Name: _____

Student #: _____

Midterm Exam, CPSC 302, WINTER 2011
Feb. 21, 2011

Instructions :

- Write down your name and student number in the designated spot at the top of this page.
- Time: 50 minutes.
- Total three questions, 50 marks: 20 for Q. 1, 20 for Q. 2, 10 for Q. 3.
- No extra material (cheat sheets, calculators, etc) is allowed.
- Write your answers in the space provided. There are blank sheets at the back for rough work.
- Show your work, but please avoid unnecessarily lengthy answers. Anything not to be marked should be clearly crossed out.

Useful formulae

The following formulae may be useful when answering the questions and may be used without proof.

Mean value theorem: If $f(x)$ is continuous on the interval $[a, b]$, and differentiable on (a, b), then there exists a point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Intermediate value theorem: If $f(x)$ is continuous on the interval $[a, b]$, and ξ is a number such that $f(a) < \xi < f(b)$, then there exists a $c \in [a, b]$ such that

$$f(c) = \xi.$$

Properties of determinants: Let $A, B \in \mathbb{R}^{n \times n}$ and $\alpha \in \mathbb{R}$. Then:

$$\begin{aligned} \det(\alpha A) &= \alpha^n \det(A), \\ \det(AB) &= \det(A)\det(B), \\ \det(A^{-1}) &= (\det(A))^{-1}, \\ \det(A^T) &= \det(A). \end{aligned}$$

1. A general floating point system can be characterized by the four values (β, t, L, U). Describe the meaning of each of these digits. What is the *mantissa* and what is the *exponent* of a floating point number?

   Describe two instances when floating point errors may be magnified.

   The following identity is given:

$$\ln(x - \sqrt{x^2 - 1}) = -\ln(x + \sqrt{x^2 - 1}).$$

   (a) Which of the above two (mathematically equivalent) expressions is more suitable for numerical computations if $x \gg 1$? Justify your answer.

   (b) Suggest a formula different from the above two that is at least as numerically stable as your answer for part (a) and is more likely to prevent an overflow.

continue answer here...

2. What is meant when we say an iterative method exhibits (i) linear and (ii) quadratic convergence? State (but don't prove) conditions under which Newton's method converges quadratically.

   A fixed point iteration has the form

   $$x_{k+1} = g(x_k).$$

   Show that, if we apply such an iteration on some interval $[a, b]$ (which you may assume is sufficiently close to a fixed point ξ of the function $g(x)$) then

   $$|x_{k+1} - \xi| \leq L|x_k - \xi|, \tag{1}$$

   where $L = \max_{x\in[a,b]} |g'(x)|$.

   What is the function $g(x)$ which gives us Newton's method?

   Consider the function

   $$f(x) = 25x^2 - 10x + 1.$$

   How fast should we expect convergence of Newton's method to be? Justify your answer.

   How well would the bisection method work for this function?

continue answer here...

3. Use Gaussian Elimination *without* pivoting to solve the following linear system:

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 3 \\ 8 & 7 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 24 \end{bmatrix}.$$

Hence write the matrix above as the product of a lower and an upper triangular matrix.

Using your decomposition, write down the determinant of the matrix.[1]

[1] You may use, without proof, the properties of determinants on page 2

continue answer here...

Extra page for working

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