CPSC 320 Sample Midterm 1
October 2010

Name: _____ Student ID: _____
Signature: _____

– You have 50 minutes to write the 4 questions on this examination.
A total of 40 marks are available.

– **Justify all of your answers.**

– You are allowed to bring in one hand-written, double-sided 8.5 x 11in sheet of notes, and nothing else.

– Keep your answers short. If you run out of space for a question, you have written too much.

– The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.

Question	Marks
1	
2	
3	
4	
Total	

– Use the back of the pages for your rough work.

– **Good luck!**

UNIVERSITY REGULATIONS:

– Each candidate should be prepared to produce, upon request, his/her library card.

– No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

– CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[12] 1. Answer each of the questions with either *true* or *false*. You **must** justify each of your answers; an answer without a justification will be worth at most 1.5 out of 4.

[4] a. $3^{n+2} + 5 \in O(3^{n-1})$.

[4] b. Let f, g be two functions from $\mathbf{N}$ into $\mathbf{R}^+$. Assuming that $\lim_{n\to\infty} f(n)/g(n)$ exists, we can use its value to determine whether or not f is in $O(g)$.

[4] c. In class, we proved an $\Omega(n \log n)$ lower bound on the worst-case running time of any algorithm that can be used to sort a sequence of n values.

[8] 2. Show how to construct an input for which the prefix tree generated by Huffman's algorithm will have all of its leaves on either the bottom-most level, or the level just above it. Your construction should work for any alphabet size $m \geq 2$. Sketch a proof that it results in a prefix tree with the requested property (you do not need to write a complete proof).

[8] 3. Show the tree constructed by Kruskal's minimum spanning tree algorithm for the graph in the following figure. Label each edge of your tree by a number that indicates the order in which the edges were added to the tree (so the first edge added will be labeled "1", the second edge added will be labeled "2", etc).
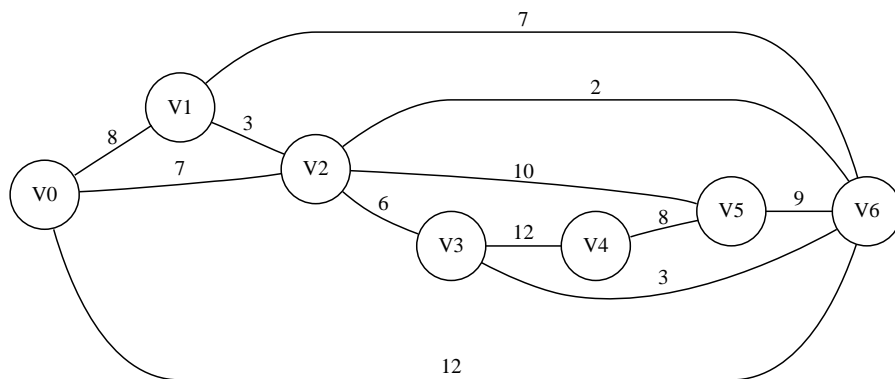
[12] 4. The Vancouver aquarium is undergoing major renovations. For the duration of the renovations, only one water tank will be available to the public. The aquarium's president decides to put as many different species of fish in that one tank so the public can see as many of them as possible. He asks for your help. He provides you with the following information:

- A list $F_1, \ldots, F_n$ of the n types of fish that the aquarium owns, and
- A list (F_i, F_j) of pairs of kinds of fish that can not be put in the same water tank as one would eat the other.

[9] a. Design a greedy algorithm that takes this information as input, and returns a list of species of fish that can cohabit peacefully. Your algorithm should try to maximize the number of species in the list.

[3] b. Explain briefly why your algorithm from part (a) is greedy.