Midterm Solutions (Sample Answers)

1. a) $B - A - C$
   b) $A \cap B \cup C$

2. a) onto $\Rightarrow$ all locks are accounted for
   $\therefore$ yes
   b) one-to-one — No (2 keys open one given lock)
   c) m keys & $n = m-1$ locks

   worst-case:
   try key 1 : n choices (opens $\#_n$)
   2 : $n-1$ " $_{n-1}$
   ...
   $m-1$ 1
   m n (optional)

$$\sum = \frac{n(n+1)}{2} + n = \frac{1}{2}n^2 + \frac{1}{2}n + \frac{1}{2} + n$$

$$\Rightarrow O(n^2)$$

or $O(m^2)$

3. # of different subsets $= 2^8 = 256$ ways

4. $f(n) \in \Omega(g(n)) \Rightarrow \exists$ constants $c_f \in \mathbb{R}^+$, $n_f \in \mathbb{P}$ s.t. $f(n) \geq c_f\, g(n)$ $\forall n \geq n_f$

   $h(n) \in \Omega(g(n)) \Rightarrow \exists$ constants $c_h \in \mathbb{R}^+$, $n_h \in \mathbb{P}$ s.t. $h(n) \geq c_h\, g(n)$ $\forall n \geq n_h$

$k(n) \in \Omega(g(n)) \Rightarrow \exists$ constants $c_k \in \mathbb{R}^+$,
$n_k \in \mathbb{P}$ s.t. $k(n) \geq c_k g(n)$
$\forall\ n \geq n_k$

$\therefore f(n) + h(n) + k(n) \geq c_f g(n) + c_h g(n) + c_k g(n)$
$= \underbrace{(c_f + c_h + c_k)}\, g(n)$
$= c_3 g(n) \quad c_3 \in \mathbb{R}^+$
$\forall\ n \geq \max\{n_f, n_h, n_k\}$

$\Rightarrow f(n) + h(n) + k(n) \in \Omega(g(n))$ // QED

5. a) $\binom{6}{3}$

b) $\binom{6}{3}\binom{6}{4} + \binom{6}{4}\binom{6}{3} + \binom{6}{5}\binom{6}{2} + \binom{6}{6}\binom{6}{1}$
G B

6.
```
int count = 0;
if (curr == NULL)
    return 0;
if (LeftOrRight == 'L')
{
    curr = curr->prev;
```

```
        while (curr != NULL)
        {
            cout << curr->name << endl;
            count++;
            curr = curr->prev;
        }
        return count;
    }
    curr = curr->next;
    while (curr != NULL)
    {
        cout << curr->name << endl;
        count++;
        curr = curr->next;
    }
    return count;
}
```

7. b, c
8. not for marks
9. d (we accepted either answer T/F for b)
10. while loop $\Rightarrow \Theta(\log_{32} N) \Rightarrow \Theta(\lg N)$

    for loop (outer) $\Rightarrow \Theta(N)$
    for loop (inner) $\Rightarrow \Theta(\lg N * N)$

    overall: $\Theta(N) * \Theta(N \lg N) + \Theta(\lg N)$

    $= \Theta(N^2 \lg N)$

    or $\Theta(n^2 \lg n)$

11.
```
int recursive_product (int m, int n)
{
    if (n==1)
        return m;
    return recursive_product(m, n-1) +
                              m;
}
```

12. $1+2+3+4+5=15$ solutions

have no choice $\Longrightarrow x_1+x_2+x_3+x_4+x_5=3$

$\Longrightarrow \binom{3+5-1}{5-1}=\binom{7}{4}$ solutions

13. <u>Base case</u>: $n=1$ (we assume the preconditions are true: $x\in\mathbb{R}$, $n\in\mathbb{Z}^+$)

Then, product $=1$

$k=1$

we enter the loop

product $=1*x=x$

$k=2$

break from loop

The <u>loop invariant</u> is that product $=x^k$ after the end of the k'th iteration of the loop.

Since product $=x^1=x$ at the end of 1 iteration, the loop invariant holds. $\therefore$ Base case holds.

Inductive Hypothesis:

Assume product $= x^i$ after $i = 1, 2, \ldots, j$ iterations of the loop where $j < n$.

Inductive Step

Show that product $= x^{i+1}$ after $i+1$ iterations

By the Inductive Hypothesis, assume product $= x^i$ after the i-th iteration, which is just before the start of the $(i+1)$ iteration.

During the $i+1$ iteration:

$$\begin{aligned} \text{product}_{new} &= \text{product}_{old} * x \\ &= x^i * x \\ &= x^{i+1} \end{aligned}$$

and $k_{new} = k_{old} + 1$

$\therefore$ At the end of iteration $i+1$, the loop invariant holds (i.e., $\text{product}_{new} = x^{i+1}$).

Termination?

Does the loop terminate after computing the n'th iteration?

The variable $k = n$ during the n'th iteration. Before completing the iteration, $k_{new} = k_{old} + 1 = n + 1$. Upon testing the condition for the while loop at the top, it fails since $k \nleq n$. The loop is exited and the answer (product $= x^n$) is returned.