# CPSC 414
# Final Exam

December 13, 2002

> Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: _____

Student Number: _____

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This exam has 8 questions, for a total of 60 points.

1. Geometric Transformations

    (a) (2 points) Sketch the transformed house that is drawn by the following code. The house already drawn in the figure shows its untransformed state.

```
glLoadIdentity();
glRotatef(90,0,0,1)
glTranslatef(-2,0,0);
DrawHouse();
```
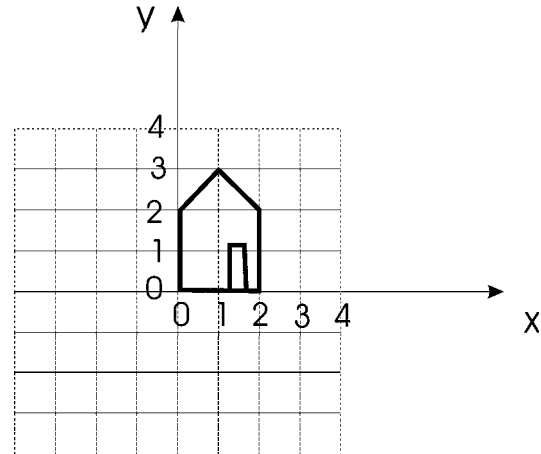
    (b) (2 points) Suppose we wanted to accomplish the same final transformation as above using the sequence of transformations given below. Provide the necessary values of $x$, $y$, and $\theta$.

```
glLoadIdentity();
glTranslatef(x,y,0);
glRotatef(θ,0,0,1)
DrawHouse();
```

    (c) (2 points) Sketch the transformed houses that are drawn by the following code. The house already drawn in the figure shows its untransformed state.
    Label the two houses that are drawn using the labels "A" and "B".

```
glLoadIdentity();
glPushMatrix();
glTranslatef(2,0,0);
glPushMatrix();
glRotatef(-45,0,0,1);
DrawHouse(); // house A
glTranslatef(0,1,0);
glPopMatrix();
glScalef(-1,1,1);
glTranslatef(-3,1,0);
DrawHouse(); // house B
glPopMatrix();
```
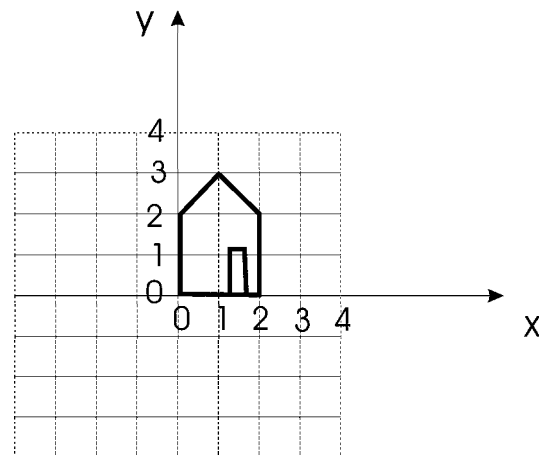
(d) (3 points) The following figure shows a house in its untransformed state, $A$, and its transformed state, $B$. Draw $i$ and $j$, the basis vectors for the $x$ and $y$ axes, before and after the transformation. Use these to help determine the $3 \times 3$ transformation matrix $M$ that produces the transformation from $A$ to $B$.
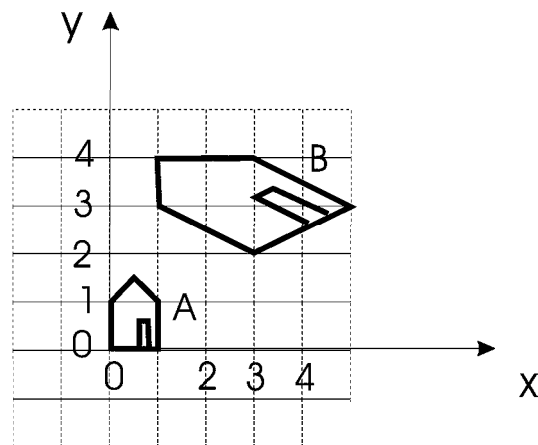
2. Colour Representation

    (a) (1 point) Suppose a monitor has RGB phosphors with the CIE coordinates given in the figure below. Sketch the monitor gamut.
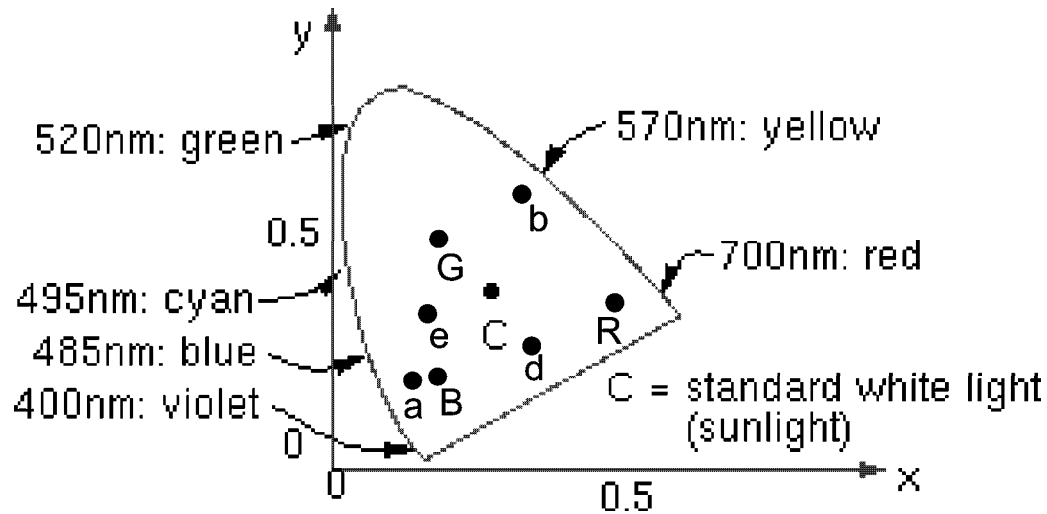


    (b) (2 points) Suppose you wished to build a 4-phosphour monitor and you had a number of types of phosphours to choose from. If you had to select one of $a, b, d, e$ (refer to the figure above) to be used as a fourth phosphour in addition to $RGB$, which would it be? Why?

    (c) (1 point) What would be the disadvantages of adding a fourth phosphor to a monitor?

    (d) (1 point) Suppose that when you power on a computer its monitor displays predominantly variations of black and magenta. You suspect that one of the three wires (RGB) in your monitor cable is no longer properly connected. Which one is it? Show your reasoning.

    (e) (2 points) What are rods and cones? How are they used to perceive colour?

3. Clipping and Culling

  (a) (2 points) Sketch a side view ($yz$-plane) of the perspective view frustum, in VCS, that is specified by the following parameters:

$$\begin{array}{lll} \text{near} = 2 & \text{top} = 1 & \text{right} = 1 \\ \text{far} = 6 & \text{bot} = \text{-1} & \text{left} = \text{-1} \end{array}$$

  (b) (2 points) Give the general form for an implicit plane equation. Give a parametric line equation that passes through two points, $P_1$ and $P_2$.

  (c) (1 point) For the view frustum given earlier, determine the implicit plane equation for the top plane, such that $F(P) > 0$ for points that are inside the view volume.

  (d) (3 points) Determine if the line $P_1(0, 4, -2)P_2(1, 1, -6)$ requires clipping for the view volume given in part (a). If it does require clipping, compute the the vertices of the clipped line segment.

  (e) (2 points) A 2D triangle is clipped to a 2D rectangular viewport. What is the maximum number of vertices that the resulting polygon can have? Sketch an example.

  (f) (2 points) Sketch a polygon having a VCS normal $N(0, -1, -0.1)$ in the view volume of part (a). Should such a polygon always be culled if we apply back-face culling? Why or why not?
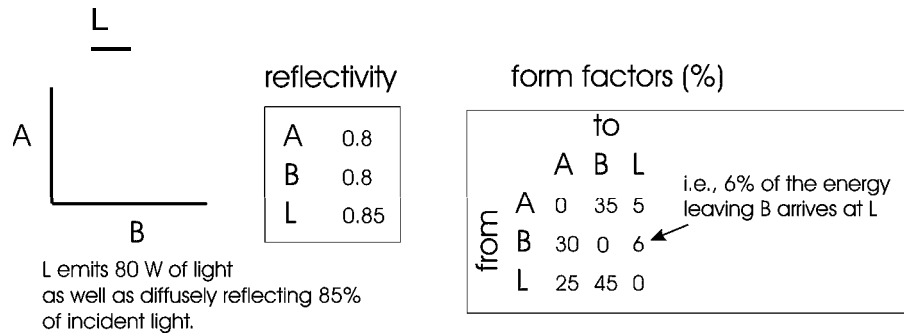
4. Visibility

(a) (3 points) Build a BSP tree for the following scene. Use the polygons as cutting planes and insert them into the tree in alphabetical order. Represent the '+' side of each cutting plane as the right child, and the '-' side as the left child.
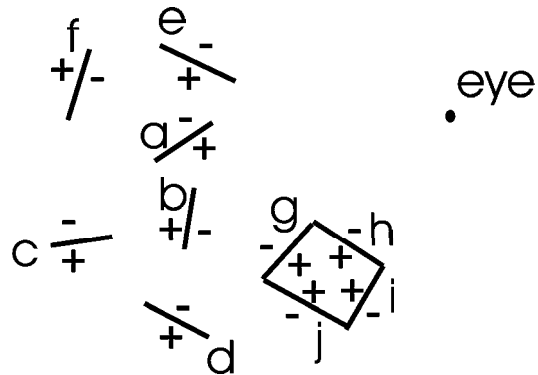


(b) (3 points) Use your BSP tree to produce a back-to-front ordering from the given eye point. Show your work.

5. Global Illumination

Consider the following scene:

L

reflectivity

| A | 0.8 |
| B | 0.8 |
| L | 0.85 |

A

B

L emits 80 W of light
as well as diffusely reflecting 85%
of incident light.

form factors (%)

|  | to | | |
|---|---|---|---|
| | A | B | L |
| A | 0 | 35 | 5 |
| B | 30 | 0 | 6 |
| L | 25 | 45 | 0 |

from

i.e., 6% of the energy
leaving B arrives at L

(a) (3 points) Write the set of linear equations that would need to be solved in order to determine the amount of power (radiant flux) leaving each surface. Use the notation $e_A$, $e_B$, and $e_L$ to represent the unknowns, i.e., the power or radiant flux that leaves surfaces $A$, $B$, and $L$, respectively. It is not necessary to solve these equations.

(b) (1 point) Suppose that surface $B$ receives a total of $b$ watts of incident flux (i.e., light arriving at $B$) from all the other surfaces. Give an expression for the amount of power that ends up being radiated from $B$ into the environment, i.e., power that does not end up arriving at the other surfaces.

(c) (3 points) Suppose that we want a solution that also includes colour and that we model colour in terms of an RGB triple. Describe how each of the following would change, if at all:
(i) the reflectivities
(ii) the form factors
(iii) the system(s) of equations to be solved

6. Local Illumination

(a) (6 points) Sketch the illumination that would be computed for the following scene using the Phong illumination model. The scene is lit from above using a directional light source that is coming directly from above. Use 4 sketches, one for each of ambient, diffuse, specular, and total illumination.

The Phong illumination model is given by:
$$I = k_a I_a + k_d I_d (N \cdot L) + k_s I_s (R \cdot V)^n$$
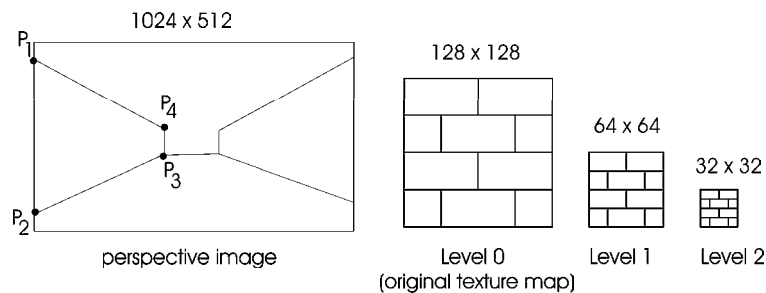and the values of the various parameters are:
$$I_a = I_d = I_s = 1.0, k_a = 0.2, k_d = 0.8, k_s = 0.7, n = 100.$$
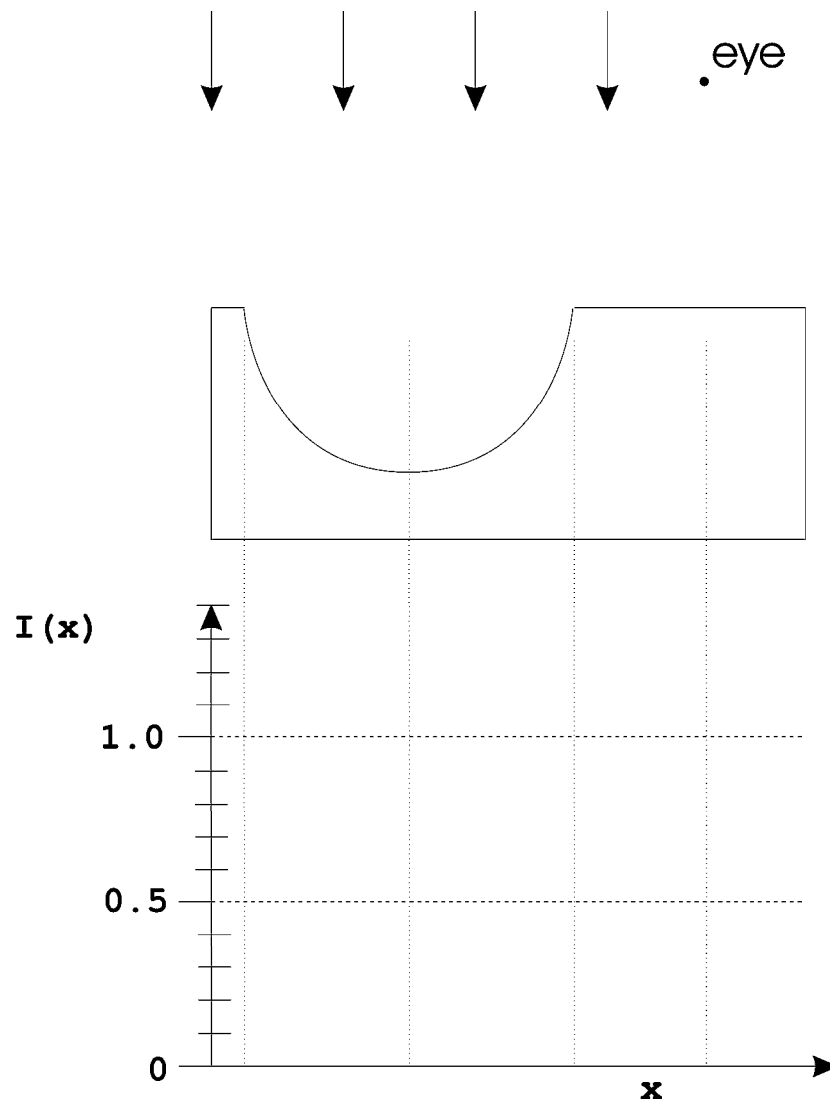
(b) (2 points) Where are local illumination models evaluated in the projective rendering graphics pipeline? Why?

7. Texture Mapping

Suppose we have a brick wall that forms the left-hand wall of a corridor in a maze game, as shown in the image below, and it is defined (in world coordinates) by points $P_1 P_2 P_3 P_4$. Assume that the brick wall is to be 16 bricks high and 200 bricks long.



1024 x 512

perspective image

128 x 128

Level 0
(original texture map)

64 x 64

Level 1

32 x 32

Level 2

(a) (2 points) Using the height of the brick wall as seen in the image, estimate how many texels a screen pixel covers, both for near points on the wall, i.e., on the edge $P_1 P_2$, and at distant points on the wall, i.e., on the edge $P_3 P_4$.

(b) (1 point) On the perspective image, sketch approximately what regions of the wall will use each of the levels of the MIP-map image pyramid on the right.

(c) (2 points) Provide the OpenGL calls that would be used to generate the texture-mapped wall. You can assume that the correct texture map is already loaded and enabled.

8. Ray Tracing

    (a) (3 points) For the following scene, sketch all the ray paths and shadow rays that would be generated by a raytracer in order to compute the color for the given pixel.

eye

image plane

wood

light

pixel(i,j)

highly specular surface

water

sand
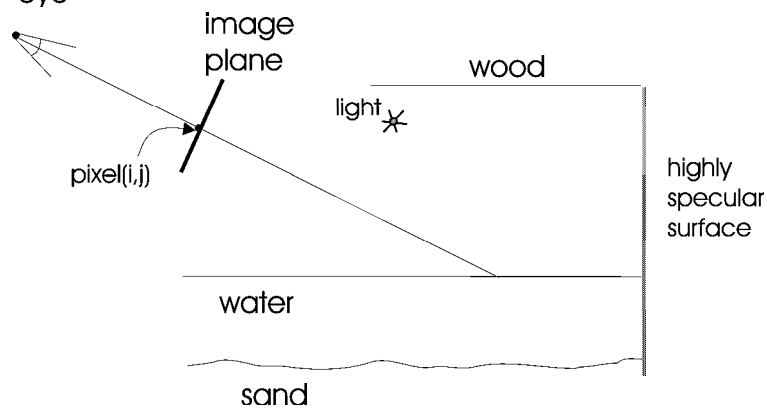
    (b) (3 points) Draw the ray tree corresponding to the above ray paths. Draw the reflected paths to the right and the transmitted paths to the left. Also indicate where shadow rays are generated. Add labels (e.g., A, B, C, ...) to both your ray tree diagram and the ray segments sketched in part (a) so that the correspondences can be seen.