

CPSC 320 Sample Midterm 1
October 2006

Name: _____ Student ID: _____
Signature: _____

- You have 50 minutes to write the 4 questions on this examination.
A total of 45 marks are available.

- **Justify all of your answers.**

- You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

Question	Marks
1	
2	
3	
4	
Total	

- Use the back of the pages for your rough work.

- **Good luck!**

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her library card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
 2. Speaking or communicating with other candidates.
 3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[12] 1. Short Answers

[3] a. We proved in class that algorithm `RandomizedQuickSelect` runs in $O(n)$ *expected* time. Why does this make sense, intuitively?

[3] b. In class, did we prove an $\Omega(n \log n)$ lower bound on the running time of every sorting algorithm? Explain why or why not.

[3] c. Can we use the limit theorem discussed in class to determine that the function $f(n)$ defined by

$$f(n) = \begin{cases} \sqrt{n} & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd} \end{cases}$$

is in $O(n)$? Explain why or why not.

[3] d. A computer scientist showed how to build a decision tree for a specific algorithm \mathcal{A} that decides whether or not a list contains two elements that have the same value. What does the height of this decision tree represent?

[12] 2. Prove or disprove each of the following two facts:

[6] a. $n^3 2^n \in O(3^n)$

[6] b. Let f , g , h and k be functions from \mathbf{N} into \mathbf{R}^+ . If $f(n) \in \Omega(g(n))$ and $h(n) \in \Omega(k(n))$, then $f(n)h(n) \in \Omega(g(n)k(n))$.

[11] 3. Consider the recurrence relation

$$T(n) = \begin{cases} 4T(n-1) + 4^n & \text{if } n \geq 2 \\ 1 & \text{if } n = 1 \end{cases}$$

[3] a. Explain briefly why we can not use the Master theorem directly to solve this recurrence relation.

[8] b. Prove upper and lower bounds on the function $T(n)$. Your grade will depend on the quality of the bounds you provide (that is, showing that $T(n) \in \Omega(1)$ and $T(n) \in O(100^n)$, while true, will not give you many marks).

[10] 4. In this question, we consider the problem of computing the smallest gap between two elements in an array of real numbers. That is, we want to find the smallest difference between two distinct elements of the array. For instance, for the array $(3.8, 8.1, 1, 6.2, 2.5)$, the smallest gap is 1.3 because $3.8 - 2.5 = 1.3$, and no other pair of elements has a difference smaller than 1.3.

[7] a. Design a divide and conquer algorithm to compute the smallest gap in the array that is passed as parameter. Your grade will depend on both the elegance and the efficiency of your algorithm.

[3] b. Analyze the running time of the algorithm you described in your answer to part (a).