CPSC 320 Sample Midterm 2
March 2009

Name: _____ Student ID: _____
Signature: _____

– You have 50 minutes to write the 4 questions on this examination. A total of 40 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

– Use the back of the pages for your rough work.

– **Good luck!**

Question	Marks
1	
2	
3	
4	
Total	

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her library card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
2. Speaking or communicating with other candidates.
3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

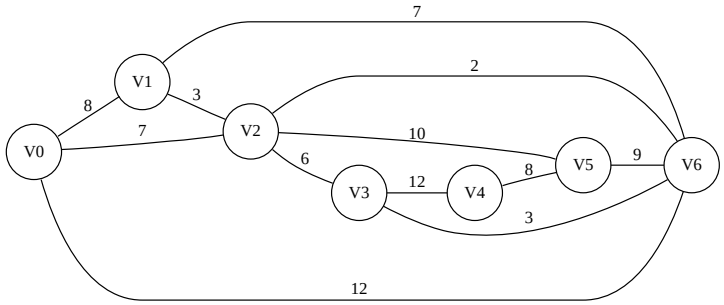
[12] 1. Skip Lists

[4] a. When we augment a skip list to support efficient Order-Statistics queries, we store a `count` value with each pointer of the skip list. What does `count` represent, and how is it used by `SkipListSelect`?

[4] b. How does algorithm `UpdatedSkipListInsert` use the `Rank` array filled by algorithm `UpdatedSkipListSearch` to recompute the `count` values associated with the pointers stored in the new node?

[4] c. When we analyzed the expected running time of algorithm `SkipListSearch` we looked at the path taken by the algorithm through the skip list "backwards". That is, we analyzed the expected number of left moves of the reverse path (from the node we are searching for to the head of the skip list) on each given level, instead of analyzing the expected number of right moves on each level. Why?

[8] 2. Show the tree constructed by Dijkstra's algorithm for the graph in the following figure, assuming that the first vertex added to the tree is vertex $V0$. Label each edge of your tree by a number that indicates the order in which the edges were added to the tree (so the first edge added will be labeled "1", the second edge added will be labeled "2", etc). Show the cost associated with each node at each stage of the algorithm.



[8] 3. After attending tutorials and learning that algorithm `DeterministicSelect` also works if we use groups of 7 elements, but not if we use groups of 3 elements, a student decides to implement a version of the algorithm in which he makes $n/\lfloor\sqrt{n}\rfloor$ groups of $\lfloor\sqrt{n}\rfloor$ elements each. Instead of finding the median of each group by sorting the group, he decides to find it by using algorithm `DeterministicSelect`.

Write a recurrence relation describing the running time of this student's version of algorithm `DeterministicSelect`. You **must** explain where each term of your recurrence relation comes from. You may ignore floors and ceilings.

[12] 4. Consider the following problem:

> You are in charge of hiring security guards for a number of Olympic events. You have been given a list of events' starting and ending times, and need to hire as few guards as possible (they are expensive) so that every event is covered by one guard in its entirety.

The problem can be modeled as a graph-coloring problem: each vertex (event) is an interval on the X-axis (starting time, ending time), an edge joins two vertices if the corresponding events (intervals) overlap, and you want to "color" each vertex using the name of a specific guard you are hiring.

[9] a. Describe a greedy algorithm that is guaranteed to give you the minimum number of guards (colors). Hint: consider the endpoints of the intervals one at a time, in a specific order.

[3] b. What is the running time of your algorithm as a function of the number of events that need a guard?

[2] c. **Bonus**: Prove the correctness of your algorithm.