Question 2

(a) Give a linear-programming formulation of the maximum bipartite matching problem.

$\max \sum x$ where x is a vector representing the edges with 1 being selected, and 0 being not selected.

Sum of all adjacent edges is less than 1.

$x_{uv} + x_{vw} \leq 1$ for all $(u, v), (v, w) \in E$.

(b) Now dualize the linear program from part (a). What do the dual variables represent? What does the objective function represent?

Primal problem

Maximize $c^T x$ subject to $Ax \leq b, x \geq 0$.

$c = [11 \ldots 1]$ for all values in x.

A is the $n \times n$ matrix formed by $[\ldots, x_{uv}, \ldots, x_{vw}, \ldots]$ for all $(u, v), (v, w) \in E$.

$b = [11 \ldots 1]$.

Dual problem

Minimize $b^T y$ subject to $A^T y \geq c, y \geq 0$.

Two edges are an adjacency pair if they are connected by a vertex.

A^T_{ij} is 1 for each adjacency pair j that edge i is in.

$A^T y$ computes the number of times an adjacency pair is picked for every edge. Since $A^T y \geq c$, that means that every edge has to be selected as part of an adjacency pair at least once. Minimizing $b^T y$, minimizes the number of pairs that are selected.