Applications of Network Flows

Max matching in Bipartite Graphs

Claim:

1. If M is a matching in G then there exists flow f in F (flow network from G) such that size(f) = |M|. $size(f*) \geq |M*|$
2. If f is an integer valued flow in F, then there exists a matching M in G of size size(f). $size(f*) \leq |M*|$.

Let $M = \{(u, v) \in G | f(u, v) = 1\}$ M forms a matching. Why? (capacities of edges from s are 1 and from t are 1.

Pennant Race Problem

Given a team A, a list of other teams $T_1, T_2, \ldots, T_n$, win-loss record for each team, and list of games remaining to be played.

Determine if it is possible for team A to win at least as many games as any other team by the end of the season.

Output is yes/no.

	Wins
A	3
T_1	4
T_2	6
T_3	5
T_4	4

Remaining games: (A, T1) (T1, T2) (T2, T3) (A, T3) (T2, T4) (A, T4) (T1, T2)

We can remove all games that A plays since we can say that A wins all of them.

Assume team A wins all remaining games. Let w = # A's wins (after A wins all remaining games). Let w_i = # wins of T_i.

If $w < w_i$ for some i then A has no hope.

Greedy Approach

Doesn't work. Can't just do an O(n) summation or anything similar.

Runtime of Ford-Fulkerson

Bipartite matching $O(|V| * |E|)$.

There's a more modern and faster way of doing this, but way more complicated. From 2012. That lets you calculate the network flow in a flow network with V vertices and E edges, $O(|V| * |E|)$.