CPSC 421 - HW7

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Question 1

Give a formal description of a Turing machine–and explain how your machine works–that recognizes the language

$$L = \{0^n 10^n | n \text{ is a non-negative integer}\}$$

You should explicitly write your choice of $Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject}$.

1. The string is provided on the first tape.
2. Copy the string on the first tape to the second in reverse order.
3. Scan both strings and make sure that the symbols on both are 0 until it hits symbol = 1. If both = 1, continue, else reject.
4. Continue scanning both strings making sure the symbols on both are 0. If it encounters a 1 reject. If it hits blanks on both, accept, else reject.

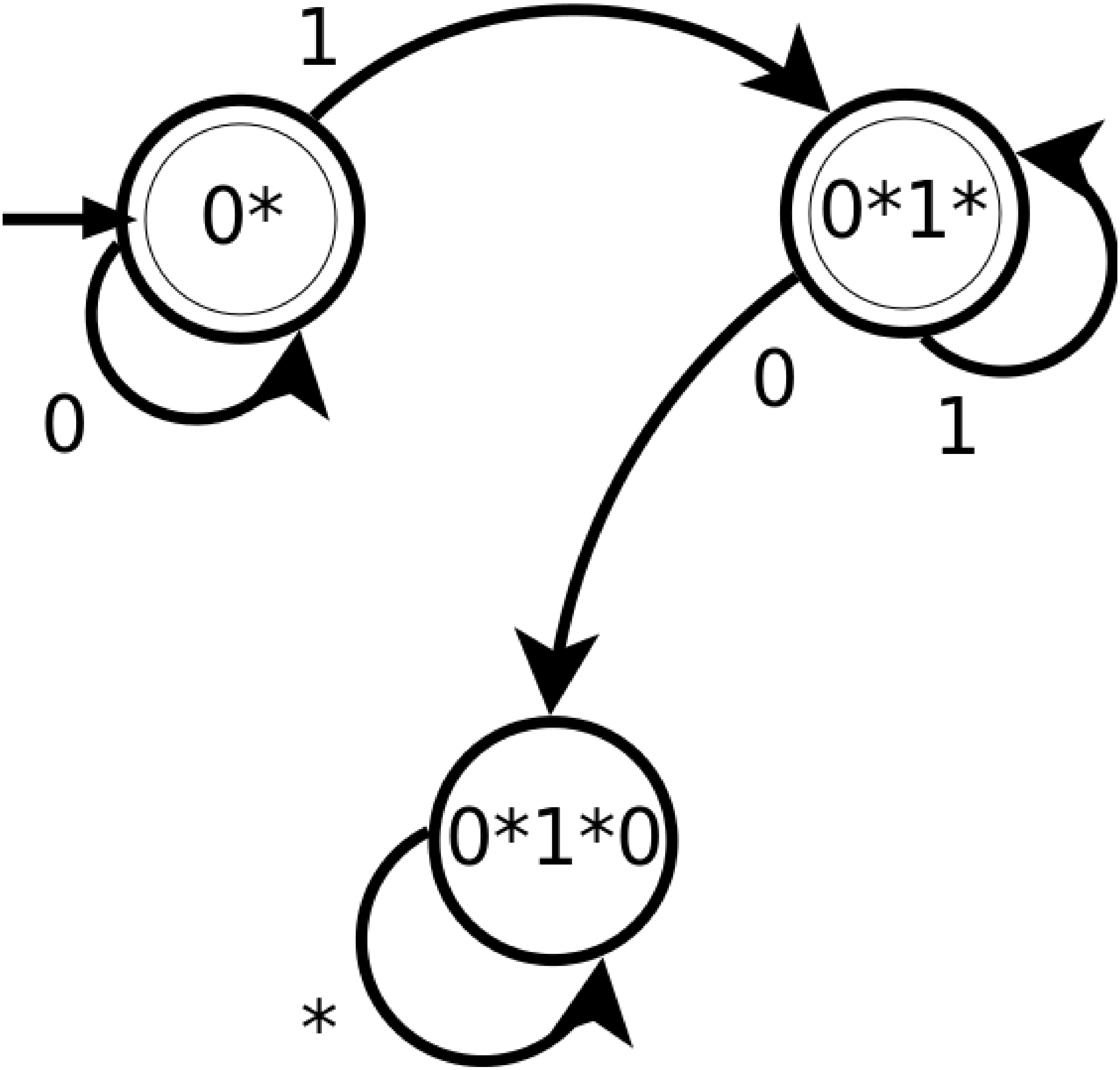
- $Q =$ reverse scan, reverse copy, reverse reset, looking for 1, seen 1, accept, reject
- $\Gamma =$ 0, 1, blank, beginning
- $\Sigma =$ 0, 1
- $q_0 =$ reverse scan
- $q_{accept} =$ accept
- $q_{reject} =$ reject

```
delta(Q, Sym1, Sym2) {
  if Q == reverse scan {
    if Sym1 == blank {
      Q = reverse copy
      move head 1 left
    } else {
      move head 1 right
    }
  } else if Q == reverse copy {
    if Sym1 == beginning {
      move head 1 right
      Q = reverse reset
    } else {
      Sym2 = Sym1
      move head 1 left
      move head 2 right
    }
  } else if Q == reverse reset {
    if Sym2 == beginning {
      Q = looking for 1
      move head 2 right
    } else {
      move head 2 left
    }
  } else if Q == looking for 1 {
    if Sym1 != Sym2 {
      Q = reject
    } else if Sym1 == blank {
      Q = reject
    } else if Sym1 == 1 {
      Q = seen 1
      move head 1 right
      move head 2 right
    } else {
      move head 1 right
      move head 2 right
```

```
    }
  } else if Q == seen 1 {
    if Sym1 != Sym2 {
      Q = reject
    } else if Sym1 == blank {
      Q = accept
    } else if Sym1 == 1 {
      Q = reject
    } else {
      move head 1 right
      move head 2 right
    }
  }
}
```

Question 2

(a)



(b)

Let $L = \{0^n 1 0^n | n \text{ is a non-negative integer}\}$. What is $Future(L, 0001)$? Explain.

$Future(L, 0001)$ is the set of possible inputs such that the language L will accept the string $00001a$ where $a \in$ the possible inputs.

In this case, the only valid thing we can append to 0001 for it to become a valid string for language L is 000.

Thus,

$$Future(L, 0001) = \{000\}$$

(c)

We can use the Myhill-Nerode theorem to show that L isn't regular by showing that there is an infinite number of accepting futures.

Claim: L can be recognized by a DFA.

$$Future(L, 1) = \epsilon$$
$$Future(L, 01) = 0$$
$$Future(L, 0^2 1) = 0^2$$
$$Future(L, 0^n 1) = 0^n$$

Since n can by any non-negative integer. That means that there are an infinite number of accepting futures and thus, the DFA representing L would have an infinite number of states. Contradiction. A DFA must have a finite number of states, and thus language L can't be recognized by any DFA.

Question 3

Claim: If $C_1, C_2, \ldots$ are countable sets, then $C_1 \cup C_2 \cup \ldots$ is countable.

Since each C_i is countable, we may write each C_i as $\{c_{i1}, c_{i2}, \ldots\}$. Then the sequence that lists each c_{ij} in order of increasing $i + j$ contains each element of $C_1 \cup C_2 \cup \ldots$.

Since it contains each element, we have an injective map $\mathbb{S} \to \mathbb{N}$. Thus, the union of countable sets is countable.

Question 4

Let Σ be a finite, nonempty alphabet.

(a)

Claim: Σ^* is infinite but countable.

Σ^* is trivially infinite since there's an infinite number of sequence lengths.

Since Σ is countable we may write it as $\Sigma_1, \Sigma_2, \ldots$. Thus, the sequence that lists each $\Sigma_{i_1} \Sigma_{i_2} \ldots$ in order of increasing $\sum i$ contains each element of Σ^*.

Since it contains each element, we have an injective map $\mathbb{S} \to \mathbb{N}$. Thus, Σ^* is countable.

(b)

No. It's not countable.

We know that any subset of a countable set is countable. The set of all subsets isn't countable however, since that's the definition of the power set. The power set of an countably infinite set is not countable. Thus, the set of subsets of Σ^* isn't countable.

(c)

Part (a) shows that there are an infinite number of programs but countably infinite. Part (b) shows that since the number of possible languages is uncountable that means that there are some languages that aren't recognizable by any program.

Question 5

$$L = \{x \in \{0,1\}^* | x \text{ contains 01 as a substring}\}$$

Using the Myhill-Nerode theorem we see there are the following states.

$$AcceptingFuture(L, \epsilon) = L$$
$$AcceptingFuture(L, 0) = 1\{0,1\}^* \cup L$$
$$AcceptingFuture(L, 00) = 1\{0,1\}^* \cup L$$
$$AcceptingFuture(L, 01) = \{0,1\}^*$$
$$AcceptingFuture(L, 001) = \{0,1\}^*$$
$$AcceptingFuture(L, 010) = \{0,1\}^*$$
$$AcceptingFuture(L, 1) = L$$
$$AcceptingFuture(L, 11) = L$$
$$AcceptingFuture(L, 10) = 1\{0,1\}^* \cup L$$

Thus, there are 3 distinct $AcceptingFuture$s and thus there are 3 state for the DFA corresponding to L. Since we can construct a DFA for L, the language L must be regular.

Question 6