Network Flows

Syntax notes:

f(a,b): flow from a to b cap(a,b): capacity from a to b

Lemma: For any flow f and any cut (S,T), $size(f) \leq cap(S,T)$.

1. The flow across the cut $(S,T) \leq cap(S,T)$.
2. The flow across cut (S,T) = flow across cut (S-1, T+1).
3. The flow across cut({s}, V-{s}) = size(f)

Correctness of Ford-Fulkerson

Theorem: If residual network G_f has no augmenting path, then f is a max size flow.

Proof: Let S be the set of vertices reachable by directed path in G_f.

Let T=V S

All edges from S to T are saturated (and don't exist in the residual network). Can't reach from S to T via residual network.

This implies the flow across cut (S,T) = cap (S, T).

size(f) = f(S,T) = cap(S,T) because f(u,v) = cap(u,v) for all u in S, v in T

size of any flow $\leq$ cap(S,T).