Computer Science 221

Practice Questions - Set 4
Data Structures and Graph Theory

Here are some more sample questions to help you prepare for the final exam.

1. Consider 6-tuples of the form (band_name, rating, lyrics, lifestyle, generosity, friendship) where band_name is a string, and the other 5 attributes are integers having values ranging from 1-100 (1=bad, 100=good). For these 5 other attributes, rock music fans have collectively decided upon the following statistics for the band: (a) how much the fans like the band, (b) how clean the band's lyrics are, (c) how clean the lifestyles of the band members are, (d) how generous the band members are in giving their money to charity (Variety Club Children's fund, Union Gospel Mission, etc.), (e) the quality of friends that band members hang around with.

Algorithm A

```
u = 0
Open the ROCK-tree
While (data exists in the update file) do
     Read a 6-tuple from the update file
     u++
     Try to insert the 6-tuple into the ROCK-tree
     If (insert failed for any reason) then
        print "insert failed for tuple ", u
     else
        print "insert was successful for tuple ", u
```

Algorithm B

```
// Perform a range query to see which bands are most similar to a
// given band (i.e., the closest in terms of the 5 numeric
// attributes for the band).
Print "Enter the name of a band to search for its neighbours"
Read targetBandName
Print "Enter a radius for the range search (e.g.,10, 500, 25000)"
Read radius
Search the ROCK-tree to find all other bands that lie within a
hypersphere of radius units of the query band.  Specifically,
return:
      (a) head = a pointer to a list of bands
          (i.e., a pointer to a list of addresses)
      (b) n = the number of bands falling within the radius
Print out all the tuples in the results list
```

The following information was taken from the ROCK-tree programmer's manual:

a) It takes O(1) time to open any file, including the ROCK-tree.
b) A ROCK-tree can handle k-tuples of any size k. Smaller k means faster searches. A k-tuple consists of k attributes—namely 1 unique string, and k-1 integers. All tuples must have the same k.
c) It takes O(k lg n) time to insert a k-tuple into a ROCK-tree having n entries.
d) Given a unique string (e.g., band name), it takes O(lg n) time to locate a target k-tuple in the ROCK-tree, and then $O(kd^2 + x)$ expected time to create a list of the addresses of all x k-tuples that lie within radius d of that target k-tuple.
e) It takes O(k) time to print a k-tuple, given its address.
f) You can assume that the update file is much smaller in size than the data that is currently contained in the ROCK-tree.

Determine the worst-case time complexity of Algorithm A, and the expected time complexity of Algorithm B. Explain your reasoning.

2. Consider the same 6-tuples from Question 1, assuming a band name length of 30 bytes, and a pointer length of 10 bytes. This time, instead of using a ROCK-tree, we'll use a B+ tree. Answer the following questions, and be sure to show your work.

   a) Estimate the number of these data entries that can be stored in a B+ tree leaf node of size 4K, assuming we're only storing key-pointer pairs.

   b) Estimate the minimum amount of space that you would need to store 10,000 bands in the index.

   c) Suppose that we place the whole record/tuple (band name and all of its 5 4-byte integer ratings) directly in the index. In other words, instead of having key-pointer fields, we have key-record fields. Now repeat parts (a) and (b) with this change. You can assume that the key is part of the record, so you don't have to count space for it twice.

3. A rock band wants to visit the following cities: Vancouver, Surrey, Burnaby, Los Angeles, San Francisco, New York, and Toronto. The band is currently arguing about the number of ways it can visit all the airports exactly once.

   Assume that there are only 5 airports (Vancouver, Los Angeles, San Francisco, New York, Toronto) that service these 7 cities. You can assume that the Vancouver airport is actually located in Vancouver, etc. Each of these 5 airports

has flights to the other 4. Recall that Surrey and Burnaby are suburbs of Vancouver. Assume that the band lives in the Vancouver area.

In answering the following questions, show your work. If you need to make any reasonable assumptions, be sure to document them.

a. How many possible orderings are there for the band to visit all 7 of these cities, assuming they want to visit each airport exactly once? Assume that an airport visit is made up of an inbound trip plus an outbound trip (or in Vancouver's case, an outbound trip, followed later by an inbound trip).
b. Suppose the band wants to visit just the 5 airport cities, visiting each of them exactly once (i.e., they fly in, then they give their concert in the city, then they get questioned at police headquarters about all kinds of things, and then they head out to the airport to leave the city—being warned not to return again). Is such a cycle possible among the airport cities? If so, give one such ordering. If not, explain why no such ordering is possible.
c. Is a Hamilton path possible among the 7 cities? If so, give one such ordering. If not, explain why no such ordering is possible.

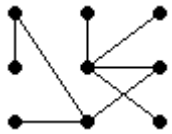
4. After completing their North American tour in (3), the band members notice that the more they fly, the more frequent flyer points they get. So, they decide that they want to fly on all of the flights that Euler Airlines offers between all the following cities. They plan to travel first class, get free beverages on board each flight, mingle with the flight staff, and collect their frequent flyer points.

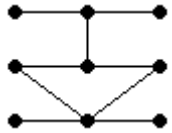
   Euler Airlines has flights between these cities. A "1" means that a single one-way flight exists from the source city to the destination city.

to->> from	Vancouver	Toronto	Los Angeles	New York	San Francisco
Vancouver	0	0	1	0	1
Toronto	0	0	1	1	0
Los Angeles	1	1	0	1	1
New York	0	1	1	0	0
San Francisco	1	0	1	0	0

   Assuming the legal authorities actually permit the band to travel to these cities, is there an Euler cycle that the band can complete using each of the one-way flights in the table? If so, give one such cycle. If an Euler cycle is not possible, then: (a) state why not, and (b) determine whether an Euler *path* is possible (giving an example if there is one, or an explanation if there isn't one).

5. A rock band has time to kill between flights, and wants to know how to turn the following graphs into trees.

a. 

b. 

Re-draw, if possible, each graph as an `m`-ary (max. `m` children per parent) rooted tree, where `m=4`. Also, re-draw, if possible, each graph as a balanced tree (where "balanced" means that all of the leaves appear on no more than 2 adjacent levels: level x and (possibly) level x-1, where x is the furthest level from the root.