CPSC 320 Sample Midterm 2
November 2012

Name: _____ Student ID: _____

Signature: _____

– You have 50 minutes to write the 5 questions on this examination. A total of 40 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[12] 1. Short answers

[3] a. When should you use the "guess and test" method to solve a recurrence relation?

[3] b. The Prim-Jarník minimum spanning tree algorithm maintains a value *Cost(v)* for each vertex v that is still in the heap. What does this value represent?

[3] c. Does amortized analysis help us **design** better algorithms and data structures? Explain why or why not.

[3] d. For each positive integer n, there are graphs G with n vertices and $\Theta(n^2)$ edges such that Kruskal's algorithm will look at **every** edge of G, and hence run in $\Theta(n^2 \log n)$ time. Describe what such a graph might look like.

[8] 2. You have been asked to prove that an arbitrary sequence of n `grow` and `cut` operations on a `Bosquet` data structure runs in $O(n\sqrt{n})$ time. You thus define a potential function Φ, and then work on computing the amortized costs of the `grow` and `cut` operations.

[4] a. The running time of a `grow` operations is $x + 5$, where x is the number of trees involved in the operation (each `Bosquet` consists of a number of trees). Knowing that the amortized cost of a `grow` operation is in $O(1)$, what can you say about the impact of the `grow` operation on the Bosquet's potential?

[4] b. Determine the amortized cost of a `cut` operation as precisely as possible from the information given above.

[5] 3. Describe in words a sequence of $2n - 1$ operations on a Disjoint Sets data structure such that a subsequent `find` operation will run in $\Theta(\log n)$ time, even if path compression is used. Hint: start by calling `makeSet` n times.

[7] 4. Write a recurrence relation that describes the worst-case running time of the following algorithm as a function of n. Note: I do not believe that this algorithm computes anything useful, so don't waste any time trying to understand what it does.

```
Algorithm Elephant(A, first, n)

 if n < 3 then
   return (A[first] + A[first + n - 1]) / 2

 x ← Elephant(A, first + n/3, 2*n/3)

 while (n > 1) do
   n ← n / 2
   x ← x + Elephant(A, first, n) * Elephant(A, first + n, n)
   first ← first + n/2
 endwhile

 return x
```

[8] 5. Prove an upper bound on the function $T(n)$ defined by

$$T(n) = \begin{cases} T(3n/4) + T(n/2) + 3T(n/4) + n^2 & \text{if } n \geq 4 \\ 1 & \text{if } n \leq 3 \end{cases}$$

Your grade will depend on the quality of the bounds you provide (that is, showing that $T(n) \in \Omega(1)$ and $T(n) \in O(100^n)$, while true, will not give you many marks).