Computer Science 221

Sample Solutions to: Practice Questions - Set 5

1. a) Use one stack (say Stack 1) to contain the data, and the other stack is a work stack (Stack 2). To enqueue: just push onto Stack 1 (this is O(1)). To dequeue: pop all elements from Stack 1 (this is O(n)) and push all except the last one (the one to be dequeued) onto Stack 2 (this is O(n)), and finally pop all from Stack 2 (this is O(n)) and push them onto Stack 1 (again, O(n)). Overall, this is O(n).

b) To reverse a queue: rename Stack 1 to Stack 2. Then, pop all from Stack 2 and push them all onto Stack 1. For example:

Stack 1: 1, 2, 3
Stack 2:

becomes ...
Stack 2: 1, 2, 3
Stack 1:

becomes ...
Stack 1: 3, 2, 1
Stack 2

This is done in O(n) steps.

2. Divide the rectangle into 4 rectangles of size 3 cm by 4 cm. Recall that we have 5 points. There are 4 "quadrants" in the big rectangle. By the pigeonhole principle, one of the 3 cm by 4 cm rectangles must contain 2 points. The 2 points within this rectangle are within (hypotenuse) 5 cm of each other.

3.

a) $\Theta(n^2)$ for Insertion Sort
b) Input:

01010101...	There are $n/2$ such "01" pairs.
or	
111...1 00...0	Each of these 2 strings contains $n/2$ entries.

c) $\Theta(n^2)$ for QuickSort
d) Input:

000...	There are n such entries.
or	
111...	There are n such entries.

4. Suppose that it is *not* possible that a box has at least 12 balls, but that we do have 100 balls. Then, we have 9 boxes * 11 balls/box (maximum). This is 99 balls in all. But this contradicts the statement about there being 100 balls. Therefore, at least one box must have at least 12 balls.

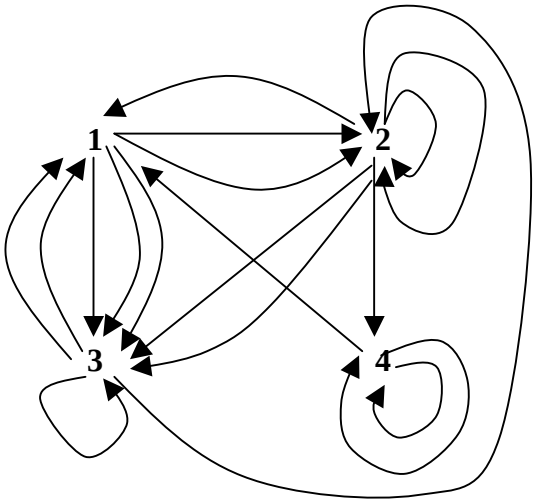
5. For the maximum height, choose either end of the longest path as the root. For the minimum height, choose the vertex at the half-way point of the path.

6. i) paths?
   a) yes
   b) yes
   c) no (there's no d to b edge)
   d) no

   ii) simple paths?
   a) no (b repeats)
   b) no
   c) no
   d) no

   iii) cycles?
   a) no
   b) yes
   c) no
   d) no

7.



8.