

## Linear Programming Duality

$$\max x_1 + 6x_2$$

subject to  $x_1 \leq 200, x_2 \leq 300$  ... (same thing as last time)

### Duality Theorem

If LP has a bounded optimum then so does its dual and the two optimum values are the same.

### Primal

$$I = \{1, 2, \dots, m\}$$

$$N = \{1, 2, \dots, n\}$$

$$\max c_1x_1 + \dots + c_nx_n$$

such that  $a_{i1}x_1 + \dots + a_{in}x_n \leq b_i$  for  $i \in I$

$$a_{i1}x_1 + \dots + a_{in}x_n = b_i \text{ for } i \notin I$$

$$x_j \geq 0 \text{ for } j \in N$$

### Dual

$$\min b_1y_1 + \dots + b_my_m$$

such that  $a_{j1}x_1 + \dots + a_{jm}x_m \geq c_j$  for  $j \in N$

$$a_{j1}x_1 + \dots + a_{jm}x_m = c_j \text{ for } j \notin N$$

$$y_i \geq 0 \text{ for } i \in I$$

## Two Player Zero-Sum Games

### Rock paper scissors

	rock	paper	scissors
rock	0	-1	1
paper	1	0	-1
scissors	-1	1	0

If Row plays "rock" then Col can win every time.

A mixed strategy is a probability distribution on the actions.