CPSC 121 :: 2011S :: Final Exam :: 2011.07.30

NAME:

SIGNATURE:

STUDENT NUMBER:

- There are 16 pages in total.

- You have 150 minutes (2.5 hours)

- A total of 100(+2) marks are available.

- As a suggestion, allocate ≈1.5 minutes per mark.

- You may want to complete what you consider to be the easiest questions first!

- No notes or electronic devices are permitted.

- Use the backs of pages if you require additional space, and clearly identify when you have done so.

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her university-issued ID.

- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.

- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
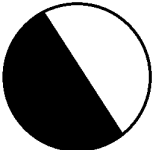
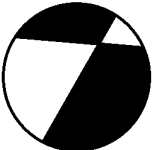| Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL

Question 1. [7 marks]

Prove (using induction) that if you divide a region with n straight lines to create smaller sub-regions, you only need two colours to paint your regions so that no two adjacent regions have the same colour.
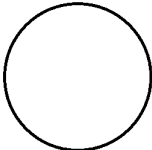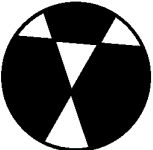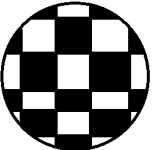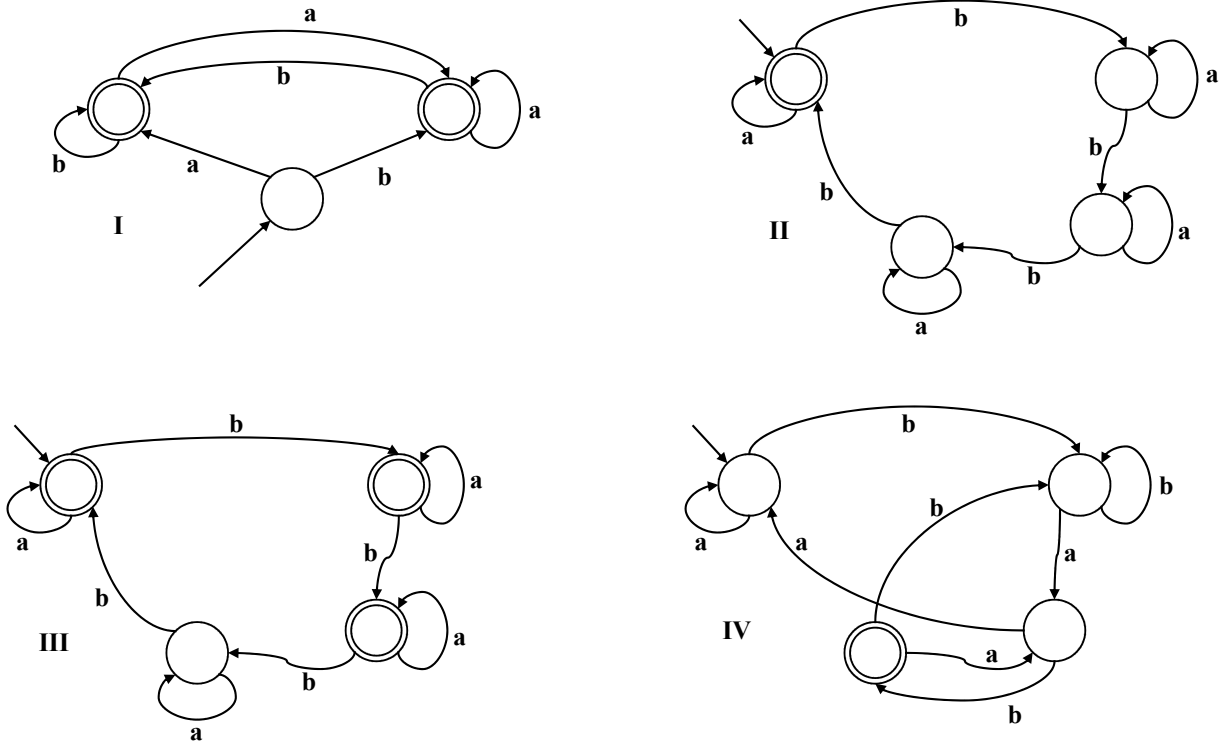
n=0 **n=1** **n=2** **n=3** **n=9**

Question 2. [10 marks]

I

II

III

IV

For each of the above DFAs, with the input language $\{a, b\}$, give an example of a 6-character string using at least one a and one b that it would accept, and a 6-character string using at least one a and one b that it would reject (or *n/a* if one does not exist). In addition, give an *equivalent* regular expression for the DFA. A correct equivalent regular expression may or may not be amongst those listed below; if it is, you can just identify it by number.

DFA	Accept (6)	Reject (6)	Regular Expression
I			
II			
III			
IV			

1. `[ab]`   2. `[ab]+`   3. `[ab]*`   4. `[ab]?`
5. `[ab]*ab`   6. `[ab]*ba`   7. `[ab]*aba`   8. `[ab]*bab`
9. `((bbbb)|a)*`   10. `((ba*ba*ba*b)|a)*`   11. `((bababab)|a)*`   12. `.*`

Question 3. [7 marks]

part a. [2 marks]
What unsigned integer (in decimal notation) is represented by the 9-bit binary number 100111011?

part b. [2 marks]
What signed integer (in decimal notation) is represented by the 9-bit binary number 100111011?

part c. [3 marks]
A professional programmer wrote a program that takes a floating point number x, sets its value to 0, and then adds 0.1 to x repeatedly until it becomes equal to 10, printing each value of x as it goes. Why does the program produce the output

```
0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.70000005  0.8000001  0.9000001
1.0000001  1.1000001  1.2000002
```

(and so on) instead of

```
0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0  ?
```

Question 4. [7 marks]

Prove using induction that for every positive integer n,

$$\sum_{j=1}^{n} j(j+1) = \frac{n(n+1)(n+2)}{3}$$

Question 5. [4 marks]

Marge has come up with a new rule of inference:

$$(p \lor q) \to (r \lor s)$$
$$p \to r$$
$$\overline{}$$
$$\therefore q \to s$$

But Flanders was not sure if it is a valid rule, and so to verify it he produced a truth table for some formula X. What was his formula X? Can you identify a variable assignment (row in the truth table) where X is false? *Note: you do not necessarily need to do the entire truth table.* If such a row exists, what does that mean about Marge's rule?

Question 6. [9 marks]

A decimal (base 10) integer is divisible by 3 if and only if the sum of its digits is itself divisible by 3. Design a DFA that accepts all decimal integers divisible by 3. You can assume that the only valid input characters are the digits 0 to 9 and that 0 (and the empty string) is divisible by 3. For example, it should accept 111, 12, 18, 111331218 and reject 77, 91, 13, 779113.

Question 7. [5 marks]

In your own words, give a one or two sentence description of the following stages or components of the working computer you used in lab 9:

Registers

Fetch

Decode

Execute

ALU

Question 8. [9 marks]

part a. [3 marks]
What is the difference between what a half-adder does, and what a full-adder does?

part b. [3 marks]
Describe what a flipflop is.

part c. [3 marks]
What is the difference between regular induction and strong induction?

Question 9. [8 marks]

Given the following 4 premises, prove that p is true. Justify your answer *fully*.

1. $(\sim p \vee q) \to r$
2. $r \to (s \vee t)$
3. $\sim s \wedge \sim u$
4. $\sim u \to \sim t$

Question 10. [5 marks]

A friend of yours says she has a nifty new way of looking at 2's complement numbers that Dave never discussed in class. She says that to convert an n-bit signed integer to decimal, you proceed exactly the same way as you do for unsigned integers, *except* for the leftmost bit. She says that if the leftmost bit is one, you *subtract* $2^{(n-1)}$ instead of adding it. In other words, if x is a signed integer represented with the bits:

$$b_{(n-1)}, b_{(n-2)}, \ldots b_1, b_0$$

then the value of x is:

$$x = -b_{(n-1)} \cdot 2^{(n-1)} + b_{(n-2)} \cdot 2^{(n-2)} + \ldots + b_1 \cdot 2^1 + b_0 \cdot 2^0$$

Do you agree with her new way of looking at 2's complement numbers? You do not need to prove or fully justify that her representation is valid or invalid, but you should give some evidence and an example that supports your answer.

Question 11. [11 marks]
Given:

- A is the set of all animals in the world

- $H(x) \Leftrightarrow x$ is a human (x is an animal, and humans are animals)

- $P(x) \Leftrightarrow x$ is a pig

- $D(x) \Leftrightarrow x$ is a dog

- $E(x, y) \Leftrightarrow x$ eats y

Rewrite the following statements formally without defining any new domains or predicates:

part a. [2 marks]
Bob is a human, Porky is a pig, and Pluto is a dog.

part b. [2 marks]
If there are no humans in our world, then there are no pigs in our world.

part c. [2 marks]
Natalie is a vegetarian (She does not eat any animals).

part d. [3 marks]
If Bob is the only human alive, then he would eat any dog but Pluto.

part e. [2 marks]
It's a dog eat dog world. (All dogs eat all *other* dogs).

Question 12. [6 marks]

Explain briefly how you would prove a statement of the form:

$$\forall x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+, \exists z \in \mathbb{Z}^+, P(x, y, z) \rightarrow Q(x, y, z).$$

In other words, what would your strategy be, and what steps would you take?

Question 13. [12 marks]

Let $A = \{2, 3, 5\}$, $B = \{1, 2, 3, 4\}$.

part a. [3 marks]
What are the elements of the power set of A?

part b. [3 marks]
List every element of $(A \cup B) \times (A \cap B)$.

part c. [3 marks]
Consider the function $f : A \to B, f(x) = x - 1$, and indicate (circle) whether it is:

Injective/one-to-one: YES   NO   NOT ENOUGH INFORMATION

Surjective/onto: YES   NO   NOT ENOUGH INFORMATION

part d. [3 marks]
Consider $f : X \to M$ defined by $f(x) =$ Dave's x^{th} favorite movie, where M is the set of all movies that Dave has ever seen. So, $f(17)$ would be Dave's seventeenth favourite movie *("Groundhog Day")*. Dave has a single, clear preference order amongst the movies he has seen. If you are told that f is bijective, what can you conclude about the contents of set X?

Question Bonus. [$0 \ldots 2$ **marks]**

Describe your favourite (or most memorable) part of CPSC 121 this term. Be creative – you can draw a picture, write a poem, etc.

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