

Open Pit Mining

1. Create edge from s to all negative value vertices with capacity $-v$.
2. Create an edge from all of the positive value points to t with capacity v .
3. All inner edges have infinite capacity.
4. Solve the flow network, so you can look at minimum cuts.

Infinite edges aren't part of the minimum cut, since it's going the wrong direction.

Want to maximize profit for initial set u .

$$profit = \sum_{u \in U, w(u) > 0} w(u) + \sum_{v \in U, w(v) < 0} w(v)$$

Want to minimize

$$-profit = -\sum_{u \in U, w(u) > 0} w(u) - \sum_{v \in U, w(v) < 0} w(v)$$

Capacity of cut associated with initial set U .

$$T = U \cup \{T\}$$

$$S = V - T$$

$$cap(S, T) = \sum_{u \notin U, w(u) > 0} w(u) + \sum_{v \in U, w(v) < 0} -w(v)$$

These is different than the profit calculation.

Linear Programming Duality

[max flow = min cap cut]

1. maximize $x_1 + 6x_2$
2. $x_1 \leq 200$
3. $x_2 \leq 300$
4. $x_1 + x_2 \leq 400$
5. $x_1, x_2 \geq 0$

$$(x_1, x_2) = (100, 300)$$

$$value = 1900$$

multiply inequality constraints by multipliers y_1, y_2, y_3 and add them up.

1. y_1 's must be non-negative (otherwise, inequality flips)
2. sum weighted inequalities $(y_1 + y_3)x_1 + (y_2 + y_3)x_2 \leq 200y_1 + 300y_2 + 400y_3$ $(y_1 + y_3) = 1, (y_2 + y_3) = 6$
3. Choose "best" multipliers minimize $200y_1 + 300y_2 + 400y_3$ subject to $y_1 + y_3 \geq 1$ $y_2 + y_3 \geq 6$ $y_1, y_2, y_3 \geq 0$