Finite Element Method

Final Project

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一、 公式整理

1. Bar Problem

Shape Function (4 node rect element)

$$N_1^e(x) = \frac{1}{4}(1-\xi)(1-\eta)$$
$$N_2^e(x) = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_3^e(x) = \frac{1}{4}(1+\xi)(1+\eta)$$
$$N_4^e(x) = \frac{1}{4}(1-\xi)(1+\eta)$$

$$\boldsymbol{B}^e(\xi,\eta) = \boldsymbol{J}^{-1}\boldsymbol{G}\boldsymbol{N}^e(\xi,\eta) \tag{1}$$

Stiffness Matrix

$$\boldsymbol{K}^e = \int_\Omega (\boldsymbol{B}^e)^T \boldsymbol{D}\boldsymbol{B}^e d\Omega = \int_{-1}^{1}\int_{-1}^{1} (\boldsymbol{B}^e)^T \boldsymbol{D}\boldsymbol{B}^e |J| d\xi d\eta \tag{2}$$

Force Vector

$$\boldsymbol{f}^e = \oint_{\Gamma_t} (\boldsymbol{N}^e)^T \boldsymbol{t} d\Gamma + \int_\Omega (\boldsymbol{N}^e)^T \boldsymbol{b} d\Omega \tag{3}$$

二、 程式說明

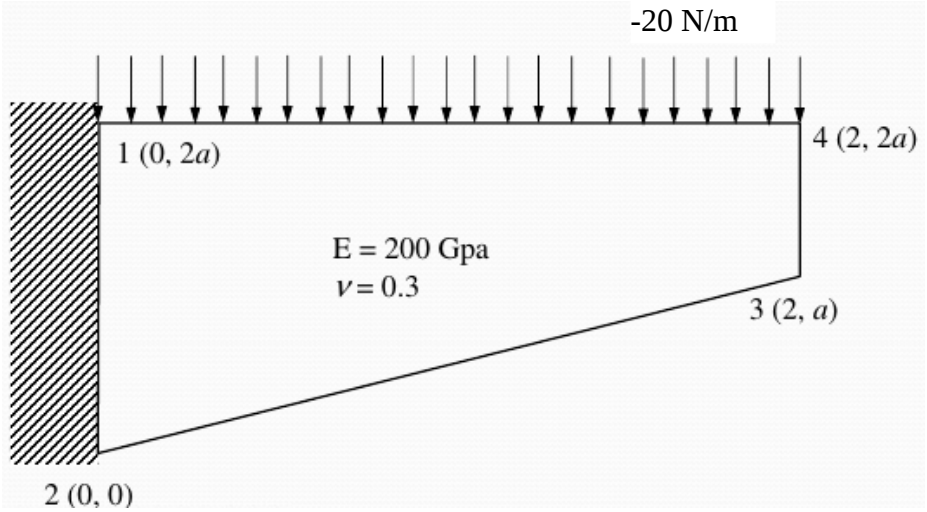
1. Programming Language – Matlab

MATLAB 是一種用於演算法開發、資料視覺化、資料分析以及數值計算的高階技術計算語言和互動式環境。除了矩陣運算、繪製函式/資料圖像等常用功能外，MATLAB 還可以用來建立使用者介面及與呼叫其它語言（包括 C,C++,Java,Python 和 FORTRAN）編寫的程式。一維基百科

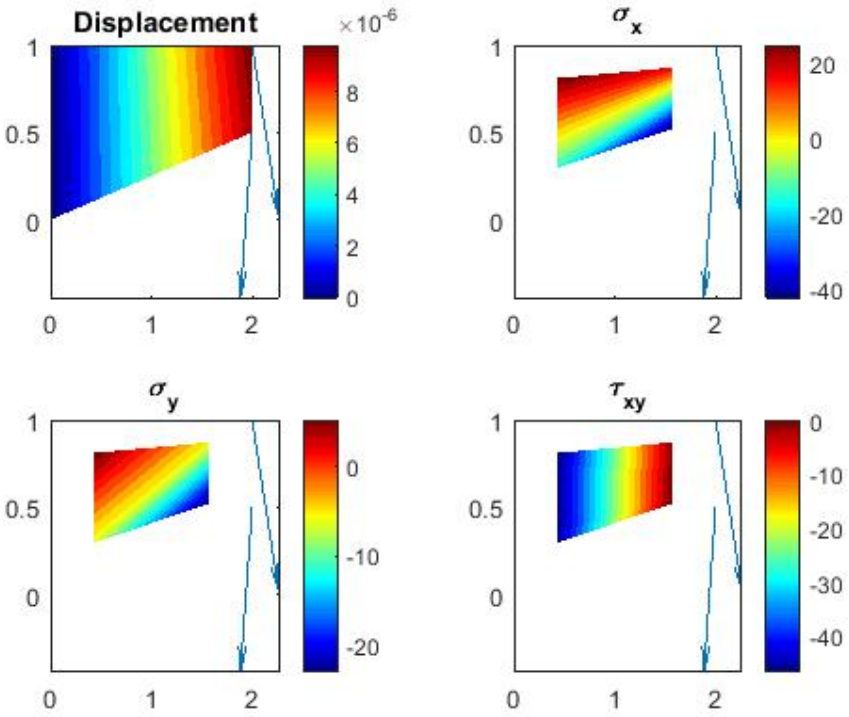
A. Functions

```
% Return meshed points
% X, Y: Boundary points
% rN, cN: Number of row and column
[x y] = MESH(X, Y, rN, cN)

% Return global K matrix with rN*cN mesh
k = Kg(rN, cN)

% Return local K matrix of (r, c) element
k = Ke(r, c)

% Return B matrix at (xi, eta) of (r, c) element
b = Be(xi, eta, r, c)

% Return dN/dx at (xi, eta) of (r, c) element
r = DNDX(xi, eta, r, c)

% Return Jacobian at (xi, eta) of (r, c) element
j = JAt(xi, eta, r, c)

% Return dN/dxi at (xi, eta)
b = dNAt(xi, eta)

% Return global force vector of the distributed load
f = Fg()

% Solve the equation K*d=F, return d vector as result
d = SOLVE(K, F, d)

% Calc Gaussian elimination of extend matrix A
x = gaussElim(A)

% Return stresses on the Gaussian points by res(result)
% Return: s(stresses), gp(coordinate of gaussian point)
[s gp] = STRESS(res)
```

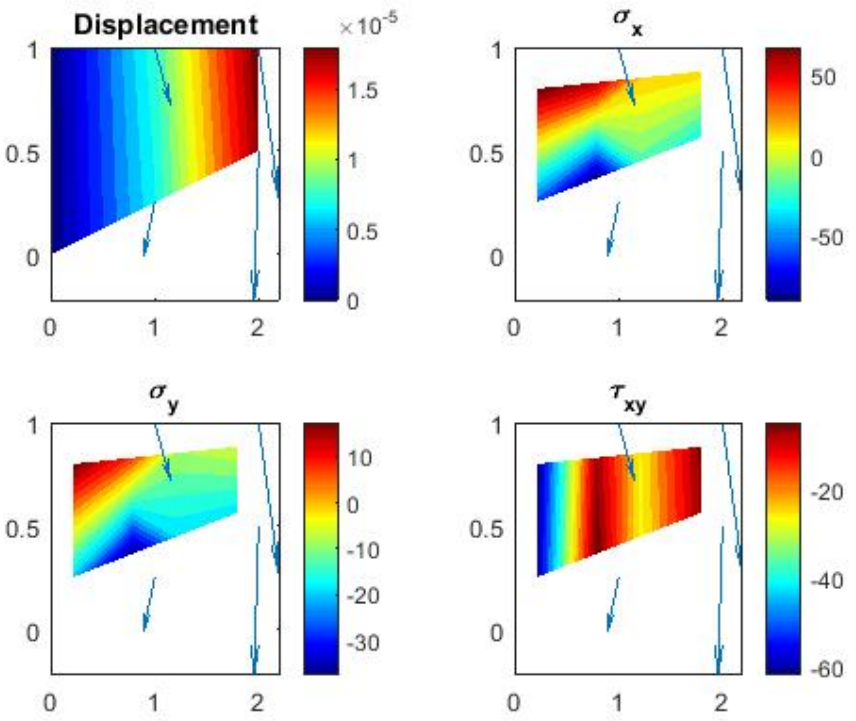
三、 討論



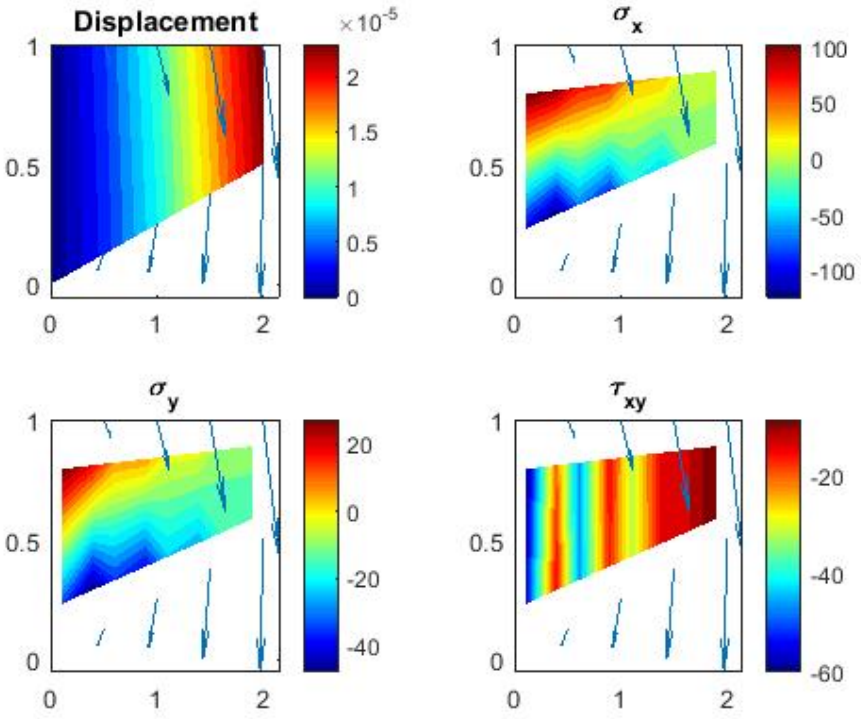
A. Result



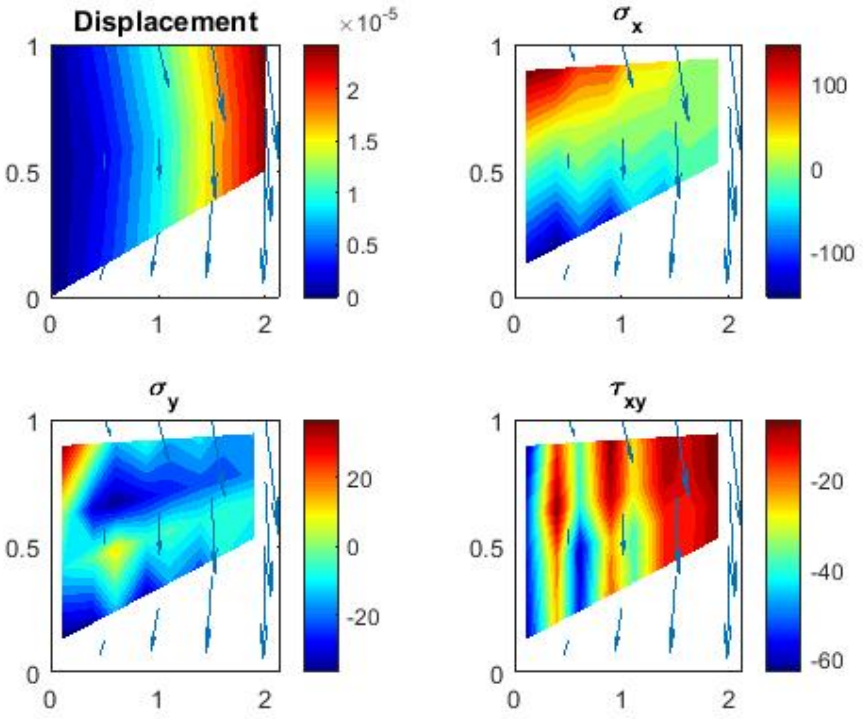
圖一 1x1 Mesh



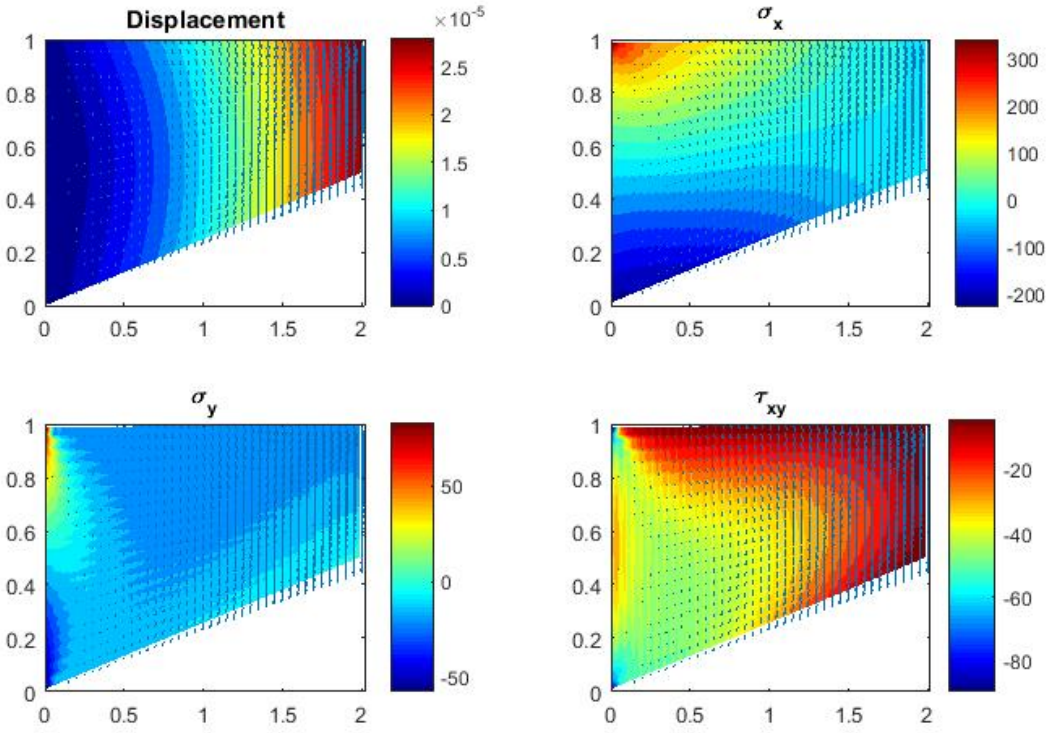
圖二 1x2 Mesh



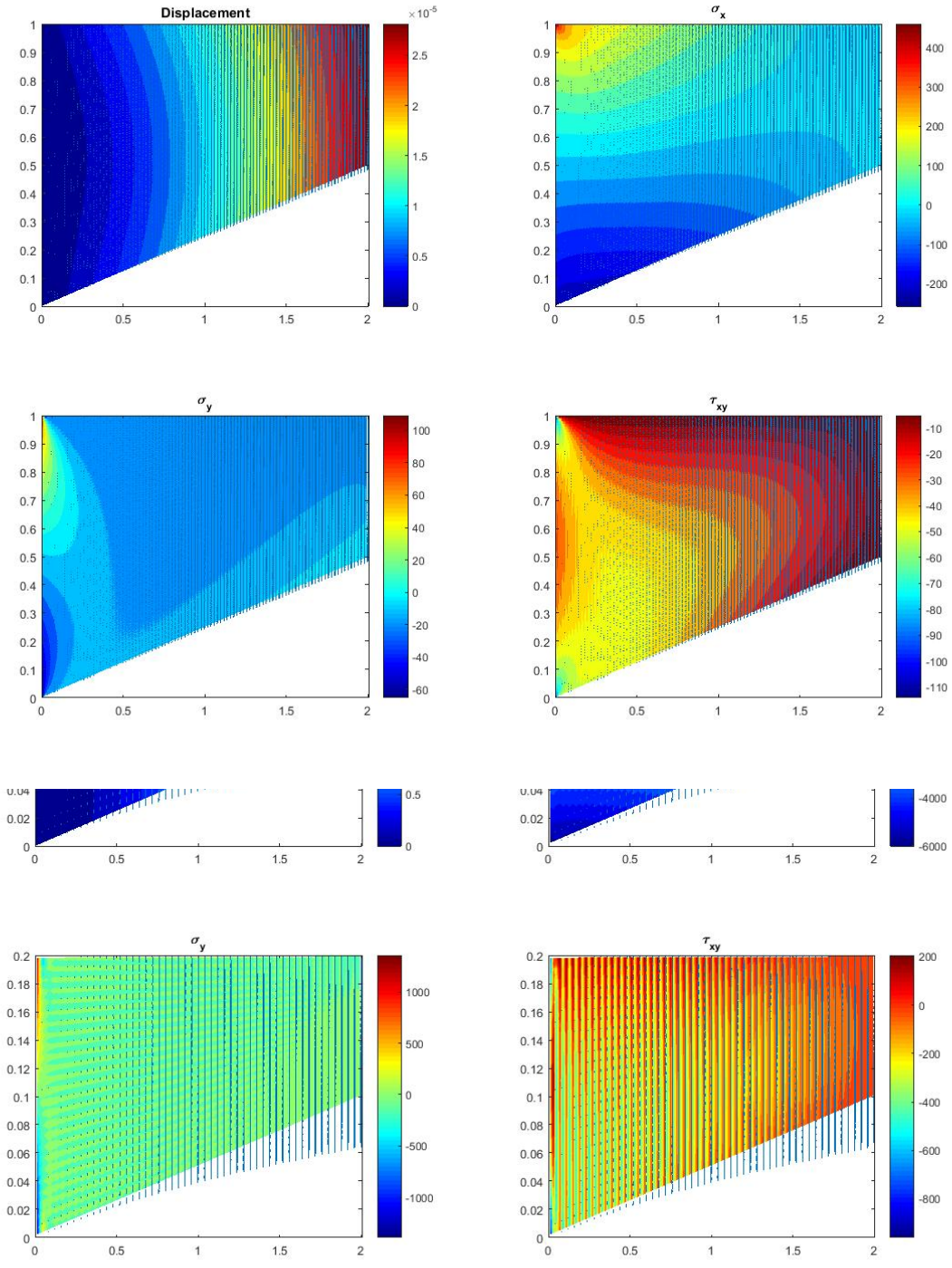
圖三 1x4 Mesh


Displacement
×10^-5
2
1.5
1
0.5
0
σx
100
0
-100
σy
20
0
-20
τxy
-20
-40
-60
0
0.5
1
0
1
2

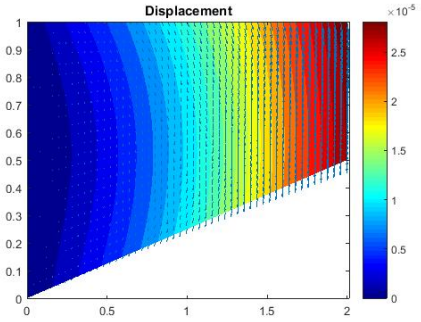
圖四 2x4 Mesh

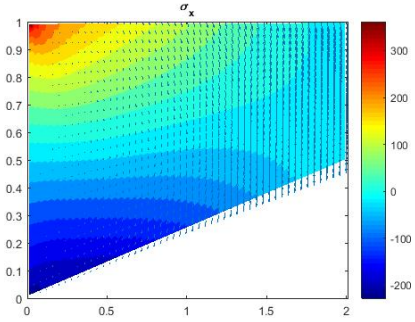


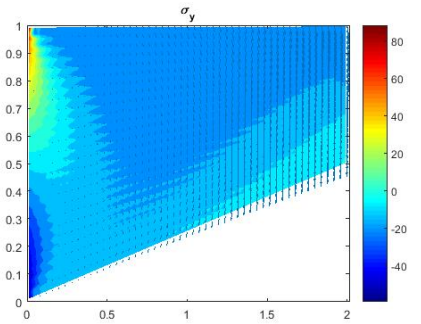
圖五 20x40 Mesh

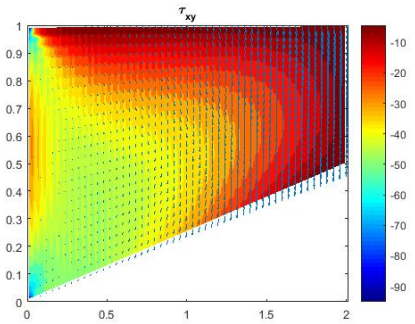


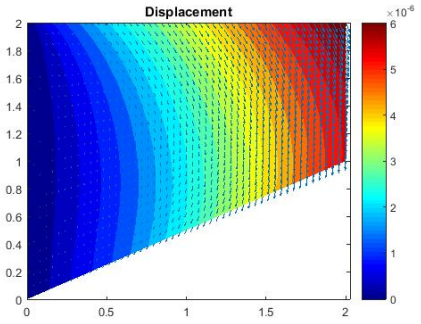
圖六 100x100 Mesh

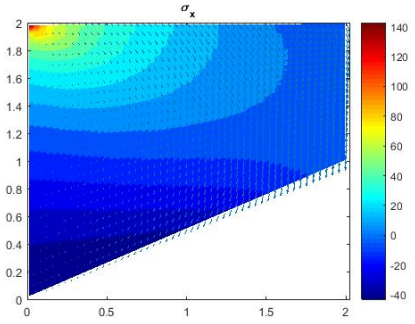

Displacement
×10^-5
2.5
2
1.5
1
0.5
0

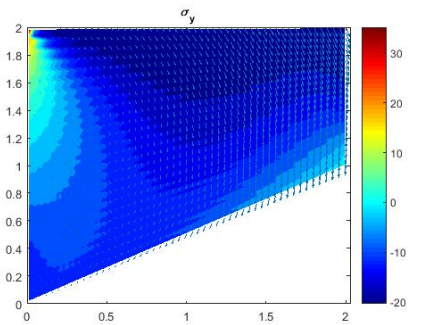

σx
300
200
100
0
-100
-200

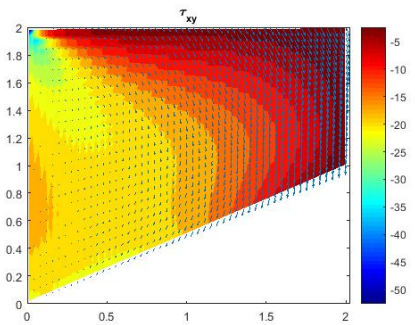

σy
80
60
40
20
0
-20
-40

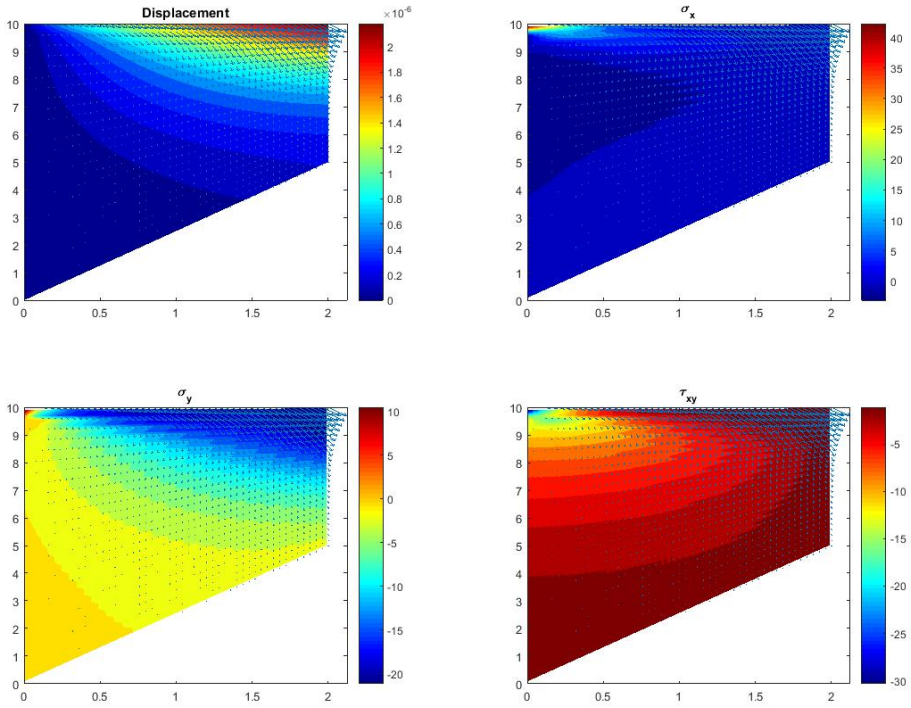

τxy
-10
-20
-30
-40
-50
-60
-70
-80
-90


Displacement
×10^-6
6
5
4
3
2
1
0

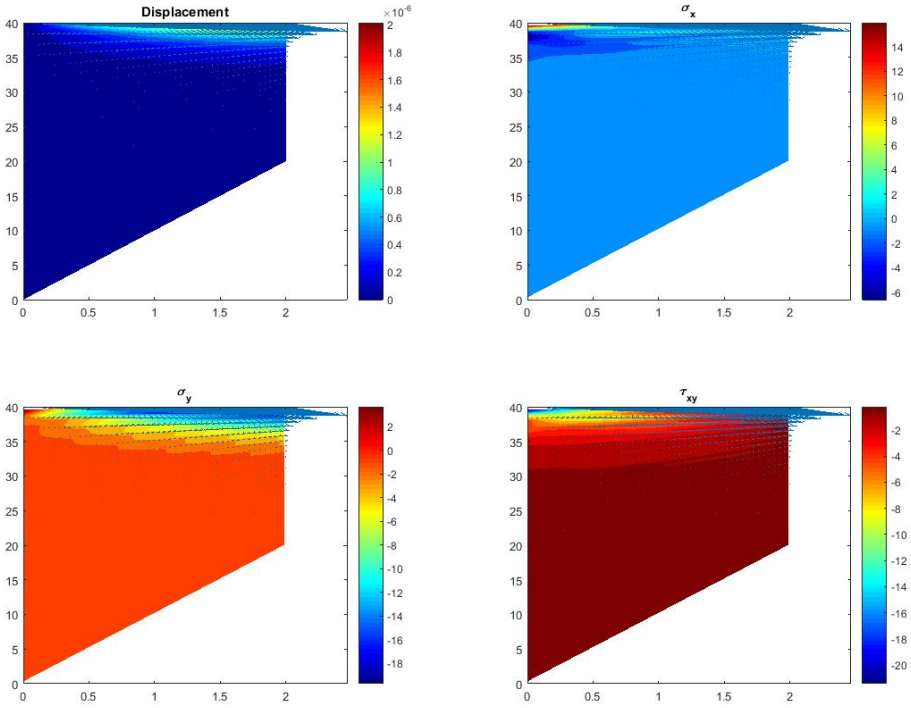

σx
140
120
100
80
60
40
20
0
-20
-40


σy
30
20
10
0
-10
-20


τxy
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50



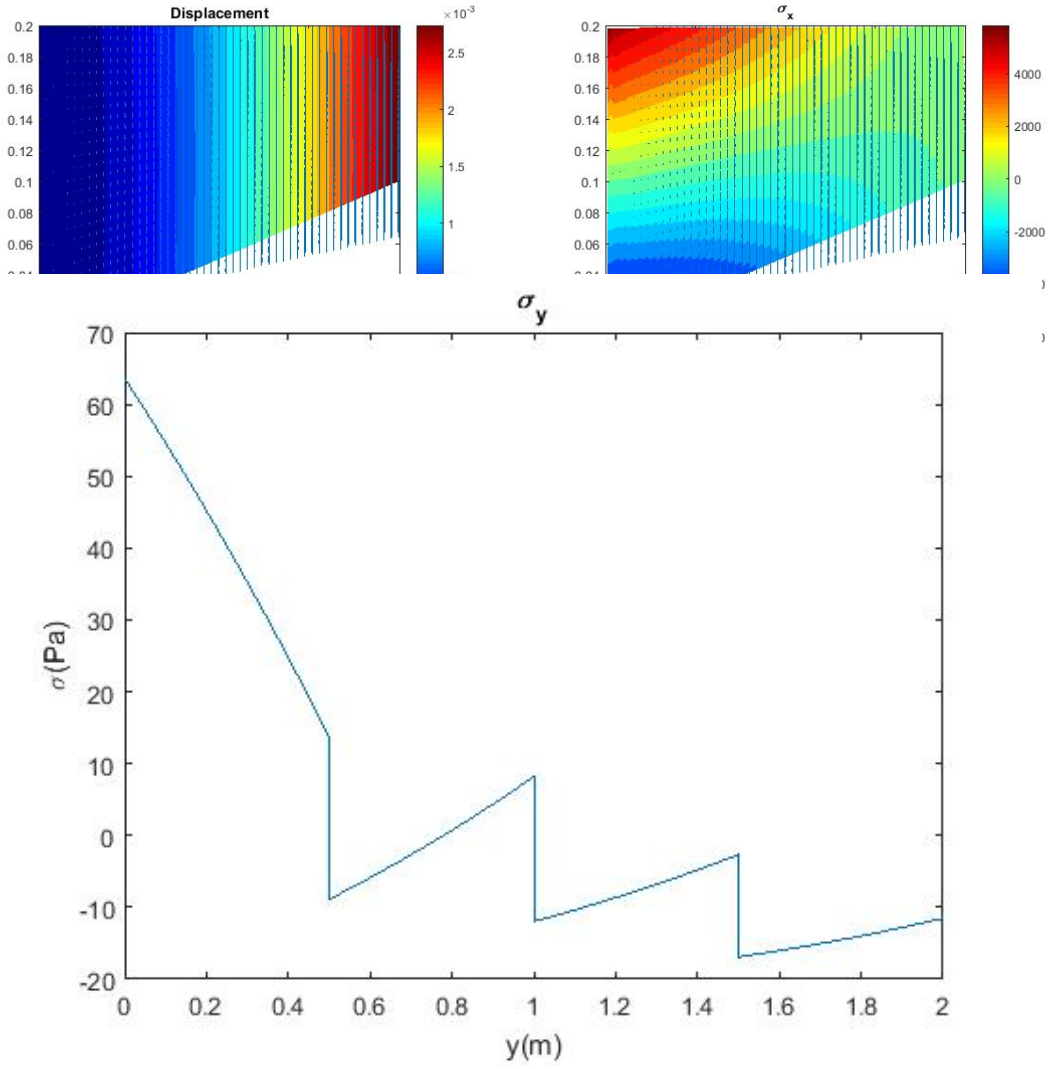
B. Effect of mesh density



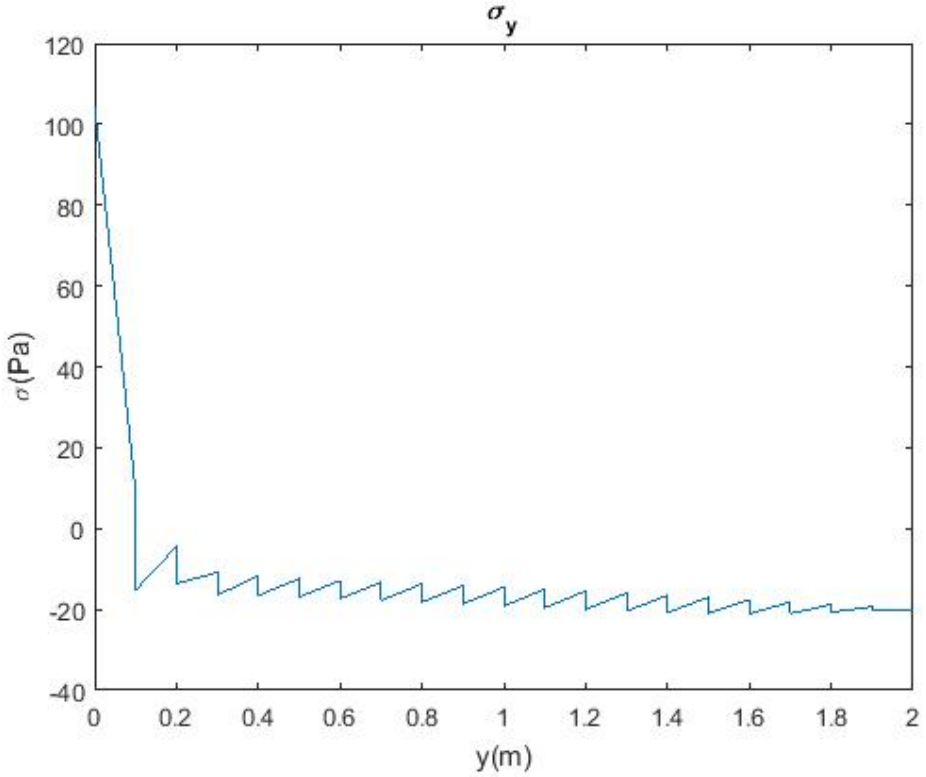
從圖一～圖六可發現（箭頭為位移的方向及大小，四張圖皆同），Mesh row 的數量會影響 x 方向位移的準確度，而 column 的數量會影響 y 方向位移的準確度，在應力方面 Mesh 的密度越高分佈越平滑。

C. Effect of parameter a

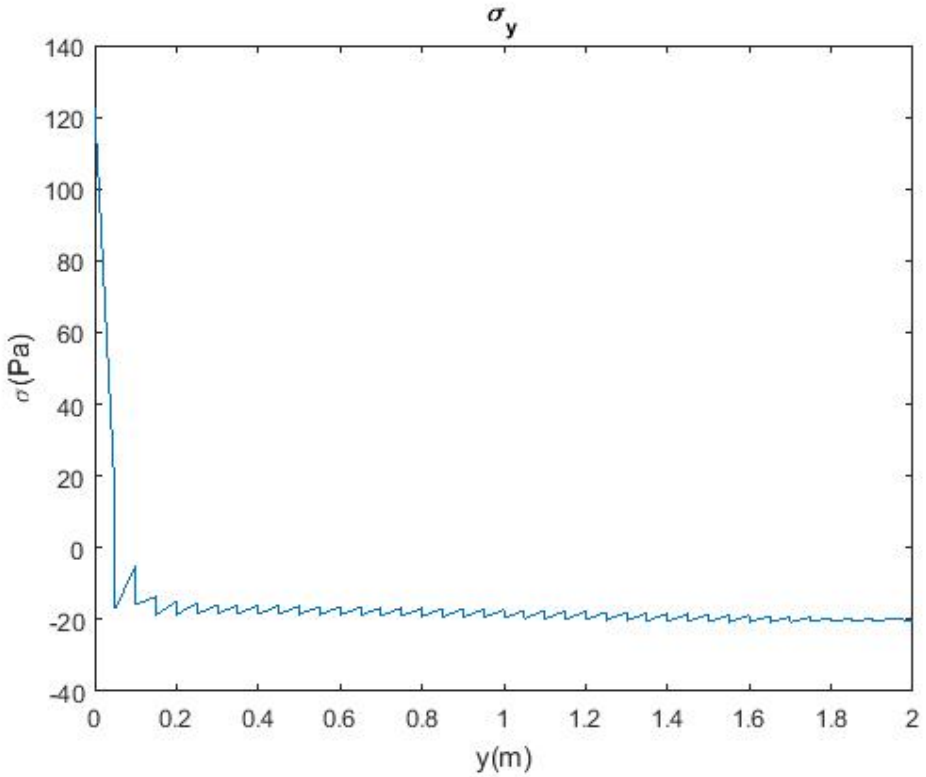
以下為不同的 a 所計算出的結果（皆為 25x50 mesh）



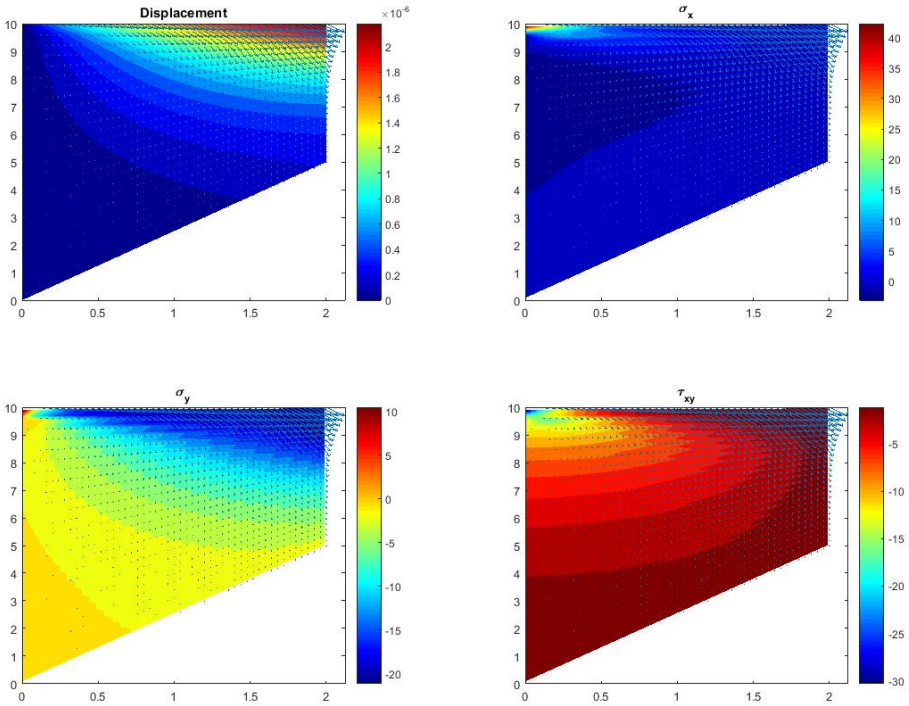
圖七 a=0.1


σ_y
120
100
80
60
40
20
0
-20
-40
σ(Pa)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
y(m)

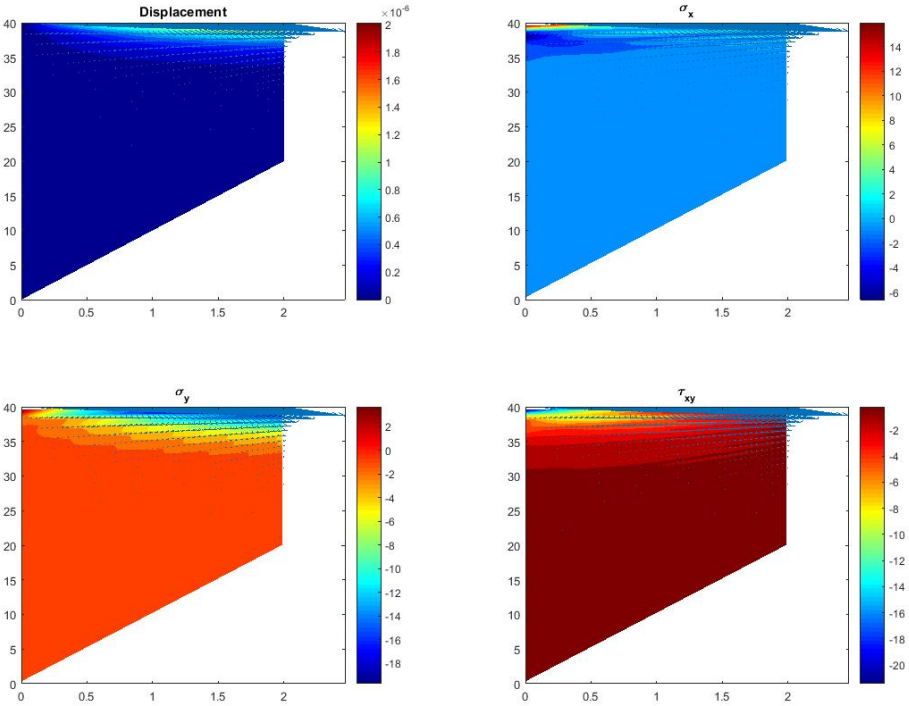
圖八 a=0.5


σ_y
140
120
100
80
60
40
20
0
-20
-40
σ(Pa)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
y(m)

圖九 a=1


Displacement
σ_x
σ_y
τ_{xy}

圖十 a=5


Displacement
σ_x
σ_y
τ_{xy}

圖十一 a=20