

Quadratic Time

At least $\Theta(n^2)$

- Say $f = \Theta(g) \{f, g : \mathbb{N} \rightarrow \mathbb{R}_{n \geq 0}$

$f(n) = \Theta(g(n))$ if $f(n) = O(g(n)), g(n) = O(f(n))$

Asymptotic Equality

$f \sim g$ if $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 1$

$$2n^3 - n \sim 2n^3$$

$$n! \sim \sqrt{2\pi n} (n/e)^n$$

$$f(n) = 2n^2 + n \log n$$

$$g(n) = 3n^2 - \sqrt{n} + 1$$

Claim: $f(n) \sim g(n)$?

$$f(n) / g(n)$$

$$\lim_{n \rightarrow \infty} \frac{f(n)^{3/2}}{g(n)} = 1$$

Last time:

$$\frac{3}{2}f(n) = 3n^2 + o(n^2)$$

We can ignore the smaller thing.

Fact: $f(n) = f_1(n) + o(f_1(n))$

$$g(n) = g_1(n) + o(g_1(n))$$

$$f = O(g) \iff f_1 = O(g_1) \quad f = o(g) \iff f_1 = o(g_1)$$

Since we can do this, we can divide both sides of the lim by $3n^2$.

$$\lim_{n \rightarrow \infty} \frac{1 + (\frac{3/2n \log n}{3n^2})}{1}$$

Omega

$f(n) = \Omega(g(n))$ if $c_1 g(n) \leq f(n) \forall n \geq n_0$ for some c_1, n_0 .

Asymptotic ratio:

$$1/2 + 2/4 + 3/8 + 4/16 + 5/32 + \dots + \frac{n}{2^n}$$

Asymptotic ratio f: $\lim_{n \rightarrow \infty} \frac{f(n+1)}{f(n)}$

$$\rho(f) := \lim_{n \rightarrow \infty} f(n+1)/f(n)$$

$\rho := \text{ratio}$

if $|p| < 1$ converges, $|p| > 1$ diverges

$$\rho(f) = \lim_{n \rightarrow \infty} \frac{f(n+1)}{f(n)} = \lim_{n \rightarrow \infty} \frac{(n+1)/2^{n+1}}{n/2^n} = \lim_{n \rightarrow \infty} \frac{1}{2} \frac{n+1}{n} = \frac{1}{2}$$

$$f(n) = f_1(n) + o(f_1(n)), \rho(f) = \rho(f_1)$$

Counting the number of walks in a directed graph

Directed: V, f, h, t

Each edge has a head and a tail. V, f are sets (usually finite).

$$h : E \rightarrow V, t : E \rightarrow V$$

Next time: How many ways are there to walk in n steps from v_1 to v_2 .