Computer Science 221

Sample Solutions to Final Exam Practice Questions - Set 7

1. Degree 0: draw 5 vertices with no edges: . . . . .

   Degree 1: not possible since $\Sigma_{v\in V}$ deg(v) is odd (recall that 2| E | is even)

   Degree 2: draw a pentagon

   Degree 3: not possible since $\Sigma_{v\in V}$ deg(v) is odd

   Degree 4: draw $\mathbf{K_4}$—the complete graph with 4 vertices

   Degree 5: not possible since $\Sigma_{v\in V}$ deg(v) is odd

2. # of name combinations = 3 * 2 * 3 = 18

   # of women = 19

   By the Pigeonhole Principle, k = ceiling(19/18) = 2

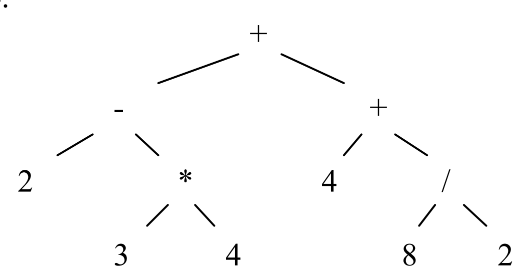
   Therefore, 2 women have the same name.

3. A single entry $c_{i,j}$ in the result requires n multiplications.

   There are $n \times n$ $c_{i,j}$'s in the matrix.

   ⇨ $\Theta(n^3)$ complexity

4.



5.

For $n = 3$, we solve: $x_1 + x_2 + x_3 = 3$ for non-negative integers, less these 3 cases: (3, 0, 0), (0, 3, 0), and (0, 0, 3)—which cannot occur.

The solution to the $n = 3$ case is: $C(3 + 3 - 1, 3 - 1) - 3 = C(5, 2) - 3$.
(same as $C(3 + 3 - 1, 3) - 3 = C(5, 3) - 1$)

... and in general: $C(n + n - 1, n - 1) - n = C(2n - 1, n - 1) - n$.

6. $\Sigma_{v \in V} \deg(v) \;=\; 2\,|E|$

$$3\,|V| \;\leq\; 34$$

$$|V| \;\leq\; 11.3333...$$

Therefore, 11 vertices is the maximum. Note that 12 vertices of degree 3 => 36/2 = 18 edges (at least).

7.

- Try to uniformly disperse the keys using your hash function.
- Try to minimize collisions and clustering—both can cause a search to generate to $O(n)$ from expected $O(1)$
- Make a wise choice about the table size when considering the expected load factor (i.e., table size N not too big, and not too small, for n expected entries)
- Have a good collision resolution policy, and use relative primes (i.e., for table size N, try every r-th location) to cover all cells in a hash table

8. There are two disjoint cases to consider:

a) if the last digit is odd, then there are 2 choices for the last digit, only 4 for the first digit, and we permute the remaining 7 digits

b) if the last digit is even, then there are 2 choices for the last digit, but 5 for the first digit, and we permute the remaining 7 digits

$= (4 * 7! * 2) + (5 * 7! * 2) = 18 * 7!$

9. $C(7, 2) - C(5, 2)$—you can simplify this to 11, if you like