Question 4

Assumption: The input sequence M is sorted.

We need to find M+1 points S such that the midpoint of $s_i s_{i+1}$ is M_i. We can model this problem as a linear system with M+1 variables. For each value M define a linear equation in the format $0.5S_i + 0.5S_{i+1} = M_i$.

The only trick is setting the initial value. Since the points M are based off of the midpoint, that means the distance between m_1 and m_2 will be less than or equal to the distance between s_1 and m_1. Thus we can pick an arbitrary point s_1 within that range and see if it forms a valid solution.

We can then structure this into matrices of the form $Ax = b$ and solve it using matrix inversion with a runtime of $O(n^3)$ where n is the number of output points or M+1.

This seems rather inefficient, but does solve it and can be effectively run on GPUs.