Solutions sample hashing and binary tree questions

1. Suppose T is an empty binary tree, then its height is -1 and $2^{-1+1}-1=0$, which is the number of nodes it contains.

   I.H. Suppose every binary tree of height h has at most $2^{h+1}-1$ nodes.

   Let T be a binary tree of height $h+1$. The left (right) subtree of T's root has height $\leq h$ and thus, by I.H., at most $2^{h+1}-1$ nodes. So T has at most

   $$1 + 2^{h+1} - 1 + 2^{h+1} - 1 = 2^{h} - 1 \text{ nodes.}$$

2. Suppose T is a binary tree with $n=1$ node, then it contains $n-1=0$ edges.

   I.H. Suppose every binary tree with $n \geq 1$ nodes has $n-1$ edges

   Let T be a binary tree with $n+1$ nodes. Remove any leaf from T (why does T contain a leaf?) and the edge to its parent. This creates a tree T' with n nodes and $n-1$ edges (by I.H.) so T has n edges

   (This is true of any tree.)

3. Let T be a full binary tree. Every node has 0 or 2 children so the number of nodes that have a parent is $2 \cdot m$ (where m is the number of internal nodes in T). Every node except the root has a parent, so the total number of nodes in T is $2 \cdot m + 1$ which is odd.

4.

0	7
1	14
2	21
3	28
4	
5	
6	

5.

0	7
1	14
2	28
3	
4	21
5	
6	

The first probe sequence doesn't visit all the table entries.
The second one does.

6. $365^{23} - P(365,23)$ = sequences with ≥ 2 identical birthdays

   365^{23} = total sequences

   $$1 - \frac{P(365,23)}{365^{23}} \approx 0.51 = \text{probability of } \geq 2 \text{ identical bdays.}$$