CPSC 421 - HW 2

Tristan Rice, 25886145, q7w9a

A.8

Find the asymptotic ratios, when they exist, of the following sequences:

A.8.1

$$a_n = 5^n + 1$$

$$1 = o(5^n)$$

Thus, 5^n has the same asymptotic ratio as $5^n + 1$.

$$\lim_{n\to\infty} \frac{5^{n+1}}{5^n} = 5$$

A.8.2

$$a_n = 3n^2 - 4$$

$$-4 = o(3n^2)$$

$$\lim_{n\to\infty} \frac{3(n+1)^2}{3n^2} = (\frac{n+1}{n})^2 = 1$$

A.8.3

$$a_n = 7n^6 + 5n^2 + n$$

$$\lim_{n\to\infty} \frac{5n^2}{7n^6} = \frac{5}{7n^4} = 0$$
$$\lim_{n\to\infty} \frac{n}{7n^6} = \frac{1}{7n^5} = 0$$

$$\lim_{n\to\infty} \frac{7(n+1)^6}{7n^6} = (\frac{n+1}{n})^6 = 1$$

A.8.4

$$a_n = 2^{n^2} + 2^n + n^2 + 3$$

$$\lim_{n\to\infty} \frac{3}{2^{n^2}} = 0$$

$$\lim_{n\to\infty} \frac{n^2}{2^{n^2}} = 0$$

$$\lim_{n\to\infty} \frac{\log(2^n)}{\log(2^{n^2})} = \frac{n}{n^2} = \frac{1}{n} = 0$$

Thus,

$$\lim_{n\to\infty} \frac{2^n}{2^{n^2}} = 0$$

$$\begin{aligned} \lim_{n\to\infty} \frac{2^{(n+1)^2}}{2^{n^2}} &= \lim_{n\to\infty} 2^{(n+1)^2-n^2} \\ &= \lim_{n\to\infty} 2^{2n+1} \\ &= \infty \end{aligned}$$

Thus, this doesn't have an asymptotic ratio.

A.9

$$\frac{a_{n+1}}{a_n} = \rho$$

A.9.1

$$b_n = 3a_n$$

$$\lim_{n\to\infty} \frac{b_{n+1}}{b_n} = \lim_{n\to\infty} \frac{3a_{n+1}}{3a_n} = \frac{a_{n+1}}{a_n} = \rho$$

A.9.2

$$b_n = a_{n+1}$$

$$\lim_{n\to\infty} \frac{b_{n+1}}{b_n} = \lim_{n\to\infty} \frac{a_{n+2}}{a_{n+1}} = \frac{a_{n+1}}{a_n} = \rho$$

A.9.3

$$b_n = (a_n)^3$$

$$\lim_{n\to\infty} \frac{b_{n+1}}{b_n} = \lim_{n\to\infty} \frac{(a_{n+1})^3}{(a_n)^3} = (\frac{a_{n+1}}{a_n})^3 = \rho^3$$

A.11

A.11.1

If n is odd, $f(n) = 0$ since you start and end at the center.

When you're at an edge vertex there's only one path and when you're at the center there's 7 paths. Thus, you're at the center every other step. Thus, $1^{\frac{n}{2}} * 7^{\frac{n}{2}} = 7^{\frac{n}{2}}$.

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ 7^{\frac{n}{2}} & \text{if } x \text{ is even} \end{cases}$$

A.11.2

This is similar to question 1, but there's one extra step. If n is even, you can't get to v_1 from c. Thus, $f(n) = 0$.

Similarly if you go to an edge vertex you need to come back to c before going to v_1. Thus the number of walks is equivalent to 1 but with $n-1$.

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 7^{\frac{n-1}{2}} & \text{if } x \text{ is odd} \end{cases}$$

A.11.3

This is the same as (2), except in the reverse order.

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 7^{\frac{n-1}{2}} & \text{if } x \text{ is odd} \end{cases}$$

A.11.4

At the beginning of all walks you have to go from v_1 to c, and at the end of all walks you have to go from c to v_1. Thus, all that matters is the number of walks from c to c that are length $n-2$.

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ 7^{\frac{n-2}{2}} & \text{if } x \text{ is even} \end{cases}$$

A.11.5

This is just the combination of

$$c \to c, c \to v_2, c \to v_3, v_1 \to c, v_1 \to v_2, v_1 \to v_3, v_2 \to c, v_2 \to v_2, v_2 \to v_3$$

This boils down to:

- 1x case 1
- 4x case 2/3
- 4x case 4

$$f(x) = \begin{cases} 4 * 7^{\frac{n-1}{2}} & \text{if } x \text{ is odd} \\ 7^{\frac{n}{2}} + 4 * 7^{\frac{n-2}{2}} & \text{if } x \text{ is even} \end{cases}$$

Bonus: A.12

Bonus: A.14