

## Question 4

(a) Give an example of a matrix  $M$  that is not rearrangeable even though every column and every row contains at least one entry that equals 1.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Describe an efficient algorithm that determines if  $M$  is rearrangeable.

We can use bipartite matching to determine if  $M$  is rearrangeable. One way to do this is to create a graph with one vertex for every column and one vertex for every row. You then add edges for every element in  $M$  that equals 1 connecting the  $i$ th row vertex with the  $j$ th column vertex. If the number of bipartite matches equals  $n$ ,  $M$  is rearrangeable.

Since the number of vertexes is  $2n$ , and the worst case number of edges is  $O(n^2)$  since every element in the matrix could be 1. Finding a bipartite matching requires solving a max flow problem which is  $O(|V| * |E|)$ . Thus, this algorithm has a worst case of  $O(n^3)$  which is in polynomial time.