Feedforward Control Homework 1

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1 Problem 1

Consider the model of the flexural stage (Eq. (2) in the attachment by Hector Perez)

(a) What is the relative degree of the system?

By definition,

$$\text{relative degree} = \text{order of denominator–order of numerator} \tag{1}$$

In this case, the transfer function:

$$\text{TF} = \frac{11.88s^4 + 4.977s^3 + 539.6s^2 + 129.9s + 5625}{s^6 + 1.169s^5 + 50.3s^4 + 45.94s^3 + 685.3s^2 + 391.7s + 1952} \tag{2}$$

Where

$$\begin{aligned} \text{order of denominator} &= 6 \\ \text{order of numerator} &= 4 \\ \text{relative degree} &= 6 - 4 = 2 \end{aligned}$$

(b) Find a state-space model (in control canonical form shown below) of this flexural stage.

$$A = \begin{pmatrix} -1.17 & -50.3 & -45.9 & -685.0 & -392.0 & -1966.0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1.0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 11.9 & 4.98 & 540.0 & 130.0 & 5622.0 \end{pmatrix}$$

(c) Find the inverse system model.

$$
\begin{aligned}
\dot{x}_{\text{inv}} &= A_{\text{inv}}x_{\text{inv}} + B_{\text{inv}}y_d^{(r)} \\
&= [A - BK_y]x + [BB_y]y_d^{(r)} \\
&= \begin{pmatrix} -0.419 & -45.4 & -10.9 & -473.0 & -5.68\,10^{-14} & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \end{pmatrix} x + \begin{pmatrix} 0.0842 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} y_d^{(r)}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
u_{\text{inv}} &= C_{\text{inv}}x_{\text{inv}} + D_{\text{inv}}y_d^{(r)} \\
&= [-K_y]x + [B_y]y_d^{(r)} \\
&= [\frac{-CA^r}{CA^{r-1}B}]x + [\frac{1}{CA^{r-1}B}]y_d^{(r)} \\
&= \begin{pmatrix} 0.75 & 4.88 & 35.0 & 212.0 & 392.0 & 1966.0 \end{pmatrix} x + 0.0842\ y_d^{(r)}
\end{aligned} \tag{4}
$$

(d) Simulate the inverse system model using MATLAB(lsim command) to find the inverse feedforward input. The desired acceleration profile is shown below.