N1066 Feedforward Control 2018

Homework #1

Due: Mar. 19 (Mon.) in class

Note: 1. Hand in the hard copy in class. 2. Zip your codes in a file and email to TA.

Problem 1:

Consider the model of the flexural stage (Eq. (2) in the attachment by Hector Perez)

(a) What is the relative degree of the system?

(b) Find a state-space model (in control canonical form shown below) of this flexural stage.

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

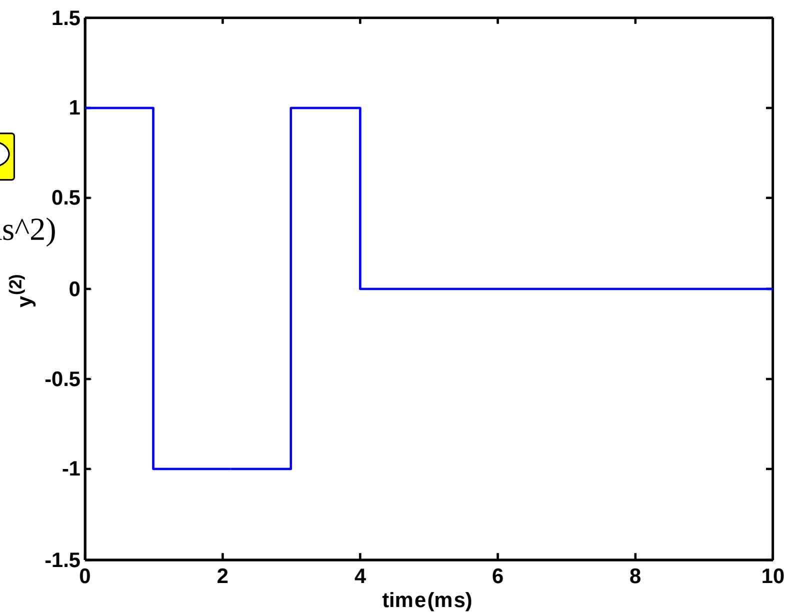
$$\dot{x} = \begin{bmatrix} -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & \cdots & 0 & b_0 & \cdots & b_m \end{bmatrix} x$$

(c) Find the inverse system model.

(d) Simulate the inverse system model using MATLAB(lsim command) to find the inverse feedforward input. The desired acceleration profile is shown below.

(V/ms^2)



(e) Apply this feedforward input to the system model found in part (b) to verify that the output tracking of y is achieved (use lsim command again)**(note: the problem only provide acceleration $y^{(2)}$,read the "findy.m" and " writing your own function.pdf" on the course website, then figure out how to use matlab commend "ode45" to reconstruct the output y.)**

(f) Investigate the effect of 5% variation in the DC gain of the system; in particular simulate the response of the system (using the original inverse input) when the numerator constant 5625 has changed by 5% and -5%. What is the maximum error in the output? How could this error be reduced?

(g) Design a feedback controller and investigate the effect of the 5% error in the parameter for the closed loop system.

(h) Compare the performance of the system with inverse plus feedback input and without inverse input (i.e., only the feedback input). Compare the tracking error and the magnitude of the total input (feedback and feedforward) applied to this system.

Problem 2:

Consider a general transfer function for a linear system with relative degree r=n-m (n greater than or equal to m).

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

Find its description in control canonical form. Show that you need to differentiate the output r-times to find the inverse input.

Problem 3:

Find the poles of the inverse system of problem 2 explicitly—prove your results. You should verify your results using some values for the constants in the transfer function (for example, use the system in problem 1 to support your results).

Attachment

Modeling of Piezo-Based Flexural Positioning System

By Hector Perez

The Piezo-based Positioning System (Piezo-stage) is a linear system that can be used in applications that require small (tens of micrometers) displacements. The piezo-stage consists of a piezo-actuator, flexural mechanism and a sensor as shown in Fig. 1. An input voltage is applied to a piezo-actuator, which expands and pushes against the flexural mechanism that amplifies the piezo-movement. The displacement of the piezo-stage is measured using an inductive sensor. The inductive sensor has a gain of 0.2 volts/μm and measures a output voltage (V_{out}).

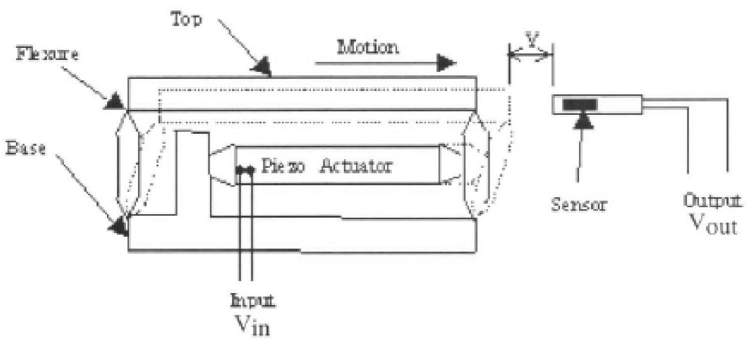


Figure 1: Schematic of piezo-stage

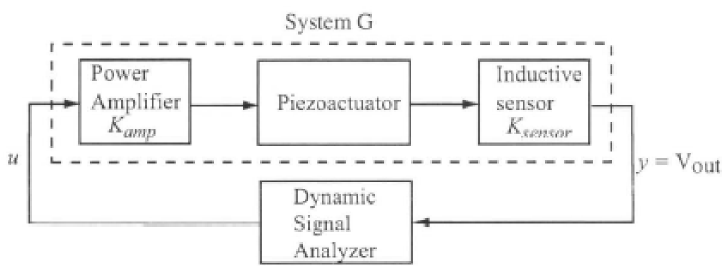


Figure 2: Modeling the vibrational dynamics using a dynamic signal analyzer

The model we obtained includes the dynamics of the amplifier, the piezo-stage and the inductive sensor, see Fig. 2. The frequency response of the system (after curve-fitting) is shown in Fig. 3, and the poles, zeros, and gain K of the system were found as (unit for 's' is rad/s)

$$poles = \begin{bmatrix} -347.0 \pm 1954i \\ -171.0 \pm 4148.8i \\ -66.5 \pm 5360.8i \end{bmatrix} \qquad zeros = \begin{bmatrix} -160.1 \pm 4208.2i \\ -49.3 \pm 5396.6i \end{bmatrix} \qquad K = 1.1883 \times 10^{7}$$

which corresponds to the transfer function,

$$TF = \frac{1.188\times10^{7}s^{4} + 4.977\times10^{9}s^{3} + 5.396\times10^{14}s^{2} + 1.299\times10^{17}s + 5.625\times10^{21}}{s^{6} + 1169s^{5} + 5.03\times10^{7}s^{4} + 4.59\times10^{10}s^{3} + 6.853\times10^{14}s^{2} + 3.917\times10^{17}s + 1.952\times10^{21}} \tag{1}$$

The DC gain of the system (in dB) equals to $20\log_{10}(5.625*10^{21}/1.952*10^{21})$=9.19dB. To avoid numerical problems in MATLAB(this is a very fast system with settling time in millisecond), we change the unit of time from second to millisecond. Therefore, we replace each s by 1000s and get the transfer function as (unit for ‘s’ is rad/ms)

$$TF = \frac{11.88s^{4} + 4.977s^{3} + 539.6s^{2} + 129.9s + 5625}{s^{6} + 1.169s^{5} + 50.3s^{4} + 45.94s^{3} + 685.3s^{2} + 391.7s + 1952} \tag{2}$$

Warning! If users use step command in MATLAB, rescale the time axis to millisecond. (see Fig. 4)

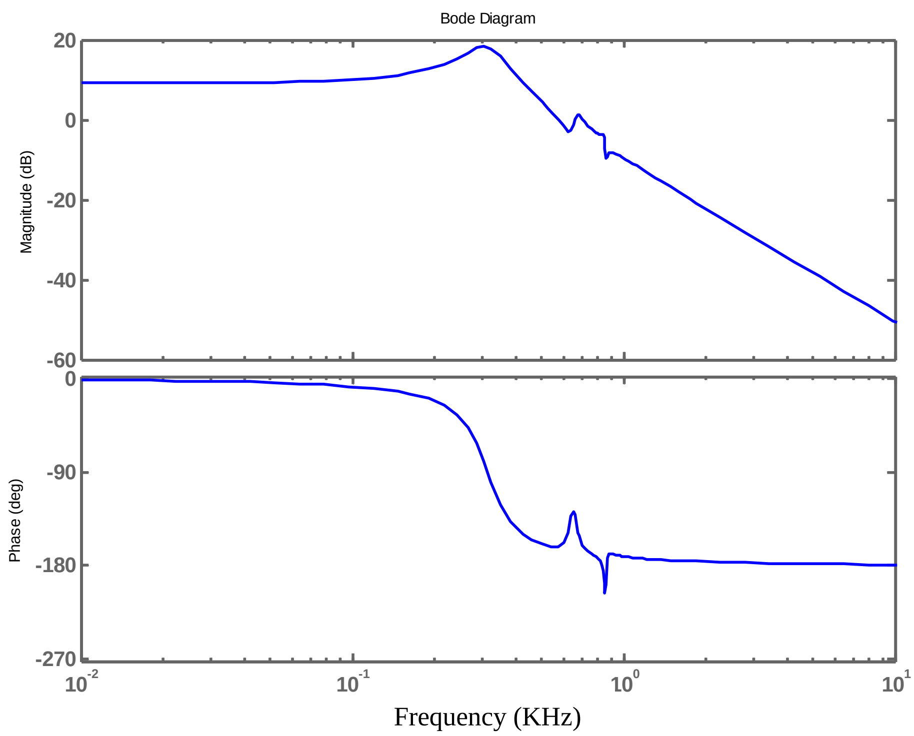


Fig. 3: Frequency response of the piezo-stage

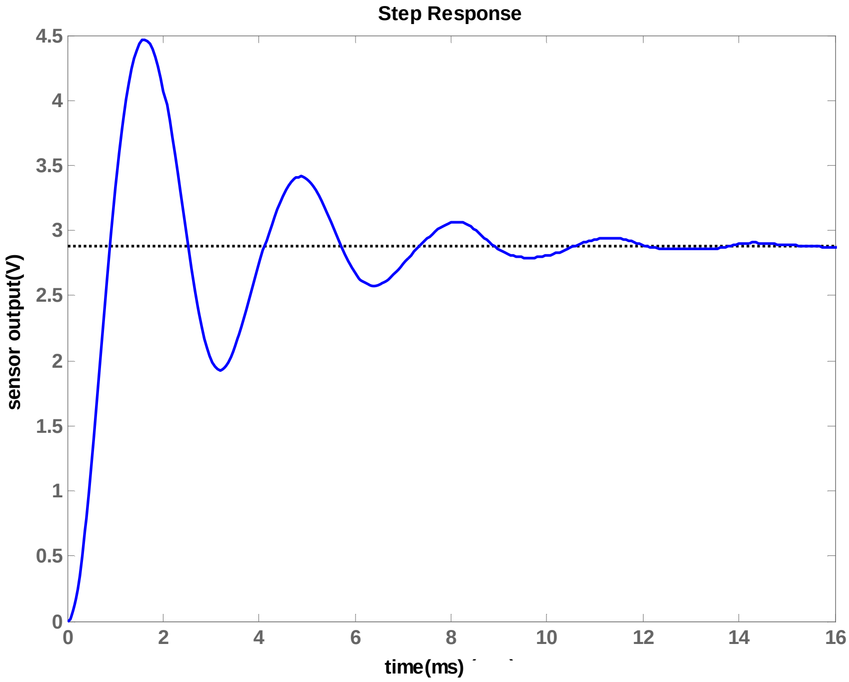


Fig. 4: Step response of the piezo-stage. Note that the steady state value is equal to 2.8816 that corresponds to 9.19dB, i.e., the DC gain of the system. Also note that the time is in millisecond. The actual displacement can be inferred by using the inductive sensor gain of 0.2 volt/ μ m.