

計算機圖學期末報告 倒單擺模擬

機械系106丙

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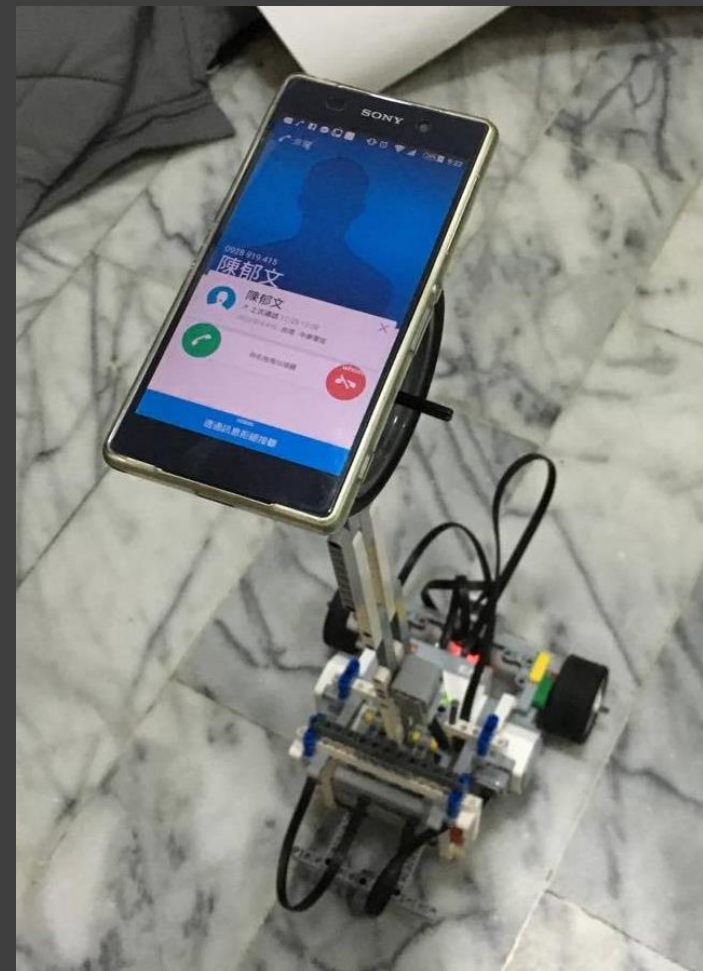
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摘要

◆研究動機

自控期末Project...

Processing模擬倒單擺



摘要

◆預期目標

模擬結果可套用到實際運用上

摘要

◆部分完成

模擬結果合理但和現實差異太大

說明

◆數學模型

Dynamic Analysis

$x_m = x_M + l \sin \theta$
 $y_m = l \cos \theta$
 $\dot{x}_m = \dot{x}_M + l \dot{\theta} \cos \theta$
 $\dot{y}_m = -l \dot{\theta} \sin \theta$
 $\ddot{x}_m = \ddot{x}_M + l \ddot{\theta} \cos \theta - l \dot{\theta}^2 \sin \theta$
 $\ddot{y}_m = -l \ddot{\theta} \sin \theta - l \dot{\theta}^2 \cos \theta$

Take FBD of pendulum:

$H = m \ddot{x}_m \quad \text{--- (1)}$
 $V - mg = m \ddot{y}_m \quad \text{--- (2)}$
 $V l \sin \theta - H l \cos \theta = I \ddot{\theta} \quad \text{--- (3)}$

FBD of Cart:

$u - H = M \ddot{x}_M \quad \text{--- (4)}$

sub (1) - (2) into (3) - (4)

$$\Rightarrow m(g + \ddot{y}_m) l \sin \theta - m \ddot{x}_m l \cos \theta = I \ddot{\theta}$$

$$u - m \ddot{x}_m = M \ddot{x}_M$$

$$\Rightarrow m(g - l \ddot{\theta} \sin \theta - l \dot{\theta}^2 \cos \theta) l \sin \theta - m \ddot{x}_m l \cos \theta = I \ddot{\theta}$$

$$u - m(\ddot{x}_M + l \ddot{\theta} \cos \theta - l \dot{\theta}^2 \sin \theta) = M \ddot{x}_M$$

$$\Rightarrow \cos \theta \ddot{x}_M + \left(\frac{I}{ml} + l\right) \ddot{\theta} = g \sin \theta$$

$$\left(\frac{M}{ml} + l\right) \ddot{x}_M + \cos \theta \ddot{\theta} = \frac{u}{ml} + \dot{\theta}^2 \sin \theta$$

find \ddot{x}_M , $\ddot{\theta}$:

$$\ddot{x}_M = \frac{A \dot{\theta}^2 \sin \theta - g \sin \theta \cos \theta + \frac{A}{ml} u}{AB - \cos^2 \theta}$$

$$\ddot{\theta} = \frac{g B \sin \theta - \dot{\theta}^2 \sin \theta \cos \theta - \frac{\cos \theta}{ml} u}{AB - \cos^2 \theta}$$

where $A = \frac{I}{ml} + l$, $B = \frac{M}{ml} + \frac{I}{l}$

説明

◆ Runge-Kutta method Numerical IVP-ODE solver

$$\begin{aligned}\dot{y} &= f(t, y) & y(t_0) &= y_0 \\ y_{n+1} &= y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ t_{n+1} &= t_n + h\end{aligned}$$

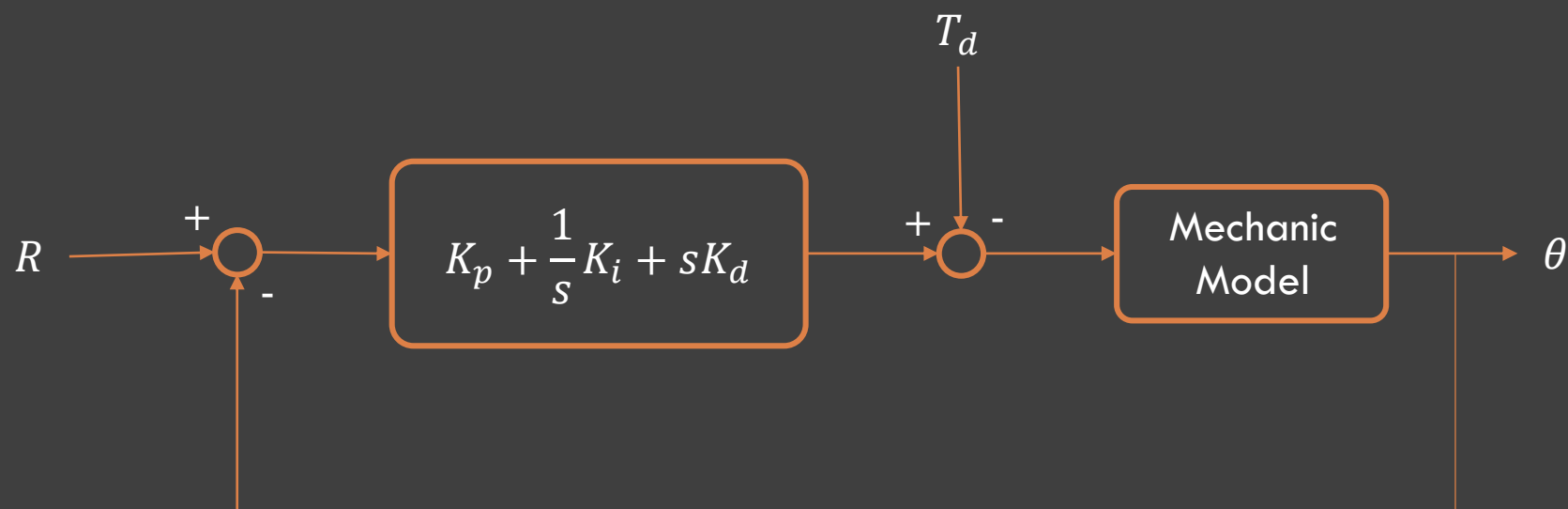
where h is step size, and for $n = 0, 1, 2, 3 \dots$

$$\begin{aligned}k_1 &= f(t_n, y_n) \\ k_2 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right) \\ k_3 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right) \\ k_4 &= f(t_n + h, y_n + hk_3)\end{aligned}$$

說明

◆控制模型

PID Control



說明

◆使用函式庫

OpenGL

Arduino PID Library

RKF45

STL

RKF45

$$\begin{aligned}k_1 &= hf(t_k, y_k), \\k_2 &= hf\left(t_k + \frac{1}{4}h, y_k + \frac{1}{4}k_1\right), \\k_3 &= hf\left(t_k + \frac{3}{8}h, y_k + \frac{3}{32}k_1 + \frac{9}{32}k_2\right), \\k_4 &= hf\left(t_k + \frac{12}{13}h, y_k + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right), \\k_5 &= hf\left(t_k + h, y_k + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right), \\k_6 &= hf\left(t_k + \frac{1}{2}h, y_k - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right).\end{aligned}$$

說明

◆困難

RKF45的使用

Select不準確（尚未解決）

模擬不出實際情況，因未考慮摩擦、馬達功率輸出轉換

功能說明

◆功能

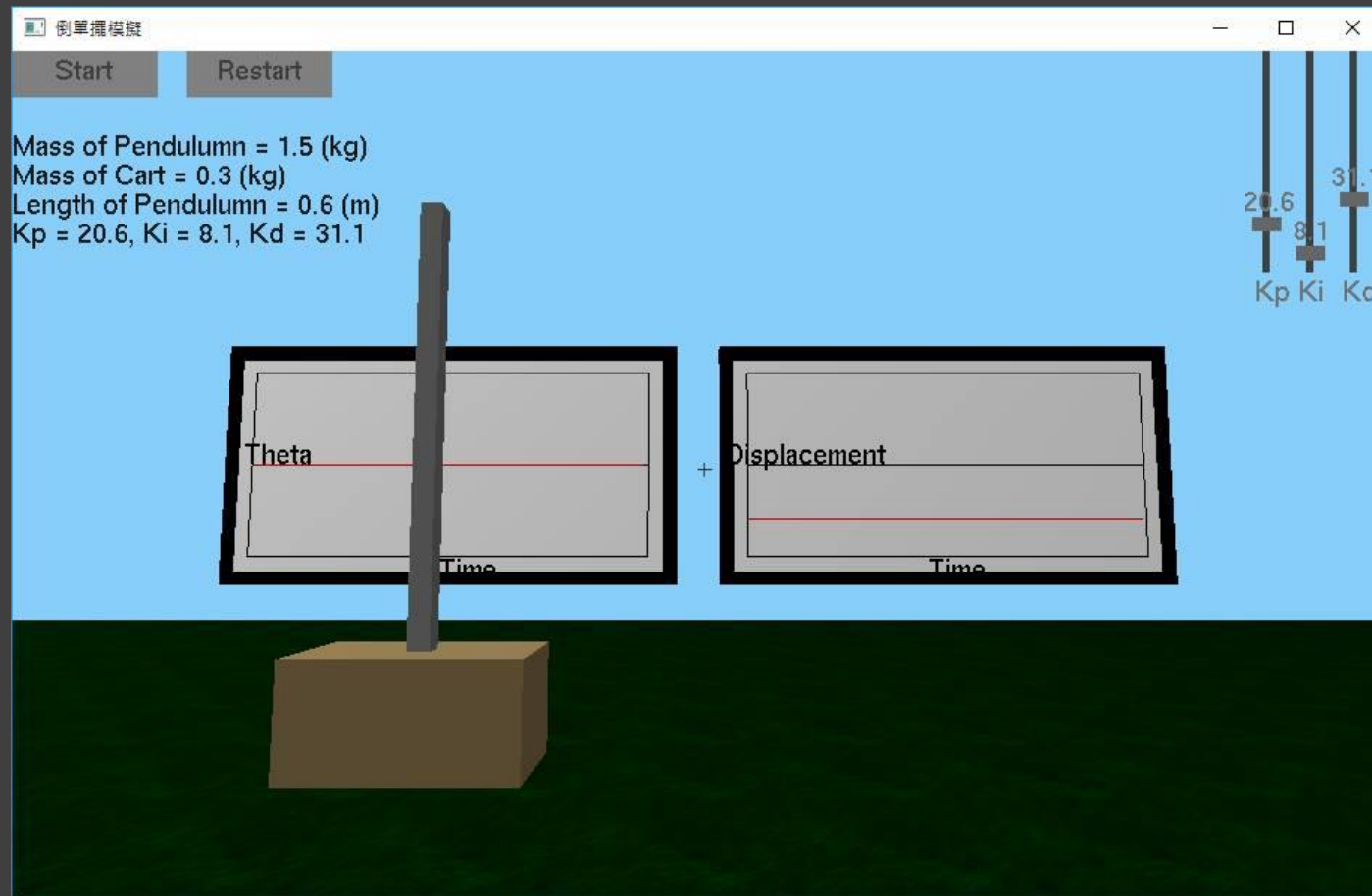
PID倒單擺模擬

視角移動

Texture Mapping

PID Sliding Bar

Dashboard



功能說明

◆車身材質

| | | | |
|------------------------------|----|---|----|
| $(\frac{2}{3}, \frac{1}{6})$ | 左側 | $(\frac{2}{3}, \frac{1}{2})$ $(\frac{2}{3}, \frac{2}{3})$ | |
| 側 | 車頂 | 後 | 車底 |
| $(\frac{1}{3}, \frac{1}{6})$ | 右側 | $(\frac{1}{3}, \frac{1}{2})$ $(\frac{1}{3}, \frac{2}{3})$ | |

Demo Time

Reference

◆數學分析

<http://rapot2014.blogspot.tw/2014/08/arduino-inverted-pendulum5-processing.html>

◆RK

https://www.wikiwand.com/en/Runge%E2%80%93Kutta_methods

http://boson4.phys.tku.edu.tw/numerical_methods/nm_units/ODE_Runge-Kutta.htm

◆RKF

https://www.wikiwand.com/en/Runge%E2%80%93Kutta%E2%80%93Fehlberg_method

<http://mathfaculty.fullerton.edu/mathews/n2003/rungekuttafehlberg/RungeKuttaFehlbergProof.pdf>

Reference

◆ RK45 in C

<http://www.ma.utexas.edu/CNA/cheney-kincaid/Ccode/CHP10/rk45.c>

◆ RKF45 in C++

http://people.sc.fsu.edu/~jburkardt/cpp_src/rkf45/rkf45.html

◆ PID

<https://github.com/geekfactory/PID>

◆ deque

<http://www.cplusplus.com/reference/deque/deque/>

◆ 視角

<http://chchwy.blogspot.tw/2011/10/8-using-viewing-and-camera-transforms.html>