The University of British Columbia

Computer Science 221: Discrete Math, Data Structures
Midterm Examination, July 14, 2004. Instructor: E.M. Knorr

Time: 55 minutes.
Closed book. No notes. No calculators (you can leave answers in combinatorial form).

Name_____ Student No_____
(PRINT) (Last) (First)

Signature_____

The examination has 6 pages, and that includes this cover sheet. Check that you have a complete paper.

Print your name and ID at the top of this page, and provide your signature. Have your student ID ready.

In the interest of time, it is sufficient to print your initials at the top of pages 2-6.

This is a closed book exam. No help sheet, and no calculators are permitted. You can leave your answers in combinatorial form.

Work quickly and do the easy questions first. Part marks are available.

The marks for each question are given in braces**. Do not spend too much time on any one question.**

To minimize disruptions during the exam, **please avoid asking the invigilators for help, tips, or explanations.** *Please write down any reasonable assumptions that you are making, if you believe that a question is ambiguous.*

Marks

Question	Max.	Achieved
1-3	9	
4-5	10	
6-9	12	
10-11	9	
12-13	10	
Total	50	

Questions 1-3. {9 marks} Multiple choice questions. Each question is worth 3 marks. **Circle** ALL correct answers because there may be more than 1 correct answer for each question! Part marks are available.

1. Which of the following statements about stacks, queues, and deques are true?
   a) Stacks and queues can be implemented using arrays.
   b) A deque allows a user to retrieve nodes using the LIFO principle, in O(1) time.
   c) A deque allows a user to retrieve nodes using the FIFO principle, in O(1) time.
   d) Suppose we have a circular array holding only integers, and having a capacity (maximum size) of n. If the array is a circular array, then we can store more than n entries in it at any given time.
   e) Using a linked list, it is not possible to keep the nodes sorted.

2. Which of the following statements are true about sorting algorithms?
   a) Quicksort runs in O(lg n) average time, and $O(n^2)$ worst case time.
   b) In the worst case, Mergesort runs in O(n lg n) time.
   c) There are no pivot elements in Mergesort.
   d) Quicksort and Insertion Sort perform equally well (when sorting a list of numbers).
   e) It is not possible to perform a comparison-based sort in less than Ω(n lg n) time.

3. Which of the following statements are general design goals of an *efficient algorithm*?
   a) It must be correct (i.e., it must work).
   b) It can easily be turned into program code by a programmer of average ability.
   c) It should run in less than a minute on today's PC's.
   d) It shouldn't use more than 64K of memory.
   e) It probably shouldn't take $\Theta(k^n)$ steps, where $k > 1$, and n is the size of the input.

4. {5 marks} Let $f(x) = (1+x) + (1+x)^2 + (1+x)^3 + ... + (1+x)^{100}$. What is the coefficient of x^{98} in f(x)?

5. {5 marks} Consider 5-digit integers of the form _ _ _ _ _ where the first digit must *not* be a zero (e.g., 56292 is allowed, but 06292 is not).

a) If digits can be repeated, how many 5-digit integers are possible?

b) If digits cannot be repeated, how many 5-digit integers are possible?

c) If digits can be repeated, and the integer is divisible by 5, how many 5-digit integers are possible?

6. {3 marks} Suppose 3 separate midterms are being prepared for CPSC 221, and that 15 questions will be divided at random into 3 tests: Test A, Test B, and Test C. In how many ways can the questions be divided, assuming that each test gets exactly 5 questions, and that no question appears on more than one test?

7. {2 marks} Put these Big-Theta sets of functions into ascending order: $\Theta(n^{1/2})$, $\Theta(\lg n)$, $\Theta(n^{1/3})$, $\Theta(n)$, $\Theta(1/n)$:

8. {4 marks} Suppose we have the following sets:

$A = \{1, 2, 3, ..., 10\}$
$B = \{ x \mid x \text{ is a positive divisor of } 100\}$
$C = \{ x \mid x \text{ is a multiple of 4 where } 3 \leq x \leq 20 \}$

Compute the actual elements of the following sets:

a) $B \cap C =$

b) Is it possible that function $f:A \rightarrow C$ is an onto function? Why or why not?

9. {3 marks} The body of a parrot (a type of bird) has one of 4 colours: red, green, white, or grey. The head of a parrot is one of 5 colours: purple, red, orange, green, or blue. How many parrots must fly onto 50 trees to ensure than 2 identically coloured birds share the same tree?

10. {4 marks} Use the definition of Big-O to prove:

If f(n) = O(g(n)) and g(n) = O(h(n)), then f(n) = O(h(n))

11. {5 marks} Compute the time complexity of the following C++ function. Justify your answer by showing your work.

```
int transformation(int n)
  {
    int counter, sum;

    counter = 0;
    sum = 0;

    while (sum < n)
     {
       counter++;
       sum = sum + counter;
     }
    return counter;
  }
```

12. {5 marks} How many non-negative integer solutions are there to the following equation?

$18 \leq x_1 + x_2 + x_3 + x_4 + x_5 \leq 20$ such that $x_2 \geq 4$ and all other $x_i \geq 0$

13. {5 marks} (a) Perform Quicksort on the following array of integers. Show the result of the array just before the first recursive call (for the left partition) is made. Use the same method shown in class.

24	55	33	19	7	44	12	50	22

(b) Was the pivot element that you used for the set of iterations in part (a) the best possible choice of pivot (of the 9 numbers in the array)? If yes, just say so; if not, indicate which value would have been better.