
primes(N) when is_integer(N) ->
   do_primes([], lists:seq(2, N)).

% invariants of do_primes(Known, Maybe):
%   All elements of Known are prime.
%   No element of Maybe is divisible by any element of Known.
%   lists:reverse(Known) ++ Maybe is an ascending list.
%   Known ++ Maybe contains all primes ≤ N, where N is from p(N).
do_primes(KnownPrimes, []) -> lists:reverse(KnownPrimes);
do_primes(KnownPrimes, [P | Etc]) ->
do_primes([P | KnownPrimes],
          lists:filter(fun(E) -> (E rem P) /= 0 end, Etc)).
```


A More Efficient Sieve

- If N is composite, then it has at least one prime factor that is at most $\sqrt{N}$.
- This means that once we've found a prime that is $\geq \sqrt{N}$, all remaining elements of `Maybe` must be prime.
- Revised code:

```
% primes(N): return a list of all primes ≤ N.
primes(N) when is_integer(N) and (N < 2) -> [];
primes(N) when is_integer(N) ->
   do_primes([], lists:seq(2, N), trunc(math:sqrt(N))).

do_primes(KnownPrimes, [P | Etc], RootN)
          when (P =< RootN) ->
   do_primes([P | KnownPrimes],
      lists:filter(fun(E) -> (E rem P) /= 0 end, Etc), RootN);
do_primes(KnownPrimes, Maybe, _RootN) ->
   lists:reverse(KnownPrimes, Maybe).
```


Prime-Sieve: Parallel Version

- Main idea
  - Find primes from $1 \ldots \sqrt{N}$.
  - Divide $\sqrt{N} + 1 \ldots N$ evenly between processors.
  - Have each processor find primes in its interval.
- We can speed up this program by having each processor compute the primes from $1 \ldots \sqrt{N}$.
  - Why does doing extra computation make the code faster?

Memory Overhead

The total memory needed for P processes may be greater than that needed by one process due to replicated data structures and code.

- Example: the parallel sieve: each process had its own copy of the first $\sqrt{N}$ primes.

Overhead: Summary

Overhead is loss of performance due to extra work that the parallel program does that is not performed by the sequential version. This includes:

- Communication: parallel processes need to exchange data. A sequential program only has one process; so it doesn't have this overhead.
- Synchronization: Parallel processes may need to synchronize to guarantee that some operations (e.g. file writes) are performed in a particular order. For a sequential program, this ordering is provided by the program itself.
- Extra Computation:
  - Sometimes it is more efficient to repeat a computation in several different processes to avoid communication overhead.
  - Sometimes the best parallel algorithm is a different algorithm than the sequential version and the parallel one performs more operations.
- Extra Memory: Data structures may be replicated in several different processes.

Limited Parallelism

Sometimes, we can't keep all of the processors busy doing useful work.

- Non-parallelizable code
  The dependency graph for operations is narrow and deep.
- Idle processors
  There is work to do, but it hasn't been assigned to an idle processor.
- Resource contention
  Several processes need exclusive access to the same resource.

Non-parallelizable Code

- Finding the length of a linked list:

```
int length=0;
for(List p = listHead; p != null; p = p->next)
   length++;
```

  - Must dereference each `p->next` before it can dereference the next one.
  - Could make more parallel by using a different data structure to represent lists (some kind of skiplist, or tree, etc.)
- Searching a binary tree
  - Requires 2^k processes to get factor of k speed-up.
  - Not practical in most cases.
  - Again, could consider using another data structure.
- Interpreting a sequential program.
- Finite state machines.

Idle Processors

- There is work to do, but processors are idle.
- Start-up and completion costs.
- Work imbalance.
- Communication delays.

Resource Contention

.

- Processors waiting for a limited resource.
- It's easy to change a compute-bound task into an I/O bound one by using parallel programming.
- Or, we run-into memory bandwidth limitations:
  - Processing cache-misses.
  - Communication between CPUs and co-processors.
- Network bandwidth.

Lecture Summary

Causes of Performance Loss in Parallel Programs

- Overhead
  - Communication, slide 5.
  - Synchronization, slide 9.
  - Computation, slide 10.
  - Extra Memory, slide 15.
- Other sources of performance loss
  - Non-parallelizable code, slide 18
  - Idle Processors, slide 19.
  - Resource Contention, slide 20.

Review Questions

- What is overhead? Give several examples of how a parallel program may need to do more work or use more memory than a sequential program.
- Do programs running on a shared-memory computer have communication overhead? Why or why not?
- Do message passing program have synchronization overhead? Why or why not?
- Why might a parallel program have idle processes even when there is work to be done?

Models of Parallel Computation

Mark Greenstreet

CpSc 418 – Feb. 6, 2017

- The RAM Model of Sequential Computation
- Models of Parallel Computation
- An entertaining proof

Objectives

- Learn about models of computation
  - Sequential: Random Access Machine (RAM)
  - Parallel
    - Parallel Random Access Machine (PRAM)
    - Candidate Type Architecture (CTA)
    - Latency-Overhead-Bandwidth-Processors (LogP)
- An entertaining algorithm and its analysis
  - If a model has invalid assumptions,
  - then we can show that algorithm 1 is faster than algorithm 2,
  - but in real life algorithm 2 is faster.
  - Valiant's algorithm also provides some mathematical entertainment.

The RAM Model

RAM = Random Access Machine

- Axioms of the model
  - Machines work on words of a "reasonable" size.
  - A machine can perform a "reasonable" operation on a word as a single step.
    - such operations include addition, subtraction, multiplication, division, comparisons, bitwise logical operations, bitwise shifts and rotates.
  - The machine has an unbounded amount of memory.
    - A memory address is a "word" as described above.
    - Reading or writing a word of memory can be done in a single step.

The Relevance of the RAM Model

- If a single step of a RAM corresponds (to within a factor close to 1) to a single step of a real machine.
- Then algorithms that are efficient on a RAM will also be efficient on a real machine.
- Historically, this assumption has held up pretty well.
  - For example, `mergesort` and `quicksort` are better than `bubblesort` on a RAM and on real machines, and the RAM model predicts the advantage quite accurately.
  - Likewise, for many other algorithms
    - graph algorithms, matrix computations, dynamic programming, ...
    - hard on a RAM generally means hard on a real machine as well: NP complete problems, undecidable problems, ...

The Irrelevance of the RAM Model

The RAM model is based on assumptions that don't correspond to physical reality:

- Memory access time is highly non-uniform.
  - Architects make heroic efforts to preserve the illusion of uniform access time fast memory –
    - caches, out-of-order execution, prefetching, ...
  - – but the illusion is getting harder and harder to maintain.
    - Algorithms that randomly access large data sets run much slower than more localized algorithms.
    - Growing memory size and processor speeds means that more and more algorithms have performance that is sensitive to the memory hierarchy.
- The RAM model does not account for energy:
  - Energy is the critical factor in determining the performance of a computation.
  - The energy to perform an operation drops rapidly with the amount of time allowed to perform the operation.

The PRAM Model

PRAM = Parallel Random Access Machine

- Axioms of the model
  - A computer is composed of multiple processors and a shared memory.
  - The processors are like those from the RAM model.
    - The processors operate in lockstep.
    - I.e. for each $k > 0$, all processors perform their k^{th} step at the same time.
  - The memory allows each processor to perform a read or write in a single step.
    - Multiple reads and writes can be performed in the same cycle.
    - If each processor accesses a different word, the model is simple.
    - If two or more processors try to access the same word on the same step, then we get a bunch of possible models:
      EREW: Exclusive-Read, Exclusive-Write
      CREW: Concurrent-Read, Exclusive-Write
      CRCW: Concurrent-Read, Concurrent-Write
      ★ See slide 25 for more details.

The Irrelevance of the PRAM Model

The PRAM model is based on assumptions that don't correspond to physical reality:

- Connecting N processors with memory requires a switching network.
  - Logic gates have bounded fan-in and fan-out.
  - ⇒ any switch fabric with N inputs (and/or N outputs) must have depth of at least $\log N$.
  - This gives a lower bound on memory access time of $\Omega(\log N)$.
- Processors exist in physical space
  - N processors take up $\Omega(N)$ volume.
  - The processor has a diameter of $\Omega(N^{1/3})$.
  - Signals travel at a speed of at most c (the speed of light).
  - This gives a lower bound on memory access time of $\Omega(N^{1/3})$.

The CTA Model

CTA = Candidate Type Architecture

- Axioms of the model
  - A computer is composed of multiple processors.
  - Each processor has
    - Local memory that can be accessed in a single processor step (like the RAM model).
    - A small number of connections to a communications network.
  - There is a communication network connecting the processors.
    - The general model:
      - ★ The communication network is a graph where all vertices (processors and switches) have bounded degree.
      - ★ Each edge has an associated bandwidth and latency.
    - The simplified model:
      - ★ Global actions have a cost of λ times the cost of local actions.
      - ★ λ is assumed to be "large".
    - The exact communication mechanism is not specified.

The (Ir)Relevance of the CTA Model

- Recognizing that communication is expensive is the one, most important point to grasp to understand parallel performance.
  - CTA highlights the central role of communication.
  - PRAM ignores it.
- The general model is parameterized by the communication network
  - Can we apply results from analysing a machine with a 3-D toroidal mesh to a machine with fat trees?
  - PRAM ignores it.
- The simple model neglects bandwidth issues
  - Messages are assumed to be "small".
  - But, bigger messages often lead to better performance.
  - If we talk about bandwidth, do we mean the bandwidth of each link?
  - Or, do we mean the bisection bandwidth?

The LogP Model

- **Motivation (1993): convergence of parallel architectures**
  - Individual nodes have microprocessors and memory of a workstation or PC.
  - A large parallel machine had at most 2000 such nodes.
  - Point-to-point interconnect –
    - Network bandwidth much lower than memory bandwidth.
    - Network latency much higher than memory latency.
    - Relatively small network diameter: 5 to 20 "hops" for a 1000 node machine.
- **The model parameters:**
  L the latency of the communication network fabric
  o the overhead of a communication action
  g the bandwidth of the communication network
  P the number of processors

Why does **g** stand for "bandwidth"?

Marketing!

- What if we used **b** for "bandwidth"?
- Need a catchy acronym with 'ℓ', 'o', 'b', and 'p' ...
  - got it: **BLOP**
  - but the marketing department vetoed it.

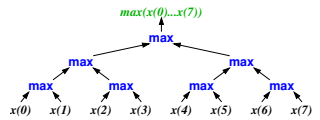
logP in practice

- The authors got some surprisingly good performance prediction for a few machines and a few algorithms by finding the "right" values for ℓ, o, g, and P for each architecture.
- It's rare to get a model that comes to within 10-20% on several examples. So, this looked very promising.
- Since then, logP seems to be a model with more parameters than simplified CTA, but not particularly better accuracy.
- Good to know about, because if you meet an algorithms expert, they'll probably know that PRAM is unrealistic.
  - Then, you'll often hear "What about logP"? – the paper has lots of citations.
  - In practice, it's a slightly fancier was of saying "communication costs matter".

Fun with the PRAM Model

Finding the maximum element of an array of N elements.

- The obvious approach
  - Do a reduce.
  - Use $N/2$ processors to compute the result in $\Theta(\log_2 N)$ time.



A Valiant Solution

L. Valiant, 1975

- Use N processors.
- The big picture:
  - Initially, we can use clumps of three processors to find the largest of three elements in $O(1)$ time – just do all three comparisons.
  - Now, we have $N/3$ elements but we still have N processors. We can perform all of the comparisons for larger clusters of elements in $O(1)$ time in a single step because we have more processors per element.
  - Valiant showed that the size of a cluster for which we can do all of the pair-wise comparisons in a single step grows as 2^{k^2} where k is the number of steps.
  - This leads to a $\log\log N$ time bound for finding the max.
- I'll sketch the proof.
- Then we'll look at why this shows that you can't actually build a PRAM.

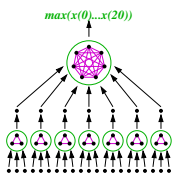
Valiant's algorithm, step 1

- Step 1:
  - Divide the N elements into $N/3$ sets of size 3.
  - Assign 3 processors to each set, and perform all three pairwise comparisons in parallel.
  - Mark all the "losers" (requires a CRCW PRAM) and move the max of each set of three to a fixed location.
- The PRAM operations in a bit more detail.
  - Initially, every element has a flag set to `1` that says "might be the max".
  - When $\binom{k}{2}$ processors perform all of the pairwise comparisons of k values,
    - Each processor sets the flag for the smaller value to `0`.
    - Note that several processors may write `0` to the same location, but the CRCW allows this because they are all writing the same value.
  - One processor for each value checks if its flag is still set to `1`.
    - The winner for the cluster is moved to a specific location;
    - The flag for that location is set to `1`
    - And now we're ready for subsequent rounds.

Valiant's algorithm, step 2

- We now have $N/3$ elements left and still have N processors.
- We can make groups of 7 elements, and have 21 processors per group, which is enough to perform all $\binom{7}{2} = 21$ pairwise comparisons in a single step.
- Thus, in $O(1)$ time we move the max of each set to a fixed location. We now have $N/21$ elements left to consider.

Visualizing Valiant



max from group of 7
(21 parallel comparisons)

group of 7 values

max from each group
(3 parallel comparisons/group)

groups of 3 values

N values, N processors

Valiant's Algorithm, the remaining steps

- On step k, we have N/m_k elements left.
- On step m_k is the "sparsity" of the problem – i.e. the number of processors per remaining element.
- We can make groups of $2m_k + 1$ elements, and have

$$m_k(2m_k+1) = \frac{(2m_k+1)((2m_k+1)-1)}{2} = \binom{2m_k+1}{2}$$

  processors per group, which is enough to perform all pairwise comparisons in a single step.
- We now have $N/(m_k(2m_k+1))$ elements to consider.
- Therefore, $m_{k+1} = 2m_k^2 + m_k$.
  - The sparsity is squared at each step.
  - It follows that the algorithm requires $O(\log\log N)$.
  - Valiant showed a matching lower bound and extended the results to show merging is $\theta(\log\log N)$ and sorting is $\theta(\log N)$ on a CRCW PRAM.
  - See slide 26 to see the details of the first few rounds.

Valiant's Algorithm, run-time

- The sparsity is roughly squared at each step.
- It follows that the algorithm requires $O(\log\log N)$.
- Valiant showed a matching lower bound and extended the results to show merging is $\theta(\log\log N)$ and sorting is $\theta(\log N)$ on a CRCW PRAM.
- See slide 27 for the details.

Take-home message from Valiant's algorithm

- The PRAM model is simple, and elegant, and many clever algorithms have been designed based on the PRAM model.
- It is also physically unrealistic:
  - As shown on slide 7, logic gates have bounded fan-in and fan-out.
  - Implementing the processor to memory interconnect requires a logic network of depth $\Omega(\log P)$.
  - Therefore, access time must be $\Omega(\log P)$.
  - Each step of the PRAM must take $\Omega(\log P)$ physical time.
- Valiant's $O(\log\log N)$ algorithms takes $O(\log N \log\log N)$ physical time
  - It's slower than doing a simple reduce.
  - And it uses lots of communication – think of all those λ penalties!
  - But it's very clever. ☺
- Valiant understood this and pointed these issues in his paper.
  - But there has still be extensive research on PRAM algorithms.
  - It's an elegant model, what can I say?

Summary

- Simplified CTA reminds us that communication is expensive, but it doesn't explicitly charge for bandwidth.
- LogP accounts for bandwidth, but doesn't recognize that all bandwidth is not the same:
  - Communicating with an immediate neighbour is generally much cheaper than communicating with a distant machine.
  - Otherwise stated, the bisection bandwidth for real machines is generally much less than the per-machine bandwidth times the number of machines.
    - We can't have everyone talk at once at full bandwidth.
    - logP uses the bisection bandwidth – this is conservative, but it doesn't recognize the advantages of local communication.
- Both are based on a 10-20 year old machine model
  - That's ok, the papers are 18-25 years old.
  - Doesn't account for the heterogeneity of today's parallel computers:
    - multi-core on chip, faster communication between processors on the same board than across boards, etc.
- We'll use CTA because it's simple.
  - But recognize the limitations of any of these models.
- Getting a model of parallel computation that's as all-purpose as the RAM is still a work-in-progress.

Preview

February 8: Parallel Sorting – The Zero-One Principle
Reading: https://en.wikipedia.org/wiki/Sorting_network
February 10: Bitonic Sorting (part 1)
Reading: https://en.wikipedia.org/wiki/Bitonic_sorter
http://www.iti.fh-flensburg.de/lang/algorithmen/sortieren/bitonic/bitonicen.htm
February 13: Family Day – no class
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March 8: CUDA Threads, Part 1
Reading Kirk & Hwu Ch. 3
Homework: HW 4 earlybird (11:59pm)
March 8: CUDA Threads, Part 2
Homework: HW4 due (11:59pm).

Review

- Compare and Contrast the main features of the PRAM, CTA, and LogP models?
- How does each model represent computation?
- How does each model represent communication?
- How might one determine parameter values for the CTA and LogP models? Describe at a high-level the kinds of experiments you could run to estimate the parameters. Hint: review the Jan. 9 lecture.
- What does the '**g**' stand for in "logP"?

For further reading

- [Valiant1975] Leslie G. Valiant, "Parallelism in Comparison Problems," *SIAM Journal of Computing*, vol. 4, no. 3, pp. 348–355, (Sept. 1975).
- [Fortune1979] Steven Fortune and James Wyllie, "Parallelism in Random Access Machines," *Proceeding of the 11th ACM Symposium on Theory of Computing* (STOC'79), pp. 114–118, May 1978.
- [Snyder1986] Lawrence Snyder, "Type architectures, shared memory, and the corollary of modest potential", *Annual review of computer science*, vol. 1, no. 1, pp. 289–317, 1986.
- [Culler1993] David Culler, Richard Karp, *et al.*, "LogP: towards a realistic model of parallel computation," *ACM SIGPLAN Notices*, vol. 28, no. 7, pp. 1–12, (July 1993).

EREW, CREW, and CRCW

- EREW: Exclusive-Read, Exclusive-Write
  - If two processors access the same location on the same step,
    - ⋆ then the machine fails.
- CREW: Concurrent-Read, Exclusive-Write
  - Multiple machines can read the same location at the same time, and they all get the same value.
  - At most one machine can try to write a particular location on any given step.
  - If one processor writes to a memory location and another tries to read or write that location on the same step,
    - ⋆ then the machine fails.
- CRCW: Concurrent-Read, Concurrent-Write
  If two or more machines try to write the same memory word at the same time, then if they are all writing the same value, that value will be written. Otherwise (depending on the model),
  - the machine fails, or
  - one of the writes "wins", or
  - an arbitrary value is written to that address.

Valiant Details

round	values remaining	group size	processors per group
1	N	$2*1+1=3$	$3=3$ choose 2
2	$\frac{N}{3}$	$2*3+1=7$	$3*7=21=7$ choose 2
3	$\frac{N}{3}\frac{1}{21}=\frac{N}{63}$	$2*21+1=43$	$21*43=903=43$ choose 2
4	$\frac{N}{63}\frac{1}{903}<\frac{N}{60,000}$	$2*903+1=1,807$	$903*1,807=1,631,721=1807$ choose 2
...	...	...	...
k	$\frac{N}{m_k}$	$2m_k+1$	$m_k(2m_k+1)=(2m_k+1)$ choose 2
$k+1$	$\frac{1}{2m_k+1}\frac{N}{m_k(2m_k+1)}$ $=\frac{N}{m_k}\frac{1}{(2m_k+1)}$ $=\frac{N}{m_{k+1}}$	$2m_{k+1}+1$	$m_{k+1}(2m_{k+1}+1)=(2m_{k+1}+1)$ choose 2

- m_k is the "sparsity" at round k:
  $$m_1 = 1 \qquad m_{k+1} = m_k(2m_k+1)$$
- Now note that $m_{k+1}+1 = m_k(2m_k+1) > 2m_k^2 > m_k^2$.
- Thus, $\log(m_{k+1}) > 2\log(m_k)$.
- For $k \geq 3$, $m_k > 2^{2^{k-1}}$.
- Therefore, if $N \geq 2$, $k > \log\log(N)+1 \;\Rightarrow\; m_k > N$.

Let's solve the run-time recurrence

- For Valiant's algorithm. Let $m_0 = 3$ denote the sparsity at the first step.
- $m_{k+1} = 2m_k^2 + m_k$
  - $\log_2 m_{k+1} = \log_2(2m_k^2 + m_k)$
  - $2\log_2 m_k + 1 < \log_2 m_{k+1} < 2\log_2 m_k + 1 + \alpha/m_k$; where $\alpha = \log_2(e)/2$.
  - $2^k \log_2 m_0 + 2^k - 1 < \log_2 m_k < 2^k m_0 + (5/4)(2^k - 1)$; because $m_k \geq 3$, $\log_2(e)/6 = 0.240449\ldots < 1/4$.
  - $(1+\log_2 3)2^k - 1 < \log_2 m_k < ((5/4)+\log_2 3)2^k - (5/4)$; because $m_0 = 3$.
- We want to find k such that $m_k \geq N$. It is sufficient if
  - $(1+\log_2 3)2^k - 1 > \log_2 N$
  - $2^k > (\log_2 N + 1)/(1+\log_2 3)$
  - $k > \log_2[(\log_2 N + 1)/(1+\log_2 3)]$
  - For $N > 2$, $(\log_2 N + 1)/(1+\log_2 3) < \log_2 N$.
- For $N > 2$, let $k = \log_2\log_2 N$. We have shown that $m_k > N$.
  - Valiant's algorithm takes $O(\log\log N)$ rounds.
  - Each round takes constant time on a CRCW PRAM.
  - ∴ Valiant's algorithm takes $O(\log\log N)$ time on a CRCW PRAM.

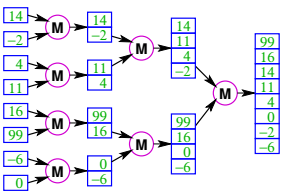
Sorting Networks

Mark Greenstreet

CpSc 418 – Feb. 8, 2017

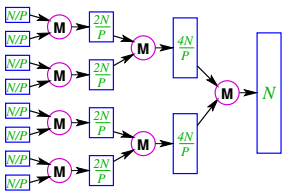
Parallelizing Mergesort

We could use reduce?



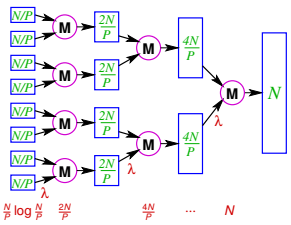
Parallelizing Mergesort

We could use reduce?



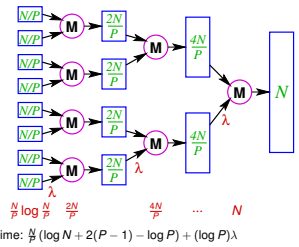
Parallelizing Mergesort

We could use reduce?



Parallelizing Mergesort

We could use reduce?

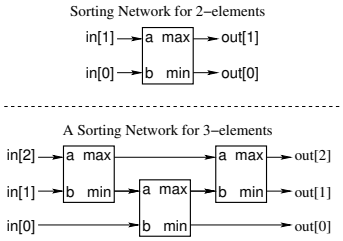


Total time: $\frac{N}{P}(\log N + 2(P-1) - \log P) + (\log P)\lambda$

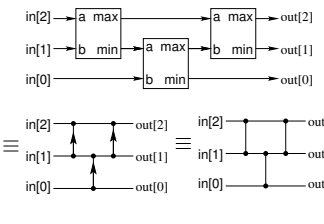
Parallelizing Quicksort

How would you write a parallel version of quicksort?

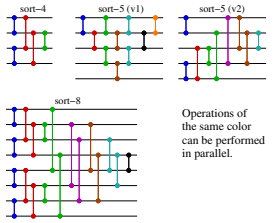
Sorting Networks



Sorting Networks – Drawing



Sorting Networks – Examples



See: http://pages.ripco.net/~jgamble/nw.html

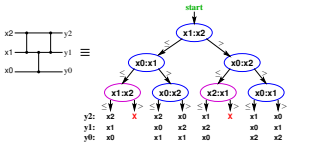
Sorting Networks: Definition

Structural version:

- A sorting network is an acyclic network consisting of compare-and-swap modules.
  - Each primary input is connected either to the input of exactly one compare-and-swap module or to exactly one primary output.
  - Each compare-and-swap input is connected either to a primary input or to the output of exactly one compare-and-swap module.
  - Each compare-and-swap output is connected either to a primary output or to the input of exactly one compare-and-swap module.
  - Each primary output is connected either to the output of exactly one compare-and-swap module or to exactly one primary input.
- More formally, a sorting network is either
  - the identity network (no compare and swap modules).
  - a sorting network, S composed with a compare-and-swap module such that two outputs of S are the inputs to the compare-and-swap, and the outputs of the compare-and-swap are outputs of the new sorting network (along with the other outputs of the original network).

Sorting Networks: Definition

Decision-tree version:



- Let v be an arbitrary vertex of a decision tree, and let x_i and x_j be the variables compared at vertex v.
- A decision tree is a sorting network iff for every such vertex, the left subtree is the same as the right subtree with x_i and x_j exchanged.

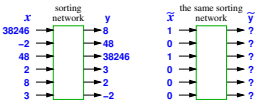
The 0-1 Principle

If a sorting network correctly sorts all inputs consisting only of 0s and 1s, then it correctly sorts inputs consisting of arbitrary (comparable) values.

- The 0-1 principle doesn't hold for arbitrary algorithms:
  - Consider the following linear-time "sort"
  - In linear time, count the number of zeros, nz, in the array.
  - Set the first nz elements of the array to zero.
  - Set the remaining elements to one.
  - This correctly sorts any array consisting only of 0s and 1s, but does not correctly sort other arrays.
- By restricting our attention to sorting networks, we can use the 0-1 principle.

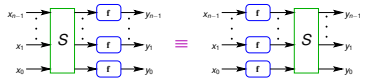
The 0-1 Principle: Proof Sketch

- We will show the contrapositive: if y is not sorted properly, then there exists an $\bar{x}$ consisting of only 0s and 1s that is not sorted properly.



- Choose $i < j$ such that $y_i > y_j$.
- Let $\bar{x}_k = 0$ if $x_k < x_i$ and $\bar{x}_k = 1$ otherwise.
  - Clearly $\bar{x}$ consists only of 0s and 1s.
  - We will show that the sorting network does not sort correctly with input $\bar{x}$.

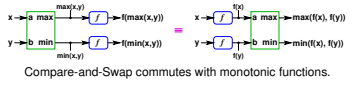
Monotonicity Lemma



Lemma: sorting networks commute with monotonic functions.

- Let S be a sorting network with n inputs an N outputs.
  - I'll write $x_0, \ldots, x_{n-1}$ to denote the inputs of S.
  - I'll write $y_0, \ldots, y_{n-1}$ to denote the outputs of S.
- Let f be a monotonic function.
  - If $x \leq y$, then $f(x) \leq f(y)$.
- The monotonicity lemma says
  - applying S and then f produces the same result as
  - applying f and then S.
- Observation: `f(X) when X < Xi -> 0; f(_) -> 1.` is monotonic.

Compare-and-Swap Commutes with Monotonic Functions



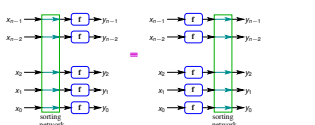
Compare-and-Swap commutes with monotonic functions.

- Case $x \leq y$:

$$\begin{aligned} f(x) &\leq f(y), && \text{because } f \text{ is monotonic.} \\ \max(f(x),f(y)) &= f(y), && \text{because } f(x) \leq f(y) \\ \max(f(x),f(y)) &= f(\max(x,y)), && \text{because } x \leq y \end{aligned}$$

- Case $x \geq y$: equivalent to the $x \leq y$ case.
- □

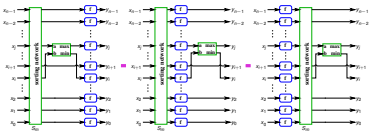
The monotonicity lemma – proof sketch



Induction on the structure of the sorting network, S.
Base case:

- The simplest sorting network, S_0 is the identity function.
- It has 0 compare-and-swap modules.
- Because S_0 is the identity function, $S_0(f(x)) = f(x) = f(S_0(x))$.

The monotonicity lemma – induction step



- Let S_m be a sorting network with n inputs and let $0 \leq i < j < n$.
- Let S_{m+1} be the sorting network obtained by composing a compare-and-swap module with outputs i and j of S_m.
- We can "move" the f operations from the outputs of the new compare-and-swap to the inputs (see slide 12).
- We can "move" the f operations from the outputs S_m to the inputs (induction hypothesis).
- Therefore, S_{m+1} commutes with f.

The 0-1 Principle

If a sorting network correctly sorts all inputs consisting only of 0s and 1s, then it correctly sorts inputs of any values.

I'll prove the contrapositive.

- If a sorting network does not correctly sort inputs of any values, then it does not correctly sort all inputs consisting only of 0s and 1s.
- Let S be a sorting network, let x be an input vector, and let $y = S(x)$, such that there exist i and j with $i < j$ such that $y_i > y_j$.
- Let $f(x) = \begin{cases} 0, & \text{if } x < y_i \\ 1, & \text{if } x \geq y_i \end{cases}$
  $\bar{y} = S(f(x))$
- By the definition of f, $f(x)$ is an input consisting only of 0s and 1s.
- By the monotonicity lemma, $\bar{y} = f(y)$. Thus,
  $$\bar{y}_i = f(y_i) = 1 > 0 = f(y_j) = \bar{y}_j$$
- Therefore, S does not correctly sort an input consisting only of 0s and 1s.
- □

Summary

- Sequential sorting algorithms don't parallelize in an "obvious" way because they tend to have sequential bottlenecks.
  - Later, we'll see that we can combine ideas from sorting networks and sequential sorting algorithms to get practical, parallel sorting algorithms.
- Sorting networks are a restricted class of sorting algorithms
  - Based on compare-and-swap operations.
  - The parallelize well.
  - They don't have control-flow branches – this makes them attractive for architectures with large branch-penalties.
- The zero-one principle:
  - If a sorting-network sorts all inputs of 0s and 1s correctly, then it sorts all inputs correctly.
  - This allows many sorting networks to be proven correct by counting arguments.

Preview

February 10: Bitonic Sorting (part 1)
Reading: https://en.wikipedia.org/wiki/Bitonic_sorter
http://www.iti.fh-flensburg.de/lang/algorithmen/sortieren/bitonic/bitonicen.htm
February 13: Family Day – no class
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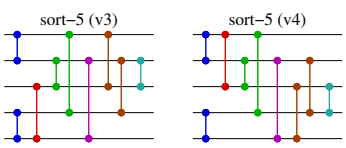
March 8: CUDA Threads, Part 1
Reading Kirk & Hwu Ch. 3
Homework: HW 4 earlybird (11:59pm)
March 8: CUDA Threads, Part 2
Homework: HW4 due (11:59pm).

Review 1

- Why don't traditional, sequential sorting algorithms parallelize well?
- Try to parallelize another sequential sorting algorithm such as heap sort? What issues do you encounter?
- Consider network sort-5(v2) from slide 6. Use the 0-1 principle to show that it sorts correctly?
  - What if the input is all 0s?
  - What if the input has exactly one 1?
  - What if the input has exactly two 1s?
  - What if the input has exactly three 1s? Note, it may be simpler to think of this the input having exactly two 0s.
  - What if the input has exactly four 1s? Five ones?

Review 2



Consider the two sorting networks shown above. One sorts correctly; the other does not.

- Identify the network that sorts correctly, and prove it using the 0-1 principle.
- Show that the other network does not sort correctly by giving an input consisting of 0s and 1s that is not sorted correctly.

Review 3

I claimed that `max` and `min` can be computed without branches. We could work out the hardware design for a compare-and-swap module. Instead, consider an algorithm that takes two "words" as arguments – each word is represented as a list of characters. The algorithm is supposed to output the two words, but in alphabetical order. For example:

```
% See: http://www.ugrad.cs.ubc.ca/~cs418/2016-2/lecture/02-08/cas.erl
compareAndSwap(L1, L2) when is_list(L1), is_list(L2) ->
    compareAndSwap(L1, L2, []).
compareAndSwap([], L2, X) ->
    {lists:reverse(X), lists:reverse(L2)};
compareAndSwap(L1, [], X) ->
    {lists:reverse(X), lists:reverse(X, L1)};
compareAndSwap([H1 | T1], [H2 | T2], X) when H1 == H2 ->
    compareAndSwap(T1, T2, [H1 | X]);
compareAndSwap(L1=[H1 | _], L2=[H2 | _], X) when H1 < H2 ->
    {lists:reverse(X, L1), lists:reverse(X, L2)};
compareAndSwap(L1, L2, X) ->
    {lists:reverse(X, L2), lists:reverse(X, L1)}.
```

Show that `compareAndSwap` can be implemented as a scan operation.

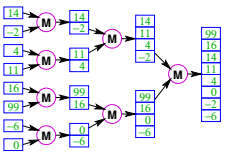
Bitonic Sort

Mark Greenstreet

CpSc 418 – Feb. 10, 2017

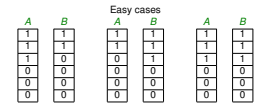
- I know that some of the links in the electronic version are broken. I know that it would be nice if I complete the final slides. I will post to piazza when this is done.

Parallelizing Mergesort



- We looked at this in the Feb. 8 lecture.
- The challenge is the merge step:
  - Can we make a parallel merge?

Merging and the 0-1 Principle



The main idea:

- Use divide-and-conquer.
  - Given two arrays, A and B, divide them into smaller arrays that we can merge, and then easily combine the results.
  - What criterion should we use for dividing the arrays?
- Observation:
  - It's easy to merge two arrays of the same size, if they both have the same number of **1s**.
  - If they have **nearly** the same number of **1s**, that's easy as well.

Dividing the problem (part 1)

- For simplicity, assume each array has an even number of elements.
  - As we go on, we'll assume that each array has an power-of-two number of elements.
  - That's the easiest way to explain bitonic sort.
  - Note: the algorithm works for arbitrary array sizes.
    - ⋆ See the lecture slides from 2013.
- Divide each array in the middle?
  - If A has N elements and N_1 are ones,
  - How many ones are in $A[0, \ldots, (N/2)-1]$?
  - How many ones are in $A[N/2, \ldots, N-1]$?
- Taking every other element?
  - How many ones are in the $A[0, 2, \ldots, N-2]$?
  - How many ones are in the $A[1, 3, \ldots, N-1]$?
- Other schemes?

Dividing the problem (part 2)

- Let A and B be arrays that are sorted into ascending order.
  - Let A_0 be the odd-indexed element of A and A_1 be the odd-indexed.
  - Likewise for B_0 and B_1.
- Key observations:

$$\begin{array}{rcl} \text{HowManyOnes}(A_0) & \le & \text{HowManyOnes}(A_1) \le \text{HowManyOnes}(A_0)+1 \\ \text{HowManyOnes}(B_0) & \le & \text{HowManyOnes}(B_1) \le \text{HowManyOnes}(B_0)+1 \end{array}$$

- With a bit of algebra, we get

$$\left|\text{HowManyOnes}(A_0 \mathbin{+\!\!+} B_1) - \text{HowManyOnes}(A_1 \mathbin{+\!\!+} B_0)\right| \le 1$$

- In English that says that
  - If we merge A_0 with B_1 to get C_0,
  - and we merge A_1 with B_0 to get C_1,
  - then C_0 and C_1 differ by at most one in the number of ones that they have.
    - ⋆ This is an "easy" case from slide 3.

Merging

- Given N that is a power of 2, and arrays A and B that each have N elements and are sorted into ascending order, we can merge them with a sorting network.
- If $N = 1$, then just do `CompareAndSwap(A, B)`.
- Otherwise, let A_0 be the odd-indexed element of A and A_1 be the odd-indexed, and likewise for B_0 and B_1.
- Merge A_0 and B_1 into a single ascending sequence, C_0.
- Merge A_1 and B_0 into a single ascending sequence, C_1.
  - Note that the number of ones in C_0 and C_1 differ by at most one.
- Merge C_0 and C_1 into a single ascending sequence.
  - This is an "easy" case from slide 3.
  - We can perform this merge using $N/2$ compare-and-swap modules.
- Complexity:
  - Depth: $O(\log N)$ – logarithmic parallel time.
  - Number of compare-and-swap modules $O(N \log N)$.
- **Pause:** If you understand this, you've got all of the key ideas of bitonic sorting.
  - The bitonic approach just improves on this simple algorithm.

Bitonic Sequences

- A sequence is **bitonic** if it consists of a monotonically increasing sequence followed by a monotonically decreasing sequence.
  - Either of those sub-sequences can be empty.
  - We'll also consider a monotonically decreasing followed by monotonically increasing sequence to be bitonic.
- Properties of bitonic sequence
  - Any subsequence of a bitonic sequence is bitonic.
  - Let A be a bitonic sequence consisting of **0s** and **1s**. Let A_0 and A_1 be the even- and odd-indexed subsequences of A.
  - The number of **1s** in A_0 and A_1 differ by at most 1.
    - ⋆ We'll examine the number of **0s** on slide 10.

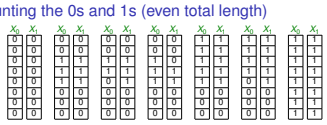
Bitonic Merge – big picture

- Bitonic merge produces a monotonic sequence from an bitonic input.
- Given two sorted sequences, A and B, note that

$$X = A \mathbin{+\!\!+} \text{reverse}(B)$$

is bitonic.
  - We don't require the lengths of A or B to be powers of two.
  - If fact, we don't even require that A and B have the same length.
- Divide X into X_0 and X_1, the even-indexed and odd-indexed subsequences.
  - X_0 and X_1 are both bitonic.
  - The number of **1s** in X_0 and X_1 differ by at most 1.
- Use bitonic merge (recursion) to sort X_0 and X_1 into ascending order to get Y_0 and Y_1.
  - $\text{HowManyOnes}(Y_0) = \text{HowManyOnes}(X_0)$, and $\text{HowManyOnes}(Y_1) = \text{HowManyOnes}(X_1)$.
  - Therefore, the number of **1s** in Y_0 and Y_1 differ by at most 1.
  - This is an "easy" case from slide 3.

Counting the 0s and 1s (even total length)

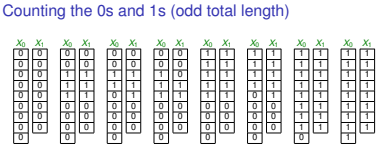


- First, we'll look at the case when length$(A \mathbin{+\!\!+} B)$ is even.
- Given two sorted sequences, A and B, let

$$\begin{array}{rcl} X_0 & = & \text{EvenIndexed}(A \mathbin{+\!\!+} \text{reverse}(B)) \\ X_1 & = & \text{OddIndexed}(A \mathbin{+\!\!+} \text{reverse}(B)) \end{array}$$

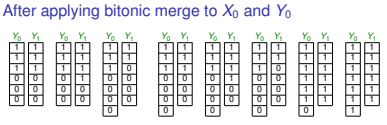
  - This means that $X[i] = X_{i \bmod 2}[i \text{ div } 2]$.
  - In English, the elements of X go left-to-right and then bottom-to-top in X_0 and X_1.
- The number of **1s** in X_0 and the number of ones in X_1 differ by at most 1.
- Likewise for the number of **0s**.

Counting the 0s and 1s (odd total length)



- Let $N = \text{length}(A \mathbin{+\!\!+} B)$, where N is odd.
- The number of **1s** in X_0 and the number of ones in X_1 differ by at most 1.
- The number of **0s** in $X_0[1, \ldots, \lfloor N/2 \rfloor]$ and the number of zeros in X_1 differ by at most 1.
- Either $X_0[0]$ or $X_0[\lfloor N/2 \rfloor]$ is the **least** element of $A \mathbin{+\!\!+} B$.

After applying bitonic merge to X_0 and Y_0



- Let $N = \text{length}(A \mathbin{+\!\!+} B)$.
-
- If N is even,
  - Any out of order elements are in the same row, i.e. $X_0[i] > X_1[i]$ for some $0 \le i < N/2$.
- If N is odd
  - Any out of order elements are of the form $X_0[i+1] > X_1[i]$ for some $0 \le i < N/2$.
  - $X_0[0]$ is the least element of X_0 and X_1.

The complexity of bitonic merge

- We'll count the compare-and-swap operations
  - Is it OK to ignore reversing one array, concatenating the arrays, separating the even- and odd-indexed elements, and recombining them later?
  - Yes. The number of these operations is proportional to the number of compare-and-swaps
  - Yes. Even better, in the next lecture, we'll show how to eliminate most of these data-shuffling operations.
- A bitonic merge of N elements requires:
  - two bitonic merges of $N/2$ items (if $N > 2$)
  - $\lfloor N/2 \rfloor$ compare-and-swap operations.
- The total number of compare and swap operations is $O(N \log N)$.

Bitonic-Sort, and it's complexity

Shuffle and unshuffle

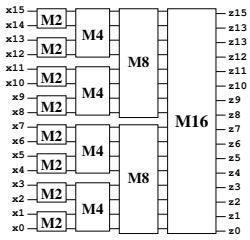
- Shuffle is like what you can do with a deck of cards:
  - Divide the deck in half
  - Select cards alternately from the two halves.
  - Shuffle is a circular-right-shift of the index bits.
    - ⋆ Assuming the number of cards in the deck is a power of two.
- Unshuffle is the inverse of shuffle.
  - Unshuffling a deck of cards is dealing to two players.
  - Unshuffle is a circular-left-shift of the index bits.

Bitonic Sort

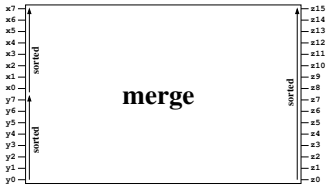
Mark Greenstreet

CpSc 418 – Feb 15, 2017

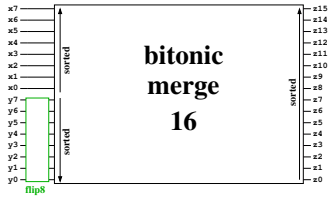
Parallelizing Mergesort



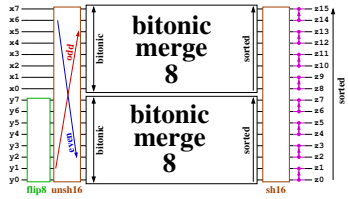
Bitonic Merge



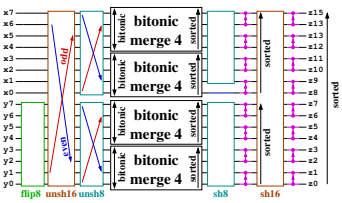
Bitonic Merge



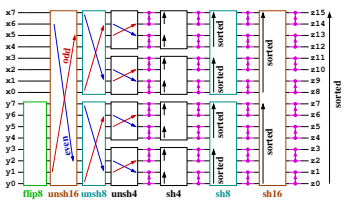
Bitonic Merge



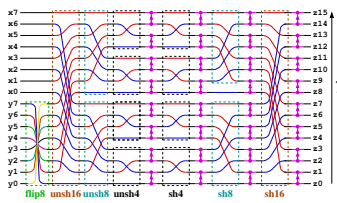
Bitonic Merge



Bitonic Merge



Bitonic Merge



Shuffle

- Given two sequences, X of length N where N is even, the **shuffle** of X is $Y = \text{shuffle}(X)$ where

$$\begin{array}{rcll} Y_i & = & X_{i/2}, & \text{if } i \text{ is even} \\ & = & X_{(i+N-1)/2}, & \text{if } i \text{ is odd} \end{array}$$

  - shuffle$([0, 1, 2, 3, 4, 5, 6, 7]) \rightarrow [0, 4, 1, 5, 2, 6, 3, 7]$.
- shuffle is like shuffling a deck of cards.
  - Split the deck in half.
  - Interleave the cards from the two halves.
- If N is a power of 2, then shuffle rotates the least-significant bit of the index to the most significant bit:

```
shuffle([000, 001, 010, 011, 100, 101, 110, 111]) ->
    [000, 100, 001, 101, 010, 110, 011, 111])
```

- If N is odd,

$$\begin{array}{rcll} Z_i & = & X_{i/2}, & \text{if } i \text{ is even} \\ & = & X_{(i+N)/2}, & \text{if } i \text{ is odd} \end{array}$$

  - shuffle$([0, 1, 2, 3, 4]) \rightarrow [0, 3, 1, 4, 2]$

Unshuffle

- The inverse of shuffle.
- Let $N = \text{length}(Y)$ and $X = \text{unshuffle}(Y)$, then

$$\begin{array}{rcll} X_i & = & Y_{2i}, & \text{if } i < N/2 \\ & = & X_{2i-N+1}, & \text{if } N/2 \le i \end{array}$$

- It's like dealing a deck of cards into two piles, and then stacking one pile on top of the other.
- If N is a power of 2, then unshuffle rotates the most significant bit of the index to the least significant bit:
- If N is odd,

$$\begin{array}{rcll} X_i & = & Y_{2i}, & \text{if } i < (N+1)/2 \\ & = & X_{2i-N}, & \text{if } (N+1)/2 \le i \end{array}$$

Bit operations: `rotr` and `rotl`

- `rotr(I, W)` % Rotate the lower `W` bits of `I` one place to the right:

```
rotr(I, 0) -> I;
rotr(I, W) when is_integer(W), W > 0 ->
    Mask = (1 bsl W) - 1,       % ones in the W least-significant bits
    Ilo = I band Mask,          % the W least-significant bits of I
    Ihi = I band (bnot Mask),   % the rest of I
    Ilsb = I band 1,            % the least-significant-bit of I
    % Ilor is Ilo rotated one place to the right
    Ilor = (Ilsb bsl (W-1)) bor (Ilo bsr 1),
    Ihi bor Ilor.
```

  - `rotr(6,3) -> 5;`
  - `rotr(6,4) -> 12;`
- `rotr(I, W)` rotates the lower `W` bits of `I` 1 place to the left.
- Note: `rotr(I,1) -> I`, and `rotl(I,1) -> I`.

Shuffle, Unshuffle, and Bit-Operations

- If `K` is a power of 2, `x[0..(K-1)]` is the input of a `shuffle_K` module, and `y[0..(K-1)]` is the output, then
  - the `shuffle_K` operation moves `x[i]` to `y[rotl(i, log2(k))]`.
  - equivalently: `y[j] = x[rotr(j, log2(k))]`.
- If `K` is a power of 2, `x[0..(K-1)]` is the input of a `unshuffle_K` module, and `y[0..(K-1)]` is the output, then
  - the `unshuffle_K` operation moves `x[i]` to `y[rotr(i, log2(k))]`.
  - equivalently: `y[j] = x[rotl(j, log2(k))]`.

The Initial Unshuffles

- Bitonic merge for `K` elements starts with an `unshuffle_K`, followed by a `unshuffle_`$\frac{K}{2}$, followed by a `unshuffle_`$\frac{K}{4}$, ..., followed by a `unshuffle_1`.
- If we let `x[0..(K-1)]` be the input to this network (I'm assuming we've already done the flip for inputs `x[0..((K/2)-1)]`), and `y[0..(K-1)]` be the output then:
  - `y[j] = x[rotl(rotl(...rotl(rotl(j, 1), 2), ..., log2(K)-1), log2(K))]`
  - and we note that:
    `rotl(rotl(...rotl(rotl(j, 1), 2), ..., log2(K)-1), log2(K)) = bitrev(j, log2(K))`
    where `bitrev(j, W)` is the bit-reverse of the lower `W` bits of `j`.
- More specifically, for the 16-way bitonic merge, $K = 16$ and $\log_2(K) = 4$.
  - If we write array indices as four bits, b_3, b_2, b_1, b_0,
  - Then `y[`b_3, b_2, b_1, b_0`] = x[`b_0, b_1, b_2, b_3`]`.

The first compare-and-swap

The first compare-and-swap operates on `y[`$b_3, b_2, b_1, 0$`]` and `y[`$b_3, b_2, b_1, 1$`]`, for all 8 choices of b_3, b_2, and b_1.

- This corresponds to a compare-and-swap of `x[`$b_0, b_1, b_2, 0$`]` with `x[`$b_0, b_1, b_2, 1$`]`.
- I'll call the result of the compare-and-swap `z` where
  - `z[`$b_3, b_2, b_1, 0$`] = min(y[`$b_3, b_2, b_1, 0$`], y[`$b_3, b_2, b_1, 1$`]);`
  - `z[`$b_3, b_2, b_1, 1$`] = max(y[`$b_3, b_2, b_1, 0$`], y[`$b_3, b_2, b_1, 1$`]);`
- And I'll write $\widetilde{z}$ for `z` with "`x` indexing":
  - $\widetilde{z}$`[`b_3, b_2, b_1, b_0`] = z[`b_0, b_1, b_2, b_3`];`
  - $\widetilde{z}$`[0,`b_2, b_1, b_0`] = min(x[0,`b_2, b_1, b_0`], x[1,`b_2, b_1, b_0`]);`
  - $\widetilde{z}$`[1,`b_2, b_1, b_0`] = max(x[0,`b_2, b_1, b_0`], x[1,`b_2, b_1, b_0`])`
  - These are comparisons with a "stride" of 8 (for `x`).

The first shuffle

- The first shuffle takes `z` as an input and I'll call the output `w`.
- The first shuffle is a `shuffle_4`; so
  - `w[i] = z[rotl(i,2)].`
  - Equivalently, `w[`b_3, b_2, b_1, b_0`] = z[`b_3, b_2, b_0, b_1`].`
- Let

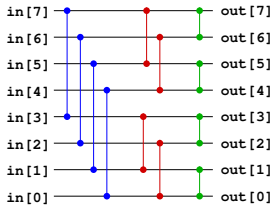
$$\begin{aligned}\widetilde{w}[b_3, b_2, b_1, b_0] &= w[b_0, b_1, b_3, b_2] \\ &= z[b_0, b_1, b_2, b_3] \\ &= \widetilde{z}[b_3, b_2, b_1, b_0]\end{aligned}$$

- The second stage of compare-and-swap modules operates on
  - `w[`$b_3, b_2, b_1, 0$`]` and `w[`$b_3, b_2, b_1, 1$`]`
  - Equivalently, $\widetilde{w}[b_3, b_2, b_1, 0]$ and $\widetilde{w}[b_3, b_2, b_1, 1]$.
  - These are comparisons with a stride of 4 for $\widetilde{z}$ and $\widetilde{w}$.

The rest of the merge

- In the same way, the third stage of compare-and-swap modules operates has a stride of 2 for `x` indices,
- And the final stage has a stride of 1.
- More generally, to merge two sequences of length 2^L:
  - Flip the lower sequence
  -
  - Or, just sort it in reverse in the first place.
  - Perform compare-and-swap operations with stride L.
  - Perform compare-and-swap operations with stride $L/2$ – note that these operate on pairs of elements whose indices differ in the $L/2$ bit, and all of their other index bits are the same.
  - Perform compare-and-swap operations with stride $L/4, \ldots$
  - Perform compare-and-swap operations with stride 1 – this compares the element at `2*i` with the element at `2*i+1` for $0 \le i < 2^L$.
- Done!

The "Textbook" Diagram

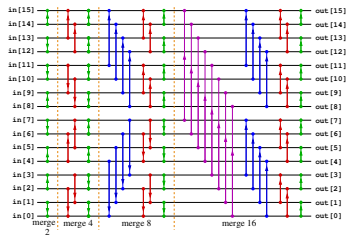


Flipping Out

What should we do about the flips?

- Push them back (right-to-left) through the network
  - Keep track of how many flips we've accumulated.
  - Sort up for an even number of flips.
  - Sort down for an odd number of flips.
- Flip the wiring in the bottom half of each unshuffle.
- In practice:
  - Do the one that's easier for your implementation.

Bitonic Sort



Bitonic Sort in practice

- Sorting networks can be used to design practical sorting algorithms.
- To sort N values with P processors:
  - Divide input into $2P$ segments of length $\frac{N}{2P}$.
  - Each processor sorts its pair of segments into one long segment.
    - The sorted segments are the inputs to the sorting network.
  - Now, follow the actions of the sorting network:
    - Processor l handles rows $2l$ and $2l+1$ of the sorting network.
    - Each compare-and-swap is replaced with "merge two sorted sequences and split into top half and bottom half."
    - When the sorting network has a compare-and-swap between rows $2l$ and $2l+1$, each processor handles it locally.
    - When the sorting network has a compare-and-swap between rows $2l$ and $2l+K$ for $K > 1$, then processor l sends the upper half of its data to processor $l+(K/2)$, and processor $l+(K/2)$ sends the lower half of its data to processor l. Both perform merges.
    - Note, if the compare-and-swap was flipped, then flip "upper-half" and "lower half".

Practical performance

- Complexity
  - Total number of comparisons: $O(N(\log N \log^2 P))$.
  - Time: $O\left(\frac{N}{P}(\log N + \log^2 P)\right)$, assuming each processor sorts N/P elements in $O((N/P)\log(N/P))$ time and merges two sequences of N/P elements in $O(N/P)$ time.
- Remarks:
  - The idea of replacing compare-and-swap modules with processors that can perform merge using an algorithm optimized for the processor, is an extremely powerful and general one. It is used in the design of many practical parallel sorting algorithms.
  - Sorting networks are cool because they avoid branches:
    - Ideal for SIMD machines that can't really branch.
    - Need to experiment some to see the trade-offs of branch-divergence vs. higher asymptotic complexity on a GPU.

Related Algorithms

- Counting Networks
  - How to match servers to requests.
- FFT
  - The Platonic Ideal of a Divide-and-Conquer Algorithm
  - Used for speech processing, signal processing, and lots of scientific computing tasks.

Map-Reduce

Mark Greenstreet & Ian M. Mitchell

CPSC 418 – February 27, 2017

Portrait of a Data Centre

- Sketch of a big data center:
  - Tens of thousands of machines, each with its own disk(s).
  - Distributed file system—what is that?
  - Commodity networks and routers.
    - Each machine has a network interface (e.g. 10Gb ethernet)
    - Cross-section bandwidth is **way** smaller than the number of machines times the per-machine bandwidth.
  - Scale is so big that there **will be failures**: chips, cores, memory, disks, network interfaces, switches, ...
    - If the average lifetime of a machine is five years, then 10,000 machine data center will have a failure every four hours.
    - Even without failure, maintenance will take machines offline.
- Need to analyse the huge data sets stored on these machines.

Problem Domain: Large-Scale Data Analysis

- Data analysis requires:
  - Fetching the relevant records.
  - Performing analysis of related records.
  - Summarizing the results.
- Example: word frequency in documents
- Example: core curriculum
  - How do 200-level courses impact success in 400-level courses?
  - Look at all transcripts.
  - Analyze relationships for *(2XX, 4YY)* pairs.
- Google noted that each such problem was getting its own custom code.
  - All of that code development is expensive.
  - Often required similar rewrites when underlying system changed.

The MapReduce Pattern

Slight generalization of description from [Dean & Ghemawat 2008].

- All data is represented as collections of *(Key, Value)* pairs.
- **map**
  - For each *(Key1, Value1)* pair of the input, user code produces a collection of *(Key2, Value2)* pairs for the output.
- **shuffle**
  - All *(Key2, Value2)* pairs with the same *Key2* are combined into a *(Key2, [list of Value2])* result and sorted by *Key2*.
- **reduce**
  - For each *(Key2, [list of Value2])*, user code produces a *(Key2, Value3)* result (where *Value3* might be a list itself).

Apache Hadoop is an open-source implementation of this basic framework.

MapReduce: Word-Count

Example from [Dean & Ghemawat 2008] revised.

- Input data:
  - *Key1* is the document name.
  - *Value1* is document text.
- map:
  - *Key2* is a word.
  - *Value2* is the count of times it appears in the document.
- shuffle:
  - Collect counts from all documents for each word *(Word, [list of Counts])*.
- reduce:
  - *Value3* is the sum of all counts; in other words, the total number of times the word appears in all documents.

MapReduce: Curriculum

- Input data *(Key1, Value1)*:
  - *Key1* is the student number for the transcript
  - *Value1* is a list of *(CourseNumber, Grade)* pairs.
- map: For each 200-level course and for each 400-level course in the transcript:
  - *Key2* is the pair *(Course200, Course400)*.
  - *Value2* is the pair *(Grade200, Grade400)*.
- shuffle:
  - Collect matching course pairs from all students *((Course200, Course400), [(Grade200, Grade400), (Grade200, Grade400), ...])*.
- reduce:
  - *Value3* is (for example) the sample Pearson correlation coefficient r for the data set *[(Grade200, Grade400), (Grade200, Grade400), ...]*.
  - More complex analyses could be performed.

Wait a Minute Now...

But didn't we already study "reduce"?

- The course library's `wtree:reduce()` in Erlang had `leaf()`, `combine()` and `root()`.
- MapReduce at Google has *(Key1, Value1)*, *(Key2, Value2)*, and shuffle?

These patterns have similar names but seek to solve different problems.

- **Reduce** is a generic name for a functional programming pattern which takes a collection of data and produces some kind of summary information.

Many flavours of Reduce

- In Erlang, `wtree:reduce()` is designed to spread the computation of a reduction across many workers.
  - Implementation maximizes parallelism for a single reduce operation.
  - Collection and combination of data occurs in `combine()` functions.
  - Note that the `leaf()` function can perform a map operation before the reduction, so no loss of generality.
- In MapReduce, many independent reductions (one for each *Key2*) are spread across many workers, but each reduction is performed by a single worker.
  - Implementation emphasizes fault tolerance and disk-based data storage.
  - Collection (but not combination) of data occurs in shuffle step.
  - If reduction is too big for a single worker, user must change the intermediate *(Key2, Value2)* representation and/or break the problem into multiple MapReduce steps.

Programming Model

The user creates a **MapReduce specification object** which provides:

- The *map* and *reduce* functions.
- The names of the input and output files.
- Optionally other tuning parameters; for example:
  - Number of map and reduce workers to use.
  - Custom function to combine intermediate results within a map worker to reduce size of intermediate data.
  - A custom hashing function to help with shuffle step.

The user then invokes the `MapReduce` function.

Execution (Part 1)

- The `MapReduce` function spawns M map worker, R reduce workers, and one master.
- Each map worker:
  - Is assigned fragment(s) of the input file by the master – these fragments are called **splits**.
  - Reads a *(Key1, Value1)* record from an assigned split.
  - Runs user `map` code on that record.
  - Writes the *(Key2, Value2)* result to a temporary file.
  - Repeats until all records in the split are completed.
  - Repeats until all assigned splits are completed.
  - Notifies the master when it is done.
- Result is a bunch of temporary files holding *(Key2, Value2)* pairs.

Execution (Part 2)

- Start from temporary files holding *(Key2, Value2)* pairs.
- Shuffle:
  - Each reduce worker is assigned *Key2* value(s).
  - Corresponding *Value2* lists are taken from map worker's temporary output files and written to reduce worker's temporary input files.
  - Reduce worker receives *(Key2, [list of Value2])* pairs sorted by *Key2*.
- Each reduce worker:
  - Reads a *(Key2, [list of Value2])* record from a temporary file.
  - Runs user `reduce` code on that record.
  - Writes the *(Key2, Value3)* result to a file.
  - Notifies the master when it is done.
- When all the reduce computations are complete, the master sends a message to wake up the user process, and the `MapReduce` function returns.

Do the MapReduce Shuffle

How do the intermediate results get from the map workers to the reduce workers? Simple version described in [Dean & Ghemawat, 2008]:

- Map workers know the number of reduce workers R.
- Each *(Key2, Value2)* is written to a different file according to *hash(Key2) mod R*.
- The master tells the reduce worker which file to read from each map worker.

Later versions of MapReduce utilized more complex or even user-specified hashing; for example to:

- Better balance size of reduce problems.
- Reduce network traffic and/or simplify sorting during shuffle step.
- Cluster certain *Key2* tuples onto the same reduce workers.

Fault Tolerance

Bad things happen: Failed disks, partitioned networks, power shortages, ...

- Key Idea:
  - The `map` and `reduce` operations are based on functional programming ideas: referential transparency and no side-effects.
  - If a worker crashes, it is as if it never existed.
  - The master can restart the task on another machine.
- The master periodically pings the tasks, and restarts dead ones.
  - Map tasks produce only temporary files, so if a completed map task fails before informing the master then it must be re-executed.
  - Reduce tasks produce files in the distributed file system (redundant and fault-tolerant), so no need to re-execute.

Semantics

- If the *map* and *reduce* functions are deterministic, then the result of `MapReduce` is the same as a sequential execution.
  - **This is really cool!**
  - There **is** a sequential implementation of `MapReduce`:
    - Read all of the *(Key1, Value1)* pairs from the input file.
    - Write all of the *(Key2, [list of Value2])* tuples to an intermediate file.
    - Sort the intermediate file by the *Key2* values.
    - Perform the reduce operation for each *Key2* value and write the results to the output file.
- If the *map* and *reduce* functions are not deterministic, then
  - It's a bit more complicated, but it's still reasonable.
  - If the reduce tasks are non-deterministic, then the result for each *Key2* is the result from some sequential implementation.
  - The paper doesn't talk about non-determinism for *map*, but it is probably similar.

Work Stealing

- Sometimes a worker is slow – **stragglers**.
- If the `MapReduce` task is near completion, the master assigns straggler tasks to idle processors.
- These are called **backup tasks**.
- Either the original or the backup process can complete the task.
- In practice, this **work stealing** by backup tasks:
  - Only adds a few percent to the total compute resources used.
  - Can result in substantial performance improvements: The paper reported a 44% slow-down when the sort benchmark was run without backup tasks.

Performance Issues

- The master attempts to schedule map tasks on the processor whose local disk holds the split being processed, or are nearby (by the network connections).
- Shuffle moves data from many map tasks to many reduce tasks.
  - Easily saturates the cross-section bandwidth of the network.
- For good performance, the map tasks should be filters that output much less data than they read.
  - Often not true of the "natural" intermediate representation (such as curriculum problem above).
  - Fewer distinct *Key2* values means less parallelism in reduce tasks.
- Can often reduce intermediate data size by partial reduction in the map workers.
  - May change the semantics of MapReduce (but not if reduce is associative and commutative).

MapReduce is Designed for BIG Data

- Communication between standard linux machines with generic networks takes milliseconds.
- Reading large disk files takes seconds.
- The task needs to be big enough to justify these overheads:
  - Equivalent to increasing λ by a few orders of magnitude.
  - MapReduce makes sense if the task is disk-limited and harnassing a few thousand disks provides the necessary disk bandwidth.
    - Think of it as "disk parallelism" instead of "CPU parallelism".
    - Note: big-data companies like Amazon, Facebook and Google are moving to using FLASH memory and DRAM instead of disks, exactly because of these I/O bottlenecks.
  - Or if you have a really huge data set the compute time may dominate all of these overheads.

Results (Part 1)

Achieves impressive performance on massive data sets 2008–2013(?)

- Report in [Dean & Ghemawat, 2008]: Good performance on $\sim$ 2000 machines: `grep` and `sort` work through 10^{10} 100-byte records (1TB) in minutes.
- Google estimates $\sim$ 20PB / day in total MapReduce processing in January 2008.
- Google research blog reports sorting 10^{13} 100-byte records (1PB) on $\sim$ 4000 machines (and $\sim$ 48, 000 disks) in six hours in November 2008, then 33 minutes for 1 PB or 6 hours for 10 PB on 8000 machines in September 2011.
- Open source implementation in Hadoop widely available as a cloud service.
- Many example algorithms documented; for example, search for "map reduce" on `http://scholar.google.ca`.

Results (Part 2)

Big data processors are now moving away from MapReduce.

- Framework emphasizes batch processing of data residing in distributed file system, which limits flexibility and efficiency.
- "We don't really use MapReduce anymore" [Urs H olzle, senior vice president of technical infrastructure at Google speaking at Google I/O conference in 2014]
- Machine learning project Apache Mahout is shifting away from MapReduce algorithms to alternatives such as Apache Spark.
  - Worth noting: Spark also uses a functional programming model with referential transparency.

Summary

- MapReduce is a parallel programming pattern
  - Data are represented by collections of *(Key, Value)* pairs.
  - User provides **map** to take input *(Key1, Value1)* pairs to intermediate *(Key2, Value2)* pairs.
  - Shuffle step collects intermediate data into *(Key2, [list of Value2])* pairs and sorts it by *Key2*.
  - User provides **reduce** to take sorted *(Key2, [list of Value2])* pairs into *(Key2, Value3)* pairs.
- Details of the parallel implementation are handled by the MapReduce API:
  - Creating workers processes, delivering input and output files, shuffling intermediate data between map and reduce workers.
  - Handling failures and slow nodes.
- Performance is often bandwidth limited.
  - Locality matters: perform *map* on the machine with the data.
  - If *map* is an effective filter (bytes out $\ll$ bytes in), then we can reduce the impact of network congestion.
  - In practice, user must choose *(Key2, Value2)* representation wisely to trade-off shuffle effort for reduce parallelism.

Review: Properties of MapReduce

- How does MapReduce distribute work between map tasks?
- How does MapReduce distribute work between reduce tasks?
- How does MapReduce handle machine, network or other failures?
- How does MapReduce handle slow (i.e. straggler) machines?
- What are the requirements for the type-signatures of the *map* and *reduce* functions in a map-reduce?

Review: Example of a MapReduce Problem

I want to fly from Vancouver to Timbuktu. There are no direct flights, so I want to find the fastest route with one stop. How could I do this using map reduce?

- Input data is a table of airline flights of the form:

```
(DepartCity, DepartTime, ArriveCity, ArriveTime)
```

  - Hint: use the intermediate city as *Key2*.
  - For simplicity, assume that all times are GMT (no need for time-zone conversion).
  - How does *map* filter out irrelevant flights?

Introduction to GPGPUs

Mark Greenstreet

CpSc 418 – Mar. 3, 2017

- GPUs
  - Early geometry engines.
  - Adding functionality and programmability.
  - GPGPUs
- CUDA
  - Execution Model
  - Memory Model
  - A simple example

Before the first GPU

Early 1980's: bit-blit hardware for simple 2D graphics.

- Draw lines, simple curves, and text.
- Fill rectangles and triangles.
- Color used a "color map" to save memory:
  - bit-wise logical operations on color map indices!

1989: The SGI Geometry Engine

- Basic rendering: coordinate transformation.
  - Represent a 3D point with a 4-element vector.
  - The fourth element is 1, and allows translations.
  - Multiply vector by matrix to perform coordinate transformation.
- Dedicated hardware is much more efficient that a general purpose CPU for matrix-vector multiplication.
  - For example, a 32×32 multiplier can be built with $32^2 = 1024$ one-bit multiplier cells.
    - A one-bit multiplier cell is about 50 transistors.
    - That's about 50K transistors for a very simple design. 30K is quite feasible using better architectures.

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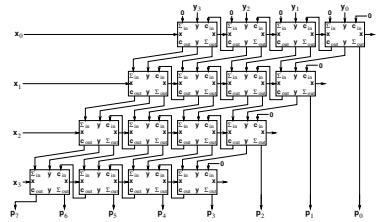
  - For example, a 32×32 multiplier can be built with $32^2 = 1024$ one-bit multiplier cells.
    - A one-bit multiplier cell is about 50 transistors.
    - That's about 50K transistors for a very simple design. 30K is quite feasible using better architectures.
  - The 80486DX was also born in 1989.
    - The 80486DX was 1.2M transistors, 16MHz, 13MIPs.
    - That's equal to 24 dedicated multipliers.
    - 16 multiply-and-accumulate units running at 50MHz (easy in the same 1μ process) produce 1.6GFlops!

Why is dedicated hardware so much faster?

A simple multiplier



Latency and period are $2N$.

Building a better multiplier

- Simple multiplier has latency and period of $2N$.
- Pipelining: add registers between rows.
  - The period is N, but the latency is N^2.
  - The bottleneck is the time for carries in each row.
- Use carry-lookahead adders (compute carries with a scan)
  - period is $\log N$, the latency is $N \log N$.
  - but the hardware is more complicated.
- Use carry-save adders and one carry-lookahead at the end
  - each adder in the multiplier forwards its carriers to the next adder.
  - the final adder resolves the carries.
  - period is 1, latency is N.
  - and the hardware is **way** simpler than a carry-lookahead design
- Graphics and many scientific and machine learning computations are very tolerant of latency.

Why is dedicated hardware so much faster?

Example: matrix-vector multiplication

- addition and multiplication are "easy".
- it's the rest of CPU that's complicated and the usual performance bottleneck
  - memory read and write
  - instruction fetch, decode, and scheduling
  - pipeline control
  - handling exceptions, hazards, and speculation
  - etc.
- GPU architectures amortize all of this overhead over a lot of execution units.

The fundamental challenge of graphics

Human vision isn't getting any better.

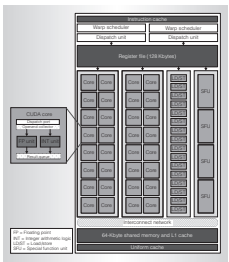
- Once you can perform a graphics task at the limits of human perception (or the limits of consumer budget for monitors), then there's no point in doing it any better.
- Rapid advances in chip technology meant that coordinate transformations (the specialty of the SGI Geometry Engine) were soon as fast as anyone needed.
- Graphics processors have evolved to include more functions. For example,
  - Shading
  - Texture mapping
- This led to a change from hardwired architectures, to programmable ones.

The GPGPU

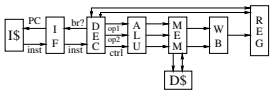
General Purpose Graphics Processing Unit

- The volume market is for graphics, and the highest profit is GPUs for high-end gamers.
  - Most of the computation is floating point.
  - Latency doesn't matter.
  - Abundant parallelism.
- Make the architecture fit the problem:
  - SIMD – single instruction, multiple (parallel) data streams.
    - Amortize control overhead over a large number of functional units.
    - They call it SIMT (. . . , multiple threads) because they allow conditional execution.
  - High-latency operations
    - Allows efficient, high-throughput, high-latency floating point units.
    - Allows high latency accesses to off-chip memory.
  - This means lots of threads per processor.

The Fermi Architecture



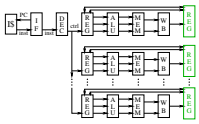
What does a core look like?



A RISC Pipeline

- RISC pipeline: see Jan. 23 slides (e.g. slides 5ff.)
  - Instruction fetch, decode and other control takes much more power than actually performing ALU and other operations!
- SIMD: Single-Instruction, Multiple-Data
- What about memory?

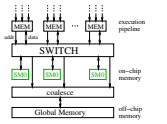
What does a core look like?



A SIMD Pipeline

- RISC pipeline: see Jan. 23 slides (e.g. slides 5ff.)
- SIMD: Single-Instruction, Multiple-Data
  - Multiple execution pipelines execute the same instructions.
  - Each pipeline has its own registers and operates on separate data values.
  - Commonly, pipelines access **adjacent** memory locations.
  - Great for operating on matrices, vectors, and other arrays.
- What about memory?

What does a core look like?



Memory Architecture

- RISC pipeline: see Jan. 23 slides (e.g. slides 5ff.)
- SIMD: Single-Instruction, Multiple-Data
- What about memory?
  - On-chip "shared memory" switched between cores: see Jan. 25 slides (e.g. slide 3)
  - Off-chip references are "coalesced": the hardware detects reads from (or writes to) consecutive locations and combines them into larger, block transfers.

More about GPU Cores

- Execution pipeline can be very deep – 20-30 stages.
  - Many operations are floating point and take multiple cycles.
  - A floating point unit that is deeply pipelined is easier to design, can provide higher throughput, and use less power than a lower latency design.
- No bypasses
  - Instructions block until instructions that they depend on have completed execution.
  - GPUs rely on extensive multi-threading to get performance.
- Branches use **predicated execution**:
  - Execute the then-branch code, disabling the "else-branch" threads.
  - Execute the else-branch code, disabling the "then-branch" threads.
  - The order of the two branches is unspecified.
- Why?
  - All of these choices optimize the hardware for graphics applications.
  - To get good performance, the programmer needs to understand how the GPGPU executes programs.

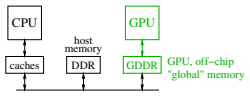
Lecture Outline

- GPUs
  - been there, done that.
- CUDA – we are here!
  - Execution Model
  - Memory Model
  - Code Snippets

Execution Model: Functions

- A CUDA program consists of three kinds of functions:
  - Host functions:
    - callable from code running on the host, but not the GPU.
    - run on the host CPU;
    - In CUDA C, these look like normal functions.
  - Device functions.
    - callable from code running on the GPU, but not the host.
    - run on the GPU;
    - In CUDA C, these are declared with a __device__ qualifier.
  - Global functions
    - called by code running on the host CPU,
    - they execute on the GPU.
    - In CUDA C, these are declared with a __global__ qualifier.

Execution Model: Memory



- Host memory: DRAM and the CPU's caches
  - Accessible to host CPU but not to GPU.
- Device memory: GDDR DRAM on the graphics card.
  - Accessible by GPU.
  - The host can initiate transfers between host memory and device memory.
- The CUDA library includes functions to:
  - Allocate and free device memory.
  - Copy blocks between host and device memory.
  - BUT host code can't read or write the device memory directly.

Structure of a simple CUDA program

- A __global__ function to called by the host program to execute on the GPU.
  - There may be one or more __device__ functions as well.
- One or more host functions, including `main` to run on the host CPU.
  - Allocate device memory.
  - Copy data from host memory to device memory.
  - "Launch" the device kernel by calling the __global__ function.
  - Copy the result from device memory to host memory.
- We'll do the `saxpy` example from the paper.
  - `saxpy` = "Scalar `a` times `x` plus `y`".

`saxpy`: device code

```
__global__void saxpy(uint n, float a, float *x, float *y) {
   uint i = blockIdx.x*blockDim.x + threadIdx.x; // nvcc built-ins
   if(i < n)
      y[i] = a*x[i] + y[i];
}
```

- Each thread has `x` and `y` indices.
  - We'll just use `x` for this simple example.
- Note that we are creating one thread per vector element:
  - Exploits GPU hardware support for multithreading.
  - We need to keep in mind that there are a large, but limited number of threads available.

`saxpy`: host code (part 1 of 5)

```
int main(int argc, char **argv) {
   uint n = atoi(argv[1]);
   float *x, *y, *yy;
   float *dev_x, *dev_y;
   int size = n*sizeof(float);
   x = (float *)malloc(size);
   y = (float *)malloc(size);
   yy = (float *)malloc(size);
   for(int i = 0; i < n; i++) {
      x[i] = i;
      y[i] = i+1;
   }
   ...
}
```

- Declare variables for the arrays on the host and device.
- Allocate and initialize values in the host array.

`saxpy`: host code (part 2 of 5)

```
int main(void) {
   ...
   cudaMalloc((void**)&dev_x), size);
   cudaMalloc((void**)&dev_y), size);
   cudaMemcpy(dev_x, x, size, cudaMemcpyHostToDevice);
   cudaMemcpy(dev_y, y, size, cudaMemcpyHostToDevice);
   ...
}
```

- Allocate arrays on the device.
- Copy data from host to device.

`saxpy`: host code (part 3 of 5)

```
int main(void) {
   ...
   float a = 3.0;
   saxpy<<<ceil(n/256.0),256>>>(n, a, dev_x, dev_y);
   cudaMemcpy(yy, dev_y, size, cudaMemcpyDeviceToHost);
   ...
}
```

- Invoke the code on the GPU:
  - `add<<<ceil(n/256.0),256>>>(...)` says to create $\lceil n/256 \rceil$ blocks of threads.
  - Each block consists of 256 threads.
  - See slide 22 for an explanation of threads and blocks.
  - The pointers to the arrays (in device memory) and the values of `n` and `a` are passed to the threads.
- Copy the result back to the host.

`saxpy`: host code (part 4 of 5)

```
...
for(int i = 0; i < n; i++) { // check the result
   if(yy[i] != a*x[i] + y[i]) {
      fprintf(stderr, "ERROR: i=%d, a[i]=%f, b[i]=%f, c[i]=%f\n
              i, a[i], b[i], c[i]);
      exit(-1);
   }
}
printf("The results match!\n");
...
}
```

- Check the results.

`saxpy`: host code (part 5 of 5)

```
int main(void) {
   ...
   free(x);
   free(y);
   free(yy);
   cudaFree(dev_x);
   cudaFree(dev_y);
   exit(0);
}
```

- Clean up.
- We're done.

Threads and blocks

- Our example created $\lceil \frac{n}{256} \rceil$ **blocks** with 256 **threads** each.
- The GPU hardware has a pool of running threads.
  - Each thread has a "next instruction" pending execution.
  - If the dependencies for the next instruction are resolved, the "next instruction" can execute.
  - The hardware in each streaming multiprocessor dispatches an instruction each clock cycle if a ready instruction is available.
  - The GPU in `lin25` supports 1024 such threads.
- What if our application needs more threads?
  - Threads are grouped into "thread blocks".
  - Each thread block has up to 1024 threads (the HW limit).
  - The GPU can swap thread-block in and out of main memory
    - This is GPU system software that we don't see as user-level programmers.

But is it fast?

- For this example, not really.
  - Execution time dominated by the memory copies.
- But, it shows the main pieces of a CUDA program.
- To get good performance:
  - We need to perform many operations for each value copied between memories.
  - We need to perform many operations in the GPU for each access to global memory.
  - We need enough threads to keep the GPU cores busy.
  - We need to watch out for thread divergence:
    - If different threads execute different paths on an if-then-else,
    - Then the else-threads stall while the then-threads execute, and vice-versa.
  - And many other constraints.
- GPUs are great if your problem matches the architecture.

Preview

March 6: Intro. to CUDA
Reading Kirk & Hwu Ch. 2
March 8: CUDA Threads, Part 1
Reading Kirk & Hwu Ch. 3
March 10: CUDA Threads, Part 2
March 13: CUDA Memory
Reading Kirk & Hwu Ch. 4
March 15: CUDA Memory: examples
March 17: CUDA Performance
Reading Kirk & Hwu Ch. 5
March 20: Matrix multiplication with CUDA, Part 1
March 22: Matrix multiplication with CUDA, Part 2
March 24 – April 3: Other Topics
- more parallel algorithms, e.g. dynamic programming?
- reasoning about concurrency, e.g. termination detection
- other paradigms, e.g. Scala and futures?

April 5: Party: 50th Anniversary of Amdahl's Law

Review

- What is SIMD parallelism?
- How does a CUDA GPU handle branches?
- How does a CUDA GPU handle pipeline hazards?
- What is the difference between "shared memory" and "global memory" in CUDA programming.
- Think of a modification to the `saxpy` program and try it.
  - You'll probably find you're missing programming features for many things you'd like to try.
  - What do you need?
  - Stay tuned for upcoming lectures.

Introduction to CUDA

Mark Greenstreet

CpSc 418 – Mar. 6, 2017

- GPU Summary: slide 2
- CUDA
  - Data parallelism: slide 6
  - Program structure: slide 8
  - Memory: slide 10
  - A simple example: slide 12
  - Launching kernels: slide 19

GPU Summary: architecture

- Lots of cores:
  - Up to 90 or more SIMD processors.
  - Each SIMD processors has 32 pipelines.
  - This is the nVidia architecture – other GPUs are similar.
- Deep, simple, execution pipelines
  - Optimized for floating point.
  - No bypassing: use multi-threading for performance.
  - Branches handled by predicated execution

"When you come to a fork in the road, take it."
(Often attributed to Yogi Berra.)

- Limited on-chip memory.
  - 1 or 2 MBytes total. Big CPUs have 32-64MB of L3 cache.
  - The programmer manages data placement.

GPU Summary: Performance

- Today's processors are constrained by how much performance can you get using $\sim$ 200 watts.
  - Moving bits around takes lots of energy.
  - Performing operations as quickly as possible takes lots of energy.
  - $E \sim dt^{-\alpha}$, where E is energy, d is distance, t is time per operation, and $1 < \alpha < 2$ depending on design details.
    - Corollary: $P \sim d^{\alpha+1}$. Power grows someplace between quadratically and cubically with clock frequency.
- How GPUs optimize performance/power
  - SIMD: instruction fetch and decode moves lots of bits. Amortize over many cores.
  - Simple pipelines: bypassing means moving bits quickly. GPUs omit bypasses.
  - High latency: avoid pipeline stages that must do a lot in a hurry.
  - Expose the memory hierarchy: let the programmer control moving data bits around.

GPU Summary: Economics

- GPUs are designed for the high-volume, consumer graphics market.
  - Amortize high design cost over a large number of units sold.
- This means GPUs aren't really optimized for scientific computing:
  - More on-chip memory would certainly help scientific computing, but not needed for graphics rendering.
  - Comparison: An nVidia GPU has about 2 MBytes of on-chip memory, an Intel Xeon can have 40MBytes or more.
  - Cache memory is about 60-70 transistors per byte.
  - A high-end nVidia GPU has 7 billion transistors, 1 or 2% for memory.
  - What if the chip were 30-40% memory?
    - better for general purpose computing
    - little pay-off for graphics
    - smaller distinction with Intel CPUs
- Cheap is good
  - It's the economics of cheap-computing that drives Moore's Law and all the other exponential growth-rate trends that make computing a field of intense, ongoing innovation.
  - That keeps the field in transition – deal with it.

Programming GPUs: CUDA

- Data Parallelism
- CUDA program structure
- Memory
- Launching kernels

Data Parallelism

- When you see a `for`-loop:
  - Is the loop-index used as an array index?
  - Are the iterations independent?
  - If so, you probably have data-parallel code.
- Data-Parallel problems:
  - Run well on GPUs because each element (or segment) of the array can be handled by a different thread.
  - Data parallel problems are good candidate for most parallel techniques because the available parallelism grows with the problem size.
  - Compare with "task parallelism" where the problem is divided into the same number of tasks regardless of its size.

Which of the following loops are data parallel?

```
for(int i = 0; i < N; i++)
   c[i] = a[i] + b[i].
```

```
dotprod = 0.0;
for(int i = 0; i < N; i++)
   dotprod += a[i]*b[i];
```

```
for(int i = 1; i < N; i++)
   a[i] = 0.5*(a[i-1] + a[i]);
```

```
for(int i = 1; i < N; i++)
   a[i] = sqrt(a[i-1] + a[i]);
```

```
for(int i = 0; i < M; i++) {
   for(int j = 0; j < N; j++) {
      sum = 0.0;
      for(int k = 0; k < L; k++)
         sum += a[i,k]*b[k,j];
      c[i,j] = sum;
} }
```

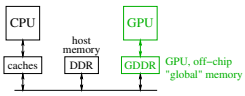

CUDA Program Structure

- A CUDA program consists of three kinds of functions:
  - Host functions:
    - callable from code running on the host, but not the GPU.
    - run on the host CPU;
    - In CUDA C, these look like normal functions – they can be preceded by the `__host__` qualifier.
  - Device functions.
    - callable from code running on the GPU, but not the host.
    - run on the GPU;
    - In CUDA C, these are declared with a `__device__` qualifier.
  - Global functions
    - called by code running on the host CPU,
    - they execute on the GPU.
    - In CUDA C, these are declared with a `__global__` qualifier.

Structure of a simple CUDA program

- A `__global__` function to called by the host program to execute on the GPU.
  - There may be one or more `__device__` functions as well.
- One or more host functions, including `main` to run on the host CPU.
  - Allocate device memory.
  - Copy data from host memory to device memory.
  - "Launch" the device kernel by calling the `__global__` function.
  - Copy the result from device memory to host memory.

Execution Model: Memory



- Host memory: DRAM and the CPU's caches
  - Accessible to host CPU but not to GPU.
- Device memory: GDDR DRAM on the graphics card.
  - Accessible by GPU.
  - The host can initiate transfers between host memory and device memroy.
- The CUDA library includes functions to:
  - Allocate and free device memory.
  - Copy blocks between host and device memory.
  - BUT host code can't read or write the device memory directly.

More Memory

- GPUs support fairly large off-chip memory bandwidth: 200-400GB/s.
  - But this isn't fast enough to keep 1000 processors busy at 1Gflop/s each!
- The GPU has on-chip memory to help:
  - Shared memory: 16KBytes or 48KBytes.
  - Registers: 128Kbytes (256KBytes on more recent GPUs).
  - Note that we need to use each value from memory for 20 or more instructions or else the memory bandwidth will limit performance.
- GPUs also have L2 caches, around 1.5MByte in the most recent chips.
  - But I haven't found a good way to understand them from the textbook, or from other CUDA manuals.
  - The coherence/consistency guarantees seem to be pretty weak.

Example: `saxpy`

`saxpy` = "Scalar `a` times `x` plus `y`".

- The device code.
- The host code.
- The running `saxpy`

`saxpy`: device code

```
__global__ void saxpy(uint n, float a, float *x, float *y) {
   uint i = blockIdx.x*blockDim.x + threadIdx.x; // nvcc built-ins
   if(i < n)
      y[i] = a*x[i] + y[i];
}
```

- Each thread has `x` and `y` indices.
  - We'll just use `x` for this simple example.
- Note that we are creating one thread per vector element:
  - Exploits GPU hardware support for multithreading.
  - We need to keep in mind that there are a large, but limited number of threads available.

`saxpy`: host code (part 1 of 5)

```
int main(int argc, char **argv) {
   uint n = atoi(argv[1]);
   float *x, *y, *yy;
   float *dev_x, *dev_y;
   int size = n*sizeof(float);
   x = (float *)malloc(size);
   y = (float *)malloc(size);
   yy = (float *)malloc(size);
   for(int i = 0; i < n; i++) {
      x[i] = i;
      y[i] = i*i;
   }
   ...
}
```

- Declare variables for the arrays on the host and device.
- Allocate and initialize values in the host array.

`saxpy`: host code (part 2 of 5)

```
int main(void) {
   ...
   cudaMalloc((void**)(&dev_x), size);
   cudaMalloc((void**)(&dev_y), size);
   cudaMemcpy(dev_x, x, size, cudaMemcpyHostToDevice);
   cudaMemcpy(dev_y, y, size, cudaMemcpyHostToDevice);
   ...
}
```

- Allocate arrays on the device.
- Copy data from host to device.

`saxpy`: host code (part 3 of 5)

```
int main(void) {
   ...
   float a = 3.0;
   saxpy<<<ceil(n/256.0),256>>>(n, a, dev_x, dev_y);
   cudaMemcpy(yy, dev_y, size, cudaMemcpyDeviceToHost);
   ...
}
```

- Invoke the code on the GPU:
  - `add<<<ceil(n/256.0),256>>>(...)` says to create $\lceil n/256 \rceil$ blocks of threads.
  - Each block consists of 256 threads.
  - See slide 20 for an explanation of threads and blocks.
  - The pointers to the arrays (in device memory) and the values of `n` and `a` are passed to the threads.
- Copy the result back to the host.

`saxpy`: host code (part 4 of 5)

```
...
for(int i = 0; i < n; i++) { // check the result
   if(yy[i] != a*x[i] + y[i]) {
      fprintf(stderr,
              "ERROR: i=%d, a[i]=%f, b[i]=%f, c[i]=%f\n",
              i, a[i], b[i], c[i]);
      exit(-1);
   }
}
printf("The results match!\n");
...
}
```

- Check the results.

`saxpy`: host code (part 5 of 5)

```
int main(void) {
   ...
   free(x);
   free(y);
   free(yy);
   cudaFree(dev_x);
   cudaFree(dev_y);
   exit(0);
}
```

- Clean up.
- We're done.

Launching Kernels

- Terminology
  - Data parallel code that runs on the GPU is called a **kernel**.
  - Invoking a GPU kernel is called **launching** the kernel.
- How to launch a kernel
  - The host CPUS invokes a `__global__` function.
  - The invocation needs to specify how many threads to create.
  - Example:
    - `add<<<ceil(n/256.0),256>>>(...)`
    - creates $\lceil \frac{n}{256} \rceil$ **blocks**
    - with 256 **threads** each.

Threads and Blocks

- The GPU hardware combines threads into **warps**
  - Warps are an aspect of the hardware.
  - All of the threads of warp execute together – this is the SIMD part.
  - The functionality of a program doesn't depend on the warp details.
  - But understanding warps is critical for getting good performance.
- Each warp has a "next instruction" pending execution.
  - If the dependencies for the next instruction are resolved, it can execute for all threads of the warp.
  - The hardware in each streaming multiprocessor dispatches an instruction each clock cycle if a ready instruction is available.
  - The GPU in `lin25` supports 32 such warps of 32 threads each in a "thread block."
- What if our application needs more threads?
  - Threads are grouped into "thread blocks".
  - Each thread block has up to 1024 threads (the HW limit).
  - The GPU can swap thread-blocks in and out of main memory
    - This is GPU system software that we don't see as user-level programmers.

Compiling and running

```
lin25$ nvcc saxpy.cu -o saxpy
lin25$ ./saxpy 1000
The results match!
```


But is it fast?

- For the `saxpy` example as written here, not really.
  - Execution time dominated by the memory copies.
- But, it shows the main pieces of a CUDA program.
- To get good performance:
  - We need to perform many operations for each value copied between memories.
  - We need to perform many operations in the GPU for each access to global memory.
  - We need enough threads to keep the GPU cores busy.
  - We need to watch out for thread divergence:
    - If different threads execute different paths on an if-then-else,
    - Then the else-threads stall while the then-threads execute, and vice-versa.
  - And many other constraints.
- GPUs are great if your problem matches the architecture.

Preview

March 8: CUDA Threads, Part 1
Reading Kirk & Hwu 3rd ed., Ch. 3 (Ch. 4 in 2nd ed.)
March 10: CUDA Threads, Part 2
March 13: CUDA Memory
Reading Kirk & Hwu 3rd ed., Ch. 4 (Ch. 5 in 2nd ed.)
March 15: CUDA Memory: examples
March 17: CUDA Performance
Reading Kirk & Hwu 3rd ed., Ch. 5 (Ch. 6 in 2nd ed.)
March 20: Matrix multiplication with CUDA, Part 1
March 22: Matrix multiplication with CUDA, Part 2
March 24 – April 3: Other Topics
- more parallel algorithms, e.g. dynamic programming?
- reasoning about concurrency, e.g. termination detection
- other paradigms, e.g. Scala and futures?

April 5: Party: 50th Anniversary of Amdahl's Law

Review

- What is SIMD parallelism?
- What is the difference between "shared memory" and "global memory" in CUDA programming.
- Think of a modification to the `saxpy` program and try it.
  - You'll probably find you're missing programming features for many things you'd like to try.
  - What do you need?
  - Stay tuned for upcoming lectures.

CUDA Threads

Mark Greenstreet & Ian M. Mitchell

CPSC 418 – March 8 & March 10, 2017

- Kernel organization: grids, blocks & threads.
- Hardware organization: SMs, SPs & warps.

Compute Capability

- Lots of nVidia jargon here.
- Lots of very specific constraints on hardware capabilities.
- Values of those constraints depend on the *compute capability*: essentially a version number for the GPU hardware.
  - CS department lab ({`lin01`, `lin02`, ..., `lin25`}`.ugrad.cs.ubc.ca`) has GeForce GTX 550 Ti which feature compute capability 2.1.
  - Examples of recent GPUs:
    - Compute capability 3.5: GT 730 & GTX 780.
    - Compute capability 5.0: GTX 750, 8xxM & 960M.
    - Compute capability 5.2: GTX 9xx, 965M.
    - Compute capability 6.1: GTX 10xx.
  - More details at the CUDA wikipedia page.

Thread organization: Grids, Blocks and Threads

- When a kernel is launched, it creates a collection of threads.
- This collection is called a **grid**.
  - A grid is organized as an array of **blocks**
  - Each block is an array of **threads**
  - Array sizes are fixed once a kernel is launched.
- Why so many details?
  - Switching between blocks is done (I infer) by software in the GPU.
  - Switching between threads in a block is done by hardware.
  - By distinguishing blocks from threads, the CUDA model exposes the performance issues to the programmer.

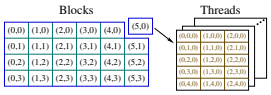
A grid is an array of blocks

(0,0)	(1,0)	(2,0)	(3,0)	(4,0)	(5,0)
(0,1)	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)
(0,2)	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)
(0,3)	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)

A grid

- Blocks are scheduled by the GPU **software**.
- Blocks can be arranged as 1D, 2D or 3D array.
  - Dimensions are called "`x`", "`y`" and "`z`".
- There can be **lots** of blocks:
  - Each dimension can be up to $2^{16} - 1 = 65535$.
  - CC 3.0+ allows `x` dimension up to $2^{31} - 1$ blocks.

Each block is an array of threads



Where do they put all those threads?

- Threads are scheduled by the GPU **hardware**.
- Threads can be arranged as a 1D, 2D, or 3D array
  - Grid and block dimensions and sizes may be different.
- There can be a moderate number of threads in each dimension:
  - `x` or `y` up to 1024 threads.
  - `z` up to 64 threads.
- However, total number of threads per block (product of all dimensions) is also capped at 1024.

Threads and blocks: launching a kernel

- Let's say we have:
  ```
  __global__ void kernel_fun(args)
  ```
- To launch this kernel, we execute a statement like:
  ```
  kernel_fun<<<dimGrid, dimBlock>>>(actuals);
  ```
  where
  - *dimGrid* specifies the dimension(s) of the grid (an array of blocks):
    - *dimGrid* can be an `int`, in which case the array is 1D.
    - *dimGrid* can be a `dim3`, for example:
      ```
      dim3(6,4,1)
      ```
  - *dimBlock* specifies the dimension(s) of each block (an array of threads):
    - *dimBlock* can be an `int` or a `dim3`.

Threads and Blocks within a Kernel's Grid

- Within a running kernel, CUDA-C provides four built-in variables to determine the position of a thread within the grid: `blockDim`, `blockIdx`, `threadDim`, and `threadIdx`.
- There is a naming pattern:
  - Each of these structures has three fields: `x`, `y` and `z` corresponding to the three possible dimensions.
  - `blockDim.?` gives the size of the grid in each dimension `x`, `y` or `z`.
  - `threadDim.?` gives the size of each block in each dimension.
  - `blockIdx.?` gives the indices of the thread's block within the grid.
  - `threadIdx.?` gives the indices of the thread within its block.
- For dimensions which are absent:
  - `blockDim` or `threadDim` will be 1.
  - `blockIdx` or `threadIdx` will be 0.

Threads and Blocks: Where are We?

- Note the constraints:

$$0 \le \texttt{blockIdx.x} < \texttt{blockDim.x}$$
$$0 \le \texttt{blockIdx.y} < \texttt{blockDim.y}$$
$$0 \le \texttt{blockIdx.z} < \texttt{blockDim.z}$$
$$0 \le \texttt{threadIdx.x} < \texttt{threadDim.x}$$
$$0 \le \texttt{threadIdx.y} < \texttt{threadDim.y}$$
$$0 \le \texttt{threadIdx.z} < \texttt{threadDim.z}$$

- Because the size of blocks are limited, it is common to use code such as:
  ```
  uint my_idx = blockDim.x*blockIdx.x + threadIdx.x;
  ```
  to combine the block and thread indices into a single index.

Bounds checking: launching kernels

- Consider executing `kernel_fun` on an array of `n` elements.
- Because `n` might be large, we'll use `n/256` blocks of 256 threads.
  - THINK: what if `n` is not a multiple of 256?
  - We'll round up to make sure we have enough threads.
- The kernel launch looks like:
  ```
  kernel_fun<<<ceil(n/256.0), 256>>>(n, myArray);
  ```
  - Why divide by `256.0` instead of `256`?
  - Why use `ceil`?

Bounds checking: in the kernel

- The kernel launch looks like:
  ```
  kernel_fun<<<ceil(n/256.0), 256>>>(n, myArray);
  ```
- THINK: what if `n` is not a multiple of 256?
  - We'll launch more than `n` threads?
  - For example, if `n==1000`, then we'll launch 4 blocks of 256 threads. A total of 1024 threads.
  - What will the last 24 threads do?
- Add a test:
  ```
  uint my_idx = blockDim.x*blockIdx.x + threadIdx.x;
  if(my_idx < n) {
    ...
  }
  ```


SMs, SPs and Warps (oh my!)

- Each *streaming multiprocessor* (SM) has multiple *streaming processors* (SPs) and can be responsible for multiple groups of 32 threads called *warps*.
  - From the *New Oxford American Dictionary*: (the) "warp" is "the threads on a loom over and under which other threads (the weft) are passed to make cloth"
- Details, details...
  - These concepts are not part of the CUDA platform and API: Code is written in terms of a grid of blocks of threads.
  - You can write correct code without thinking about these details.
  - If you want to write fast code, you must take them into account.
  - The block vs grid structure exposes these details if you want to take advantage of them.

SMs, SPs and Warps: What are They?

- Each streaming multiprocessor (SM) in the GPU executes threads in SIMD fashion.
  - All threads in a block are assigned to the same SM.
  - Each SM has a single (or small number of?) instruction fetch unit(s) and a larger number of execution units.
- Each SM has multiple streaming processors (SPs) that actually execute an instruction.
  - The SPs are specialized: ALUs, load / store, special function units.
  - A single SP can perform a single operation on a small set of threads.
- A warp is a collection of 32 threads that execute together on the same SP.

SMs, SPs and Warps: Why do We Care?

- Fill your warps: Ensure the number of threads in a block is a multiple of the warp size to avoid idle hardware.
- Have lots of warps: If one warp is waiting on a long latency operation, the SM can find another warp to execute.
  - Provides *latency tolerance* or *latency hiding*.
- Watch out for hardware limits (per SM).
  - Maximum number of resident blocks (8 in 2.x, 32 in 6.x).
  - Maximum number of resident warps (48 in 2.x, 64 thereafter).
  - Maximum number of resident threads (1536 in 2.x, 2048 thereafter).
  - Exceeding these limits will not crash the system, but will result in slower execution.
- Watch out for thread divergence.
  - If different threads in the same warp are following different code paths, all possible paths will be executed sequentially and those threads not on the current path will be idle.
  - Execution is still correct, but much slower.

A Warped Example: Reduce (part 1)

- Consider a reduce of an array `data` containing `n` elements using `n/2` threads (assume `n` is power of 2).
- Simple code:
  ```
  for(int stride = 1; stride < n; stride += stride) {
    if((my_idx & (stride-1)) == 0)
      data[2*my_idx] += data[2*my_idx + stride];
    __syncthreads();
  }
  ```
- The `__syncthreads()` call ensures that every thread has completed an iteration of the loop before any thread starts the next iteration.
  - More discussion on slide 18.

A Warped Example: Reduce (part 2)

- Consider `n == 16`
  - First iteration, for `i` in 0, ..., 7:
    ```
    data[2*i] += data[2*i]+1
    ```
    Now, all the even indexed elements have their sum with their odd counterpart.
  - Second iteration, for `i` in 0, 2, 4, 6:
    ```
    data[2*i] += data[2*i]+2.
    ```
    All elements with indices that are multiples of four, have their sum with the next three elements.
  - Third iteration leads with `data[0]` and `data[8]` holding sums for their halves of the array.
  - The fourth iteration puts the complete sum into `data[0]`.
- There are at most 8 threads working, so everything fits within a single warp.

A Warped Example: Reduce (part 3)

- What if `n==1024`?
  - We have 512 threads: 16 warps of 32 threads.
  - In the first iteration all threads are active.
  - In the next iteration each warp has 16 active threads, so the GPU has to execute the code for all 16 warps even though half the threads do nothing.
  - In subsequent iterations, the warps are more and more poorly utilized.
- This solution is correct, but much of the parallel hardware will sit idle much of the time.
- We would like to pack the busy threads into the minimum number of warps.

Warp Speed!

```
for(int stride = n/2; stride > 0; stride >>= 1) {
  if(my_idx < stride)
    data[my_idx] += data[my_idx] + stride;
  __syncthreads();
}
```

- Consider `n == 1024` again.
  - In the first iteration, there are 16 active warps – all threads in each warp are busy.
  - In the second iteration, there are 8 active warps – all threads in each active warp are busy.
  - Similarly, for the 3rd through 5th iterations
- The number of active warps decreases after each iteration, but all threads in each active warp are busy.
  - The inactive warps have no pending instructions, so they will not be scheduled and will not occupy processing resources.

Synchronization

- The reduce example used `__syncthreads()`: all the threads in the block must execute this statement before any can continue beyond it.
  - Be **very** careful about thread divergence: All threads in the block must meet at the **same** barrier.
  - That means the **same** line of code.
  - In loops, that means the **same** iteration.
  - Executing different `__syncthreads()` commands will cause the kernel to hang.
- Also, `__syncthreads()` only synchronizes between threads within a single block.
  - Note that threads within a warp already stay synchronized because they are executed together.
  - The only way to synchronize between threads in different blocks is to finish the kernel and launch another.
- We'll cover synchronization in more detail later.

Preview

March 10:	CUDA Threads, Part 2
March 13:	CUDA Memory
Reading	Kirk & Hwu Ch. 4
March 15:	CUDA Memory: examples
March 17:	CUDA Performance
Reading	Kirk & Hwu Ch. 5
March 20:	Matrix multiplication with CUDA, Part 1
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	• more parallel algorithms, e.g. dynamic programming?
	• reasoning about concurrency, e.g. termination detection
	• other paradigms, e.g. Scala and futures?
April 5:	Party: 50th Anniversary of Amdahl's Law

Review

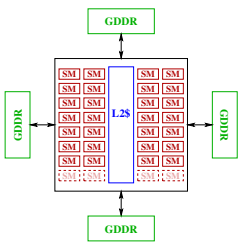
- In CUDA, what is a grid, a block, and thread?
- Why does CUDA allow millions of thread blocks but only 1024 threads per block?
- How does a programmer specify the number of blocks and number of threads when launching a CUDA kernel?
- How does a thread determine its position within the grid?
- Why do threads need to check their indices against array bounds?
- What is a warp? Why does it matter?

CUDA: Memory

Mark Greenstreet

CpSc 418 – March 13 & 15, 2017

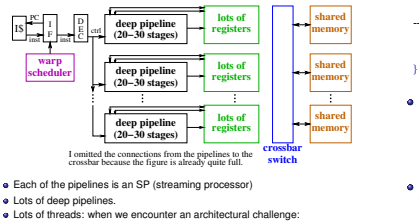
First, GPU Architecture Review



A "typical", high-end GPU.

- 28 SMs:
  - 128 SPs/SM.
  - That's 3584 SPs on the chip.
  - Each SM can schedule 4 warps in a single cycle.
- ∼ 1.6GHz clock frequency.
- 11 GBytes of GDDR memory, ∼ 484GBytes/sec. memory bandwidth.

A Streaming Multiprocessor (SM)



I omitted the connections from the pipelines to the crossbar because the figure is already quite full.

- Each of the pipelines is an SP (streaming processor)
- Lots of deep pipelines.
- Lots of threads: when we encounter an architectural challenge:
  - Raising throughput is easy, lowering latency is hard.
  - Solve problems by increasing latency and adding threads.
  - Make the programmer deal with it.

Why do we need a memory hierarchy

```
__global__ void saxpy(uint n, float a, float *x, float *y) {
  uint myId = blockDim.x*blockIdx.x + threadIdx.x;
  if(myId < n)
    y[myId] = a*x[myId] + y[myId];
}
```

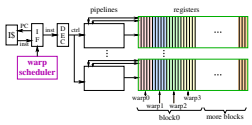
- A GPU with 3584 SPs, and a 1.6GHz clock rate (see slide 2 can perform over 5700 single-precision GFlops.
  - With a main memory bandwidth of 484 GBytes/sec., and 4 bytes per `float`, a CUDA kernel needs to perform $\frac{3584*1.6*4}{484} \approx 48$ floating point operations per memory read or write.
  - Otherwise, memory bandwidth becomes the bottleneck.
- Registers and shared memory let us use a value many times without going to the off-chip, GDDR memory.
  - But, we need to program carefully to make this work.
- Is `saxpy` a good candidate for GPU execution?

Matrix Multiplication and Memory

```
for(int i = 0; i < M; i++)
  for(int j = 0; j < N; j++)
    for(int k = 0; k < L; k++)
      c[i,j] += a[i,k]*b[k,j];
```

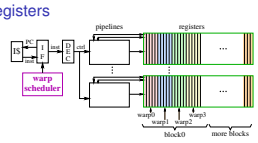
- Focus on the innermost loop: `for(k...)`
  - Why?
- How many floating point operations per iteration?
- How many memory reads?
- How many memory writes?
- What is the "Compute-to-Global-Memory-Access" ratio (CGMA)?

Registers



- Each SP has its own register file.
- The register file is partitioned between threads executing on the SP.
- Local variables are placed in registers.
  - The compiler in-lines functions when it can
    - A kernel with recursive functions or deeply nested calls can cause register spills to main memory – this is **slow**.
  - Local array variables are mapped to global memory – **watch out**.

More Registers



- In recent versions of CUDA, threads in the same warp can swap registers.
  - Provides very efficient intra-warp communication.
  - But not available in CUDA 2.1. ☹
- Performance trade-offs
  - A thread can avoid slow, global memory accesses by keeping data in registers.
  - But, using too many registers reduces the number of threads that can run at the same time.

Registers and Memory Bandwidth

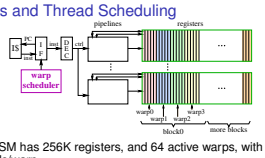
The GPU on slide 2 has 28 SMs, each with 128 SPs.

- Each SP has access to a register file.
- I'll guess two register reads and one write per clock cycle, per SP.
- I'll assume 4-byte registers.
- We get

$$28\text{SM} * 128\frac{\text{SP}}{\text{SM}} * 3\frac{\text{RW}}{\text{SP} * \text{cycle}} * 1.6 \times 10^9 \frac{\text{cycle}}{\text{sec.}} * 4\frac{\textit{Byte}}{\textit{RW}} = 68813\frac{\textit{GByte}}{\text{sec.}}$$

- 142 times faster than main memory bandwidth!
- ymmv: the GPUs in the linXX box are older.

Registers and Thread Scheduling



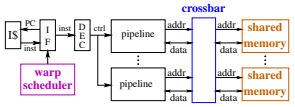
- Each SM has 256K registers, and 64 active warps, with 32 threads/warp.
  - That's 32 4-byte registers per thread.
- If a thread uses more registers
  - The SM cannot fully use its warp scheduler, or
  - Registers will spill to main memory – **slow**
- The numbers are smaller for older GPUs.
  - The GTX 550 Ti GPUs in the linXX boxes support 21 registers/thread.

Registers and Matrix Multiply

```
for(int i = 0; i < M; i +=2) {
   for(int j = 0; j < N; j +=2) {
      sum00 = sum01 = sum10 = sum11 = 0.0;
      for(int k = 0; k < L; k +=2) {
         a00 = a[i,k]; a01 = a[i,k+1];
         a10 = a[i+1,k]; a11 = a[i+1,k+1];
         b00 = b[k,j]; b01 = b[k,j+1];
         b10 = b[k+1,j]; b11 = b[k+1,j+1];
         sum00 += a00*b00 + a01*b10;
         sum01 += a00*b01 + a01*b11;
         sum10 += a10*b00 + a11*b10;
         sum11 += a10*b01 + a11*b11;
      }
      c[i,j] = sum00; c[i,j+1] = sum01;
      c[i+1,j] = sum10; c[i+1,j+1] = sum11;
} }
```

- use a register to accumulate `c[i,j]`
- hold blocks of each matrix in main memory.
- can use each value loaded from `a[i,k]` or `b[k,j]` three or four times.
- What is the CGMA for the example above?

Shared Memory



- On-chip, one bank per SP.
- Banks are interleaved by:
  - Early CUDA GPUs: 4-byte word
  - Later GPUs: programmer configurable 4-byte or 8-byte words
  - Why?
- Shared memory is a limited resource: 48KBytes to 96Kbytes/SM.
  - Each SM has more registers than shared-memory.
  - Shared memory demands limit how many blocks can execute concurrently on a SM.

Shared Memory Example: Reduce

```
__shared__ float v[1024];

__device__ float f(float x) {
  return(5/2)*(x*x*x - x));
}

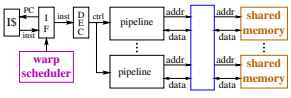
__device__ void compute_and_reduce(uint n, uint m, float *x) {
  uint myId = threadIdx.x;
  if(myId < n) {
     float y = x[myId];
     for(uint i = 0; i < m; i++)
        y = f(y); % y=f^i*(x[myId])
     v[myId] = y; % shared memory is much faster than global memory
     for(uint m = n >> 1; m > 0; m = n >> 1) { % reduce
        n -= m;
        __syncthreads();
        if(myId < m)
           v[myId] += v[myId+n];
     }
     x[myId] = v[myId]; % move result to global memory
  }
}
```

See notes on the next slide.

Notes on Reduce

- We calculate $f^m(x[myId])$ locally using the register variable `y`.
- For the reduce, we use the **shared** array `v`
  - This avoids the penalty of off-chip, global memory access for each step of the reduce.
  - All threads in a warp can access shared memory on the same cycle.
  - We can have multiple blocks running on multiple SMs
    - Each SM has its own shared memory.
    - Blocks running on different SMs in parallel can **all** access their shared memories in parallel.
    - **But**, threads in one block do not share shared-memory with threads in other blocks.
  - To perform a reduce across blocks:
    - Each block writes its subtotal to the **global** memory.
    - The results from the blocks are combined on the host CPU or by launching a new kernel.
- At the end, we copy our value from shared memory to the global memory so the CPU or a subsequent kernel can access it.

Shared Memory: Collisions



- When one thread in a warp accesses shared memory, **all** active threads in the warp access shared memory.
- If each thread accesses a different bank, then all accesses are performed in a single cycle.
  - Otherwise, the load or store can take multiple cycles.
  - Multiple accesses to the same bank are called **collisions**.
  - The **worst-case** occurs when **all threads access the same bank**.
- The programmer needs to think about the index calculations to avoid collisions.
  - When programming GPUs, the programmer needs to think about index calculations a lot.

Shared Memory Example: Matrix Multiply

Running example from the textbook: $C = A\,B$

- Each thread-block loads a 16×16 block from A and B.
  - The threads to these loads "cooperatively":
  - Read $A_{I,K}$ and $B_{K,J}$ from global memory with "coalesced" loads.
  - Write these blocks to shared-memory in a way that avoids bank conflicts.
- Compute: $C_{I,J} \mathrel{+}= A_{I,K} B_{K,J}$.
  - This takes $16^3 = 4096$ fused multiply-adds.
  - Loading $A_{I,K}$ fetches $16^2 = 256$ floats from global memory.
  - Likewise for $B_{K,J}$. Total of 512 floats fetched.
  - CGMA $= 4096/512 = 8$.
- Note: the L2 cache may help here: A and B are read-only.
  - Need to try more experiments.

Global Memory

- Off-chip DRAM
  - GDDR supports higher-bandwidth than than regular DDR.
  - A GPU can have multiple memory interfaces.
  - Total bandwidth 80 to 484+ GBytes/sec
- Memory accesses can be a big bottleneck.
  - CGMA: compute to global memory access ratio

Now for a word about DRAM

The memory that you plug into your computer is mounted on DIMMs (dual-inline memory modules).

- A DIMM typically has 16 or 18 chips
- E.g. each chip of an 8Gbyte DIMM holds 512MBytes = 4Gbits.
- Each chip consists of many "tiles",
  - a typical chip has 1Mbit/tile
  - that's 4096 tiles for a 4Gbit chip.
- Each tile is an array of capacitors.
  - each capacitor holds 1 bit.
  - a typical tile could have 1024 rows and 1024 columns.

Writing and reading DRAM

- Writing: easy
  - drive all 1024 column-lines to the values you want to write.
  - open up all the valves for one row.
  - the drinking cups for each column in that row get filled or emptied.
  - note: you end up writing **every** column in the row; so writes are often preceded by reads.
- Reading: hard
  - drive all 1024 column-lines to "half-way", and let them "float".
  - open up all the valves for one row.
  - if the level in the pipe goes up a tiny amount, that cup held a 1.
  - if the level in the pipe goes down a tiny amount, that cup held a 0.
  - it's a delicate measurement – it takes time to set it up.
  - This is why DRAM is slow.
- But: we just read 1024 bits, from each chip of the DIMM.
  - That's 16Kbits = 2Kbytes total.
  - Conclusion: DRAM has awful latency, but we can get very good bandwidth.
    - The bandwidth bottleneck is the wires from the DIMM to the CPU or GPU.
    - But I'm pretty sure that Ian won't let me give a lecture on transmission lines, phase-locked loops, equalizers, and all the other cool stuff in the DDR (or GDDR) interface.

GPUs meet DRAM

- DRAM summary: terrible latency (60-200ns or more), fairly bandwidth.
- The GPU lets the program take advantage of high bandwidth.
  - If the 32 loads from a warp access 32 consecutive memory location,
    - The GPU does **one** GDDR access,
    - and it transfers a large block of data.
  - The same optimization is applied to stores, and to loads from the on-chip caches.
- In CUDA-speak, if the loads from a warp access consecutive locations, we say that the memory accesses are **coalesced**.
- It's a big deal to make sure that your memory accesses are coalesced.
  - Note that the memory optimizations are exposed to the programmer.
  - You can get the performance by considering the memory model.
  - But, it's not automatic.

Example: Matrix Multiplication

- In C, matrices are usually stored in row-major order.
  - `A[i,k]` and `A[i,k+1]` are at adjacent locations, but
  - `B[k,j]` and `B[k+1,j]` are N words apart (for $N \times N$ matrices).
- For matrix multiplication, accesses to `A` are naturally coalesced, but accesses to `B`.
- The optimized code loads a block of `B` into shared memory.
  - This allows accesses to be coalesced.
  - But we need to be careful about how we store the data in the shared memory to avoid bank conflicts.

Other Memory

- Constant memory: cached, read-only access of global memory.
- Texture memory: global memory with special access operations.
- L1 and L2 caches: only for memory reads.

Summary

- GPUs can have thousands of execution units, but only a few off-chip memory interfaces.
  - This means that the GPU can perform 10-50 floating point operations for every memory read or write.
  - Arithmetic operations are very cheap compared with memory operations
- To mitigate the off-chip memory bottleneck
  - GPUs have, limited on-chip memory
  - Registers and the per-block, shared-memory will be our main concerns in this class.
- Moving data between different kinds of storage is the programmer's responsibility.
  - The programmer explicitly declares variables to be stored in shared memory.
  - The programmer needs to be aware of the per-thread register usage to achieve good SM utilization.
  - The only way to communicate between thread blocks is to write to global memory, end the kernel, and start a new kernel (ouch!)

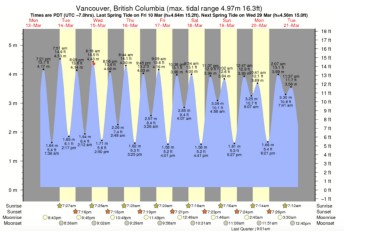
Preview

March 15: CUDA Memory: examples	
March 17: CUDA Performance	
Reading	Kirk & Hwu 3rd ed., Ch. 5 (Ch. 6 in 2nd ed.)
Mini Assignment	Mini Assignment 5 due at 10am.
March 20: Matrix multiplication, Part 1	
March 22: Matrix multiplication, Part 2	
March 24: Complete CUDA	
March 27 – April 3: this may change	
March 27: Using Parallel Libraries	
March 29 – April 3: Verification of/and Parallel Programs	
April 5: Party: 50th Anniversary of Amdahl's Law	

Review

- What is CGMA?
- On slide 15 we computed the CGMA for matrix-multiplication using 16×16 blocks of the A, B, and C matrices.
  - How many such thread-blocks can execute concurrently on an SM with 48KBytes of memory?
  - How does the CGMA change if we use 32×32 blocks?
  - If we use the larger matrix-blocks, how many thread blocks can execute concurrently on an SM with 48Kbytes of memory?
  - If we use the larger matrix-blocks, how many thread blocks can execute concurrently on an SM with 96Kbytes of memory?
- What are bank conflicts?
- How can increasing the number of registers used by a thread improve performance?
- How can increasing the number of registers used by a thread degrade performance?
- What is a "coalesced memory access"?

Beware the Tides of March



From https://www.tide-forecast.com/locations/Vancouver-British-Columbia/tides/latest

CUDA: Performance Considerations

Mark Greenstreet & Ian M. Mitchell

CPSC 418 – March 17, 2017

Thread Divergence

- If threads in a warp are following different code paths, execution will be much slower.
- See "A Warped Example" from March 13 slides.

Try to minimize thread divergence within warps.

Remarks about floating point

- When working on my solution to last year's HW3, Q1,
  - I first wrote:
    ```
    x = alpha*x*(1.0 - x);
    ```
  - and the performance was disappointing.
  - After many frustrating attempts to track down the problem, I added one, little `f`:
    ```
    x = alpha*x*(1.0f - x);
    ```
  - and my code ran 5.5× faster.
- What happened?

Floats, doubles, and GPUs

- GPUs are optimized for single-precision floating point arithmetic.
- For the GeForce GTX 550 Ti, double precision arithmetic is way slower than single precision.
- In C, `1.0` is a **double precision** constant, and `1.0f` is single precision.
- When I wrote `x = alpha*x*(1.0-x)`, the compiler generated code that:
  - computes the product `alpha*x`.
    - both operands are single precision.
    - the computation is done using single precision arithmetic.
  - computes the difference `1.0-x`
    - 1.0 is double precision, x is single precision.
    - the computation is done using double precision arithmetic
    - and the result is double precision.
  - computes the product `alpha*x*(1.0-x)`.
    - the computation is done using double precision arithmetic
    - and the result is double precision.
- When I wrote `x = alpha*x*(1.0f-x)`, everything stays in single-precision, and it's **much** faster.

Fused multiply adds

- Calculating $ax + b$ is very common
  - Example: dot product.
- The multiplier hardware is just a pipeline of adders.
  - When multiplying `a*x`, the hardware can start the pipeline from `b` instead of from `0`.
  - We get the sum for "free".
  - This is called a **fused** multiply-add.
- The marketing people like to count the fused multiply-add as **two** floating point operations.
  - This helps make some performance claims make sense.
- For the obsessive compulsive:
  - Rounding with a fused-multiply add can be slightly different than when doing two, separate operations.
  - Compilers usually let the users specify "strict" floating point (no fusing) or "fast" floating point (with fusing).
  - `nvcc` uses fused multiply add unless you give it an option not to.

Memory Access

Memory is slow.

- It takes a long time to identify, access and deliver data to/from a memory address.
- Delivery rate is limited by clock rate of the memory interface.

How can we get enough data to/from our thousands of threads? **Parallelism!**

- Retrieve lots of data at once.
- Use multiple memory interfaces.
- Build with lots of independent memory components.

All standard techniques in the CPU world, but

- CPU design philosophy: Try to achieve maximum performance even if the programmer uses the RAM model.
- GPU design philosophy: Expose (almost) everything and let the programmer figure it out.

Memory System Parallelism 1: Get Lots of Data

Memory is addressed per byte, but you retrieve a bunch of (sequential) bytes at once.

- GDDR5 DRAM: 32-bit bus per chip and transfers are in 16 word bursts (so 64 bytes per access per chip).
- GPU global memory (from GDDR DRAMs): Accessed by 32-, 64- or 128-byte transactions.
  - Transactions must be "naturally" aligned: First address must be a multiple of the transaction size.
  - CC 2.x: L1 cache (1 per SM) serviced by 128-byte transactions, L2 cache (shared by SMs) by 32-byte transactions.
  - CC 6.x: Same as 2.x, but L1 cache rules are complicated.
- GPU shared memory (on-chip SRAM): Access in 32-bit words.

Amortize addressing overhead and thereby increase bandwidth.

Memory System Parallelism 2: Multiple Interfaces

If one memory component cannot give you enough bandwidth, use a bunch (see March 13 slides).

- Global memory: K&H(3) calls these "channels" (March 13 slide 2).
- Shared memory: Mark called these "banks" (March 13 slide 11) and NVidia documentation does too.
  - Do not confuse with K&H(3) "banks" (see next slide).

Consecutive chunks are placed into components in a round-robin fashion, where "chunk" means

- 32-bytes (64 more recently?) in global memory.
- 32- or 64-bits in shared memory.

Separate subsystems can all provide data at their native rate and thereby increase bandwidth.

Memory System Parallelism 3: Independent Memory Components

Even after memory address is delivered, it still takes time for the DRAM to return the data.

- Rather than let the memory bus sit idle while waiting, pipeline a bunch of memory requests to different memory components.
  - K&H(3) calls these "banks".
  - Mark called these "tiles".
- Consecutive memory chunks are assigned first to channels / banks (see previous slide).
  - These subsystems allow concurrent access because they have independent communication lines.
- Then assign next set of consecutive chunks to banks / tiles.
  - These subsystems allow sequential but pipelined access because they share communication lines

Pipelining increases throughput (although latency remains).

- Only relevant for global memory.
- Shared memory achieves dramatically lower latency with SRAM.

Is This on the Final?

No (sort of): This is not a course on memory system design and implementation.

- Mark and I are far from experts.
- Details depend on the particular GPU chip and card, and change regularly.
- Program correctness does not depend on getting it right.

Yes (sort of): By following some simple rules, speed can be improved dramatically.

- Design global memory access pattern to allow accesses from threads in the same warp to be **coalesced** into a single memory transaction.
- Design memory access pattern to avoid channel / bank / tile **collisions**.

Implications for Shared Memory

See CUDA Toolkit Documentation C Programming Guide Figure 17 and Figure 18.

- Consider shared memory address bits:
  - 48KB / thread block requires 16 bits to address.
  - Bottom two bits specify the byte within a 32-bit word of data.
  - Next five bits specify which of 32 banks.
  - Top nine bits specify which word within the bank.
- Key takeaway: If two threads in a warp access a memory location in the same bank (same middle five bits of address):
  - If threads access the same location (same top nine bits), then broadcast (on read) or one value wins (on write).
  - If threads access different location, access is serialized (slower but still correct).

Implications for Global Memory

Try to get memory access addresses from threads in a warp to be very close together.

- Accesses to consecutive (or nearly so) addresses are coalesced into a single transaction on the off-chip memory bus.
  - You should already be doing this for your CPU designs so that your caches can take advantage of spatial locality.
- Best coalescing occurs when the set of addresses is naturally aligned.
  - For two and higher dimensional arrays, that may mean padding thread block and array width allocation in memory to be a multiple of the warp size.
- Possibility of channel / bank collisions would argue for avoiding addresses with the same "middle" bits.
  - I could not find NVidia documentation of these details.
  - How do caches interact with channels / banks?

Comments from Mark?

SMs and Thread Occupancy

- Occupancy: how many warps are available for the SM
  - Why we care: the SP pipelines have long latencies.
  - The CUDA approach is to run lots of threads simultaneously to keep the pipelines busy.
- Limits to occupancy
  - How many blocks per SM.
  - How much shared-memory per block.
  - How many threads per block.
  - How many registers per thread.
- Figuring it out
  - `nvcc -O3 -c --ptxas-options -v examples.cu`
  - The nVidia occupancy calculator: CUDA_Occupancy_calculator.xls
  - But we can do it manually?

Occupancy with CUDA 2.1

- Different GPUs at level CUDA 2.1 have differing numbers of SMs.
  - But the SMs all look the same, even for different GPUs.
- CUDA 2.1 SMs
  - An SM has warps of 32 threads
  - An SM can simultaneously execute up to 1536 threads (48 warps).
  - An SM has 32K (2^{15}) 32-bit registers (128K/bytes, 1K registers/SP).
  - An SM has 48K bytes of shared memory.
  - An SM can simultaneously execute up to 8 blocks.
  - Each block can have up to 1024 threads.

Why all these numbers?

- When designing a new generation of GPUs, the GPU architects run lots of simulations to estimate the performance for various choices of the architectural parameters.
- For example, if more warps are allowed in the scheduling pool
  - The SM will have useful instructions to dispatch more often ⇒ better performance.
  - **BUT** the on-chip circuitry to hold and manage the scheduling pool will be larger.
  - This means instruction scheduling will be slower ⇒ a longer clock period.
  - Instruction scheduling will use more power ⇒ a longer clock period, or fewer SMs, or more expensive chip cooling.
  - The real-estate on the chip could have been used for something else. Is this the **best** use of that area.
  - Note that CUDA 5 made the increase to 64 warps/SM.
- Architects explore these trade-offs to optimize performance for graphics applications, the main source of revenue.
- Architects are also risk-adverse: make the chip as much like the last one that worked as you can.
- These hard-wired constraints have a large impact on program performance.

SMs, blocks, and threads

- A SM can have simultaneously execute most 8 blocks.
- All blocks have the same number of threads.
- Thus, a SM can execute at most

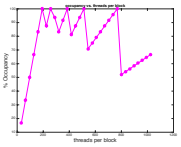
$$\min\left(8, \left\lfloor \frac{1536}{threadsPerBlock} \right\rfloor\right)$$

blocks.

- The ratio of the number of threads executing to the maximum possible is called the "thread occupancy":

$$threadOccupancy \leq \min\left(8, \left\lfloor \frac{1536}{threadsPerBlock} \right\rfloor\right) \frac{threadsPerBlock}{1536}$$

SMs, blocks, and threads – the plot



- I get 100% occupancy when $threadsPerBlock \in \{192, 384, 768\}$, but the CUDA calculator doesn't.
  - I'll have to try some experiments – stay tuned.
- This assumes the grid had enough blocks to keep the SMs busy.
  - A grid with a single block will have poor performance.

SMs, threads, and registers

- Each SM has 32K registers – that's 1K registers per SP.
- This is another constraint:

$$nblks \leq \frac{1024}{registersPerThread}$$

- An SM can run 48 warps simultaneously
  - But only if each warp uses at most 21 registers.

Hitting the register constraint

What if each thread uses 22 registers?

- $22 * 48 = 1056 > 1024 \rightarrow$ can't run 48 warps.
- $\lfloor \frac{1024}{22} \rfloor = \lfloor 46.\overline{54} \rfloor = 46$.
- Can we run 46 warps?
  - One block with 46 warps would have $46 * 32 = 1472 > 1024$ threads. Not allowed.
  - Two block with 23 warps each would each have 736 threads. That should work.
  - But, the plot with the occupancy calculator only shows warp counts that are multiples of 8.
  - Have I overlooked another architectural constraint?
    - probably
- Let's assume that with 23 registers per thread, the SM can run at most 40 warps simultaneously.
  - Then either each thread must have enough instruction-level parallelism to keep the SPs busy.
  - Or, we'll see a drop in performance.

How many registers does my thread use?

- use the `--ptxas-options -v` option for `nvcc`

```
nvcc--ptxas-options -v -O3 -c examples.cu
ptxas info :   0 bytes gmem
ptxas info :   Compiling entry function '_Z8sh_mem_2jiiPj' for 'sm_20'
ptxas info :   Function properties for _Z8sh_mem_2jiiPj
    0 bytes stack frame, 0 bytes spill stores, 0 bytes spill loads
ptxas info :   Used 17 registers, 4096 bytes smem, 56 bytes cmem[0]
ptxas info :   Compiling entry function '_Z8sh_mem_1jiiPj' for 'sm_20'
ptxas info :   Function properties for _Z8sh_mem_1jiiPj
    0 bytes stack frame, 0 bytes spill stores, 0 bytes spill loads
ptxas info :   Used 14 registers, 4096 bytes smem, 56 bytes cmem[0]
```

- Translation:
  - kernel `sh_mem_2` uses 17 registers per thread.
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How much work should a kernel do?

- Do more work within a kernel: Launching each kernel takes time.
- Do less work within a kernel: New kernels allow for changes in block and grid size, and ensure synchronization between threads even in different blocks.
- Either way: Minimize movement of data to and from the host.

How much work should a thread do?

- Do more work in a single thread: Fewer chances for memory collisions, easier synchronization, less register contention.
- Do less work in a single thread: More potential parallelism, more chance for latency hiding.
- Tradeoff will depend on GPU resources, typically SM block, thread and register limits.

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CUDA: Performance Considerations

Mark Greenstreet & Ian M. Mitchell

CPSC 418 – March 17, 2017

Thread Divergence

- If threads in a warp are following different code paths, execution will be much slower.
- See "A Warped Example" from March 13 slides.

Try to minimize thread divergence within warps.

Remarks about floating point

- When working on my solution to last year's HW3, Q1,
  - I first wrote:

```
x = alpha*x*(1.0 - x);
```

  - and the performance was disappointing.
  - After many frustrating attempts to track down the problem, I added one, little `f`:

```
x = alpha*x*(1.0f - x);
```

  - and my code ran 5.5× faster.
- What happened?

Floats, doubles, and GPUs

- GPUs are optimized for single-precision floating point arithmetic.
- For the GeForce GTX 550 Ti, double precision arithmetic is way slower than single precision.
- In C, `1.0` is a **double precision** constant, and `1.0f` is single precision.
- When I wrote `x = alpha*x*(1.0-x)`, the compiler generated code that:
  - computes the product `alpha*x`.
    - both operands are single precision.
    - the computation is done using single precision arithmetic.
  - computes the difference `1.0-x`
    - 1.0 is double precision, x is single precision.
    - the computation is done using double precision arithmetic
    - and the result is double precision.
  - computes the product `alpha*x*(1.0-x)`.
    - the computation is done using double precision arithmetic
    - and the result is double precision.
- When I wrote `x = alpha*x*(1.0f-x)`, everything stays in single-precision, and it's **much** faster.

Fused multiply adds

- Calculating $ax + b$ is very common
  - Example: dot product.
- The multiplier hardware is just a pipeline of adders.
  - When multiplying `a*x`, the hardware can start the pipeline from `b` instead of from `0`.
  - We get the sum for "free".
  - This is called a **fused** multiply-add.
- The marketing people like to count the fused multiply-add as **two** floating point operations.
  - This helps make some performance claims make sense.
- For the obsessive compulsive:
  - Rounding with a fused-multiply add can be slightly different than when doing two, separate operations.
  - Compilers usually let the users specify "strict" floating point (no fusing) or "fast" floating point (with fusing).
  - `nvcc` uses fused multiply add unless you give it an option not to.

Memory Access

Memory is slow.

- It takes a long time to identify, access and deliver data to/from a memory address.
- Delivery rate is limited by clock rate of the memory interface.

How can we get enough data to/from our thousands of threads? **Parallelism!**

- Retrieve lots of data at once.
- Use multiple memory interfaces.
- Build with lots of independent memory components.

All standard techniques in the CPU world, but

- CPU design philosophy: Try to achieve maximum performance even if the programmer uses the RAM model.
- GPU design philosophy: Expose (almost) everything and let the programmer figure it out.

Memory System Parallelism 1: Get Lots of Data

Memory is addressed per byte, but you retrieve a bunch of (sequential) bytes at once.

- GDDR5 DRAM: 32-bit bus per chip and transfers are in 16 word bursts (so 64 bytes per access per chip).
- GPU global memory (from GDDR DRAMs): Accessed by 32-, 64- or 128-byte transactions.
  - Transactions must be "naturally" aligned: First address must be a multiple of the transaction size.
  - CC 2.x: L1 cache (1 per SM) serviced by 128-byte transactions, L2 cache (shared by SMs) by 32-byte transactions.
  - CC 6.x: Same as 2.x, but L1 cache rules are complicated.
- GPU shared memory (on-chip SRAM): Access in 32-bit words.

Amortize addressing overhead and thereby increase bandwidth.

Memory System Parallelism 2: Multiple Interfaces

If one memory component cannot give you enough bandwidth, use a bunch (see March 13 slides).

- Global memory: K&H(3) calls these "channels" (March 13 slide 2).
- Shared memory: Mark called these "banks" (March 13 slide 11) and NVidia documentation does too.
  - Do not confuse with K&H(3) "banks" (see next slide).

Consecutive chunks are placed into components in a round-robin fashion, where "chunk" means

- 32-bytes (64 more recently?) in global memory.
- 32- or 64-bits in shared memory.

Separate subsystems can all provide data at their native rate and thereby increase bandwidth.

Memory System Parallelism 3: Independent Memory Components

Even after memory address is delivered, it still takes time for the DRAM to return the data.

- Rather than let the memory bus sit idle while waiting, pipeline a bunch of memory requests to different memory components.
  - K&H(3) calls these "banks".
  - Mark called these "tiles".
- Consecutive memory chunks are assigned first to channels / banks (see previous slide).
  - These subsystems allow concurrent access because they have independent communication lines.
- Then assign next set of consecutive chunks to banks / tiles.
  - These subsystems allow sequential but pipelined access because they share communication lines

Pipelining increases throughput (although latency remains).

- Only relevant for global memory.
- Shared memory achieves dramatically lower latency with SRAM.

Is This on the Final?

No (sort of): This is not a course on memory system design and implementation.

- Mark and I are far from experts.
- Details depend on the particular GPU chip and card, and change regularly.
- Program correctness does not depend on getting it right.

Yes (sort of): By following some simple rules, speed can be improved dramatically.

- Design global memory access pattern to allow accesses from threads in the same warp to be **coalesced** into a single memory transaction.
- Design memory access pattern to avoid channel / bank / tile **collisions**.

Implications for Shared Memory

See CUDA Toolkit Documentation C Programming Guide Figure 17 and Figure 18.

- Consider shared memory address bits:
  - 48KB / thread block requires 16 bits to address.
  - Bottom two bits specify the byte within a 32-bit word of data.
  - Next five bits specify which of 32 banks.
  - Top nine bits specify which word within the bank.
- Key takeaway: If two threads in a warp access a memory location in the same bank (same middle five bits of address):
  - If threads access the same location (same top nine bits), then broadcast (on read) or one value wins (on write).
  - If threads access different location, access is serialized (slower but still correct).

Implications for Global Memory

Try to get memory access addresses from threads in a warp to be very close together.

- Accesses to consecutive (or nearly so) addresses are coalesced into a single transaction on the off-chip memory bus.
  - You should already be doing this for your CPU designs so that your caches can take advantage of spatial locality.
- Best coalescing occurs when the set of addresses is naturally aligned.
  - For two and higher dimensional arrays, that may mean padding thread block and array width allocation in memory to be a multiple of the warp size.
- Possibility of channel / bank collisions would argue for avoiding addresses with the same "middle" bits.
  - I could not find NVidia documentation of these details.
  - How do caches interact with channels / banks?

Comments from Mark?

SMs and Thread Occupancy

- Occupancy: how many warps are available for the SM
  - Why we care: the SP pipelines have long latencies.
  - The CUDA approach is to run lots of threads simultaneously to keep the pipelines busy.
- Limits to occupancy
  - How many blocks per SM.
  - How much shared-memory per block.
  - How many threads per block.
  - How many registers per thread.
- Figuring it out
  - `nvcc -O3 -c --ptxas-options -v examples.cu`
  - The nVidia occupancy calculator: CUDA_Occupancy_calculator.xls
  - But we can do it manually?

Occupancy with CUDA 2.1

- Different GPUs at level CUDA 2.1 have differing numbers of SMs.
  - But the SMs all look the same, even for different GPUs.
- CUDA 2.1 SMs
  - An SM has warps of 32 threads
  - An SM can simultaneously execute up to 1536 threads (48 warps).
  - An SM has 32K (2^{15}) 32-bit registers (128K/bytes, 1K registers/SP).
  - An SM has 48K bytes of shared memory.
  - An SM can simultaneously execute up to 8 blocks.
  - Each block can have up to 1024 threads.

Why all these numbers?

- When designing a new generation of GPUs, the GPU architects run lots of simulations to estimate the performance for various choices of the architectural parameters.
- For example, if more warps are allowed in the scheduling pool
  - The SM will have useful instructions to dispatch more often ⇒ better performance.
  - **BUT** the on-chip circuitry to hold and manage the scheduling pool will be larger.
  - This means instruction scheduling will be slower ⇒ a longer clock period.
  - Instruction scheduling will use more power ⇒ a longer clock period, or fewer SMs, or more expensive chip cooling.
  - The real-estate on the chip could have been used for something else. Is this the **best** use of that area.
  - Note that CUDA 5 made the increase to 64 warps/SM.
- Architects explore these trade-offs to optimize performance for graphics applications, the main source of revenue.
- Architects are also risk-adverse: make the chip as much like the last one that worked as you can.
- These hard-wired constraints have a large impact on program performance.

SMs, blocks, and threads

- A SM can have simultaneously execute most 8 blocks.
- All blocks have the same number of threads.
- Thus, a SM can execute at most

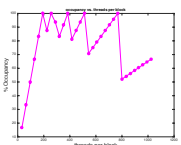
$$\min\left(8, \left\lfloor \frac{1536}{threadsPerBlock} \right\rfloor\right)$$

blocks.

- The ratio of the number of threads executing to the maximum possible is called the "thread occupancy":

$$threadOccupancy \leq \min\left(8, \left\lfloor \frac{1536}{threadsPerBlock} \right\rfloor\right) \frac{threadsPerBlock}{1536}$$

SMs, blocks, and threads – the plot



- I get 100% occupancy when $threadsPerBlock \in \{192, 384, 768\}$, but the CUDA calculator doesn't.
  - I'll have to try some experiments – stay tuned.
- This assumes the grid had enough blocks to keep the SMs busy.
  - A grid with a single block will have poor performance.

SMs, threads, and registers

- Each SM has 32K registers – that's 1K registers per SP.
- This is another constraint:

$$nblks \leq \frac{1024}{registersPerThread}$$

- An SM can run 48 warps simultaneously
  - But only if each warp uses at most 21 registers.

Hitting the register constraint

What if each thread uses 22 registers?

- $22 * 48 = 1056 > 1024 \rightarrow$ can't run 48 warps.
- $\lfloor \frac{1024}{22} \rfloor = \lfloor 46.\overline{54} \rfloor = 46$.
- Can we run 46 warps?
  - One block with 46 warps would have $46 * 32 = 1472 > 1024$ threads. Not allowed.
  - Two block with 23 warps each would each have 736 threads. That should work.
  - But, the plot with the occupancy calculator only shows warp counts that are multiples of 8.
  - Have I overlooked another architectural constraint?
    - probably
- Let's assume that with 23 registers per thread, the SM can run at most 40 warps simultaneously.
  - Then either each thread must have enough instruction-level parallelism to keep the SPs busy.
  - Or, we'll see a drop in performance.

How many registers does my thread use?

- use the `--ptxas-options -v` option for `nvcc`

```
nvcc--ptxas-options -v -O3 -c examples.cu
ptxas info :   0 bytes gmem
ptxas info :   Compiling entry function '_Z8sh_mem_2jiiPj' for 'sm_20'
ptxas info :   Function properties for _Z8sh_mem_2jiiPj
    0 bytes stack frame, 0 bytes spill stores, 0 bytes spill loads
ptxas info :   Used 17 registers, 4096 bytes smem, 56 bytes cmem[0]
ptxas info :   Compiling entry function '_Z8sh_mem_1jiiPj' for 'sm_20'
ptxas info :   Function properties for _Z8sh_mem_1jiiPj
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Bigger Kernels

```
__global__myKernel(...)  {
    do something
}
```

Unless *do something* is big, kernel launch takes most of the time.

- We can launch a big-grid
  - If we have a huge number of array elements than each need a small amount of work, this can be a good idea.
  - BUT we're likely to create a memory-bound problem.
- Or, we can make each thread do many somethings.

```
__global__myKernel(int m, ...)  {
    for(int i = 0; i < m; i++)
        do something
}
```


Loop Limitations

- It takes two or three instructions per loop iteration to manage the loop:
  - One to update the loop index
  - One or two to check the loop bounds and branch.
  - If *do something* is only three or four instructions, then 40-50% of the execution time is for loop management.
- If each iteration of *do something* depends on the previous one
  - Then the long latency of the SP pipelines can limit performance.
  - Even if we have 48 warps running.

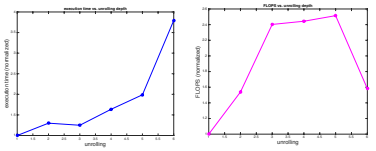
Loop Unrolling

- Have each loop iteration perform multiple copies of the loop body

```
__global__myKernel(int m, ...)  {
    for(int i = 0; i < m; i += 4) {
        do something 1
        do something 2
        do something 3
        do something 4
    }
}
```

- More "real work" for each time the loop management code is executed.
- Need to make sure that `m` is a multiple of four, or handle end-cases separately.
- Often, we need more registers.

Unrolling – the plots



This example is from last year's HW3, Q1.

Where's λ?

- Communication between the CPU and GPU
  - Kernel launch overhead
  - Transfering data between CPU memory and GPU memory
    - Is this solved with more recent GPUs that can access the CPU memory directly?
    - Not really, the data still needs to be transfered.
    - And it's one more memory level for the programmer to keep track of.
- Communication between blocks
  - Write global memory and end the kernel.
  - Launch a new kernel and read the global memory.
  - The same strategy applies if the shape for the required grid changes between phases of a larger computation.
- Communication between warps in a block
  - `__synchthreads__`
- **AND**,
  - There's a built-in energy cost of the big register file.
  - Trade-offs of energy, latency, and parallelism. large numbers of threads.

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Matrix Multiplication – Algorithms

Mark Greenstreet

CpSc 418 – Mar. 22, 2017

Outline:

- Sequential Matrix Multiplication
- Parallel Implementations, Performance, and Trade-Offs.

Objectives

Apply concepts of algorithm analysis, parallelization, overhead, and performance measurement to a real problem.

- Design sequential and parallel algorithms for matrix multiplication.
- Analyse algorithms and measure performance.
- Identify bottlenecks and refine algorithms.

Matrix representation in Erlang

- I'll represent a matrix as a list of lists.
- For example, the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \end{bmatrix}$$

is represented by the Erlang nested-list:

```
[  [1, 2,  3,  4]
   [1, 4,  9, 16]
   [1, 8, 27, 64] ]
```

- The empty matrix is `[]`.
  - This means my representation can't distinguish between a 2×0 matrix, a 0×4 matrix, and a 0×0 matrix.
  - That's OK. This package is to show some simple examples.
  - I'm not claiming it's for advanced scientific computing.

Sequential Matrix Multiplication

```
mult(A, B) ->
    BT = transpose(B),
    lists:map(
        fun(Row_of_A) ->
            lists:map(
                fun(Col_of_B) ->
                    dot_prod(Row_of_A, Col_of_B)
                end, BT)
        end, A).
dot_prod(V1, V2) ->
    lists:foldl(
        fun({X,Y},Sum) -> Sum + X*Y end,
        0, lists:zip(V1, V2)).
```

- Next, we'll use list comprehensions to get a more succinct version.

Matrix Multiplication, with comprehensions

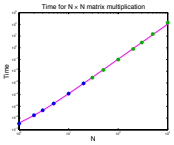
```
mult(A, B) ->
  BT = transpose(B),
  [ [ dot_prod(RowA, ColB) || ColB <- BT ] || RowA <- A].

transpose([]) -> []; % special case for empty matrices
transpose([[]|_]) -> []; % bottom of recursion, the columns are empty
transpose(M) ->
  [   [H || [H | _T] <- M ] % create a row from the first column of M
    | transpose([ T || [_H | T] <- M ]) % now, transpose what's left
  ].
```


Performance – Modeled

- Really simple, operation counts:
  - Multiplications: `n_rows_a` $*$ `n_cols_b` $*$ `n_cols_a`.
  - Additions: `n_rows_a` $*$ `n_cols_b` $*$ (`n_cols_a` -1).
  - Memory-reads: 2*#Multiplications.
  - Memory-writes: `n_rows_a` $*$ `n_cols_b`.
  - Time is $\mathcal{O}$(`n_rows_a` $*$ `n_cols_b` $*$ `n_cols_a`), If both matrices are $N \times N$, then its $\mathcal{O}(N^3)$.
- But, memory access can be terrible.
  - For example, let matrices `a` and `b` be 1000×1000.
  - Assume a processor with a 4M L2-cache (final cache), 32 byte-cache lines, and a 200 cycle stall for main memory accesses.
  - Observe that a row of matrix `a` and a column of `b` fit in the cache. (a total of ~40K bytes).
  - But, all of `b` does not fit in the cache (that's 8 Mbytes).
  - So, on every fourth pass through the inner loop, every read from `b` is a cache miss!
  - Cache miss dominates everything else.
- This is why there are carefully tuned numerical libraries.

Performance – Measured



- Cubic of best fit: $T = (107N^3 + 134N^2 + 173N - 32)$ns.
- Fit to first six data points.
- Cache misses effects are visible, for N=1000:
  - model predicts $T = 107$seconds,
  - but the measured value is $T = 142$seconds.

Tiling Matrices

An Example

- Let A, B, and $C = AB$ be 16×16 matrices.
- Let $A1 = A[1:4, 1:16]$, i.e. the first four rows of A.
  - In our Erlang represntation, `[A1,_] = lists:split(4, A).`
- Let $A2 = A[5:8, 1:16]$; $A3 = A[9:12, 1:16]$; $A4 = A[13:16, 1:16]$; and likewise for $C1$, $C2$, $C3$, and $C4$.
- Big important fact:

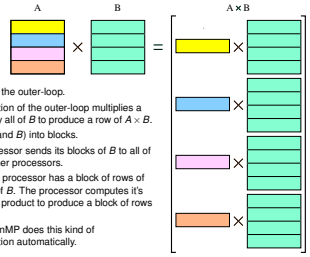
$$\begin{array}{lll} C1 = A1\,B & \quad & C2 = A2\,B \\ C3 = A3\,B & & C4 = A4\,B \end{array}$$

- In **sequential** Erlang:

```
[C1, C2, C3, C4] = [mult(AA, B) || AA <- A]
```

- To make it **parallel**, we compute each of the $C_I = A_I\,B$ with a separate process.

Parallel Algorithm 1



- Parallelize the outer-loop.
- Each iteration of the outer-loop multiplies a row of A by all of B to produce a row of $A \times B$.
- Divide A (and B) into blocks.
- Each processor sends its blocks of B to all of the the other processors.
- Now, each processor has a block of rows of A and all of B. The processor computes it's part of the product to produce a block of rows of C.
- Note: OpenMP does this kind of parallelization automatically.

Parallel Algorithm 1 in Erlang

```
% mult(W, Key, Key1, Key2) – create a matrix associated with Key
%   that is the product of the matrices associated with Key1 and Key2.
mult1(W, Key, Key1, Key2) ->
  Nproc = workers:nworkers(W),
  workers:update(W, Key,
    fun(PS, I) ->
      A = workers:get(PS, Key1), % my rows of A
      B = workers:get(PS, Key2), % my rows of B
      [WW ! {B, I} || WW <- W], % send my rows of B to everyone
      B_full = lists:append( % receive B from everyone
        [     receive {BB, J} -> BB end
          || J <- lists:seq(1, Nproc)]),
      matrix:mult(A, B_full) % compute my part of the product
    end
  ).
```


Performance of Parallel Algorithm 1 – Modeled

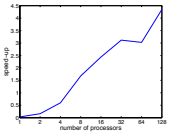
- CPU operations: same total number of multiplies and adds, but distributed around P processors. Total time: $\mathcal{O}(N^3/P)$.
- Communication: Each processors sends (and receives) $P-1$ messages of size N^2/P. If time to send a message is $t_0 + t_1 * M$ where M is the size of the message, then the communication time is

$$\begin{array}{rcll} (P-1)\left(t_0 + t_1\frac{N^2}{P}\right) & = & \mathcal{O}(N^2 + \lambda P), & \text{but, beware of large constants} \\ & = & \mathcal{O}(N^2), & N^2 > P \end{array}$$

Note: I'm assuming t_0 corresponds to λ, and that t_1 is roughly the same as a the time for "typical" sequential operations..

- Memory: Each process needs $\mathcal{O}(N^2/P)$ storage for its block of A and the result. It also needs $\mathcal{O}(N^2)$ to hold all of B.
  - The simple algorithm divides the computation across all processors, but it doesn't make good use of their combined memory.

Performance of Parallel Algorithm 1 – Measured



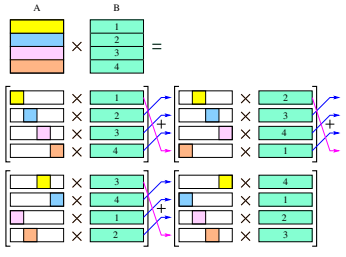
Using Memory more Efficiently

- Main idea: each process works on one "slab" of B at a time.

$$\begin{array}{rcl} C[i,j] & = & \sum_{k=1}^{N} A[i,k]\,B[k,j], \quad \text{a dot-product} \\ & = & \left(\sum_{k=1}^{N/4} A[i,k]\,B[k,j]\right) + \left(\sum_{k=(N/4)+1}^{N/2} A[i,k]\,B[k,j]\right) \\ & & + \left(\sum_{k=(N/2)+1}^{3N/4} A[i,k]\,B[k,j]\right) + \left(\sum_{k=(3N/4)+1}^{N} A[i,k]\,B[k,j]\right) \end{array}$$

- Each process does each of its four summations when it holds the corresponding slab of B.
  - Each holds one slap of A for the whole computation.
  - Each process only needs to hold one slab of B at at time.
- The algorithm generalizes to having any number of slabs for A and B in the obvious way.
  - Should be "obvious" if I've explained this clearly.
  - If it isn't obvious, that's my bad – please ask a question.

Parallel Algorithm 2 (illustrated)



Parallel Algorithm 2 (code sketch)

- Each processor first computes what it can with its rows from A and B.
  - It can only use N/P of its columns of its block from A.
  - It uses its entire block from B.
  - We've now computed one of P matrices, where the sum of all of these matrices is the matrix AB.
- We view the processors as being arranged in a ring,
  - Each processor forwards its block of B to the next processor in the ring.
  - Each processor computes an new partial product of AB and adds it to what it had from the previous step.
  - This process continues until every block of B has been used by every processor.

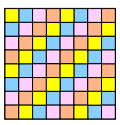
Performance of Parallel Algorithm 2

- CPU operations: Same as for parallel algorithm 1: total time: $\mathcal{O}(N^3/P)$.
- Communication: Same as for parallel algorithm 1: $\mathcal{O}(N^2 + P)$.
  - With algorithm 1, each processor sent the same message to $P-1$ different processors.
  - With algorithm 2, for each processor, there is one destination to which it sends $P-1$ different messages.
  - Thus, algorithm 2 can work efficiently with simpler interconnect networks.
- Memory: Each process needs $\mathcal{O}(N^2/P)$ storage for its block of A, its current block of B, and its block of the result.
  - Note: each processor might hold onto its original block of B so we still have the blocks of B available at the expected processors for future operations.
- Do the memory savings matter?

Bad performance, pass it on

- Consider what happens with algorithm 2 if one processor, P_{slow} takes a bit longer than the others one of the times its doing a block multiply.
  - P_{slow} will send it's block from B to its neighbour a bit later than it would have otherwise.
  - Even if the neighbour had finished its previous computation on time, it won't be able to start the next one until it gets the block of B from P_{slow}.
  - Thus, for the next block computation, both P_{slow} and its neighbour will be late, even if both of them do their next block computation in the usual time.
  - In other words, tardiness propagates.
- Solution: forward your block to you neighbour before you use it to perform a block computation.
  - This overlaps computation with communication, generally a good idea.
  - We could send two or more blocks ahead if needed to compensate for communication delays and variation in compute times.
  - This is a way to save time by using more memory.

Tiling in Real-Life



- Why? If there's time, I'll explain in class.

Summary

- Matrix multiplication is well-suited for a parallel implementation.
- Need to consider communication costs.
- In the previous algorithms, computate time grows as N^3/P, while communication time goes as $(N^2 + P)$.
- Thus, if N is big enough, computation time will dominate communication time.
- Connection of theory with actual run time is pretty good:
  - But the matrices have to be big enough to amortize the communication costs.

Preview

March 24: Matrix Multiplication in CUDA
Homework: HW4 due at 11:59pm
HW5 goes out
March 27: Using Parallel Libraries
March 29: Introduction to Model Checking
Reading: TBA
March 31: The PReach Model Checker
Reading: Industrial Strength . . . Model Checking
April 3: Distributed Termination Detection
April 5: Party: 50th Anniversary of Amdahl's Law

CUDA: Matrix Multiplication

Mark Greenstreet

CpSc 418 – Mar. 24, 2017

- A Brute Force Implementation
- Tiling

mmult1: brute-force matrix multiplication

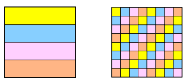
The kernel:

```
% one thread per element of the result.
% matrixMult: compute c = a*b
% For simplicity, assume all matrices are n × n.
__global__ mmult1_kernel(float *a, float *b, float *c, uint n) {
   uint i = blockDim.y*blockIdx.y + threadIdx.y;
   uint j = blockDim.x*blockIdx.x + threadIdx.x;
   if((i < n) && (j < n)) {
      float *a_row = a + n*i;
      float *b_col = b + j;
      float sum = 0.0;
      for(int k = 0; k < n; k++) {
         sum += a_row[k] * b_col[n*k];
      }
      c[i*n + j] = sum;
   }
}
```


Brute-force performance

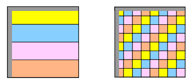
- Not very good – each loop iteration performs
  - Two global memory reads.
  - One fused floating-point add.
  - Four or five integer operations.
- Global memory is slow
  - Long access times.
  - Bandwidth shared by all the SPs.
- This implementation has a low **CGMA**
  - CGMA = Compute to Global Memory Access ratio $\approx 1/2$.
- Performance should be:
  - asymptotics: $\mathcal{O}(N^3)$
  - wall-clock: $\sim \alpha N^3$ with α determined mainly by global memory bandwidth.
  - measured: $T(1024) \approx 0.0986$s; $T(2048) \approx 0.797$s; $T(3072) \approx 2.7$s; $T(4096) \approx 6.3$s.
    $N^3/T(N) \approx 11/$ns – i.e. about 20×10^9 multiply-adds per second.
    Well below GPU peak floating point capacity. Demonstrates global memory bandwidth bottleneck (with a little help from the on-chip caches).

Tiles vs. Slabs



- Matrix multiplication: each processor (color) has tiles at (i,j) and (j,i).
  - Can compute all products for the main diagonal, and stripes at spacings of P.
  - Use a reduce to combine results to get the main diagonal and the stripes.
  - Rotate B one block to the left, and compute the next set of strips.
  - After P rounds, the computation is done.
  - Same amount of work (and communication) as the improved slab method from Wednesday.
- Other algorithms such as LU-Decomposition
  - Rows and columns are eliminated from the left and the top.
  - Tiles provide better load balancing.

Tiles vs. Slabs



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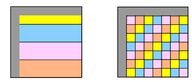

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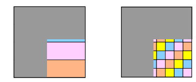

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- Other algorithms such as LU-Decomposition
  - Rows and columns are eliminated from the left and the top.
  - Tiles provide better load balancing.

Tiling the computation

- Divide each matrix into $m \times m$ tiles.
  - For simplicity, we'll assume that n is a multiple of m.
- Each block computes a tile of the product matrix.
  - Computing a $m \times m$ tile involves computing n/m products of $m \times m$ tiles and summing up the results.

A Tiled Kernel (step 1)

```
#define TILE_WIDTH 16
__global__ mmult2(float *a, float *b, float *c, int n) {
   float *a_row = a + (blockDim.y*blockIdx.y + threadIdx.y)*n;
   float *b_col = b + (blockDim.x*blockIdx.x + threadIdx.x);
   float sum = 0.0;
   for(int k1 = 0; k1 < gridDim.x; k1++) { % each tile product
      for(int k2 = 0; k2 < blockDim.x; k2++) { % within each tile
         k = k1*blockDim.x + k2;
         sum += a_row[k] * b_col[n*k]);
      }
   }
   c[ (blockDim.y*blockIdx.y + threadIdx.y)*n +
      (blockDim.x*blockIdx.x + threadIdx.x) ] = sum;
}
```

Launching the kernel:

```
int nblks = n/TILE_WIDTH;
dim3 blks(nblks, nblks, 1);
dim3 thrds(TILE_WIDTH, TILE_WIDTH, 1);
matrixMult<<<blks,thrds>>>(a, b, c, n);
```


A Tiled Kernel (step 2)

```
__global__ matrixMult(float *a, float *b, float *c, int n) {
   __shared__ a_tile[TILE_WIDTH][TILE_WIDTH];
   __shared__ b_tile[TILE_WIDTH][TILE_WIDTH+1];
   int br = blockIdx.y,     bc = blockIdx.x;
   int tr = threadIdx.y,    tc = threadIdx.x;
   float *a_row = a + (blockDim.y*br + tr)*n;
   float *b_col = b + (blockDim.x*bc + tc);
   float sum = 0.0;
   for(int k1 = 0; k1 < gridDim.x; k1++) { % each tile product
      a_tile[tr][tc] = a_row[TILE_WIDTH*k1 + tc];
      b_tile[tr][tc] = b_col[n*(TILE_WIDTH*k1 + tr)];
      __syncthreads();
      for(int k2 = 0; k2 < blockDim.x; k2++) { % within each tile
         sum += a_tile[tc][k2] * b_tile[k2][tc];
      }
      __syncthreads();
   }
   c[(blockDim.y*br + tr)*n + (blockDim.x*bc + tc) ] = sum;
}
```


Performance of mmult2

- $T(1024) = 0.027$s; $T(2048) = 0.214$s; $T(3072) = 0.742$s; $T(4096) = 1.73$s.
- Still cubic in N, of course.
  - $N^3/T(N) \approx 40/$ns – about 40 billion multiply-adds per second.
  - About four times faster than mmult1.

Performance issues for mmult2

The "checklist"

- Are global memory accesses coalesced?
- What is the CGMA?
- Do we have shared memory access conflicts?
- What is the warp-scheduler occupancy?
  - How many registers per thread?
  - How many threads per block?
  - How much shared memory per block?
- How much "other stuff" does each thread perform for each floating point operation?

Tiling is good for more than just matrix multiplication

- Other numerical applications:
  - LU-decomposition and other factoring algorithms.
  - Matrix transpose.
  - Finite-element methods.
  - Many, many more.
- A non-numerical example: revsort

```
% To sort N^2 values, arrange them as a N × N array.
repeat log N times {
   sort even numbered rows left-to-right.
   sort odd numbered rows right to left.
   sort columns top-to-bottom.
}
```

- We can get coalesced accesses for the rows, but not the columns.
- Cooperative loading can help here – e.g. use a transpose.

Summary

- Brute-force matrix multiplication is limited by global memory bandwidth.
- Using tiles addresses this bottleneck:
  - Load tile into shared memory and use them many times.
  - Each tile element is used by multiple threads.
  - The threads cooperate to load the tiles.
  - This approach also provides memory coalescing.
- Other optimizations: prefetching, double-buffering, loop-unrolling.
  - First, identify the critical bottleneck.
  - Then, optimize.
- These ideas apply to many parallel programming problems:
  - When possible, divide the problem into blocks to keep the data local.
  - Examples include matrix and mesh algorithms.
  - The same approach can be applied to non-numerical problems as well.

Preview

March 27: Using Parallel Libraries
March 29: Introduction to Model Checking
Reading: Protocol Verification as a Hardware Design Aid
March 31: The PReach Model Checker
Reading: Industrial Strength . . . Model Checking
April 3: Distributed Termination Detection
April 5: Party: 50th Anniversary of Amdahl's Law

Linear Algebra Libraries and CUDA

Mark Greenstreet & Ian M. Mitchell

CPSC 418 – March 27, 2017

- Why?
- BLAS
- Using BLAS (in general)
- Using BLAS (on CUDA GPUs)
- Other numerical libraries

Once Upon a Time. . .

- Numerical calculation has been a key application since the earliest days of computing.
  - The term "computer" has been used for centuries to refer to a person performing mathematical calculations according to a fixed set of rules.
  - One of the earliest electronic general purpose computers (1946) was the Electronic Numerical Integrator and Computer (ENIAC) designed primarily to calculate artillery firing tables for the US Army.
- High level programming language and compiler development was spurred by Fortran ("formula translator") starting in the mid-1950s.
- Turing Award for 1989 went to William Kahan for his work "making the world safe for numerical computations."
  - IEEE standard for floating point arithmetic (IEEE 754) now provides a common, reproducible and robust format across virtually all computing platforms.

Linear Algebra is Everywhere

Many numerical algorithms are designed around linear algebra operations.

- By late 1960s it was common in the numerical computing community to implement these operations as separate "subprograms"
- ACM-SIGNUM project 1973-1977 set out to design what we would now call a common API to these most common routines.
- Design process and outcomes documented in a series of papers in *ACM Trans. Mathematical Software* (*ACM-TOMS*):
  - Lawson et al, "Basic linear algebra subprograms for Fortran usage," *ACM TOMS* 5(3): 308–323 (Sept. 1979).
  - Dongarra et al, "An Extended Set of FORTRAN Basic Linear Algebra Subprograms," *ACM TOMS* 14(1): 1–17 (March 1988).
  - Dongarra et al, "A Set of Level 3 Basic Linear Algebra Subprograms," *ACM TOMS* 16(1): 1–17 (March 1990).
  - Blackford et al, "An Updated Set of Basic Linear Algebra Subprograms (BLAS)," *ACM TOMS* 28(2): 135–151 (June 2002).

Basic Linear Algebra Subprograms (BLAS)

Authors and contributors anticipated many benefits:

- Encourages "structured programming": Modularization of common code sequences.
- Code will be more self-documenting: Other programmers will recognize the subprogram names.
- Subprograms can be coded in assembly to improve efficiency, and if the majority of computational effort is within the subprograms that will significantly benefit the whole application.
- Subprograms can be coded by experts to deal with "algorithmic and implementation subtleties."
- Code becomes portable while still maintaining efficiency.

While the details may differ, similar benefits still accrue today.

Levels of BLAS

BLAS specification consists of operations at one of three "levels":

- BLAS-1: Vector-vector operations (scalar vector product, vector sum, dot product, etc.).
  - [Lawson et al, 1979].
  - Performs $\mathcal{O}(n)$ operations on $\mathcal{O}(n)$ data.
- BLAS-2: Matrix-vector operations (matrix-vector product, triangular solves)
  - [Dongarra et al, 1988].
  - Performs $\mathcal{O}(n^2)$ operations on $\mathcal{O}(n^2)$ data.
- BLAS-3: Matrix-matrix operations (matrix-matrix product, triangular solves with multiple right-hand sides)
  - [Dongarra et al, 1990].
  - Performs $\mathcal{O}(n^3)$ operations on $\mathcal{O}(n^2)$ data.

Types of Operands

- Provides for either "single precision" or "double precision" floating point arithmetic.
  - Support for complex variables (real + imaginary components).
  - Note that BLAS does not mandate IEEE FP standard: Definition of precision depends on the platform.
- Initial versions focused on dense or banded matrices.
  - Special cases for symmetric, Hermitian (complex version of symmetry) or triangular form.
- Extended in [Blackford et al, 2002]:
  - Sparse matrices.
  - Extended and mixed precision arithmetic.
  - A number of new routines whose importance was discovered during implementation of LAPACK:
    - Commonly used operations, such as matrix norm.
    - Slight generalizations of existing routines.
    - Perform two existing routines in a single call to reduce memory traffic.
- Many other extensions / implementations have been described.

Fortran? Are You Kidding Me?

- At the time of the initial design of BLAS, Fortran was by far the dominant language of numerical computing.
  - The FORTRAN 77 standard had just been adopted
  - (the first BLAS definition was non-conforming.)
- Many limitations and idiosyncracies can be avoided, such as:
  - Only Fortran bindings.
  - ALL CAPITAL LETTERS for symbols.
  - Static allocation of arrays.
  - 1-based indexing.
- Some Fortran features remain in some implementations, such as:
  - Function names and arguments are incomprehensibly short.
  - Column-major ordering of data in matrices.
  - Arguments are pass by reference (even some scalars).

Decypering BLAS Function Names

Function names in BLAS follow a pattern.

- Often a prefix, such as `BLAS_` or `cblas_`.
- One character to denote data type; for example:
  - `s`: single precision.
  - `d`: double precision.
- Operations involving a matrix add two characters to denote matrix type; for example:
  - `ge`: general dense matrix.
  - `tb`: triangular banded.
- Short mnemonic string to denote operation; for example
  - `axpy`: ax plus y.
  - `mm`: matrix multiply.
- Put them all together:
  - `BLAS_SAXPY()`: Fortran single precision vector summation.
  - `cblas_dgemm()`: C double precision dense matrix product.

Decyphering BLAS Function Arguments (part 1)

Consider matrix product $C = \alpha A^{op} B^{op} + \beta C$ implemented by

```
cblas_sgemm(enum blas_order_type layout,
            enum blas_trans_type transa,
            enum blas_trans_type transb,
            int m, int n, int k,
            float alpha,
            float *a, int lda,
            float *b, int ldb,
            float beta,
            float *c, int ldc)
```

- `layout` specifies either column-major or row-major.
- `transa` specifies whether to use A, A^T or A^H.
  - Same for `transb` and B.
- `m`, `n`, `k` specify matrix sizes: A is $m \times k$, B is $k \times n$, C is $m \times n$.
- `alpha` and `beta` specify scalar multipliers.
  - Some implementations may require pass by reference.

Decyphering BLAS Function Arguments (part 2)

Consider matrix product $C = \alpha A^{op} B^{op} + \beta C$ implemented by

```
cblas_sgemm(enum blas_order_type layout,
            enum blas_trans_type transa,
            enum blas_trans_type transb,
            int m, int n, int k,
            float alpha,
            float *a, int lda,
            float *b, int ldb,
            float beta,
            float *c, int ldc)
```

- `a` is a pointer to array for A and `lda` is the distance between the start of consecutive columns (for column-major) or rows (for row-major).
  - Same for `b`, `ldb` and B.
  - Same for `c`, `ldc` and C.

What's with the ld* Arguments?

- BLAS routines allow for data which is not stored continuously.
- These `ld*` arguments are called the *stride*.
- For vectors, striding allows access to rows or columns of a matrix.
  - Consider the data in an `m × n` column-major matrix.
  - A column has stride `1` and length `m`.
  - A row has stride `m` and length `n`.
- For matrices, striding allows access to submatrices; for example,
  - Consider the data in an `m × n` column-major matrix `a`.
  - We want the `p × q` block starting at row `i` and column `j`.
  - Data starts at `&a[i + j*m]`
  - Data has size `p` by `q`.
  - Data has stride `m`.

CUDA and BLAS

- The cuBLAS library provides an API for running BLAS routines on CUDA GPUs.
- Basic pattern of use:
  - Initialize the cuBLAS library and allocate hardware resources using `cublasCreate()`.
  - Allocate memory using `cudaMalloc()`.
  - Copy data from host to GPU using `cublasSetVector()` or `cublasSetMatrix()`.
  - Perform BLAS operations; for example `cublasSaxpy()` or `cublasSgemm()`.
  - Copy data from GPU to host using `cublasGetVector()` or `cublasGetMatrix()`.
  - Release memory using `cudaFree()`.
  - Release hardware resources using `cublasDestroy()`.
- Example(s).

Notes on cuBLAS

- Always uses column-major ordering
  - So be careful with data layout.
- Always uses 1-based indexing.
  - Usually irrelevant since you do not index into arrays.
- All cuBLAS code is called from the host.
  - You do not write any kernel code.
  - You do not have to worry about grids, blocks, shared memory, ...
- Need to link against cuBLAS library.
  - Check that environment variable `LD_LIBRARY_PATH` includes CUDA library directory.
  - (`/cs/local/lib/pkg/cudatoolkit/lib64` on `linXX` machines.)
  - Add `-lcublas` to compile command.

Efficiency of cuBLAS

Matrix product example from 2017-03-24 lecture (in seconds):

	1024	2048	3072	4096
Brute force `mmult1`	0.079	0.648	2.190	5.152
Tiled `mmult2`[†]	0.027	0.208	0.724	1.690
`cublas_sgemm`	0.007	0.052	0.176	0.421

- Brute force `mmult1` achieves $\sim$ 13 GFLOPS.
- `cublas_sgemm` achieves $\sim$ 160 GFLOPS.
- ([†]Ian's tiled implementation `mmult2` is buggy.)

Other Numerical Libraries

- Linear Algebra PACKage (LAPACK).
  - Implements the more complex linear algebra operations.
  - Designed to call BLAS for basic computational steps.
- For your CPU:
  - Intel's Math Kernel Library (MKL) implements core functions from BLAS, LAPACK, FFTs, etc.
  - Automatically Tuned Linear Algebra Software (ATLAS) generates a BLAS library tuned to a machine's memory hierarchy.
- Many other accelerated libraries available for CUDA devices.
  - For example: cuFFT, cuSPARSE, cuRAND, cuDNN, MAGMA (supports LAPACK), ...

You are almost always better off learning to use the library.

Model Checking

Mark Greenstreet

CpSc 418 – Mar. 29, 2017

Model-Checking: Motivation

- What is "model checking"?
  - Construct a "model" for a piece of hardware or software – typically a finite-state machine.
  - Give a precise, mathematical definition of properties that the design is supposed to have.
  - Show that that model satisfies the specification.
    - ⋆ For example, find all reachable states of the model.
    - ⋆ Show that every reachable state satisfies a desired property – for example, mutual exclusion.
- Why use model checking?
  - Find bugs.
  - Hardware bugs are very expensive.
  - Software bugs are very common, but
    - ⋆ Finding bugs in concurrent software is **hard**.
    - ⋆ The challenges of finding bugs motivates using more systematic approaches.
- A simple example: Dekker's Mutual Exclusion algorithm

Dekker's Algorithm

Problem statement: ensure that at most one thread is in its critical section at any given time.

thread 0:

```
PC0= 0: while(true) {
PC0= 1:   non-critical code
PC0= 2:   flag[0] = true;
PC0= 3:   while(flag[1]) {
PC0= 4:     if(turn != 0) {
PC0= 5:       flag[0] = false;
PC0= 6:       while(turn != 0);
PC0= 7:       flag[0] = true;
PC0= 8:     }
PC0= 9:   }
PC0=10:   critical section
PC0=11:   turn = 1;
PC0=12:   flag[0] = false;
PC0=13: }
```

thread 1:

```
PC1= 0: while(true) {
PC1= 1:   non-critical code
PC1= 2:   flag[1] = true;
PC1= 3:   while(flag[0]) {
PC1= 4:     if(turn != 1) {
PC1= 5:       flag[1] = false;
PC1= 6:       while(turn != 1);
PC1= 7:       flag[1] = true;
PC1= 8:     }
PC1= 9:   }
PC1=10:   critical section
PC1=11:   turn = 0;
PC1=12:   flag[1] = false;
PC1=13: }
```

See http://en.wikipedia.org/wiki/Dekker's_algorithm.

Is Dekker's algorithm correct?

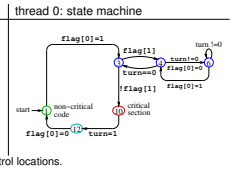
- Dijkstra (Turing Award 1972), presented the algorithm, with a proof in 1965.
- We'll use it as an example for model-checking:
  - Construct a finite-state machine model of the algorithm.
  - Determine the set of reachable states.
  - Verify that all reachable states satisfy mutual exclusion.
  - We could check other properties as well:
    - ⋆ For example, freedom from starvation: show that if a process is attempting to enter it's critical region, it will eventually succeed.

Modeling Dekker's algorithm

thread 0: code

```
PC0= 0: while(true) {
PC0= 1:   non-critical code
PC0= 2:   flag[0] = true;
PC0= 3:   while(flag[1]) {
PC0= 4:     if(turn != 0) {
PC0= 5:       flag[0] = false;
PC0= 6:       while(turn != 0);
PC0= 7:       flag[0] = true;
PC0= 8:     }
PC0= 9:   }
PC0=10:   critical section
PC0=11:   turn = 1;
PC0=12:   flag[0] = false;
PC0=13: }
```

thread 0: state machine



- Each process has six control locations.
- There are three global boolean variables: `flag[0]`, `flag[1]`, and `turn`.
- This produces a total of $6^2 \cdot 2^3 = 288$ possible states.
- We want to show that no reachable state has both processes in location 10, the critical section.

Model Checking Dekker's algorithm

- Represent each state with 9-bits:
  - three for the location of each process (6 locations)
  - three for `flag` and `turn`
  - I'll show a simple python version that uses python tuples
- Pseudo-code:

```
initialState = (1,1,0,0,0); // (loc0, loc1, flag0, flag1, turn)
workList = queue(); // initially empty
knownStates = set(); // initially empty
workList.insert(initialState);
while len(workList) > 0:
   s = workList.removeNext();
   for s' in next_states(s):
      if s' not in knownStates:
         check s' for mutual exclusion;
         add s' to workList and knownStates
```

- Model-checking finds 48 reachable states for Dekker's algorithm and verifies mutual exclusion.

A Brief History of Model Checking

- Proposed by Clarke and Emerson (1981) and independently by Sifakis (1982).
  - They shared the 2007 Turing Award.
  - Their approach was essentially the one described above.
- Symbolic methods introduced by McMillan (1987) using binary-decision diagrams, a DAG representation of boolean formulas.
- Widespread adaptation of model-checking for hardware design took place in the 1990s and continues today.
  - The murφ model checker is a landmark in this work.
- Model-checking of software is now gaining industrial acceptance
  - Based on "predicate abstraction" methods of Clarke and Grumberg, and independently Ball.
  - Enabled by advances in boolean SAT solvers and interpolation-based model checking (McMillan).

Today's Paper

Protocol Verification as a Hardware Design Aid

Mark's standard five questions:

1. What problem does the paper address?
   - Hardware designs consist of large blocks that communicate using **protocols**. Mistakes in the protocol design can cause subtle errors that only occur in rare corner cases. Such errors are hard to find by traditional simulation.
2. What is the key idea in the paper?
   - Use model checking to exhaustively verify small versions of the design.
3. How do the authors validate their idea?
   - They implemented a model checker.
   - This included defining a modeling language so that protocols can be described easily and clearly.
   - They applied their approach to two protocols from real designs in industry.
4. Is the paper convincing?
   - **Yes:** they showed that they could check important properties of two "down scaled" protocols.
   - **No:** the protocols seem down-scaled to the edge of being trivial.
   - **20/20 hindsight: Definitely!** Model-checking methods have evolved and matured and are now widely used in industry for both hardware and software.
5. Any other comments?
   - Glad you asked. See the rest of the lecture.

Overview of the paper

- How does model the hardware?
  - murφ: a **guarded command language**.
- How do we state the properties to be verified?
  - Add assertions to the murφ program.
  - Use model checking to show that these assertions hold for **all** states of **all** executions.
- How do we perform the model checking?
  - Compile the murφ program to C++.
  - Link with an efficient implementation of a model checking algorithms like the one in dekker_mc.py.
  - Run the model checker to either verify the properties or report counter-examples.

murφ: a guarded command language

- In murφ a guarded command is called a rule and is written:
  `rule` *guard* `=>` *action*
  - When *guard* is satisfied, *action* **may** be performed.
  - Example: `rule ((loc[0] == 3) and flag[1]) => loc[0] := 4`
- Rules may be quantified using the `Ruleset` construction:

```
Process:  scalarset(2);
ruleset i:  Process do
   (loc[i] == 3) and flag[1-i] => loc[i] := 4
end
```


murφ: execution model

- A program defines a fixed set of rules.
- Toss all the rules in a bag.
- Repeat indefinitely:
  - Pick a rule from the bag.
  - If it's guard is satisfied, perform it's action.
  - Put the rule back in the bag.
- For verification: an "adversary" picks the rules from the bag.

Model checking today

- Lots of progress on handling larger models:
  - Symbolic methods
  - Exploit common model properties: symmetry, commuting-actions, verifiable abstraction
  - Moore's law: faster machines, larger memories.
  - Parallelism (Friday's lecture)
- Applications
  - An essential part of cache-protocol design. Used in many other aspects of hardware design as well.
  - Software: Microsoft uses model checking to verify that driver code conforms to kernel usage rules.
  - Software: Amazon uses model-checking to verify protocols used in their cloud services.
  - Many others.

Preview

March 31:	The PReach Model Checker
Reading:	Industrial Strength ... Model Checking
April 3:	Distributed Termination Detection
April 5:	Party: 50[th] Anniversary of Amdahl's Law

Review

I'll add some review questions.

Industrial Strength, Parallel Model Checking

Mark Greenstreet

CpSc 418 – Mar. 31, 2017

Mark's Five Questions: 1–3

1. What problem does the paper address?
   - Model checking of real problems in industry requires the combined memory of many machines. Existing implementations of parallel model checkers failed to successfully run on real examples. A robust, distributed, parallel model checker is needed.
2. What is the key idea in the paper?
   - Combine murφ and Erlang.
   - Use the existing murφ code to implement the computationally intensive part of the code.
   - Use Erlang to handle the communication and coordination between worker processes.
3. How do the authors validate their idea?
   - They implemented a model checker.
   - They performed experiments to identify robustness issues and performance bottlenecks. They implemented solutions to these problems.
   - They used the PReach model checker on several benchmark examples and some real designs from Intel.

Mark's Five Questions: 4–5

4. Is the paper convincing?
   - **Yes:** they showed that they could check properties of real designs. They set new records for model-size for explicit-state model checking.
   - **No:** the run times for large examples can be several days.
   - **20/20 hindsight:** Definitely. With continued work, the performance of PReach has been improved. Brad Bingham went on to show that this explicit state approach could solve liveness verification problems that other methods cannot.
5. Any other comments?
   - PReach lacks a crash-recovery mechanism.
   - PReach doesn't take advantage of having multiple cores on a single CPU: This would require revising the base murφ code to create shared state tables, etc.

Model Checking: the algorithm

```
initialState = ...;
workList = queue(); // initially empty
knownStates = set(); // initially empty
workList.insert(initialState);
while len(workList) > 0:
   s = workList.removeNext();
   check s' for mutual exclusion;
   for s' in next_states(s):
      if s' not in knownStates:
         add s' to workList and knownStates;
```

How can we make this algorithm parallel?

- Where does the code spend most of the time?

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- Where does the code spend most of the time? `next_states(s)`
- What are the **dependencies**?
  - As written, the code is very sequential.
  - BUT, the correctness of the algorithm does not depend on the order in which states are removed from `workList`.
- What uses most of the memory?

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- What uses most of the memory? `knownStates` and `workList`

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- What are the **dependencies**?

Making it Parallel

```
initialState = ...;
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while len(workList) > 0:
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   check s' for mutual exclusion;
   for s' in next_states(s):
      if s' not in knownStates:
         add s' to workList and knownStates;
```

- Divide `knownStates` and `workList` across the worker processes? How?
  - Sending each new state, `s'` to every worker is a bad idea. Why?
  - We'll use hashing instead: send state `s'` to worker process `hash(s') mod P`.
- Each worker maintains `knownStates` for the states that hash to the worker process.
- Each worker adds each **new** state it receives to `knownStates` and `workList`.
- Each worker processes the states on its own worklist.

Parallel Model Checking: the Pseudo-Code

```
modelCheck(KnownStates, WorkList) ->
   receive
      S when member(S, KnownStates) -> % already seen
         modelCheck(KnownStates, WorkList);
      S -> modelCheck(add(S, KnownStates),
                      add(S, WorkList))
   after 0 ->
      case WorkList of
         [] -> is_everyone_else_done();
         [S | Tl] ->
            [owner(S') ! S' || S' <- next_states(S)],
            modelCheck(KnownStates, Tl)
      end
   end.
```

- See Parallelizing the murφ model checker, U. Stern and D.L. Dill.

Parallel Model Checking: Performance Analysis

- A sequential implementation of the model checking algorithm requires $\mathcal{O}(SR)$ time, where S is the number of reachable states, and R is the number of rules.
- A parallel implementation requires $\mathcal{O}(SR/P)$ compute time, and sends worst-case $\mathcal{O}(SR)$ messages.
  - In practice, the average number of successors of each state (i.e. the degree of the state-graph) is relatively small. If we assume this is a small constant, then we get $\mathcal{O}(S)$ messages.
- Consider the case where each worker process generates σ new successor states, s', per second.
  - These are send to the other processes uniformly at random (if we have a good hash function).
  - Half of these messages cross the bisection of any network.
  - That means we need a bisection bandwidth of $\sigma P/2$.
- For real-life networks, bisection bandwidth grows much more slowly than P.
  - If we scale this algorithm to a large enough number of processors, network bandwidth will be the limiting constraint.
  - This is a common performance pattern in parallel computing.

From Algorithm to Industrial Adaptation

What is needed for real-world verification?

- Lots of memory:
  - Memory and time are both concerns for model checkers, but memory tends to be the more critical concern.
  - A parallel implementation offers the combined memory of a large number of machines.
- Robustness:
  - Simple architecture and re-use stable, well-exercised code.
  - Prevent "overwhelm and crash".
  - Load balancing.
- Flexibility:
  - Solve problems that other tools cannot
  - In particular, liveness properties such as "response".

Erlang for high-performance computing (really)

- Use existing C++ code for murφ.
  - It has been carefully optimized – it's fast.
  - It has been widely used over the past 25 years – it's robust.
- Use Erlang to make it parallel
  - Erlang handles the communication between processes.
  - The code is simple: it works and it's flexible.
  - Erlang can call the C++ functions:
    - The compute intensive part is done in C++.
    - The Erlang code is not a serious bottleneck.

Memory

- The worklist is the dominant use of memory (in practice)
  - Why?
  - The worklist needs complete state descriptions.
  - The known-state set can use much smaller hashes.
- Solution: store the worklist on disk.
  - Disks are slow – is this crazy?
  - It works just fine because we can access the worklist in **any** order.
  - Keep a large piece of the worklist in main memory.
  - If the in-memory work list grows too large, then copy a large chunk to disk.
  - If the in-memory work list becomes too small, then read a large chunk from disk.
  - The disk reads and writes can be performed asynchronously.
  - See Using magnetic disks . . . in the murφ model checker, U. Stern and D.L. Dill.
- Storing the known-state set on disk is much less practical because it's a random-access structure.

Batching Messages

- If we send each new state, `s'`, one at a time to its owner process, communication overhead dominates the run time.
- **Key lesson**: pay attention to λ.
- The Erlang code maintains a separate buffer for each worker process
  - Add states that should be sent to that process to the buffer until we have enough.
  - Then send them as a batch.
  - A process that is running low on work sends requests to the other workers to ask them to flush their buffers.
  - These flush requests are bundled with state batches to avoid extra messages.
- Erlang makes the communication architecture simple and easy to extend.

Overwhelm and Crash

The dangers of using Erlang for high-performance computing

- The Erlang in-box is a list.
  - Newly received messages are prepended to the list.
  - A receive gets the oldest message that matches a pattern of the receive.
  - This means that the time for receive is linear in the number of pending message.
- This leads to a performance catastrophe
  - If a process gets slightly behind, its inbox will fill a little more than the other processes.
  - This means that a process that falls behind will slow down, and its inbox will fill even more.
  - Eventually, the process crashes.

Credits

Preventing "overwhelm and crash"

- Drain the inbox into another buffer whenever possible.
- Maintain a credit system
  - When a process X sends a message to process Y, X decrements its credit-count for Y.
  - If the credit-count is 0, X waits to send its message.
  - When Y moves a message from X out of its inbox, it sends a credit back to X.
  - Of course, these messages are piggy-backed on the new-state messages.

Load Balancing

- Not all processes have the same amount of work, and they don't all run at the same speed.
- This can lead to **idle processors**.
- Solution:
  - Processes include the length of their worklist in their messages to other workers.
  - If a worker has a short worklist, it asks for half of the worklist of the worker with the longest worklists.
- This is a very coarse-grained approach
  - PReach makes no effort to keep worklist lengths equal.
  - The coarse-grained approach requires very few messages: **avoid** λ.
  - The performance is very good.

Flexibility

- The Erlang code for PReach is simple.
  - The version described in the paper is about 1000 lines of code.
- This makes PReach a flexible platform for experiments:
  - Checking response properties: e.g. every request is eventually granted.
  - Exploiting symmetry: there are times we can verify a protocol for two or three nodes and conclude with certainty that it is correct for any number of nodes.
  - And others.
- Applications:
  - Used by Intel architects when exploring protocols for on-chip networks.
  - Used in other companies and universities.
  - It's been run on hundreds of machines with models of hundreds of billions of states.
  - Symbolic methods are faster than PReach for safety properties (showing that the model never reaches a bad state)
    - PReach is faster for handling liveness properties: showing that some condition will eventually be satisfied.

Termination

- How do we know when we're done?
- Well, times up for this lecture.
- More seriously, in PReach we need to know when
  - When every worker process has an empty worklist,
  - **And** there are no messages in flight.
- Both conditions must hold at the same time.
  - This is the topic for Monday's lecture.

Summary

- PReach shows how the ideas from this class can be used to build real-world, high-performance, large-scale, parallel systems.
- Lessons learned:
  - Erlang is a great environment for building large-scale, parallel/distributed code.
  - Use the C/C++ call interface to use native C/C++ code for the compute intensive parts of the code.
    - Erlang provides three such interfaces!
  - Watch out for overwhelm and crash
    - If you're going to send a lot of messages, you need some kind of flow control mechanism.

Preview

April 3: Distributed Termination Detection

April 5: Party: 50th Anniversary of Amdahl's Law

Review: for today's lecture

- To make a parallel implementation of a computation, we often need to identify a set (or many set(s)) of operations that can be performed in any order.
  - For model checking, what set of operations did we find that can be performed in any order? Does this exactly replicate the sequential version, or does it perform an equivalent computation?
  - Same questions as above, but for reduce.
  - Scan? Sorting? Matrix multiplication?
- Why did slow processes in PReach tend to become catastrophically slower? What was the solution?
- What is load balancing? Compare the load balancing mechanisms of PReach and Google's map-reduce.
- Is Erlang suitable for large-scale, high-performance, parallel computing? Why or why not?

Review: for March 29 lecture

- What is model checking?
- What is mutual exclusion?
- How does the model checker presented in the March 29 slides show that Dekker's algorithm guarantees mutual exclusion.
- Describe the role of the `knownStates` and `workList` data structures in the model checking algorithm.
- What is a guarded command?
- Write a murφ rule for another statement from Dekker's algorithm.

Course Summary

Mark Greenstreet

CpSc 418 – Apr. 5, 2017

- The things we have done:
  - parallel algorithms
  - parallel architectures
  - parallel performance
  - parallel paradigms
- and the things we have left undone.
- An exam, but first, a party.

Parallel Algorithms 1

- map, reduce, and scan: simple patterns
  - Easy to parallelize.
  - Learn to recognize when a problem can be solved by these simple methods.
- sorting networks
  - oblivious computation: the control flow doesn't depend on data values.
    - oblivious algorithms are good candidates for parallelism because we can determine the control flow in advance.
    - This lets us identify the data dependencies, and find a parallel solution.
  - The 0-1 principle
  - Bitonic sorting: it's merge sort with an oblivious merge.

Parallel Algorithms 2

- Matrix operations
  - matrix multiplication dividing the matrix into b
    - Dividing the matrix into blocks
    - Analysis of the compute and communication costs.
  - BLAS and cuBLAS: use a library when you can!
- Map-Reduce
- Model checking
  - We only had two lectures on the topic and no HW.
  - Won't ask any detailed questions, but if there might be some high-level questions with one sentence answers in the review questions, in which case similar questions could be on the exam.
  - Know what model checking is: verifying properties of a hardware or software design, using a finite-state-machine model, finding the reachable states.
  - The idea of distributing work by hashing values and sending each to its owner process.

Parallel Architectures

- pipelining and instruction level parallelism
- shared memory multiprocessors
  - Know what a cache coherence protocol is.
  - Explain the idea of shared-reader or exclusive writer.
  - Be able to point out that real cache coherence protocols aren't as "consistent" as the simple (e.g. MESI) model from class.
- message passing architectures
  - Rings, tori, hypercubes
  - Latency, bisection width.
- data parallel architectures
  - SIMD (and SIMT)
  - instruction execution: why so many threads
  - GPU memory hierarchy

Parallel Performance

- λ
  - Communication costs are critical to understanding parallel performance
  - This is true across all parallel architectures.
- Overheads and losses
  - Communication, synchronization, extra memory, extra computation
  - Idle processors, resource contention, non-parallelizable code
- Speed-up and Amdahl's Law.
- **Understanding real world performance requires experiments and measurements.**

Parallel Programming Paradigms

- Message passing: Erlang
- Data Parallel: CUDA
- We've mentioned shared-memory.

. . . and the things we have left undone

- More paradigms and programming frameworks
  - shared memory: Java threads, pthreads.
  - futures: e.g. Scala
  - MPI and OpenMP (for scientific computing)
  - many big-data, machine-learning, and scientific computing frameworks
- Do a bigger project.
- **The good news:**
  - You've got what you need to lean new paradigms, new frameworks, and take on realistic projects.
  - From my experience with research projects that have moved into industry in the past few years, you've got the critical knowledge and skills.
  - Writing industrial-strength, parallel-code with good performance is still more than a homework assignment, but when my students have done it, they've built on the foundation you now have.