Question 4

I looked up a definition of k-competitive algorithms online.

For a page replacement algorithm to be k-competitive, given k pages and cost function c such that for all sequences of $p = \{p_1, p_2, \ldots, p_n\}, c_{ALG}(p_i) = k * c_{OPT}(p_i)$ must hold.

We assume that both the FIFO and OPT algorithms start with the same pages in memory.

We can partition p into a number of phases such that FIFO has at most k fault on $P(0)$ and exactly k faults on $P(i), i \geq 1$. To show that FIFO is k-competitive, we must show that for each phase, OPT has at least one page fault.

For the case of $P(0)$, since both FIFO and OPT both start with the same pages in memory, the first fault for FIFO occurs at the same time as OPT.

For k page faults to occur in a phase with FIFO, there must be at least k items that are different from the request immediately before the phase starts. Assuming this is true, since there are k new items in this phase, OPT must have at least one page fault.

Assume that FIFO faults on a page twice in a phase. This implies that has been evicted once during the phase. Since the algorithm must see k new items to remove one that's been added, this cannot happen with only k page faults. Thus, during this phase, all k page faults must be distinct, and OPT must fault at least once.

Therefore, FIFO is k-competitive since for every k faults of FIFO, there must be at least one by OPT.