CPSC 320 Sample Midterm 1
October 2013

Name: ______________________ Student ID: ______________________

Signature: ______________________

– You have 50 minutes to write the 6 questions on this examination. A total of 40 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
6	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her UBC card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[9] 1. State whether each of the following statements is true or false. Justify each of your answers briefly.

[3] a. You should use decision trees if you want to prove a lower bound on the worst-case running time of an algorithm you wrote to schedule final exams.

[3] b. For any given set of preferences, the Gale-Shapley algorithm always returns the same pairing, whether men or women are the ones proposing. Assume that no one likes two or more members of the opposite gender equally,

[3] c. If the limit $\lim_{n\to\infty} f(n)/g(n)$ is equal to 0, then we can conclude that $f(n) \in O(g(n))$.

[3] 2. Why is the interval scheduling algorithm we discussed in class greedy?

[6] 3. Consider the following preference lists for four women $w_1, \ldots, w_4$ and men $m_1, \ldots, m_4$:

$$\begin{array}{lllll} w_1: & m_4 & m_2 & m_1 & m_3 \\ w_2: & m_2 & m_4 & m_1 & m_3 \\ w_3: & m_1 & m_3 & m_2 & m_4 \\ w_4: & m_2 & m_4 & m_3 & m_1 \end{array} \qquad \begin{array}{lllll} m_1: & w_3 & w_1 & w_2 & w_4 \\ m_2: & w_4 & w_3 & w_2 & w_1 \\ m_3: & w_4 & w_3 & w_1 & w_2 \\ m_4: & w_2 & w_4 & w_1 & w_3 \end{array}$$

Fill in the following table that contains each of the proposals performed by the algorithm, as well as whether the man accepted or rejected the proposal. Assume that whenever several women are not engaged, the woman with the lowest number among those will propose next. You do not need to fill out every row of the table if the algorithm terminates in fewer than 12 steps.

	Woman name		Man name		accepts/rejects
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	
Woman		proposes to man		who	

[6] 4. Consider the two functions f, g: $\mathbf{N} \rightarrow \mathbf{R}^+$ defined by

$$\begin{aligned} f(n) &= n^2 \\ g(n) &= \begin{cases} n & \text{if } n \text{ is even} \\ 5n^2 & \text{if } n \text{ is odd} \end{cases} \end{aligned}$$

Prove that $f \in \Omega(g)$, but that $f \notin \omega(g)$.

[5] 5. Mr. Isulo, a famous alien computer scientist, has designed a greedy algorithm to solve the *Clique* problem (you don't need to know what it is) on a type of graphs called *circular arc graphs* (you don't need to know what they are either). His algorithm starts by choosing the vertex with the most neighbours.

Mr. Isulo wants to prove the following lemma: "Every circular arc graph has a maximum clique that contains the vertex with the most neighbours", but he eventually finds a counter-example to his conjecture. What does this imply for Mr. Isulo's algorithm, and why?

[11] 6. Suppose that you are given a set E with n elements, and a collection $S_1, \ldots, S_m$ of subsets of E. A *Hitting Set* for $S_1, \ldots, S_m$ is a subset E' of E such that every S_i contains at least one element of E'. For instance, if $E = \{a, b, c, d, e\}$, and the subsets of E are $S_1 = \{a, b, d\}$, $S_2 = \{a, c\}$, $S_3 = \{a, d, e\}$, $S_4 = \{b, c, d\}$ and $S_5 = \{c, e\}$ then $\{b, c, e\}$ and $\{a, c\}$ are both hitting sets.

The *Hitting Set* problem consists in finding the smallest subset E' of E that is a hitting set for $S_1, \ldots, S_m$.

[7] a. Describe a greedy algorithm to solve the hitting set problem. Your algorithm does not need to always succeed at minimizing the number of the objects selected, but it should make a good attempt at it.

[4] b. Analyze the time complexity of your algorithm from part (a). Specify only as much of your implementation (for instance, data structures) as needed for your analysis. There is no need to get the most efficient possible implementation of your algorithm, but excessively pessimistic analyses (e.g. $O(n^4)$ when an algorithm could be implemented to run in $O(n \log n)$ time) will not get full marks.