Final Review - Friday next week 1:30-3:00

RMM

Claim

For all mice A (determinate or randomized) there exists $p_1, ..., p_n E[A_{COST}(p_1, p_2, ..., p_n)] > (\log m) OPT(p_1, p_2, ..., p_n)$

Proof idea

Show that a cat exists that will cause A to move $> \log m$ times more than OPT.

If cat probes at random then no matter what mouse A does, Cat finds it with probability $\frac{1}{m}$. Expected number of times A must move over sequence of t probes is $\frac{t}{m}$. How many Random Cat probes until cat examines all m spots? (coupon collector problem).

$m \ln m$.

So OPT moves once every $m \ln m$ probes while A moves $\frac{m \ln m}{m}$ times. A faults $\geq \ln m$ times OPT.

Hash Functions

Universal Hash functions

$m = $size of hash table.

A set of hash functions H that map $U \to \{0, 1, \ldots, m-1\}$ is universal if for all distinct keys $k, l \in U$ the number of hash functions $h \in H$ such that $h(k) = h(l)$ is at most $\frac{|H|}{m}$.

Chaining using universal hash functions

Hash n keys into a table T of size m using hash function $h \in_R H$

$\alpha = \frac{n}{m} =$ load factor

Theorem: For key k,

$E[n_{h(k)}] \leq \{\alpha + 1 if key \in T, \alpha if k \notin T\}$