1. (20) Multiple choice, 1pt each. For each question circle the **best** answer.

Q1. Numerical algorithms should be

a. efficient
b. accurate
c. reliable
Xd. all of the above

Q2. In numerical analysis we have to analyze the following types of errors:

a. design errors
Xb. roundoff errors
c. hardware errors
d. timing errors

Q3. The error in x has to be given in absolute form if:

a. x is unknown
Xb. x is zero
c. x can be negative
d. x can overflow

Q4. If $x = 10$ with relative error 0.01, what is the absolute error in $x^2/2$?

Xa. 1
b. 0.5
c. 0.0005
d. 0.1

Q5. Which of the following iterative methods coverges fastest under normal circumstances:

Xa. Newton's method
b. Secant method
c. Bisection method
d. Golden section search method

Q6. For some root finding method the error e_n satisfies $e_n = 0.1 \times e_{n-1}^3$. Which statement is true:

a. The order of the method is 0.1^3
b. The convergence rate is $-\log_{10}(0.1)$
c. The method is strictly convergent
Xd. The method is superlinearly convergent

Q7. The bisection method

a. converges faster than the secant method

b. is in between Newton's and the secant method in speed

c. is used for optimization

Xd. can converge when Newton's method does not

Q8. In large scale realistic applications the dominant computational cost in a root finding method usually is

a. the selection of the intial bracketing interval $[a\ b]$

Xb. evaluating the function and possibly its derivative

c. computing the termination criterium

d. swiching to the bisection method when an iteration fails

Q9. A matrix A is singular if

a. it is not square

b. Gaussian elimination with partial pivoting fails

c. it is its own inverse

Xd. it has no inverse

Q10. The p-norm of vector $(x_1, \ldots, x_n)$ is

a. $|x_1 + x_2 + \ldots + x_p|$

b. length$(x)^p$

c. $|x_1|^p + |x_2|^p + \ldots + |x_n|^p$

Xd. $(|x_1|^p + |x_2|^p + \ldots + |x_n|^p)^{1/p}$

Q11. The induced matrix norm $(||.||)$ satisfies for matrix A and vector x

a. $Ax \leq ||Ax||$

b. $||Ax|| = ||A||||x||$

c. $||A|| + ||x|| > 0$

Xd. $||Ax|| \leq ||A||||x||$

Q12. For a general nonsingular matrix A of moderate size the equation $Ax = b$ can best be solved using:

Xa. Gaussian elimination with scaled partial pivoting

b. Gaussian elimination with complete pivoting

c. Gaussian elimination with partial pivoting

d. Direct inversion

Q13. If Gaussian elimination for a matrix equation $Ax = b$ without pivoting encounters a division by zero

a. A is singular

b. the system has multiple solutions

c. the equation is ill-conditioned

Xd. nothing can be concluded about A

Q14. The operation count of solving an upper or lower triangular system of linear equations size n (i.e., $n \times n$ matrix) is about

Xa. n^2

b. $2n^2$

c. n^3

d. n

Q15. The complexity of Gaussian elimination with partial pivoting is

a. $O(n)$

b. $O(n^2)$

Xc. $O(n^3)$

d. $O(n^4)$

Q16. The complexity of computing the LU factorization of a matrix A is

a. $O(n)$

b. $O(n^2)$

Xc. $O(n^3)$

d. $O(n^4)$

Q17. If we have to solve $Ax = b$ many times for different vectors b but the same A we

a. compute the inverse of A

Xb. use LU decomposition once, then forward backward substitution for the different b's

c. use iterative methods

d. use Gaussian elimination

Q18. If A has no LU factorization then

a. A is singular

b. every matrix has a LU factorization

c. A requires complete pivoting

Xd. we can't conclude anything about A

Q19. The condition number of a matrix A is used to

a. speed up pivoting
b. determine how many steps pivoting takes
c. determine the relation between error in A and error in b
Xd. estimate the accuracy of Gaussian elimination

Q20. Would you like a free point?

Xa. Yes
b. No
c. Don't care
d. None of the above

2. (10) Consider the following finite difference approximation to the derivative of a function f at x

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}. \tag{1}$$

a. (3) Assuming that the (roundoff) absolute error in computing f is ϵ, what is the resulting AE in equation 1?
Each term in the numerator has error ϵ so the sum has error 2ϵ so we have roundof error

$$\epsilon/h$$

.

b. (4) Derive a formula for the absolute discretization error of the approximation, valid for small h.

Taylor expansion of $f(x+h)$ and $f(x-h)$ gives

$$f(x+h) \approx f(x) + f'(x)h + \frac{1}{2}f''(x)h^2 + \frac{1}{6}f'''(x)h^3 + O(h^4)$$

and

$$f(x-h) \approx f(x) - f'(x)h + \frac{1}{2}f''(x)h^2 - \frac{1}{6}f'''(x)h^3 + O(h^4).$$

Substitute in eq. refeq and we get

$$\frac{f(x+h) - f(x-h)}{2h} = f'(x) + \frac{1}{6}f'''(x)h^2 + O(h^3).$$

So discretization error is about

$$\frac{1}{6}f'''(x)h^2.$$

c. (3) Describe (in words) what happens to the overall accuracy of the approximation if we make h smaller and smaller.

$$\text{Total error} = \epsilon/h + \frac{1}{6}f'''(x)h^2.$$

If I decrease h the discretization error gets smaller but the roundoff error gets bigger. If ϵ is small the decrease in discretization error will make the total error go down at first, until h gets so small that the roundoff error becomes important, after which the error will increase.