**Sample Midterm 1 for CPSC 221**

Note that some of these questions didn't make it onto your midterm because they are confusing or tricky or hard or easy or just like ones already on it.

1. Let $f(x) = x^2$ be a function from $\{-2, -1, 0, 1, 2\}$ to $\{0, 1, 4\}$. Is $f(x)$ one-to-one? Is $f(x)$ onto? Explain your answers.

2. Express the number of operations performed by the following pseudo-code as a function of its input value $n$ (a positive integer) using Big-Theta notation.

```
for k=1 to 2n {
    for j=1 to k {
        print j * k
    }
}
```

3. Suppose we work at a railyard. Railroad cars numbered $1, 2, \ldots, n$ arrive in that order at the yard and must be sent out in a permuted order $\pi(1), \pi(2), \ldots, \pi(n)$. ($\pi(i)$ is the $i$th car sent out.) The railyard contains a *queue*. At every step, we can either ($E$) Enqueue the next arriving car, ($D$) Dequeue a car to the output, or ($P$) Pass the next arriving car to the output. For example, if four cars $1, 2, 3, 4$ arrive, then the sequence of operations $EEPDPD$ produces the cars in the order $3, 1, 4, 2$ ($\pi(1) = 3$, $\pi(2) = 1$, $\pi(3) = 4$, $\pi(4) = 2$). Any sequence of operations that attempts to dequeue from an empty queue or ends before the queue is empty is an *invalid* sequence. The sequence $EPDDEP$ is invalid because only one car is enqueued before two cars are dequeued.

   (a) Describe a simple rule (that doesn't use a queue) to determine if a sequence is invalid.

   (b) If six cars $1, 2, 3, 4, 5, 6$ arrive, give a sequence of operations that outputs them in the order $3, 1, 5, 6, 2, 4$. Can you give a sequence of operations that outputs them in the order $1, 5, 4, 6, 2, 3$ (yes or no)?

   (c) What property of an output order $\pi(1), \pi(2), \ldots, \pi(n)$ determines if it can be output by the queue railyard? (For example, for the *stack* railyard (that uses a single stack), an output order $\pi$ is achievable if and only if there does not exist $i < j < k$ such that $\pi(j) < \pi(k) < \pi(i)$.)

4. Write a C++ function that inserts a new link (`newlink`) into a doubly-linked list after a given link (`curlink`). The doubly-linked list is *not* circular.

```
struct link { int value; link *prev, *next; };
void insertAfter( link *curlink, link *newlink ) {
```

5. Here is a small function definition:

```
void f(int i, int &k) {
    i = 1;
    k = 2;
}
```

Suppose that a main program has two integer variables `x` and `y`, which are given the value 0. Then the main program calls `f(x,y)`. What are the values of `x` and `y` after the function f finishes?

(a) Both `x` and `y` are still 0

(b) `x` is now 1, but `y` is still 0

(c) `x` is still 0, but `y` is now 2

(d) `x` is now 1, and `y` is now 2

6. What is the time complexity of removing the tail (last element) from a singly-linked list of size $n$ if you are given a pointer to the head and the tail of the linked list?

7. Give Theta-notation for the following function:

$$\frac{5n^2}{n} + 2n \log n - 6n$$

and prove your answer.

8. Use the definition of Big-O and Big-Omega to prove:

If $f(n)$ is $O(g(n))$ then $g(n)$ is $\Omega(f(n))$.

9. What is an example of an input to Mergesort that causes it to perform a worst-case number of operations? How many operations is that (asymptotically)?