Hardness of Approximation

The general TSP is NP-hard to approximate.

Claim

If $P \neq NP$ then, there is no polytime c-approximation algorithm for TSP.

Proof

Suppose A is a poly time c-approximation algorithm for TSP. We use A to solve Hamiltonian Cycle.

Black box, reduction: Hamiltonian Cycle

G -> [-> X -> (G') -> A -> Y ->] -> Y/N

Transform X: Create G′ from G. G′ has all edges

$$w(u,v) = \{1if(u,v) \in G, c|V| + 1if(u,v) \notin G\}$$

Transform Y: If $|TSP_A(G')| \leq c|V|$ then output yes, otherwise no.

Why does this work?

Edges not in the original graph are so costly that there is a gap between cost of tour if G contains a Hamiltonian cycle (cost=n), otherwise if G doesn't contain it the cost has to be greater than $c|V|$.