

Question 2

Proof by induction.

Let S be a set of n points in the plane. A Voronoi diagram partitions the plane into Voronoi regions where R_i is the set of points in the plane whose closest site is s_i , that is, $R_i = \{p \mid d(p, s_i) \leq d(p, s_j) \text{ for all } j\}$.

Base case

There is only a single point in the Voronoi diagram. Since there is only one Voronoi region that encompasses all space it is trivially convex.

Inductive step

The line formed by the edge of two Voronoi regions is perpendicular to the line formed by the points of the two Voronoi regions (p_1, p_2) , since every point along the line is equal distance from p_1 and p_2 .

Assume there is a voronoi region that is convex.

When adding a new point to a voronoi diagram the new line formed partitions the old convex region into a smaller new region. When a line intersects with a convex hull, all of the edge angles are less than 180 degrees. Thus, partitioning a convex shape into two with a single line creates two new convex shapes. Therefore the new voronoi region is still convex.