CPSC 320 Sample Midterm 2
November 2011

Name: ____________________ Student ID: ____________________
Signature: ____________________

– You have 50 minutes to write the 4 questions on this examination.
A total of 40 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[16] 1. Short Answers

[4] a. Your boss asks you to design a divide-and-conquer algorithm to solve a problem whose input is an array of measurements that contain temperature data. What are the two main issues that you need to determine in order to design this algorithm?

[4] b. In the Disjoint Sets data structure used by Kruskal's minimum spanning tree algorithm, why does the union operation insert the root with the smaller rank as a child of the root with the larger rank, rather than the other way around?

[4] c. Using the method of your choice, give a tight bound on the function $T(n)$ described by the recurrence relation

$$T(n) = \begin{cases} 6T(n/3) + 4n^2 & \text{if } n \geq 13 \\ 17 & \text{if } n \leq 12 \end{cases}$$

[4] d. When should we use amortized analysis?

[6] 2. Write a recurrence relation that describes the worst-case running time of the following algorithm as a function of n. Note: I do not believe that this algorithm computes anything useful, so don't waste any time trying to understand what it does.

```
Algorithm Walrus(A, first, n)

  if (n < 4) then
      return n + 1
  endif

  i ← 0
  sum ← 0
  while (i*i < n) do
      sum ← sum + A[i * i]
      i ← i + 1
  endwhile

  j ← 1
  B ← new array
  while (j < n) do
      append A[j] to B
      j ← 2 * j
  endwhile

  x ← Walrus(A, first + n/4, n/2)
  return x * sum * Walrus(B, 0, length[B])
```

[8] 3. A Computer Science researcher has designed a new data structure called a *mélèze* that supports operations `insert`, `findSmallestGap` and `removeElement`, and decides to use amortized analysis to determine the worst-case running time of a sequence of n operations on an initially empty mélèze. She defines a potential function Φ for mélèzes, such that $\Phi(T) \geq 0$ for every mélèze T, and such that $\Phi(T) = 0$ if the mélèze T is empty. After analyzing the running times of the three operations, she has learned that

- An `insert` operation on a mélèze with n elements takes time $\log n$, and increases the mélèze's potential by $\log n$.
- A `findSmallestGap` operation on a mélèze with n elements takes time $x + \log^2 n$, where x is the number of elements examined by the operation. The potential of the mélèze goes down by $x - 2$.
- A `removeElement` operation takes time $\log n$, and increases the mélèze's potential by 3.

Give as tight a bound as possible on the worst-case running time of a sequence of n operations on an initially empty mélèze.

[10] 4. Using recursion trees, prove tight upper and lower bounds on the function $T(n)$ defined by

$$T(n) = \begin{cases} 4T(n-2) + 6T(n-3) + 3^n & \text{if } n \geq 3 \\ 1 & \text{if } n \leq 2 \end{cases}$$

Your grade will depend on the quality of the bounds you provide (that is, showing that $T(n) \in \Omega(1)$ and $T(n) \in O(100^n)$, while true, will not give you many marks).