CPSC 320 Sample Midterm 1
February 2009

Name: ____________________ Student ID: ____________________
Signature: ____________________

- You have 50 minutes to write the 5 questions on this examination.
  A total of 45 marks are available.
- **Justify all of your answers.**
- You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.
- Use the back of the pages for your rough work.
- **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
Total	

[12] 1. Answer each of the questions with either *true* or *false*. You **must** justify each of your answers; an answer without a justification will be worth at most 1.5 out of 4.

[4] a. If we can use the Master theorem to determine the solution to a recurrence relation, then we can also obtain that solution by drawing the corresponding recursion tree.

[4] b. Let f, g be two functions from $\mathbf{N}$ into $\mathbf{R}^+$. Assuming that $\lim_{n\to\infty} f(n)/g(n)$ exists, we can use its value to determine whether or not f is in $O(g)$.

[4] c. In class, we proved an $\Omega(n \log n)$ lower bound on the worst-case running time of any algorithm that can be used to sort a sequence of n values.

[9] 2. Consider an algorithm `Confusing` whose running time $T(n)$ is described by the recurrence relation $T(n) = 5T(n/8) + g(n)$, with $T(n) \in \Theta(1)$ for $n \leq 7$. Note that $\log_8 5 \approx 0.774$.

[3] (a) Suppose that $g(n) = n$. What is the running time of algorithm `Confusing`? Express your answer using Θ notation (don't forget to justify it).

[3] (b) Suppose that $g(n) = \sqrt{n}$. What is the running time of algorithm `Confusing`? Express your answer using Θ notation (don't forget to justify it).

[3] (c) Describe as precisely as possible the functions $g(n)$ that would make the running time of algorithm `Confusing` be in $\Theta(n^{\log_8 5} \log^2 n)$?

[6] 3. Prove or disprove that $3^{n+2} + 5 \in O(3^{n-1})$

[9] 4. Prove upper and lower bounds on the function $T(n)$ defined by

$$T(n) = \begin{cases} T(n/4) + T(n/9) + \Theta(\sqrt{n}) & \text{if } n \geq 9 \\ \Theta(1) & \text{if } n \leq 8 \end{cases}$$

You may ignore floors and ceilings. Your grade will depend on the quality of the bounds you provide (that is, showing that $T(n) \in \Omega(1)$ and $T(n) \in O(100^n)$, while true, will not give you many marks).

[9] 5. [7] a. Design a divide and conquer algorithm that takes as input an unordered array of elements, and returns the *second largest* element of the array. Hint: your algorithm will actually need to return two values instead of one.

[2] b. Analyze the running time of the algorithm you described in your answer to part (a) by writing a recurrence relation for it, and solving the recurrence using the Master theorem.