CPSC 320 Sample Midterm 2
March 2014

Name: _____   Student ID: _____

Signature: _____

- You have 50 minutes to write the 5 questions on this examination.
  A total of 40 marks are available.

- **Justify all of your answers.**

- You are allowed to bring in one hand-written, double-sided 8.5 x
  11in sheet of notes, and nothing else.

- Keep your answers short. If you run out of space for a question,
  you have written too much.

- The number in square brackets to the left of the question number
  indicates the number of marks allocated for that question. Use
  these to help you determine how much time you should spend on
  each question.

- Use the back of the pages for your rough work.

- **Good luck!**

Question	Marks
1	
2	
3	
4	
5	
Total	

[10] 1. State whether each of the following statements is true or false. Justify each of your answers briefly.

[3] a. If a recurrence relation can be solved using the Master theorem, then it can also be solved using recursion trees.

[3] b. When we create a new node in a skip list, we choose the level of the new node uniformly and at random between 1 and the maximum level used for the skip list (that is, it has equal probabilities of having height 1, 2, 3, etc).

[4] c. Explain the best way to find a tight upper bound on the solution of the recurrence relation

$$T(n) = \begin{cases} 4T(n/2 + 3) + 5n^2 \log n & \text{if } n \geq 8 \\ \Theta(1) & \text{if } n < 7 \end{cases}$$

and state what you believe the upper bound to be (you do not need to prove your upper bound).

[5] 2. Randomized Caching

    [2] a. How does the randomized caching algorithm discussed in class choose an item to evict when a cache miss occurs?

    [3] b. Why does the probability of a cache miss for the ith request for a recurring unmarked item increase with i?

[7] 3. Describe an efficient divide and conquer algorithm that takes as input a sorted array A of real numbers, and returns a binary search tree of minimum height that contains the elements of the array. If you want, you can add parameters other than the array A to your function.

[8] 4. Prove an upper bound on the function $T(n)$ defined by

$$T(n) = \begin{cases} 2T(n/2) + T(2n/3) + 2T(n/6) + n^2 & \text{if } n \geq 6 \\ 1 & \text{if } n \leq 5 \end{cases}$$

Your grade will depend on the quality of the bound you provide (that is, showing that $T(n) \in O(100^n)$, while true, will not give you many marks).

[10] 5. You are asked to design a data structure that supports the following two operations on a set S:

- `Insert`$(S, \ x)$: adds the element x to S.
- `Delete-Larger-Half`(S): deletes the largest $\lceil |S|/2 \rceil$ elements of S.

After pondering the problem for a few hours, you decide to store the elements of S in an `ArrayList`, since it supports $\Theta(1)$ insert operations.

Note that parts (a) and (b) are independent, and you can do them in either order.

[4] a. Explain how to implement `Delete-Larger-Half`(S) in $\Theta(|S|)$ worst-case time. Hint: your first step should be to use an algorithm from one of the assignments.

[6] b. Prove that the worst-case running time of a sequence of n operations, starting with an empty set, is in $O(n)$. Hint: use $\Phi(S_i) = u|S_i|$ for some constant u (that you can choose later on).