Computer Science 221

More Sample Final Exam Questions - Set 7

Here are some more sample questions. These are all taken from previous final exams. Other sample exam questions, including programming questions and midterms, are found in the same section where you found this document on WebCT. Some of the questions on WebCT about graph theory (in particular) vary from term to term; you can omit material that we have not covered. Check the current term's lecture slides and assignments for the currently covered materials. Your textbooks, and the current assignments and tutorials, have more sample questions.

1. Consider an undirected graph *G* with no parallel edges, and with 5 vertices in all. Suppose that all vertices have to have the same degree *d*. Draw an example of the kind of graph that is possible *for each degree d* ≤ 5. In other words, draw 6 graphs: one where all the vertices are of degree 0, one where all the vertices are of degree 1, and so on. If, for some reason, it is not possible to draw the graph, then briefly state why.

2. Consider a group of 19 women. Among them, there are 3 different first names (Linda, Karen, & Angela), 2 different middle names (Amy and Elizabeth), and 3 different last names: Bishop, Lee, and Smith. Show that among these women, there must be 2 women who have the same first, middle, and last names.

3. In matrix multiplication, the product of two $n \times n$ matrices A and B is the one $n \times n$ matrix C whose *i,j*-th entry is computed as follows:

$$c_{i,j} = \Sigma_{k=1 \text{ to n}} \; a_{i,k} * b_{k,j}$$

If we wrote an algorithm that multiplied matrices in this way, how many scalar multiplications (i.e., " * " operations) would the algorithm perform when multiplying one $n \times n$ matrix by another $n \times n$ matrix? Use Big-Theta (Θ) notation. (You do not need to find witnesses; it suffices to give us a Big-Theta expression, or some expression giving the exact number of scalar multiplications.)

4. Given the following postfix visitation/traversal of a math expression, draw a valid expression tree for it (similar to the examples from class): 2 3 4 * - 4 8 2 / + +

5. In ancient Athens, once a year, the citizens would gather to decide whether anyone was such a threat to the state that they should be exiled (sent away). Each citizen

would vote by writing another citizen's name on a piece of broken pottery. Assume that no citizen writes his or her own name on the piece of pottery, and that no two citizens have the same name.

If n is the number of citizens of Athens, how many different vote totals can there be? For example, if n=3, then (0,1,2), (0,2,1), (1,0,2), (1,1,1), (1,2,0), (2,0,1), and (2,1,0) are the seven possible vote totals—where (a,b,c) means citizen A received a votes, B received b votes, and C received c votes.

6. Suppose G is a connected graph with 17 edges. If the degree of each vertex is at least 3, what is the maximum possible value for the number of vertices in G?

7. How do we go about picking a good hash function and hash structure? Provide some general guidelines about what a hash designer needs to think about.

8. How many permutations are there of *all* the digits 1,2, ..., 9 in which the first digit is an odd digit and the last digit is one of 1,2,3, or 4? For example: 521376984 is OK, but 521376948 is not.

9. How many *more* edges are there in the complete graph $\mathbf{K_7}$ than in the complete graph $\mathbf{K_5}$?