CPSC 320 Midterm 2.5

July 20, 2007

Name: _____ Student ID: _____

Signature: _____

- You have 30 minutes on this examination. A total of 4 marks are available.
- *Justify all of your answers.*
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in the square brackets next to the question number is the # of marks allocated for that question. Use these to help determine how much time you should spend on each question.
- Use the back of the pages for your rough work.
- *Good luck!*

Question	Marks	Score
1(a)	1.5	
1(b)	2	
1(c)	0.5	
Total	4	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her library card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: Candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
    1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorized by the examiners.
    2. Speaking or communicating with other candidates.
    3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without the permission of an invigilator.

1. [4] You are given a pile of n pebbles to divide amount k people. How many different ways are there to do this? For example, if there are 2 pebbles and two people, there are three ways to split the pebbles:

   - person A gets 0 pebbles, person B gets 2 pebbles
   - person A gets 1 pebble, person B gets 1 pebble
   - person A gets 2 pebbles, person B gets 0 pebbles

   The following code computes the number of different ways recursively: It assigns i coins to the kth person, and recursively counts the number of different ways to assign $n-i$ coins to the remaining $k-1$ people.

```
Algorithm CountNumWays(n, k)
   if (n = 0) then
      return 1
   if (k = 1) then
      return 1

   return ∑_{i=0}^{n} CountNumWays(n − i, k − 1)
```

   (a) [1.5] Use memoization to make this algorithm efficient. Describe what each cell of the table (cache) represents, and give a modified recursive algorithm that uses the table (fills it in).

Let $S[i,j]$ be the number of ways to split i pebbles among j people. Set it to -1 if it is not yet computed. The dimensions of S are $(0 \ldots n) \times (1 \ldots k)$. Initialize every element of S to -1. Then call the recursive procedure below:

```
Algorithm CountNumWays(n, k)
   if (n = 0) then
      return 1
   if (k = 1) then
      return 1
   if (S[n, k] ≠ −1) then
      return S[n, k]

   S[n, k] ← ∑_{i=0}^{n} CountNumWays(n − i, k − 1)
   return S[n, k]
```

(b) [2] Write an iterative solution to this problem (i.e. do not use recursive calls — use "for" loops instead). What is the asymptotic run-time of this algorithm.

The asymptotic run-time is $O(kn^2)$.

```
for i ← 0 to n do
    S[i, 1] ← 1
for i ← 1 to k do
    S[0, i] ← 1

for k' ← 2 to k do
    for n' ← 1 to n do
        S[n', k'] ← 0
        for i ← 0 to n' do
            S[n', k'] ← S[n', k'] + S[n' − i, k' − 1]
```

(c) [0.5] Write an iterative solution that uses a one dimensional table. **Hint:** The order of the loops matters and `CountNumWays(`n, k`)` only refers to $k - 1$.

```
for n' ← 0 to n do
    S[n'] ← 1

for k' ← 2 to k do
    for n' ← n downto 1 do
        S[n'] ← 0
        for i ← 0 to n' do
            S[n'] ← S[n'] + S[n' − i]
```