Question 5

We can solve this problem by transforming the input graph into a bipartite graph and then using Ford-Fulkerson to solve the bipartite matching.

First create two new sets of vertexes, one for each of vertexes in the input graph. These will form the bipartite graph with one set on each side of the partition. The next step is for every directed edge $\{i, j\}$ in the input graph, create a new edge between the ith node in the first set, and the jth node in the second set.

Finally, once we have the graph we can solve the bipartite matching using Ford-Fulkerson as discussed in class. If there is a perfect matching, there is cycle cover of the input graph formed by the matched edges. If there is no perfect matching, report that no cycle cover exists.

The transformation step can be done in $\Theta(|V| + |E|)$ since it requires iterating through each vertex and edge once. The solving of the bipartite matching is $O(|V| * |E|)$ due to the invocation of Ford-Fulkerson.