CPSC 320 SampleMidterm 2
November 2006

Name: _____ Student ID: _____
Signature: _____

– You have 50 minutes to write the 5 questions on this examination. A total of 45 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

Question	Marks
1	
2	
3	
4	
5	
Total	

– Use the back of the pages for your rough work.

– **Good luck!**

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her library card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

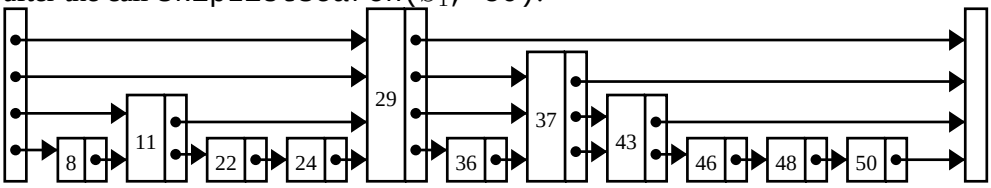
– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[12] 1. Skip Lists

[3] a. Given the skip list S_1 shown below, illustrate the contents of the `Pointer` array after the call `SkipListSearch(`S_1`, 50)`.



[3] b. Two students Cho and Michael are inserting the same elements into the skip list from part (a). Cho is inserting them in increasing order, while Michael puts them in randomly. For some strange reason, every item inserted by Cho ends up in a node with the same level as the node with the same item from Michael's list (so the only difference is the order in which items were inserted).

What differences would you expect to see between the skip lists obtained by Cho and Michael, and why?

[6] c. Explain how to modify algorithm `SkipListInsert` so that a sequence of n insertions at the *end* of the skip list can be performed in $O(n)$ expected time. Hint: you can not afford to call `SkipListSearch` to find where to insert the new node.

[12] 2. Professor Kciwtilf (a rather excentric Computer Science instructor) got bored with teaching the `DeterministicSelect` algorithm the same way every time, and so one term he modified it by choosing the $\lceil 2x/5 \rceil^{th}$ order statistics of the array of x medians as the pivot, instead of the $\lceil x/2 \rceil^{th}$ element as usual.

[5] a. How many elements of the input array are guaranteed to be smaller than the pivot, using professor Kciwtilf's version of the algorithm? Justify your answer!

[4] b. Derive a recurrence relation that describes the worst-case performance of professor Kciwtilf's algorithm.

[3] c. Will professor Kciwtilf's algorithm run in $O(n)$ time? Explain why or why not ***briefly*** (do not give a full proof; a two line justification should be about right).

[4] 3. We proved in class that algorithm `RandomizedQuickSelect` runs in $O(n)$ *expected* time. Why does this make sense, intuitively?

[6] 4. Professor Trahkcol (a colleague of professor Kciwtilf) decides to augment a skip list by associating with each pointer joining a node N_1 to a node N_2 at level i a value $\mathcal{P}(N_1, N_2)$ that depends on the elements of the skip list that are between N_1 and N_2. Suppose moreover that $\mathcal{P}(N_1, N_2)$ can be computed by just looking at the values associated with the level $i-1$ pointers in the interval from N_1 to N_2.

Briefly describe how the additional information (the $\mathcal{P}$ values) can be updated when a node of the skip list is deleted.

[11] 5. A thief breaks into a pharmacy and finds various powders that he wants to place in his backpack. He finds a large book that contains, for each powder i the pharmacy has in stock: (1) the quantity q_i of powder i in stock (measured in grams), and (2) the price p_i of powder i per gram.

[8] a. Knowing that one gram of powder takes exactly 1cm^3 of space, design a greedy algorithm that will allow the thief to walk away with the contents of his backpack (which has volume V) being worth as much as possible.

[3] b. Analyze the worst-case running time of your algorithm from part (a).