

### Question 3

Assume we have a black box algorithm that computes whether or not a undirected graph  $G$  and an integer  $k$ , contains a clique of size  $k$  as well as an independent set of size  $k$ .

We know that computing the size of the maximal independent set in a graph is NP-complete. We can use this algorithm to compute the size, by running it on the graph  $n$  times where  $k = \{1, \dots, n\}$ . If the algorithm is in polynomial time, running it  $n$  times is still in polynomial time. Likewise, if our algorithm is not in polynomial time, running it  $n$  times is still not in polynomial time. Since finding the maximal independent set is NP-complete, our black box algorithm must also be NP-complete.

Computing whether there is a clique of size  $k$  is also NP-complete. Thus, since both parts are NP-complete the overall algorithm is NP-complete.