Name: _____

Student #: _____

Midterm Exam, CPSC 302, FALL 2007
Oct. 22, 2007

Instructions :

- Make sure this exam has four pages.
- Write down your name and student number in the designated spot at the top of this page.
- Time: 50 minutes.
- The breakdown for each question is specified in square brackets on the right.
- Total three questions, 20 marks: 5 for Q. 1, 5 for Q. 2, 10 for Q. 3.
- **There is choice** for Q. 3: answer only five of the seven questions. If you answer more than five, a random choice will be graded.
- One page of handwritten notes can be used. No other material is allowed.
- If you need extra space you may use the back of a page.
- Show your work, but please avoid unnecessarily lengthy answers.
- G O O D L U C K !!

1. The following identity is given

$$\ln(x - \sqrt{x^2 - 1}) = -\ln(x + \sqrt{x^2 - 1}).$$

(a) State which of the above two mathematically equivalent expressions is more suitable for numerical computations, if x is known to be a large number. Justify your answer. [2 marks]

(b) Suppose that computing x^2 triggers overflow in a given floating point system. Suggest a formula different from the above two that is at least as numerically stable as the formula you selected in part (a) and is more likely to prevent an overflow. [3 marks]

2. The function $f(x) = x \cdot (x-1)^2$ has roots at 0 and at 1.

   (a) Write down the Newton iteration for this problem. [1 mark]

   (b) Suppose Newton's method is applied, once with the initial guess $x_0 = 0.1$ and once starting with $x_0 = 1.1$. You may assume that the iterations converge to $x^* = 0$ and to $x^* = 1$, respectively. State whether you expect convergence to be similar in speed, or whether convergence to one of the roots will be significantly faster. Justify your answer. [3 marks]

   (c) For which of the above two roots can the bisection method be successfully applied? Explain. [1 mark]

3. Answer **only** five (5) of the following seven questions. Determine whether each of the following statements is true or false. There is no need to justify your answer; just write 'T' or 'F' on the margin.

   (a) The rounding unit (which we denote by η) is equal to the smallest positive number in a floating point system. [2 marks]

   (b) If a function $f(x)$ has a zero at x^*, bisection will converge to this root if and only if $f'(x^*) \neq 0$. (You may assume that the initial bracketed interval is 'valid'.) [2 marks]

   (c) Ignoring roundoff errors, if during the kth step of Gaussian elimination a row of zeros is generated in an interim matrix $A^{(k)}$, then the original matrix A is necessarily singular. [2 marks]

   (d) If during Gaussian elimination without pivoting a zero pivot is encountered, then the matrix is not necessarily singular. [2 marks]

   (e) If the determinant of a matrix is close to zero then it must have a large condition number. [2 marks]

   (f) Given a linear system $A\mathbf{x} = \mathbf{b}$, the overall computational work and storage required for computing the solution using Gaussian elimination (i.e. forming the LU decomposition) is equivalent (in big O terms) to computing the inverse A^{-1} explicitly and then computing $A^{-1}\mathbf{b}$ to obtain the solution $\mathbf{x}$. [2 marks]

   (g) Let A and T be two nonsingular, $n \times n$, fully dense matrices. Furthermore, suppose we are given two matrices L and U such that L is unit lower triangular, U is upper triangular, and $TA = LU$. Then solving $A\mathbf{x} = \mathbf{b}$ for any given vector $\mathbf{b}$ can be done in $O(n^2)$ complexity. [2 marks]