CPSC 421 - Homework 6

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1

For languages A and B, let the shuffle of A and B be the language:

$$L = \{w | w = a_1 b_1 \dots a_k b_k, \text{ where } a_1 \dots a_k \in A \text{ and } b_1 \dots b_k \in B, \text{ each } a_i, b_i \in \Sigma^*\}$$

Claim: The class of regular languages is closed under shuffle.

Every regular language will have a corresponding DFA recognizing it. The shuffle of two regular languages means that we don't know which DFA will be running given an input. Thus, we need to construct a compound NFA which can decide which DFA to run given an input $\in A$ or $\in B$.

The states are the cartesian product of the states of A and the states of B.

The finish states are the cartesian product of the finish states of A and the finish states of B.

Initial state is (A_{start}, B_{start}).

For a given state (s_A, s_B) we update to be $(new(s_A), s_B)$ if the token is in A, and otherwise update it to be $(s_A, new(s_B))$.

This can be shown to be correct by iterating through some input a and seeing that in (s_A, s_B), s_A iterates through all the states that the A DFA would normally have iterated and get to a finish state. We see the same thing using some input b, s_B iterates to the finish state as well. Since s_A, s_B change independently we know that this holds for inputs $a \in A, b \in B$.

This can be formally proven through induction.

Thus, since an NFA can be constructed for the shuffle of A and B, we know that the shuffle of A, B must be regular. Thus, the class of regular languages is closed under shuffle.

2

$$F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

Claim: F is not regular.

Proof:

We can use the Myhill-Nerode theorem to show that F is not regular by looking at

$$AcceptingFuture(F, s) := \{t | st \in F\}$$

Some of the possible accepting futures:

$$AcceptingFuture(F, \epsilon) = \{w \in F\} = F$$
$$AcceptingFuture(F, b) = b * c*$$
$$AcceptingFuture(F, bc) = c*$$
$$AcceptingFuture(F, aa) = a * b * c*$$
$$AcceptingFuture(F, aab) = b * c*$$
$$AcceptingFuture(F, aabc) = c*$$
$$AcceptingFuture(F, ab) = b^n c^{n+1}$$
$$AcceptingFuture(F, abc) = \varnothing$$
$$AcceptingFuture(F, ab^2c) = c$$

If we take this last one and parameterize it we get:

$$AcceptingFuture(F, ab^n c) = c^{n-1}$$

Thus, there are an infinite of distinct accepting futures. Thus, a DFA matching this language has an infinite number of states. Thus, the language F is not regular since it doesn't have a corresponding DFA.

3

$$F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$
$$L = ab^* c^*$$

$$F \cap L = \{ab^j c^k | j, k \geq 0 \text{ and } j = k\}$$

Pumping Lemma: Say that $F \cap L$ is regular and accepted by a DFA of p states or fewer.

Then if $s \in F \cap L$ and $|s| \geq p$ we can write $s = xyz$ such that

1. xz, xyz, xy^2z, xy^3z
2. $y \neq \epsilon$
3. $|xy| \leq p$

Claim: $F \cap L$ is not regular.

Proof: Say $F \cap L$ is regular and accepted by a DFA of p states.

Now consider $s = ab^{p-1}c^{p-1} \in F \cap L$. Then $s = xyz$ such that 1-3 above hold. Since $s = ab^{p-1}c^{p-1} = xyz$ and $|xy| \leq p$ we have $x = ab^i, y = b^j, z = b^{p-i-j-1}c^{p-1}$.

So $xy, xyz, xy^2, xy^3z, \ldots, xy^nz \in L$. From condition 2, $y \neq \epsilon$ so $j \geq 1$ and $ab^iz \in L, ab^{i+j}z \in L, ab^{i+2j}z \in L, ab^{i+kj}z \in L.$

Thus, $ab^{i+kj}b^{p-i-j-1}c^{p-1} \in L.$

This is impossible since according to $F \cap L$,

$$i + kj + p - i - j - 1 = p - 1$$
$$p - 1 + (k-1)j = p - 1$$

Thus, since this must hold for all $k \in \mathbb{N}$ according to the Pumping Lemma, $F \cap L$ is not regular.

4

4.b

We can set $p = 1$. $x = \epsilon, y = b^p, z = c^p$

This satisfies the three conditions for the pumping lemma.

1. $|y| \geq 1$ holds since $|b| \geq 1.$
2. $|xy| \leq p$ holds since $|b| \leq 1.$
3. $xy^nz \in L$ for all $n \geq 0$ holds since $b^nc \in L.$

4.c

Parts (a), (b) don't contradict the Pumping Lemma since it depends on carefully picking the string $s = xyz$. The Pumping Lemma is mostly only used for showing that languages are irregular via contradiction. To show that a language is regular using the Pumping Lemma requires a proof for all strings in l of length at least p. (b) only shows one possible string that appears to be regular and not all thus, doesn't prove that the language is regular.

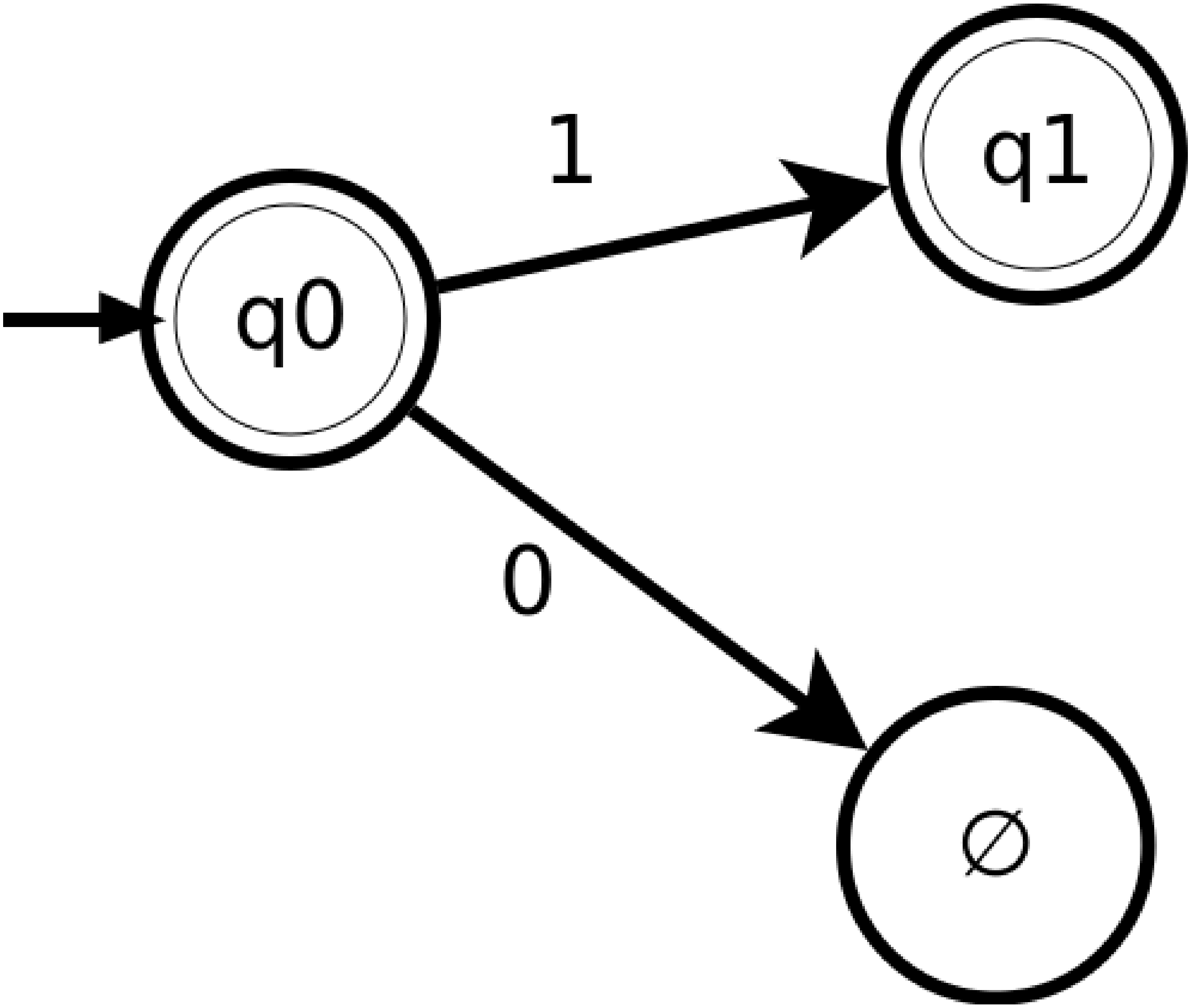
5

5.a

$$AcceptingFuture(L, \epsilon) = (1 \cup 10 \cup 1001)^*$$
$$AcceptingFuture(L, 0) = \varnothing$$
$$AcceptingFuture(L, 1) = (\epsilon \cup 0 \cup 001)(1 \cup 10 \cup 1001)^*$$

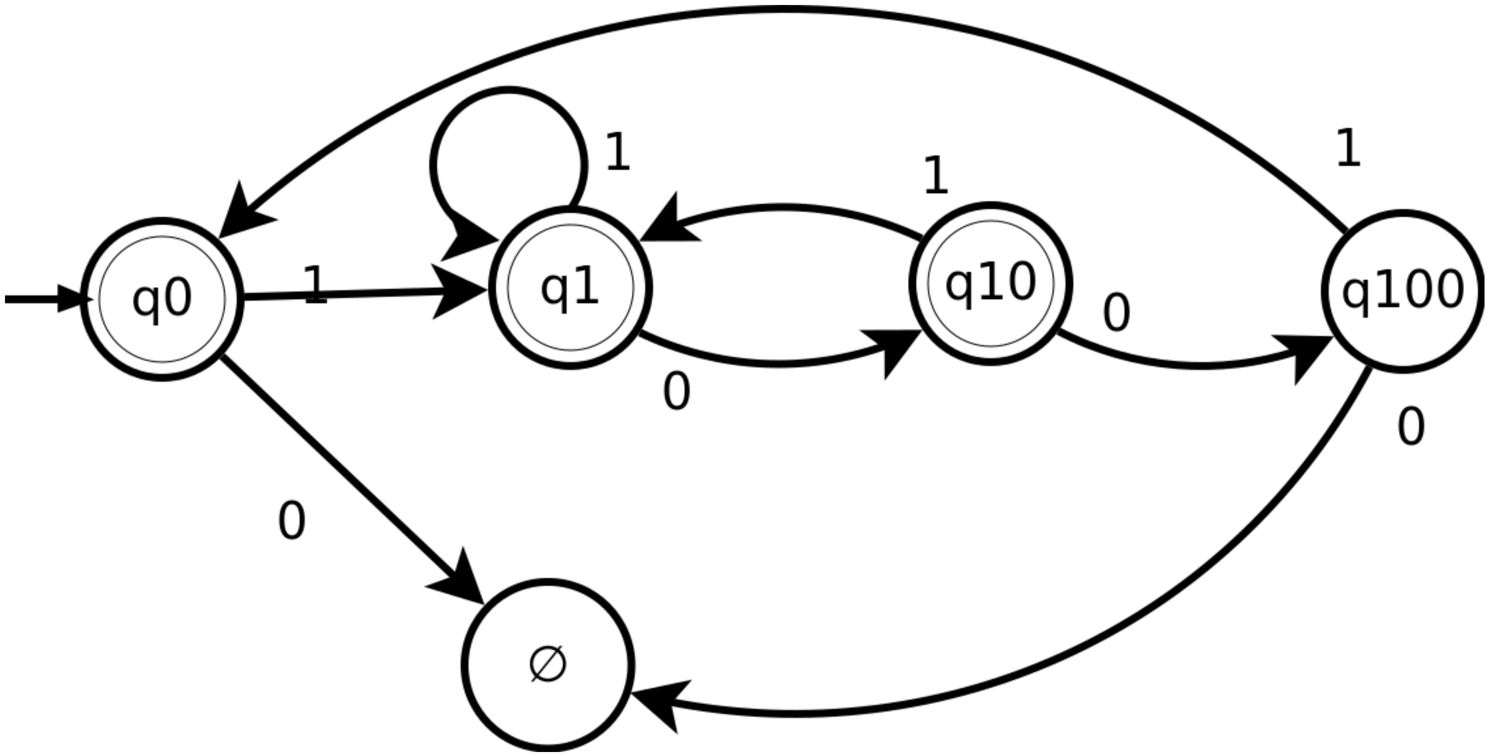
5.b

- q0: $AcceptingFuture(L, \epsilon)$
- q1: $AcceptingFuture(L, 1)$
- $\varnothing$: $AcceptingFuture(L, 0)$



5.c

$$AcceptingFuture(L, \epsilon) = (1 \cup 10 \cup 1001)^*$$
$$AcceptingFuture(L, 0) = \varnothing$$
$$AcceptingFuture(L, 1) = (\epsilon \cup 0 \cup 001)(1 \cup 10 \cup 1001)^*$$
$$AcceptingFuture(L, 10) = (\epsilon \cup 01)(1 \cup 10 \cup 1001)^*$$
$$AcceptingFuture(L, 100) = 1(1 \cup 10 \cup 1001)^*$$
$$AcceptingFuture(L, 1000) = \varnothing$$
$$AcceptingFuture(L, 1001) = (1 \cup 10 \cup 1001)^*$$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q1
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