

Open Pit Mining

Input: Directed acyclic graph $G = (V, E)$ where V = set of tasks.

$E = \{(u, v) | u \text{ must be done before } v\}$.

a function $w(v)$ that specifies profit from the task.

Find the most profitable set of tasks to perform.

An initial set is a set of vertices that has no edge coming into it from the outside.

Convert problem to a network flow problem so that 1. any infinite capacity cut corresponds to an initial set and, 2. a minimum capacity cut corresponds to max profit initial set.

If we connect all the nodes with infinite capacity, and then reverse it, we get finite capacities.

Claim: In this "network", any finite capacity cut (S, T) defines an initial set $T - \{t\}$.

Proof: If cut (S, T) has finite capacity then no original edge is directed into T from S . Thus $T - \{t\}$ is an initial set.

If set U is an initial set, then $T = U \cup \{t\}$, $S = V - T$ is a cut with no original edge entering T thus finite capacity.