Homework 1

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A.1(3,4,6)

A.1.3

Fibonacci: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10| 11| 12| 13| 14| 15| |—|—|—|—|—|—|—|—|—|—|—|—|—|—|—| | 1 | 1 | 2 | 3 | 5 | 8 | 13| 21| 34| 55| 89|144|233|377|610|

$F_nF_{n+3} - F_{n+1}F_{n+2}$ 1. $1*3-1*2=1$ 2. $1*5-2*3=-1$ 3. $2*8-3*5=1$ 4. $3*13-5*8=-1$ 5. $5*21-8*13=1$

Theorem: The output of $f(n) = F_nF_{n+3} - F_{n+1}F_{n+2}$ is 1 when n is even and -1 when n is odd.

Proof by induction.

Case: n is odd.

Base cases: - $n=1$: $1*3-1*2=1$ - $n=2$: $1*5-2*3=-1$

Induction step ($n \geq 3$):

$f(n) = f(n+2)$

$F_nF_{n+3} - F_{n+1}F_{n+2} = F_{n+2}F_{n+5} - F_{n+3}F_{n+4}$

$= F_{n+2}(F_{n+1} + F_{n+5})$

$= (F_{n+1} + F_n)(F_{n+3} + F_{n+2} + F_{n+3}) - (F_{n+2} + F_{n+1})(F_{n+3} + F_{n+2})$

$$= 2*F_{n+1}F_{n+3} + F_{n+1}F_{n+2} + 2*F_nF_{n+3} + F_nF_{n+2} - F_{n+2}F_{n+3} - F_{n+2}F_{n+2} - F_{n+1}F_{n+3} - F_{n+1}F_{n+2}$$

$$= F_{n+1}F_{n+3} + 2*F_nF_{n+3} + F_nF_{n+2} - F_{n+2}F_{n+3} - F_{n+2}F_{n+2}$$

$$= F_{n+1}F_{n+3} + 2*F_nF_{n+3} + F_nF_{n+2} - F_{n+1}F_{n+3} - F_nF_{n+3} - F_{n+1}F_{n+2} - F_nF_{n+2}$$
$$= F_nF_{n+3} - F_{n+1}F_{n+2}$$

Thus, since $f(n) = f(n+2)$ we prove via induction that the output will be 1 for all odd values of n and -1 for all even values of n.

A.1.4

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In [15]: def F(n):
    ...:         return int(0.5+((1+sqrt(5))**n-(1-sqrt(5))**n)/(2**n*sqrt(5)))
In [16]: [(n, F(n) * F(n+8) - F(n+1)*F(n+7)) for n in range(1,6)]
Out[16]: [(1, 13), (2, -13), (3, 13), (4, -13), (5, 13)]
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For all even values of n, $f(n) = -13$. For all odd values of n, $f(n) = 13$.

Proof by induction.

Base cases: * $n=1$: $f(1) = 13$ * $n=2$: $f(2) = -13$

Induction step:

We must show that for all $n > 2$, $f(n) = f(n+2)$.

$$F_nF_{n+8} - F_{n+1}F_{n+7} = F_{n+2}F_{n+10} - F_{n+3}F_{n+9}$$

$$= F_{n+1}F_{n+10} + F_nF_{n+10} - F_{n+2}F_{n+9} - F_{n+1}F_{n+9}$$

$$= F_{n+1}F_{n+9} + F_{n+1}F_{n+8} + F_nF_{n+9} + F_nF_{n+8} - F_{n+0}F_{n+9} - F_{n+1}F_{n+9} - F_{n+1}F_{n+9}$$

$$= F_{n+1}F_{n+8} + F_nF_{n+8} - F_{n+1}F_{n+9}$$

$$= F_{n+1}F_{n+8} + F_nF_{n+8} - F_{n+1}F_{n+8} - F_{n+1}F_{n+7}$$

$$= F_nF_{n+8} - F_{n+1}F_{n+7}$$

Thus, by the property of induction it holds for all $n \geq 1$, $f(n) = 13$ for odd numbers and $f(n) = -13$ for even numbers.

A.1.6

$$x^2 = x + 1$$

$$x = \{\frac{1}{2}(1-\sqrt{5}), \frac{1}{2}(1+\sqrt{5})\}$$

A.2(1,2)

A.2.1

Prove $\binom{\sum_{m=1}^{n} m=}{n+12}$ for any $n \in \mathbb{N}$.

A.2.2

A.3(1)

A.4(1b,1d,2d)

A.4.1b

A.4.1d

A.4.2d

A.5(2)