Relación de problemas 3

P6

$$H(z) = \frac{z(2z-1)}{z^2+z+1} = \frac{2z^2-z}{z^2+z+1}$$

- Calcular la ecuación de recurrencia equivalente

$$H(z) = \frac{2z^2-z}{z^2+z+1} \cdot \frac{z^{-2}}{z^{-2}} = \frac{2-z^{-1}}{1+z^{-1}+z^{-2}} = \frac{Y(z)}{X(z)}$$

$$(2-z^{-1})X(z) = (1+z^{-1}+z^{-2})Y(z)$$

$$2 \cdot X(z) - X(z)z^{-1} = Y(z) + Y(z)z^{-1} + Y(z)z^{-2}$$

Transformada z inversa

$$\boxed{2x[n] - x[n-1] = y[n] + y[n-1] + y[n-2]}$$

- Dibujar el diagrama de polos y ceros

$2z^2-z \rightarrow$ Ceros $\rightarrow$ $2z^2-z=0$

$z(2z-1)=0$

$z=0 \qquad z=\frac{1}{2}$

$z^2+z+1 \Longrightarrow$ Polos $\rightarrow z^2+z+1=0$

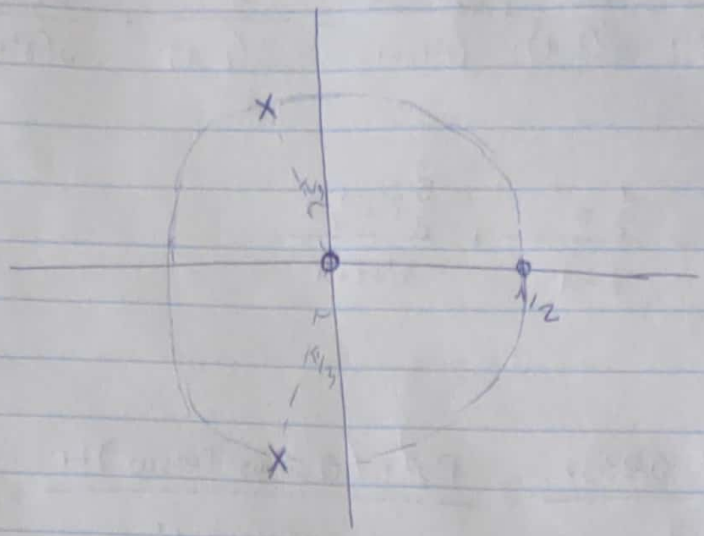
$$z=\frac{-1\pm\sqrt{1-4}}{2} = \frac{-1\pm\sqrt{-3}}{2} = \frac{-1+i\sqrt{3}}{2}, \quad \frac{-1-i\sqrt{3}}{2}$$

Módulo: $\sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{-\sqrt{3}}{2}\right)^2} = 1$

Ángulo:

$$\operatorname{arctg}\frac{\sqrt{3}/2}{-1/2} = \frac{-\pi}{3}$$

$$\operatorname{arctg}\frac{-\sqrt{3}/2}{-1/2} = \frac{\pi}{3}$$



· Hallar la respuesta al escalón r[n] del sistema (se calcula haciendo x[n]=u[n] y sin condiciones iniciales).

$$H(z)=\frac{Y(z)}{X(z)}=\frac{Y(z)}{\frac{z}{z-1}}=\frac{2z^2-z}{z^2+z+1}$$

$$Y(z)=\frac{2z^2-z}{z^2+z+1}\cdot\frac{z}{z-1}$$

$$\frac{Y(z)}{z}=\frac{2z^2-z}{z^2+z+1}\cdot\frac{1}{z-1}$$

$$\frac{Y(z)}{z}=\frac{2z^2-z}{(z^2+z+1)(z-1)}=\frac{A}{(z-1)}+\frac{Bz+C}{z^2+z+1}=\frac{Az^2+Az+A+Bz^2-Bz+Cz-C}{(z^2+z+1)(z-1)}$$

$$2z^2-z=Az^2+Az+A+Bz^2-Bz+Cz-C=z^2(A+B)+z(A-B+C)+(A-C)$$

$$\begin{cases} 2=A+B \rightarrow B=2-A \\ -1=(A-B+C) \rightarrow -1=A+A-2+A \rightarrow 1=3A \rightarrow A=1/3 \\ 0=A-C \rightarrow A=C \end{cases}$$

$$\downarrow$$

$$C=1/3$$

$$\downarrow$$

$$B=2-1/3=5/3$$

8(c)
$$\frac{2z^2+z}{(z^2+z+1)(z-1)} = \frac{1}{3(z-1)} + \frac{\frac{5}{3}z+\frac{1}{3}}{(z^2+z+1)} = \frac{1}{3(z-1)} + \frac{5z+1}{3(z^2+z+1)}$$

$$Y(z) = \frac{\frac{1}{3}z}{z-1} + \frac{\frac{5}{3}z^2+\frac{1}{3}z}{z^2+z+1}$$

$$\frac{1{,}667\,z^2+0{,}833z}{z^2+z+1} = \frac{Pz^2+(Q\,\mathrm{sen}\,\omega-P\cos\omega)zr}{z^2-2zr\cos\omega+r^2} = Pr^n\cos(\omega n)+Qr^n\,\mathrm{sen}(\omega n)$$

$$r^2=1 \rightarrow r=\pm 1$$

$$-2zr\cos\omega = z$$

$r=1$: $2\cos\omega = 1$; $r=-1$: $\cos\omega = +\frac{1}{2}$

$\cos\omega = -1/2$; $\omega = \frac{\pi}{3}$

$\omega = 2\pi/3$

$$Pz^2 = 1{,}667z^2$$
$$P = 1{,}667$$

$$\left(Q\,\mathrm{sen}\frac{\pi}{3} - 1{,}667\cos\frac{\pi}{3}\right)z = 0{,}333z$$

$$Q\,\mathrm{sen}\frac{\pi}{3} - 1{,}667\cos\frac{\pi}{3} = 0{,}333$$

$$Q_1 = \frac{7/6}{\mathrm{sen}\,\pi/3} = \frac{7\sqrt{3}}{9} \qquad Q_2 = \frac{-7\sqrt{3}}{9}$$

$$\boxed{\begin{aligned} y_1[n] &= \left(\frac{1}{3}u[n] + \frac{5}{3}\cancel{1^n}^{1}\cos\left(\frac{\pi}{3}n\right) + \frac{7\sqrt{3}}{9}\cancel{1^n}^{1}\,\mathrm{sen}\left(\frac{\pi}{3}n\right)\right)u[n] \\ y_2[n] &= \left(\frac{1}{3}u[n] + \frac{5}{3}(-1)^n\cos\left(\frac{2\pi}{3}n\right) - \frac{7\sqrt{3}}{9}(-1)^n\,\mathrm{sen}\left(\frac{2\pi}{3}n\right)\right)u[n] \end{aligned}}$$

- Comprobar que el resultado es correcto, comparando los 4 primeros valores de $y[n]$ para la ecuación de recurrencia (tomando $x[n] = u[n]$) con los valores de la respuesta al escalón $r[n]$,

$$y[n] = 2x[n] - x[n-1] - y[n-1] + y[n-2]$$
$$y_1[n] = \left(\frac{1}{3}u[n] + \frac{5}{3}\cos\left(\frac{\pi}{3}n\right) + \frac{7\sqrt{3}}{9}\operatorname{sen}\left(\frac{\pi}{3}n\right)\right)u[n]$$
$$y_2[n] = \left(\frac{1}{3}u[n] + \frac{5}{3}\cos\left(\frac{2\pi}{3}n\right) - \frac{7\sqrt{3}}{9}\operatorname{sen}\left(\frac{2\pi}{3}n\right)\right)u[n]$$

cuando $n=0$

$y[0] = 2$
$y_1[0] = 2$
$y_2[0] = 2$

cuando $n=1$

$y[1] = 2u[1] - u[0] - y[0] + y[-1] = 2 - 1 - 2 = -1$
$y_1[1] = \frac{7}{3}$
$y_2[1] = \emptyset$

cuando $n=2$

$y[2] = 2 - 1 - (-1) - 2 = 0$
$y_1[2] = \frac{2}{3}$
$y_2[2] = \frac{2}{3}$