Sample Midterm #1 — Solutions

1. $f(x)$ is not one-to-one since $f(1) = f(-1) = 1$
   $f(x)$ is onto since $f(0) = 0$, $f(1) = 1$, and $f(2) = 4$.

2. let $T(n)$ = #operations as function of n

$$T(n) \le c\left[\sum_{k=1}^{2n}\left(1 + \sum_{j=1}^{k} 1\right) + 1\right]$$

- c: Maximum #operations a single line's execution takes
- 1 (inner): initialize and final test of inner loop
- 1 (innermost): print j*k
- 1 (outer): initialize and final test of outer loop

$$\le c\left[2n + \sum_{k=1}^{2n} k + 1\right] \le c\left[2n + \frac{2n(2n+1)}{2} + 1\right]$$

$$= c\left[2n^2 + 3n + 1\right]$$

$$\le c\, 6n^2 \in O(n^2)$$

$$T(n) \ge \sum_{k=1}^{2n}\sum_{j=1}^{k} 1 \quad (\text{print } j*k) = \sum_{k=1}^{2n} k = \frac{2n(2n+1)}{2} = 2n^2 + n > n^2$$

$$\in \Omega(n^2)$$

3. a) number of E's must be $\ge$ number of D's for every prefix of sequence and number of E's = number of D's in sequence

   b) EEPDEPPDD, no

   c) π is achievable iff there does not exist $i < j < k$ such that $\pi(i) < \pi(k) < \pi(j)$

4.
```
void insertAfter( link *curlink, link *newlink) {
    link *t = curlink -> next;
    curlink -> next = newlink;
    if ( t != NULL)  t -> back = newlink;
    newlink -> back = curlink;
    newlink -> next = t;
}
```

5. c

6. $\Theta(n)$

7. $\frac{5n^2}{n} + 2n\log n - 6n = 2n\log n - n \le 2n\log n \Rightarrow \in O(n\lg n)$ $c=1$, $n_0=1$
   $2n\log n - n \ge n\lg n$ for $n \ge 10$ $\Rightarrow \in \Omega(n\lg n)$ $c=1$, $n_0=10$

8. if $f(n) \in O(g(n))$ then $f(n) \le c \cdot g(n)$ $\forall n \ge n_0$ with $c > 0$, $n_0 \ge 1$
   $\Rightarrow g(n) \ge \frac{1}{c} f(n) \Rightarrow g(n) \in \Omega(f(n))$

9. All inputs to Mergesort of size n cause $\Theta(n\log n)$ operations