CPSC 418 - Homework 4

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1

a

```
0-*---------
1-|-*-------
2-|-|-*-----
3-|-|-|-*---
4-|-|-|-|-*-
5-|-|-|-|-*-
6-|-|-|-*---
7-|-|-*-----
8-|-*-------
9-*---------
```

b) Prove that either the sequence y[0, …, (N/2) − 1] is clean or that the sequence y[(N/2), …, (N − 1)] is clean (or possibly both).

Proof. There are three main cases, when there are more ones than zeros, more zeros than ones, and when the number of zeros equals the number of ones.

Case 1:

When there are more ones than zeros, it's impossible for both halves to be clean. However, one half can be clean. When the two arrays are reversed and compared, there will be overlap between the ones, leading to them not being switched. However, since the side with the maximum values also contains a one, it will become clean.

```
1
1
0
1
1
0
```

Reverse

```
1 0
1 1
0 1
```

Compare

```
1 0
1 1
1 0
```

Case 2:

Case 2 is the inverse of case 1, and the same rule applies by swapping 0 with 1.

Case 3:

When there are an equal number of ones and zeros, the number of ones in the first half will equal the number of zeros in the second half. When this sorting network is applied, the second half of the values are reversed and compared with the bottom half. Thus, both halves will be clean.

```
1
1
0
1
0
```

```
0
```

Reverse

```
1 0
1 0
0 1
```

Compare

```
1 0
1 0
1 0
```

c

In the case that both sequences are clean, both of the sequences are bitonic.

When there is an inequal number of zeros and ones, one of the arrays will be clean and thus bitonic. The other is a mix of ones and zeros. We can see that it is also bitonic by looking at how the numbers are reversed and compared. Since the sequences are reversed before being compared, the high values of one sequence are being compared to the low values of the other sequence. Thus, the values which will be the same are located in the middle by the intermediate value theorem. Therefore, the sequence is bitonic since all duplicate values are in the middle.

d

Proof by contradiction:

Assume there is some i and j such that $y[i] > y[j]$. Since we are applying the 0-1 principle, that means $y[i] = 1, y[j] = 0$. Using the earlier proof, we know that one of the output sequences will be clean and thus have all the same values. Either the top sequence will be all 1s or the bottom sequence will be all 0s. This is a contradictions since we assumed that there was a 0 in the top sequence and a 1 in the bottom sequence. Thus, $0 \leq i \leq (N/2)$ and $(N/2) \leq j \leq N, y[i] \leq y[j]$.

2

a

from sort6.c: column 0: (1,2)

b

from sort6.c: column 0: (1, 2) column 1: (0, 1) column 2: (1, 2)

c

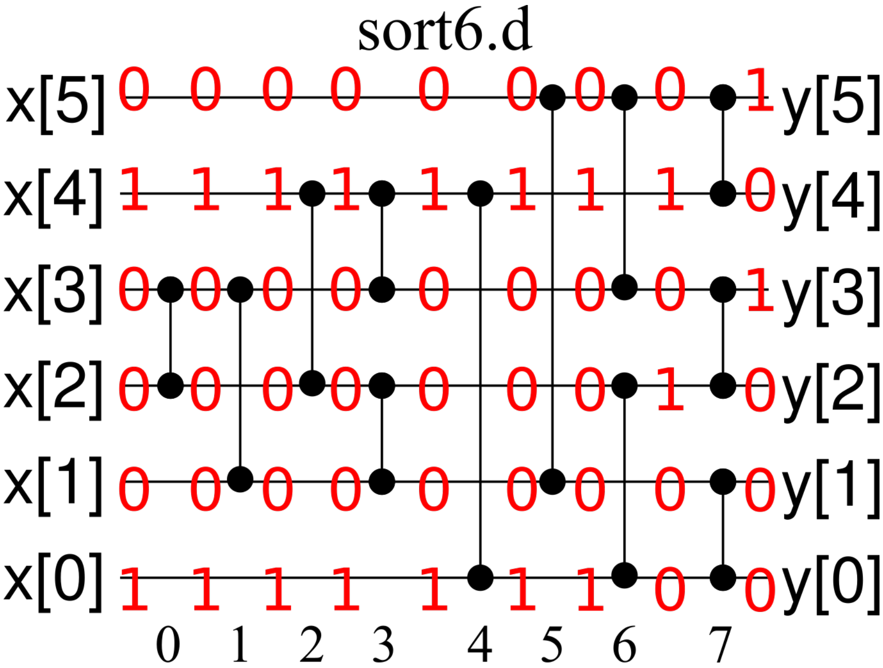
from sort6.b: column 4: (1, 2), (3, 4) column 5: (2, 4) column 6: (1, 3) column 7: (2, 3)

d

from sort6.c: column 3: (0, 5) column 4: (1, 4) column 5: (2, 3)

e

sort6.d does not sort correctly.



f

sort6.c does sort correctly.

Since it's a sorting network the output must be sorted. If there's a 1 in $x[0, \ldots, (N-1)]$, for the output to be sorted $y[5]$ must be sorted.

g

sort6.c consists of 5 subparts from previous parts of this question. There's initially two 3 input sorters which sort $x[3, \ldots, 5]$ and $x[0, \ldots, 2]$. This produces two sorted sequences. From our proofs from question one, we know the columns 3 to 5 make sure that all the values in the top half of the sorting network are greater than the bottom half. The last two subnetwork on lines 6-8, enforce that the top 3 values and the bottom 3 values are sorted. Since we know the top 3 values are larger or equal to the bottom 3 values, the entire sequence must be sorted.

h

Both sort6.a and sort6.b sort correctly.

3

Using: lin13.ugrad.cs.ubc.ca

b

```
q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 10000000 10000
f(n, ...): t_elapsed =  5.960e-01, throughput =  2.040e+09
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 1000000 10000
f(n, ...): t_elapsed =  1.400e-01, throughput =  1.007e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 256 10000
f(n, ...): t_elapsed =  8.000e-03, throughput =  3.200e+08
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 256 1000000
f(n, ...): t_elapsed =  5.200e-02, throughput =  4.923e+09
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 256 10000000
f(n, ...): t_elapsed =  2.360e-01, throughput =  1.085e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 256 100000000
f(n, ...): t_elapsed =  2.948e+00, throughput =  1.399e+09
```

```
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 512 1000000
f(n, ...): t_elapsed =  5.600e-02, throughput =  9.143e+09
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 512 5000000
f(n, ...): t_elapsed =  1.480e-01, throughput =  1.730e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 1024 25000000
f(n, ...): t_elapsed =  6.240e-01, throughput =  6.611e+09
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 1024 2500000
f(n, ...): t_elapsed =  7.200e-02, throughput =  3.556e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 65536 2500000
f(n, ...): t_elapsed =  2.364e+00, throughput =  2.670e+08
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 1024 2500000
f(n, ...): t_elapsed =  8.400e-02, throughput =  3.048e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 1024 3000000
f(n, ...): t_elapsed =  1.000e-01, throughput =  3.072e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1500000
f(n, ...): t_elapsed =  4.800e-02, throughput =  6.400e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 3000000
f(n, ...): t_elapsed =  8.000e-02, throughput =  2.311e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 3000000
f(n, ...): t_elapsed =  1.040e-01, throughput =  1.778e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 3000000
f(n, ...): t_elapsed =  8.400e-02, throughput =  2.201e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1500000
f(n, ...): t_elapsed =  4.400e-02, throughput =  6.982e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1000000
f(n, ...): t_elapsed =  2.800e-02, throughput =  7.314e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 100000
f(n, ...): t_elapsed =  1.200e-02, throughput =  1.707e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1000000
f(n, ...): t_elapsed =  4.000e-02, throughput =  5.120e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1000000
f(n, ...): t_elapsed =  3.600e-02, throughput =  5.689e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1000000
f(n, ...): t_elapsed =  2.400e-02, throughput =  8.533e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1000000
f(n, ...): t_elapsed =  3.200e-02, throughput =  6.400e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 2048 1500000
f(n, ...): t_elapsed =  5.200e-02, throughput =  5.908e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 3048 1500000
f(n, ...): t_elapsed =  5.600e-02, throughput =  4.947e+09
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 4096 1500000
f(n, ...): t_elapsed =  8.800e-02, throughput =  2.101e+10
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 4096 15000000
f(n, ...): t_elapsed =  8.840e-01, throughput =  1.482e+09
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out f 4096 1500000
f(n, ...): t_elapsed =  1.000e-01, throughput =  1.849e+10
```

Through experimentation, I found that $n = 2048$ and $m = 1000000$ had the highest throughput at 8.533e10/s iterations of the inner loop of the kernel. Each of those iterations requires 4 multiplies and 1 add which means 4 floating point operations since there's a combined multiply/add operation. Thus, we achieve a performance of 341.32GFlops.

c

```
q7w9a@lin13   ~/cs418/hw4     master     ./a.out f_cpu 2048 1000000
f_cpu(n, ...): t_elapsed =  1.336e+01, throughput =  1.533e+08
```

Speed up of: 13.36/2.400e-02=566.67

d

When writing cuda operations you want to maximize utilization of the cores as well as have a high enough ratio of floating point operations to memory operations to not be blocked by memory bandwidth. With a large M value, there's a lot of floating point operations for every memory access. It's also good to have the number of kernels be a multiple of the number of threads so you don't have any wasted computations. The number of threads should be sufficiently large as to make full use of all the cores.

4

a

Running on lin13.ugrad.cs.ubc.ca. N = 50000000

```
q7w9a@lin13   ~/cs418/hw4     master     ./a.out saxpy 50000000
saxpy(n, ...): t_elapsed =  1.000e-01
```

b

```
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out saxpy_cpu 50000000
saxpy_cpu(n, ...): t_elapsed =  1.440e-01
```

You get a 1.44 performance speedup by using the GPU and CUDA.

c

On each thread in the GPU, saxpy does a single multiply+add floating point operation. This is a very poor use of the GPU since it is mostly blocked by memory bandwidth to load those values into the GPU. In question 3, there's a much larger speedup since the computation has a much higher FMA count to memory access ratio.

If you make the implementation do way more FMA operations in the kernel, the performance differences are much more stark.

Let's do 10 iterations of the inner operation instead.

```
float yi = y[i];
float xi = x[i];
float out;
for (int j = 0; j<10; j++) {
  out = a*xi + yi;
}
y[i] = out;
```

```
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out saxpy 50000000
saxpy(n, ...): t_elapsed =  1.040e-01
 q7w9a@lin13   ~/cs418/hw4     master     ./a.out saxpy_cpu 50000000
saxpy_cpu(n, ...): t_elapsed =  8.760e-01
```

We see that the GPU and CUDA implementation only slows down by 4% whereas the CPU implementation slows down roughly 500%. This very clearly indicates that the memory accesses are the blocker.