Answers to the midterm:

1. a, b, c
2. b, c, e
3. a, e

(Each of 1, 2, 3 was worth 3 marks. 3 marks if the answer was totally correct,
2 marks if you got one thing wrong, 0 for >= 2 things wrong.)

4. C(98,98) + C(99,98) + C(100, 98) =
   C(98, 0) + C(99, 1) + C(100, 2) =
   1 + C(99, 1) + C(100, 2) = 5050
... any of the above expressions is fine

5a. 9*10*10*10*10 = 90,000

5b. 9*9*8*7*6

5c. 9*10*10*10*2 = 18,000

6. C(15,5)C(10,5)C(5,5) or C(15,5)C(10,5) or 15! / (5! 5! 5!)

7. theta(1/n), theta(lg n), theta(n^(1/3)), theta(sqrt(n)), theta(n)

8a. {4,20}

8b. yes, it's possible, since |C| <= |A|, so all of the elements in C can be "covered"
by 1 or more elements of A

9. 1001 parrots, since 50*20 could fit onto 50 trees without any duplicate colour

10. If g(n) is O(h(n)) then there exists an element c1of the set of real numbers such that g(n) <= c1 h(n)

If f(n) is O(g(n)), then there exists an element c2 of the set of real numbers such that f(n) <= c2 g(n)

Therefore, f(n) <= c2 (c1 h(n))
f(n) <= (c2 c1) h(n)
f(n) <= c3 h(n) where c3 is an element of the set of real numbers

Thus, f(n) is O(h(n)).

11. Note that the counter is being returned at the end of the function. Let's call it k.
We want to know how many times the while loop iterates (this will equal k). So, how does k relate to the sum, n? Well, we break from the loop as soon as the sum of the k's equals or exceeds n.

So, we solve k(k+1)/2 >= n

One way:
k(k+1) >= 2n
(k+1)^2 >= 2n
k >= sqrt(2) sqrt(n) -1

Since we pick the smallest such k to make the formula work, we see that k grows proportional to sqrt(n). So, T(n) = O(sqrt (n))

12. This is the same as solving 3 equations: x1 + x2 + x3 + x4 + x5 = 18 or 19 or 20

C(20 4) + C(19 4) + C(18 4) or C(20 16) + C(19 15) + C(18 14)

13a. Using the version of Quicksort from the (Summer 2004) lectures, and taking 22 as the pivot, we get the following result:

19 12 7 22 33 44 55 50 24

Using Jon Bentley's version of Quicksort, and taking the first element, 24, as the pivot, we get:

22 19 7 12 24 44 55 50 33.

13b. No, for the 1st set of iterations, 24 would have been better since we'd get an even split. (24 is the median.)