CPSC 540 Assignment 1

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Question 1 - Very-Short Answer Questions

Give a short and concise 1-sentence answer to the below questions.

1. Why is the IID assumption important in supervised learning?

Supervised learning works on the principle that the training data is a good representation of the test data and the IID assumption states that.

2. Suppose we have a supervised learning problem where we think the examples xi form clusters. To deal with this, we combine our training and test data together and fit a k-means model. We then add the cluster number as an extra feature, fit our supervised learning model based on the training data, then evaluate it on the test data. What have we done wrong?

We used the test data during training which violates the golden rule.

3. What is the difference between a validation set error and the test error?

Validation set error is the error you compute via the validation set where as test error is the theoretical error if you had infinite test data.

4. Describe a setting where using a validation set to choose hyper-parameters can lead to overfitting.

If you are using the validation set to optimize a large number of hyper-parameter options it's very easy to overfit since you're optimizing for the validation set error instead of the test error.

5. What is the effect of the number of features d that our model uses on the training error and on the approximation error?

Increasing d, exponentially increases the problem space and thus with the same amount of data can cause overfitting on the training error (lowering it) and making the approximation error much larger.

6. What is wrong with with using … as the validation error of a regression model?

In regression it's very unlikely that the two numbers will be an exact match and thus the validation error will be always very close to 0 and thus useless for telling us how accurate the model is.

7. Describe a situation where it could be better to use gradient descent than the normal equations to solve a least squares problem.

If you have a huge amount of data solving the normal equations isn't feasible since it requires a quadratic+ amount of computational resources due to very large matrices.

8. How does λ in an L0-regularizer (like BIC) affect the sparsity pattern of the solution, the training error, and the approximation error?

Increasing λ causes the solution to be sparser since there's a higher penalty on non-zero values, the training error to increase since it can't overfit as much and lowers the approximation error since the model is likely more general.

9. Minimizing the squared error with L0-regularization is NP-hard, what does this imply?

It's computationally infeasible to find the model with the lowest training error and approximations are the best we can do.

10. For a fixed target y, what is the likely effect of increasing the condition number of X (in a least squares problem) on the approximation error?

Increasing the condition number of X means the problem is less stable and will have a higher approximation error.

11. For supervised training of a linear model ..., why do we use the logistic loss intead of the squared error?

12. What is the key difference between "one vs. all" logistic regression and training using the softmax loss?

13. Give a supervised learning scenario where you would use the Laplace likelihood and a scenario where you would use a Laplace prior.

14. What do we use the backpropagation algorithm for?

Updating weights in a neural network.

15. What are the two key properties of a problem that let us use dynamic programming?

16. Consider a deep neural network with 1 million hidden units. Explain whether this is a parametric or a non-parametric model.

This is a parametric model since the amount of memory it uses is constant with a varying number of training examples.

17. What are two reasons that convolutional neural networks overfit less than classic neural networks?

Question 2 - Calculation Questions

2.1

2.1.1

$$f(w) = \frac{1}{2}(w-u)^T\Sigma(w-u)$$

$$\Sigma = G^TG$$

$$f(w) = \frac{1}{2}(G(w-u))^2$$

$$f'(w) = G(w-u)G$$

$$0 = G(w-u)G$$

$$GwG = GuG$$

$$w = u$$

2.1.2

$$f'(w) = \frac{||Xw-y||}{\sigma^2}\frac{Xw-y}{||Xw-y||} + \lambda||w||\frac{w}{||w||}$$

$$f'(w) = \frac{Xw-y}{\sigma^2} + \lambda w$$

$$f'(w) = (\frac{X}{\sigma^2} - \lambda I)w - \frac{y}{\sigma^2}$$

$$0 = (\frac{X}{\sigma^2} - \lambda I)w - \frac{y}{\sigma^2}$$

$$w = (\frac{X}{\sigma^2} - \lambda I)^{-1}\frac{y}{\sigma^2}$$

2.1.3

$$f'(w) = \sum_{i=1}^{n} v_i(w^T x^i - y^i)x^i + \Lambda w - \Lambda u$$

$$f'(w) = w^T \sum_{i=1}^{n} v_i {x^i}^2 - \sum_{i=1}^{n} v_i y^i x^i + \Lambda w - \Lambda u$$

$$f'(w) = (\sum_{i=1}^{n} v_i {x^i}^2 + \Lambda)w - \sum_{i=1}^{n} v_i y^i x^i - \Lambda u = 0$$

$$w = (\sum_{i=1}^{n} v_i {x^i}^2 + \Lambda)^-1(\sum_{i=1}^{n} v_i y^i x^i + \Lambda u)$$

2.2

2.2.1

$$||w||_\infty \leq ||w||_2$$

$$max_i w_i \leq \sqrt{w_0^2 + w_1^2 + ...}$$

$$\sqrt{(max_i w_i)^2} \leq \sqrt{w_0^2 + w_1^2 + ...}$$

$$(max_i w_i)^2 \leq w_0^2 + w_1^2 + ...$$

Since the max element of w is included in both sides and all the squared terms are positive, the right side must be greater or equal to the left hand side.

$$||w||_2 \leq ||w||_1$$

$$\sqrt{w_0^2 + w_1^2 + ...} \leq |w_0| + |w_1| + ...$$

$$w_0^2 + w_1^2 + ... \leq (|w_0| + |w_1| + ...)^2$$

$$w_0^2 + w_1^2 + ... \leq w_0^2 + 2|w_0||w_1| + w_1^2 + ...$$

Thus, both sides contain the w_i^2 elements and thus the right side must be greater or equal due to the $2|w_i||w_j|$ elements which are all positive.

2.2.2

$$||w||_1 \leq \sqrt{d}||w||_2$$

$$|w_0| + |w_1| + ... \leq \sqrt{d}\sqrt{w_0^2 + w_1^2 + ...}$$
$$(|w_0| + |w_1| + ...)^2 \leq d(w_0^2 + w_1^2 + ...)$$

$$w_0^2 + 2|w_0||w_1| + w_1^2 + ... \leq d(w_0^2 + w_1^2 + ...)$$

$$2|w_0||w_1| + ... \leq (d-1)(w_0^2 + w_1^2 + ...)$$

We see on the left side there are $\frac{d(d-1)}{2}$ terms.

$$2|w_0||w_1| + ... \leq (d-1)(w_0^2 + w_1^2 + ...)$$

We can complete the square, since there are $d-1$ occurences of each variable on the left hand side and $d-1$ squared terms of it on the right.

$$0 \leq \sum_{i=1}^{d} \sum_{j=i+1}^{d} (w_i - w_j)^2$$

Since all the terms on the right are squared, they must be positive and the inequality holds.

$$\sqrt{d}||w||_2 \leq d||w||_\infty$$

$$d||w||_2^2 \leq d^2||w||_\infty$$
$$||w||_2^2 \leq d||w||_\infty^2$$

$$w_1 + w_2 + ... \leq d \max_i w_i^2$$

Every $w_i^2 \leq \max_i w_i^2$, thus, this inequality holds.

2.2.3

$$\frac{1}{2}((w_1+u_1)^2+(w_2+u_2)^2+...)\leq ||w||^2+||u||^2$$
$$w_1^2+2w_1u_1+u_1^2+w_2^2+2w_2u_2+u_2^2+...\leq 2(||w||^2+||u||^2)$$

The squared terms on the left equal ($||w||^2+||u||^2$).

$$2w_1u_1+2w_2u_2+...\leq ||w||^2+||u||^2$$

$$2w_1u_1+2w_2u_2+...\leq w_1^2+u_1^2+w_2^2+u_2^2+...$$

$$0\leq (w_1^2-2w_1u_1+u_1^2)+(w_2^2-2w_2u_2+u_2^2)+...$$

Completing the square gets us

$$0\leq (w_1-u_1)^2+(w_2-u_2)^2+...$$

Since all the terms on the right hand side are squared they must be positive and thus ≥ 0. Thus, the inequality holds.

2.3

2.3.1 Laplace distribution

$$f(x|\mu,b)=\frac{1}{2b}\,\text{exp}(-\frac{|x-\mu|}{b})$$

Gaussian distribution

$$f(x|\mu,\sigma)=\frac{1}{\sqrt{w\pi\sigma}}\,\text{exp}(-\frac{-\frac{(x-\mu)^2}{2\sigma^2}}{b})$$

$$f(x)=-\sum_{i=1}^{n}\text{log}(p(y^i|x^i,w))-\sum_{j=1}^{n}\text{log}(p(w^j))$$

$$f(x)=-\sum_{i=1}^{n}(\text{log}(\frac{1}{\sqrt{2\pi\sigma_i^2}})-\frac{(w^Tx^i-y^i)^2}{2\sigma_i^2})-\sum_{j=1}^{n}(\text{log}(\frac{\lambda}{2})-\lambda|w_j|)$$

We can then discard any constants.

$$f(x)=-\sum_{i=1}^{n}(\text{log}(\frac{1}{\sqrt{2\pi\sigma_i^2}})-\frac{(w^Tx^i-y^i)^2}{2\sigma_i^2})+\sum_{j=1}^{n}\lambda|w_j|)$$

2.3.2

$$f(x)=-\sum_{i=1}^{n}\text{log}(p(y^i|x^i,w))-\sum_{j=1}^{n}\text{log}(p(w^j))$$

$$f(x)=-\sum_{i=1}^{n}\text{log}(\frac{1}{\sqrt{\nu}\mathrm{B}(\frac{1}{2},\frac{\nu}{2})}(1+\frac{(w^Tx^i-y^i)^2}{\nu})^{-\frac{\nu+1}{2}})+\sum_{j=1}^{n}\frac{\lambda(w_j-u_j)^2}{2}$$

$$f(x)=-\sum_{i=1}^{n}-\frac{\nu+1}{2}\,\text{log}(1+\frac{(w^Tx^i-y^i)^2}{\nu}))+\sum_{j=1}^{n}\frac{\lambda(w_j-u_j)^2}{2}$$

2.3.3

$$f(x) = -\sum_{i=1}^{n} \mathsf{log}(p(y^i|x^i, w)) - \sum_{j=1}^{n} \mathsf{log}(p(w^j))$$

$$f(x) = -\sum_{i=1}^{n} \mathsf{log}(\mathsf{exp}(v^i w^T x^i)\, \mathsf{exp}(-y^i\, \mathsf{exp}(w^T x^i))) + \sum_{j=1}^{n} \frac{\lambda_j w_j^2}{2}$$

$$f(x) = -\sum_{i=1}^{n} (v^i w^T x^i - y^i\, \mathsf{exp}(w^T x^i)) + \sum_{j=1}^{n} \frac{\lambda_j w_j^2}{2}$$

2.4

2.4.1

$$f(w) = w^T u + u^T A w + \frac{1}{2} w^T w + w^T A w$$

$$\nabla f(w) = u + u^T A + w + (A + A^T) w$$
$$\nabla^2 f(w) = I + A + A^T$$

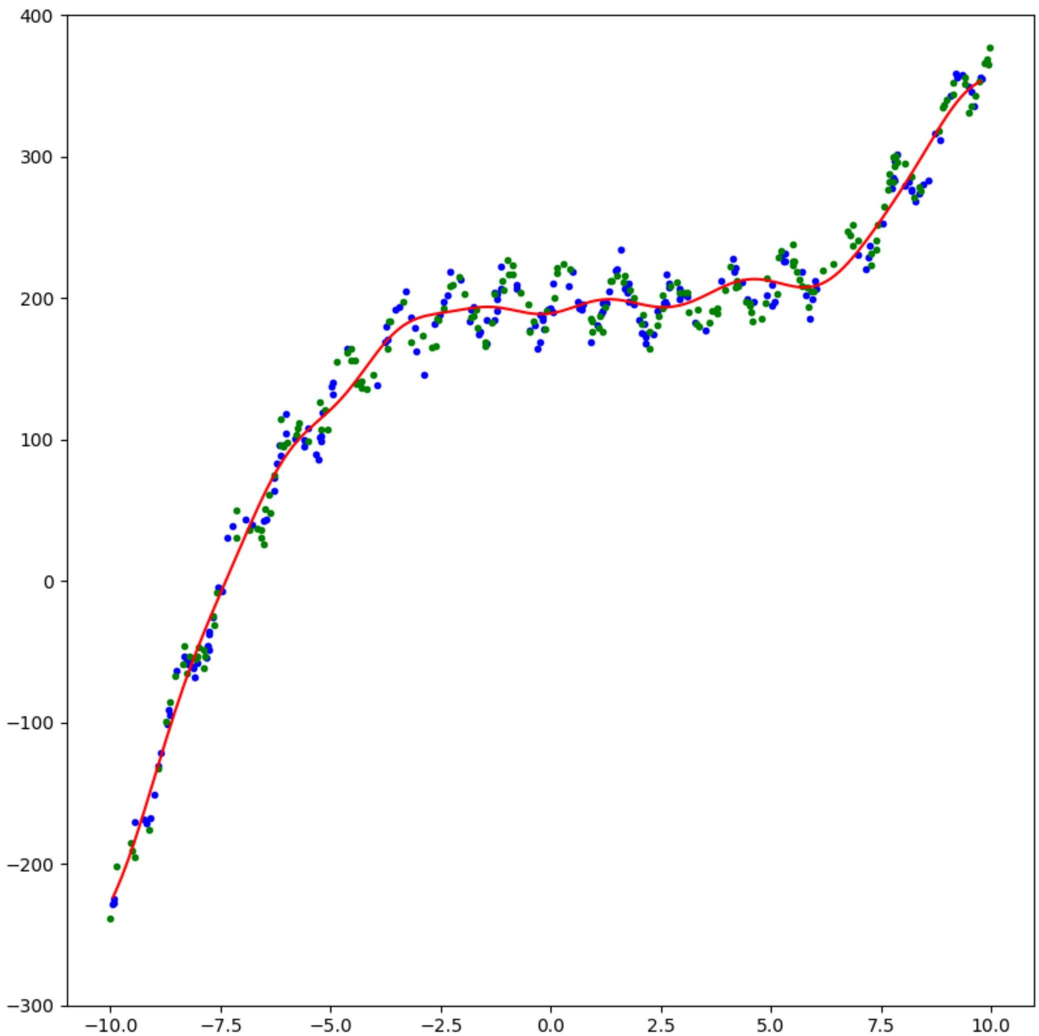
2.4.2 TODO

2.4.3 TODO

Question 3 - Coding Questions

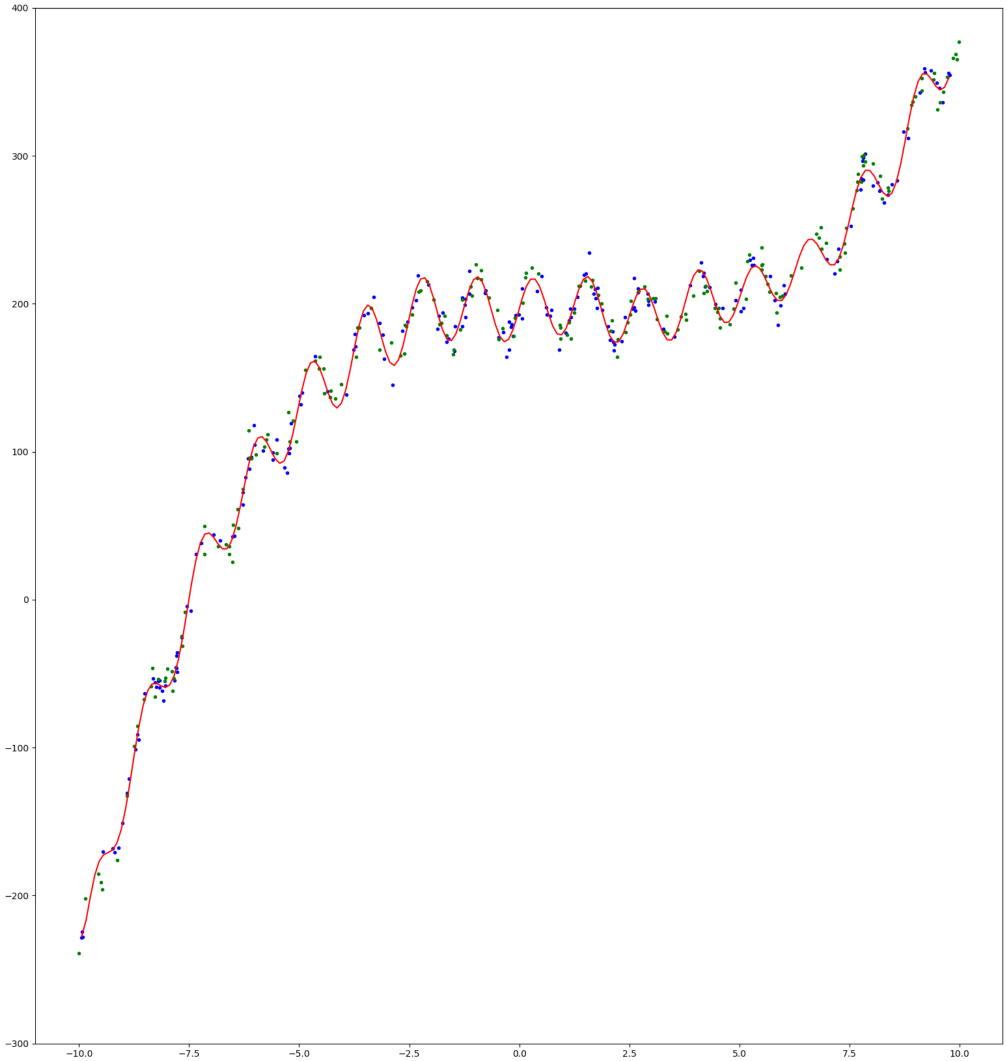
3.1

TODO code



3.1.1

3.1.2



3.1.3

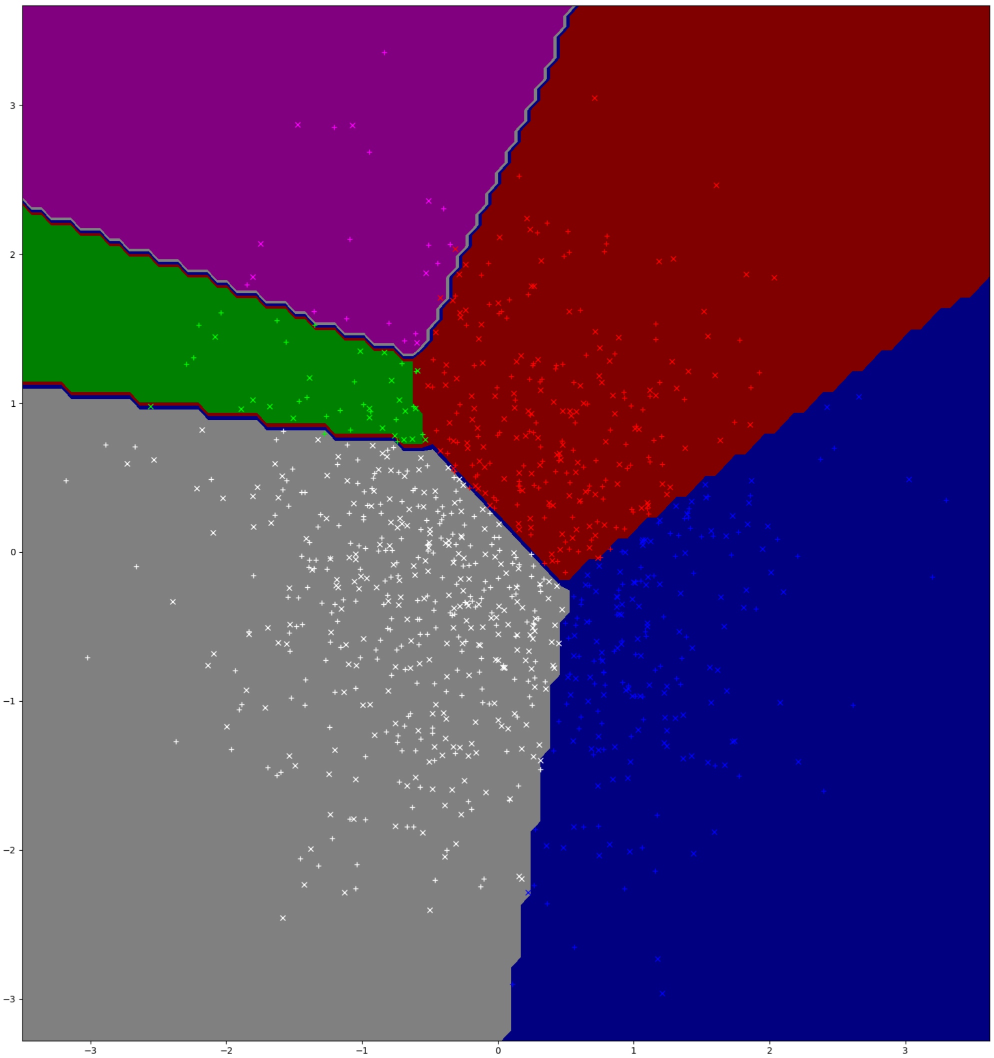
3.1.4 If the period of the oscillations isn't constant, we can transform the data before hand and put it on a non-linear scale before making using the Gaussian RBF. That way the RBF sees a constant period, but our data isn't.

If we haven't evenly sampled the training data, but we know the distribution we can weigh the examples to counterbalance our uneven sampling. This would be fairly simple to implement by adding a weight to the distance calculated for each RBF point.

3.2

```
trainError = 0.004
```

```
validError = 0.026
```

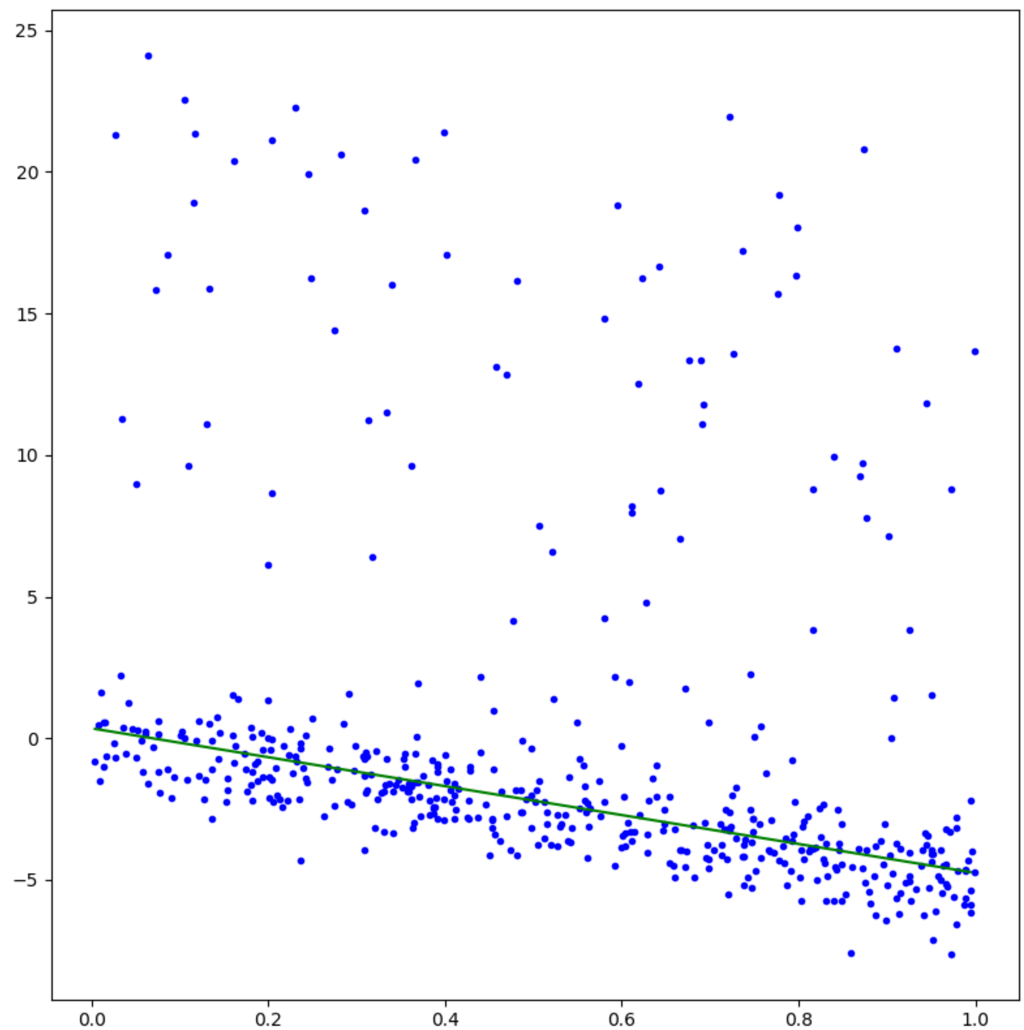


3.3

3.3.1 leastAbsolutes.jl

```
using MathProgBase, GLPKMathProgInterface

include("misc.jl")
```

```julia
function leastAbsolutes(X, y)
  (n, d) = size(X)
  X = [ones(n, 1) X]
  d += 1
  y = y[:,1]
  c = [ones(n); zeros(d)]
  A = [eye(n) -X; eye(n) X]
  b = [-y; y]

  display(size(y))
  display(size(c))
  display(size(A))
  display(size(b))

  l = [zeros(n); ones(d) * -Inf]

  sol = linprog(c, A, '>', b, l, Inf, GLPKSolverLP())
  w = sol.sol[n+1:n+d]
  if sol.status == :Optimal
      println("Optimal objective value is $(sol.objval)")
      println("Optimal solution vector is: $(w)")
  else
      println("Error: solution status $(sol.status)")
  end
  predict(Xtilde) = [ones(size(Xtilde,1),1) Xtilde]*w

  # Return model
  return LinearModel(predict, w)
end
```



3.3.2 Expanding the L-infinity norm gets an objective function of:

$$\text{min}\, w \sum_{i=1}^{n} \max_{j} \{|w_j x_j^i - y^i|\}$$

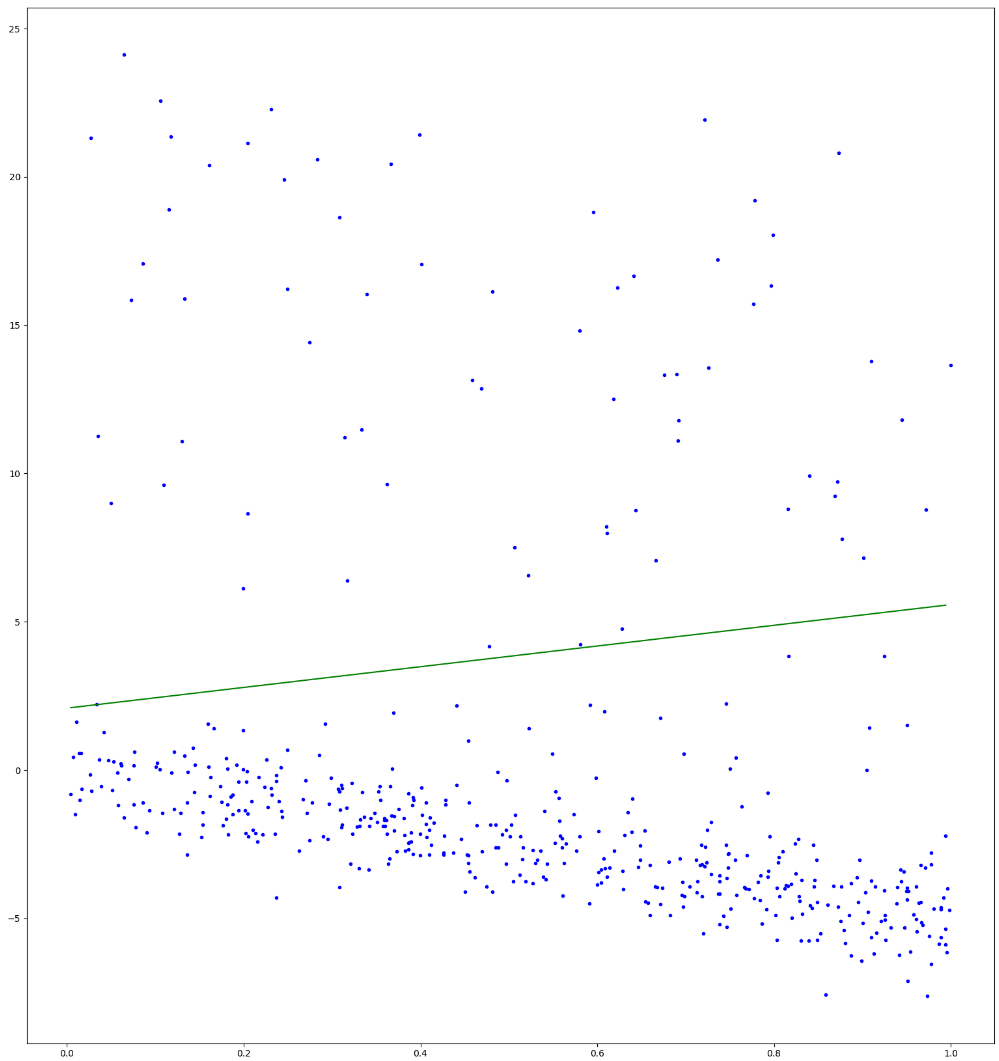
We can replace the max with d constraints and a dummy variable that must be larger than all the possible values of $w_j x_j^i - y^i$.

$$\text{min}\, w, r \sum_{i=1}^{n} r_i, r_i \geq |w_j x_j^i - y^i| \forall i, j$$

$$\min w, r \sum_{i=1}^{n} r_i, r_i \geq w_j x_j^i - y^i, r_i \geq y^i - w_j x_j^i \forall i, j$$

3.3.3 leastMax.jl

```
using MathProgBase, GLPKMathProgInterface

include("misc.jl")

function leastMax(X, y)
  (n, d) = size(X)
  X = [ones(n, 1) X]
  d += 1
  y = y[:,1]
  c = [ones(n); zeros(d)]
  A = zeros(0, d + n)
  b = Float64[]

  for j = 1:d
    colj = zeros(d, d)
    colj[j,j] = 1
    A = [A; eye(n) X*colj; eye(n) -X*colj]
    b = [b; -y; y]
  end

  display(size(y))
  display(size(c))
  display(size(A))
  display(size(b))

  l = [zeros(n); ones(d) * -Inf]

  sol = linprog(c, A, '>', b, l, Inf, GLPKSolverLP())
  w = sol.sol[n+1:n+d]
  if sol.status == :Optimal
      println("Optimal objective value is $(sol.objval)")
      println("Optimal solution vector is: $(w)")
  else
      println("Error: solution status $(sol.status)")
  end
  predict(Xtilde) = [ones(size(Xtilde,1),1) Xtilde]*w

  # Return model
  return LinearModel(predict, w)
end
```


25
20
15
10
5
0
−5
0.0
0.2
0.4
0.6
0.8
1.0