

Question 2

Let the candidate set be all vertices in G . Pick a vertex v in the candidate set, add it to the independent set, remove all neighbors of v from the candidate set. Repeat until the candidate set is empty. This can be done in $O(n)$ which makes it very efficient.

The tricky part is showing that this has the correct size.

This algorithm will pick at least one vertex from the set of a vertex and its neighbors of degree d . If it picks the central vertex with degree d , that means of that local set $\frac{1}{d+1} * (d + 1)$ is included. In the case of a star, the maximum independent set will be all of the edge vertices with size d . The worst case as above is picking the middle, which is $\frac{1}{d}$ the optimal size. Thus in the limit.