CPSC 121 Midterm 1
Thursday, February 3rd, 2011

[1] 1. Do you want your tutorial attendance to count toward your grade? If you answer 'yes', then 1% of your final course mark will be based on your tutorial attendance (# attended divided by total number minus 2, but no more than full credit). Online quizzes would be worth only 5% rather than 6%. If you answer 'no', then tutorial attendance is worth nothing in your mark, and online quizzes remain a full 6% of your final course mark.

[6] 2. Translate the following statements from English into propositional logic. Give an appropriate definition for any variables you use. No variable is allowed to correspond to a proposition that contains logical operators (for instance, for part (a), you can not use "r: Either I wear my wedding ring on my left hand or I wear it on my right hand"). You may circle portions of the printed statement and write the variable name beside the circle to reduce writing time.

[2] a. Either I wear my wedding ring on my left hand or I wear it on my right hand

> **Solution :** We define
>
> l: I wear my wedding ring on my left hand.
> r: I wear my wedding ring on my right hand.
>
> and then we can write $l \oplus r$.

[2] b. Naomi did not like the movie, but she really liked the popcorn.

> **Solution :** We define
>
> m: Naomi liked the movie.
> p: Naomi liked the popcorn.
>
> and then we can write $\sim m \wedge p$.

[2] c. If a voting system is Pareto efficient and exhibits Independence of Irrelevant Alternatives, then it must be a dictatorship.

> **Solution :** We define
>
> p: A voting system is Pareto efficient.
> i: A voting system exhibits Independence or Irrelevant Alternatives.
> d: A voting system is a dictatorship.
>
> and then we can write $(p \wedge i) \rightarrow d$.

[6] 3. Translate the following statements from propositional logic back into English. Use the following definitions:

w: Werewolves are better than ghosts.   g: Ghosts are better than werewolves.
s: Steve is a werewolf.   p: Patrice is a ghost.
r: Werewolves are real.

[2] a. $w \oplus g$

Solution : Either werewolves are better than ghosts, or ghosts are better than were-wolves, but not both.

[2] b. $s \to (r \land w)$

Solution : If Steve is a werewolf, then werewolves are real and are better than ghosts.

[2] c. $p \leftrightarrow g$

Solution : Patrice is a ghost if and only if they are better than werewolves.

[7] 4. Determine what the following four premises:

$$w \oplus g$$
$$s \to (r \land w)$$
$$p \leftrightarrow g$$
$$w$$

indicate about the truth of p (whether Patrice is a ghost!). That is, prove p, prove $\sim p$, or show that both are consistent with the premises. You may skip steps involving only the commutative, associative, and double negation laws, but you should show all other steps. You need not write the names of the rules you use.

Note: you may use any of the following logical equivalences for exclusive OR and the biconditional, but no others. If you would like to refer to them, use the names E1 to E4.

E1 : $x \oplus y \equiv (x \land \sim y) \lor (\sim x \land y)$ **E3 :** $x \leftrightarrow y \equiv (x \to y) \land (y \to x)$
E2 : $x \oplus y \equiv (x \lor y) \land \sim (x \land y)$ **E4 :** $x \leftrightarrow y \equiv (x \land y) \lor (\sim x \land \sim y)$

Solution : First let us number the premises:

1. $w \oplus g$
2. $s \to (r \land w)$
3. $p \leftrightarrow g$
4. w

And now we can prove the following:

5. $(w \lor g) \land \sim (w \land g)$ E2 from (1).
6. $\sim (w \land g)$ simplification from (5).
7. $\sim w \lor \sim g$ DeMorgan's law from (6).
8. $w \to \sim g$ Definition of $\to$ from (7).
9. $\sim g$ Modus ponens from (4) and (8).
10. $(p \to g) \land (g \to p)$ E3 from (1).
11. $p \to g$ Simplification from (10).
12. $\sim p$ Modus tollens from (9) and (11).
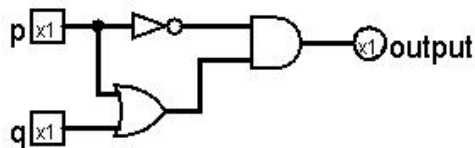
Hence Patrice is not a ghost[1].

[9] 5. Translate the following three circuits into propositional logic statements. (Do not simplify the propositional logic statements; leave them in the same form as the circuits.)
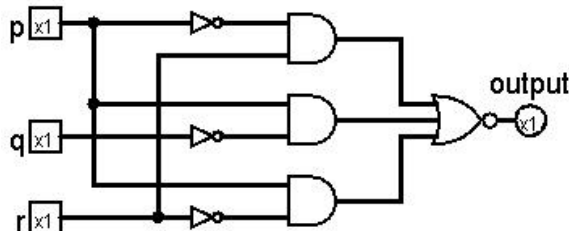
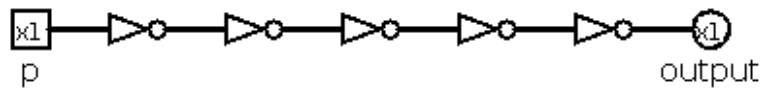[2] a.
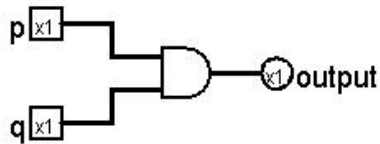
Solution :   output $\equiv p \wedge q$.

[3] b.

Solution :   output $\equiv {\sim}p \, \wedge \, (p \vee q)$.

[4] c.

Solution :   output $\equiv {\sim}(({\sim}p \wedge r) \vee (p \wedge {\sim}q) \vee (p \wedge {\sim}r))$.

[6] 6. Consider the following circuit:

[1] a. Write the logical expression that corresponds to this circuit. (Do not simplify the propositional logic statement; leave it in the same form as the circuit.)
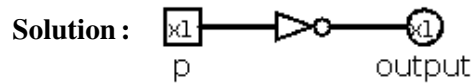
Solution :   output $\equiv {\sim}{\sim}{\sim}{\sim}{\sim}p$.

[1] b. Now simplify this logical expression as much as possible.

Solution :   output $\equiv {\sim}p$.

[1] c. Draw the circuit corresponding to the simpler expression.

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[1] To his wife's relief

Solution :

[3] d. What does our model of combinational circuits (i.e., propositional logic) fail to account for that causes these two circuits to behave differently?

Solution : Propositional logic does not explain the fact that the output of the original circuit will take longer to react to a change in the input then the output of the simplified circuit, because it does not model gate delays.

[6] 7. Consider the following predicates:

V(x,y): person x voted for person y during the last elections.
E(x,y): person x likes to eat food y.

where the domain for people and foods are denoted by P and F respectively. Translate each of the following quantified statements into English:

[2] a. $\forall x \in P, V(x, \text{"Stephen Harper"}) \rightarrow E(x, \text{"Carrots"})$

Solution : "Every person who voted for Stephen Harper likes carrots."

[2] b. $\exists x \in F, \forall y \in P, V(y, \text{"Michael Ignatieff"}) \rightarrow \sim E(y, x)$

Solution : "There is a food that everyone who voted for Michael Ignatieff does not like."

[2] c. $\forall x \in P, \exists y \in F, E(x, y) \wedge \sim E(\text{"Jack Layton"}, y)$

Solution : "Every person likes some food that Jack Layton does not like." Note that this proposition is false if we assume that Jack Layton is a person, since if it were true, there would be a food that Jack Layton both likes and dislikes. However that does not make our translation incorrect.

[6] 8. Prove that $\sim q \vee \sim (p \vee r) \equiv \sim (p \wedge q) \wedge (q \rightarrow \sim r)$ using logical equivalences (NOT truth tables). You may skip steps involving only the commutative, associative, and double negation laws, but you should show all other steps. You need not write the names of the rules you use.

Solution :

$$
\begin{aligned}
\sim(p \wedge q) \wedge (q \rightarrow \sim r) \ &\equiv\ (\sim p \vee \sim q) \wedge (q \rightarrow \sim r) && \text{by the DeMorgan's law.} \\
&\equiv\ (\sim p \vee \sim q) \wedge (\sim q \vee \sim r) && \text{by the definition of } \rightarrow. \\
&\equiv\ \sim q \vee (\sim p \vee \sim r) && \text{by the commutative and distributive laws.} \\
&\equiv\ \sim q \sim (p \vee r) && \text{by the DeMorgan's law.}
\end{aligned}
$$

[12] 9. This question considers the problem of determining whether an unsigned binary integer x is divisible by another unsigned binary integer y (recall that x is divisible by y if x/y is an integer). Assume for this question that no integer is divisible by 0, and that 0 is divisible by every other integer.

[4] a. Build a circuit that takes a 3-bit unsigned binary number x and determines whether it is divisible by 2. You may use inverters (NOT gates), and AND, OR, NAND, and NOR gates with any number of inputs.

Solution :
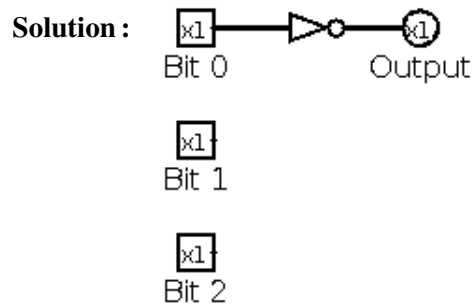
[4] b. Build a circuit that takes a 3-bit unsigned binary number x and determines whether it is divisible by 3. You may use inverters (NOT gates), and AND, OR, NAND, and NOR gates with any number of inputs.

Solution :
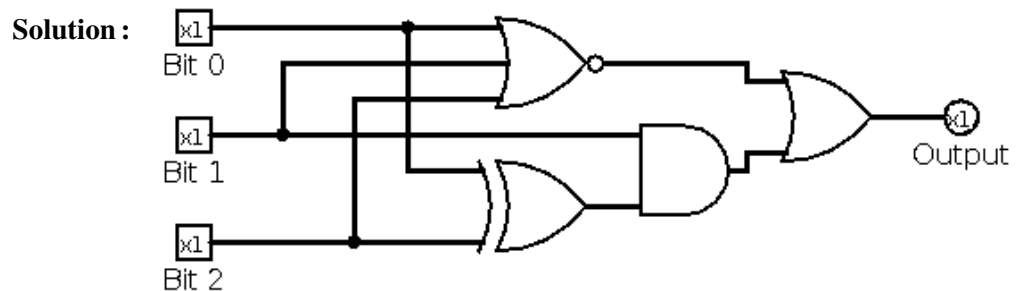
[4] c. Complete the following circuit that takes a 3-bit unsigned binary number x and a 2-bit unsigned binary number y and determines whether x is divisible by y. Assume the "divisible by 2" and "divisible by 3" subcircuits work (correctly) as described in the previous two problems. If you use one of these, represent it by a rectangle containing the label db2 or db3.

(Reminder: assume for this question that no integer is divisible by 0, and that 0 is divisible by every other integer.)

Solution :
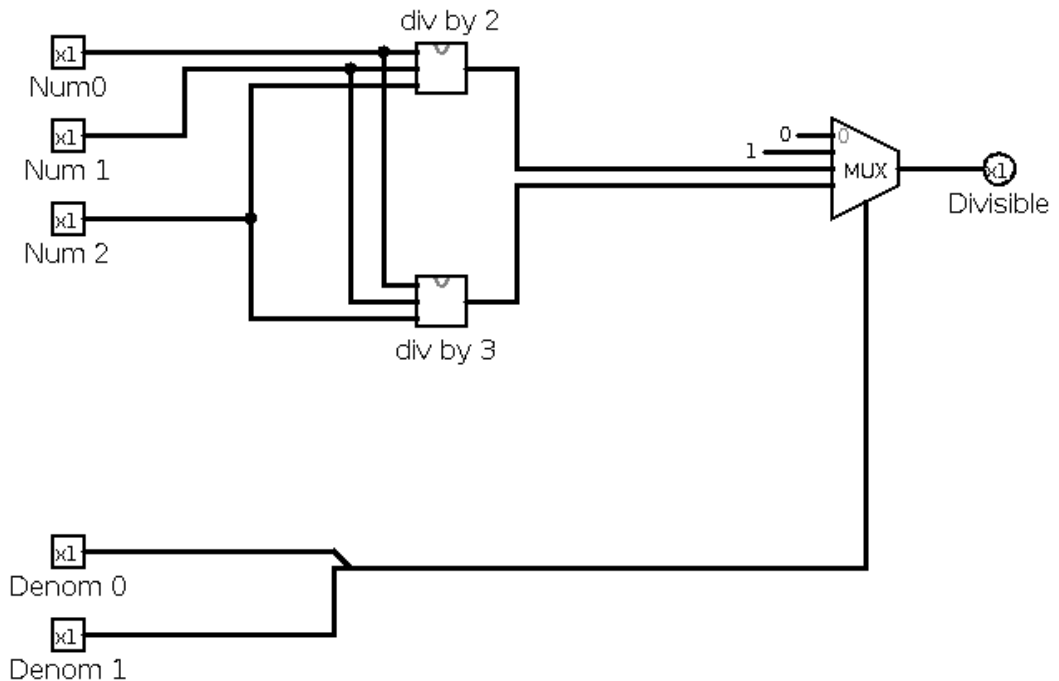
[6] 10. The human ear can detect sounds that range from frequencies of about 20 Hz to about 20000 Hz. However, it takes larger absolute differences in frequency toward the higher frequency values for us to differentiate between sounds. For example, the difference between 40 Hz and 80 Hz is MUCH clearer than the difference between 10000 Hz and 10040 Hz, even though both are 40 Hz apart.

A friend proposes to represent the frequency of a sound (which is intended to be used for playing that sound to a human) as a 12-bit signed binary number.

Explain at least two different important problems with this representation choice.

Solution: First, there was no reason to use a signed integer, since frequencies can not be negative. Using an unsigned binary number thus doubles the number of frequencies that can be represented.

Second, because we have more difficulty differentiating higher frequencies, we should use a non-linear scale, where the gap between two consecutive binary integers gets bigger as the values of the integers grow. For instance, the integers 0 to 100 might represent frequencies 0 to 100, the integers 101 to 200 might represent frequencies 102 to 300, etc.