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(P2)

$$H(z) = \frac{1}{4} + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}$$

Filtro no recursivo (FIR) porque tiene la forma

$$\boxed{H(z) = \sum_{i=0}^{n} h_i z^{-1} = h_0 + h_1 z^{-1} + h_2 z^{-2} + \dots + h_n z^{-n}}$$

1 • Calcular la ecuación de recurrencia

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4} + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}$$

$$Y(z) = \frac{1}{4} X(z) + \frac{1}{2} z^{-1} X(z) + \frac{1}{4} z^{-2} X(z)$$

Transformada Z inversa

$$y[n] = \frac{1}{4} x[n] + \frac{1}{2} x[n-1] + \frac{1}{4} x[n-2]$$

2 • Diagrama de polos y ceros

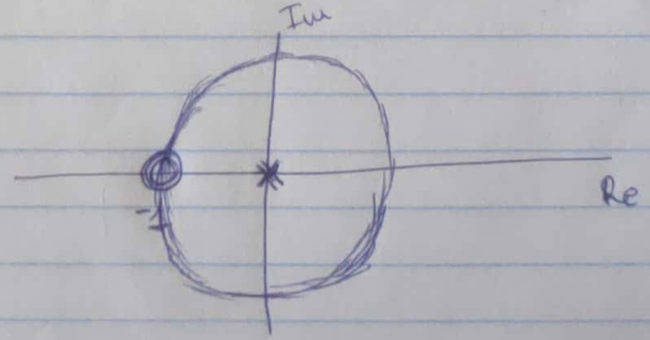
$$H(z) = \frac{1}{4} + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2} = \frac{1 + 2z^{-1} + z^{-2}}{4} = \frac{1 + 2z^{-1} + z^{-2}}{4} \cdot \frac{z^2}{z^2} =$$

$$= \frac{z^2 + 2z + 1}{4z^2} = \frac{1}{4} \frac{z^2 + 2z + 1}{z^2}$$

Ceros: $Y(z) = 0 \rightarrow z^2 + 2z + 1 = 0 \quad z = \frac{-2 \pm \sqrt{4-4}}{2} = \frac{-2}{2} = -1$

$(z+1)^2 = 0$ dos raíces

polos: $X(z) = 0 \rightarrow z^2 = 0 \; ; \; z = 0$ doble raíz



3 · Calcula el valor del módulo del filtro para:

$$H(z)=\frac{1}{4}\,\frac{z^2+2z+1}{z^2}$$

$$z=e^{j\omega}=\cos(\omega)+j\,\mathrm{sen}(\omega)$$

- $\omega=0 \longrightarrow z=\cos(0)+j\,\mathrm{sen}(0)=1$

$$\cancel{H(0)=\frac{1}{4}\frac{0+0+1}{0}}$$

$$H(0)=\frac{1}{4}\,\frac{1+2+1}{1}=\underline{1}$$

$$\underline{|H(0)|=1}$$

- $\omega=45^\circ=\frac{\pi}{4} \longrightarrow z=\cos\left(\frac{\pi}{4}\right)+j\,\mathrm{sen}\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}$

$$H\left(\frac{\pi}{4}\right)=\frac{\left(\frac{\sqrt2}{2}+j\frac{\sqrt2}{2}\right)^2+2\cdot\left(\frac{\sqrt2}{2}+j\frac{\sqrt2}{2}\right)+1}{4\cdot\left(\frac{\sqrt2}{2}+j\frac{\sqrt2}{2}\right)^2}=\frac{j+\sqrt2+j\sqrt2+1}{4j}\cdot\frac{j}{j}=$$

$$\cancel{=\frac{1}{4}+\frac{\sqrt2}{4j}+\frac{\sqrt2}{4}+\frac{1}{4j}}=\frac{-1+j\sqrt2+\sqrt2+j}{-4}=\frac{1}{4}-j\frac{\sqrt2}{4}+\frac{\sqrt2}{4}-\frac{1}{4}j=$$

$$=\frac{1}{4}+\frac{\sqrt2}{4}+j\left(-\frac{\sqrt2}{4}-\frac{1}{4}\right)=\frac{1+\sqrt2}{4}+j\left(\frac{-1-\sqrt2}{4}\right)=\frac{1}{4}\left((1+\sqrt2)+j(-1-\sqrt2)\right)=$$

$$\left|H\left(\frac{\pi}{4}\right)\right|=\frac{1}{4}\sqrt{(1+\sqrt2)^2+(-1-\sqrt2)^2}=\frac{1}{4}\sqrt{3+2\sqrt2+3+2\sqrt2}=$$

$$=\frac{1}{4}\cdot\sqrt{6+4\sqrt2}=\underline{0{,}8536}$$

$$- \omega = 90^\circ = \pi/_2 \longrightarrow z = \cos(\pi/_2) + j\,\text{sen}(\pi/_2) = j$$

$$H\left(\frac{\pi}{2}\right) = \frac{1}{4}\,\frac{j^2 + 2j + 1}{j^2} = \frac{1}{4}\,\frac{-1+2j+1}{-1} = \frac{1}{4}\,\frac{2j}{-1} = \frac{-2j}{4} = -\frac{1}{2}j$$

$$\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\left(\frac{1}{2}\right)^2} = \frac{1}{2} = 0{,}5$$

$$- \omega = 120^\circ = \xrightarrow{2\pi/3} z = \cos\left(\frac{2\pi}{3}\right) + j\,\text{sen}\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + j\,\frac{\sqrt{3}}{2}$$

$$H\left(\frac{2\pi}{3}\right) = \frac{\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)^2 + 2\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right) + 1}{4\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)^2} = \frac{-\frac{1}{2} - j\frac{\sqrt{3}}{2} - 1 + j\sqrt{3} + 1}{4\cdot\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)} =$$

$$\cancel{= \frac{1/2 + j\frac{\sqrt{3}}{2}}{-2 - j2\sqrt{3}} = \frac{\frac{1}{2}\left(-1+j\sqrt{3}\right)}{-2\left(1+j\sqrt{3}\right)} = -\frac{1}{4}\,\frac{-1+j\sqrt{3}}{1+j\sqrt{3}}\;\frac{-1+j(\sqrt{3})}{-1+j(\sqrt{3})} =}$$

$$\cancel{= -\frac{1}{4}\,\frac{-2-2\sqrt{3}j}{-\sqrt{3}-1}}$$

$$= \frac{\left(-1/2 + j\frac{\sqrt{3}}{2}\right)\left(-1/2 + j\frac{\sqrt{3}}{2}\right)}{4\left(-1/2 - j\sqrt{3}/2\right)\left(-1/2 + j\frac{\sqrt{3}}{2}\right)} = \frac{-1/2 - j\sqrt{3}/2}{4\left(\frac{1}{4} + \frac{3}{4}\right)} = \frac{-1/2 - j\sqrt{3}/2}{4} =$$

$$= -\frac{1}{8} - j\,\frac{\sqrt{3}}{8} \longrightarrow \left|H\left(\frac{2\pi}{3}\right)\right| = \sqrt{\left(\frac{1}{8}\right)^2 + \left(\frac{\sqrt{3}}{8}\right)^2} = \sqrt{\frac{1}{64} + \frac{3}{64}} = \sqrt{\frac{4}{64}} =$$

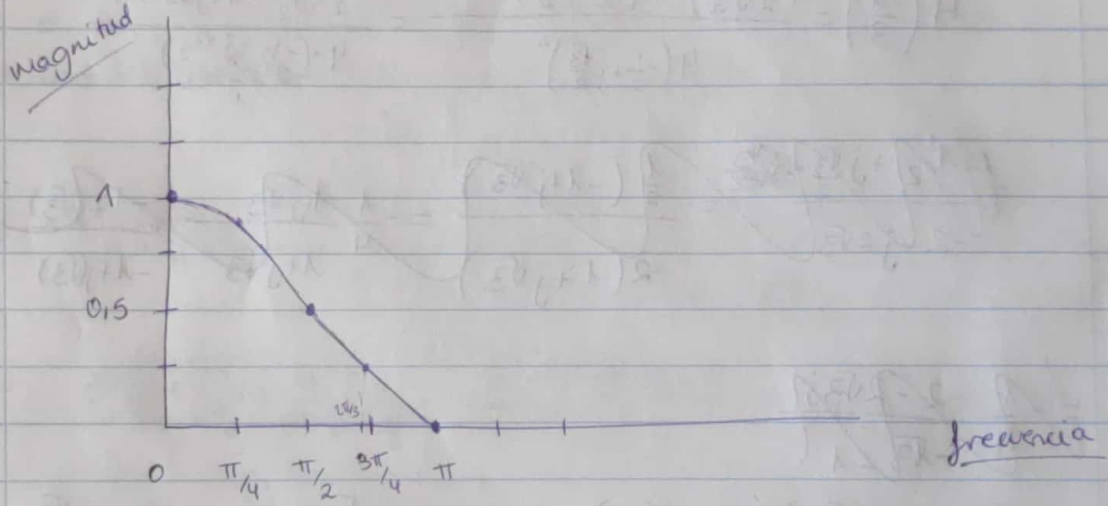
$$= \sqrt{\frac{2^2}{8^2}} = \frac{1}{4}$$

$-\omega = 180° \simeq \pi \rightarrow z = \cos(\pi) + j \operatorname{sen}(\pi) = -1$

$$H(\pi) = \frac{(-1)^2 + 2\cdot(-1) + 1}{4\cdot(-1)^2} = \frac{0}{4} = \emptyset$$

$$|H(\pi)| = \emptyset$$

4 • Gráfica con valores del filtro



• Máxima ganancia y mayor atenuación

- El punto de mayor ganancia se produce en $\emptyset$ radianes con una ganancia de 1.

- El punto de mayor atenuación se produce en π radianes con un valor de $\emptyset$