CIS 575. Introduction to Algorithm Analysis
Material for April 22, 2024

The Edmonds-Karp Algorithm

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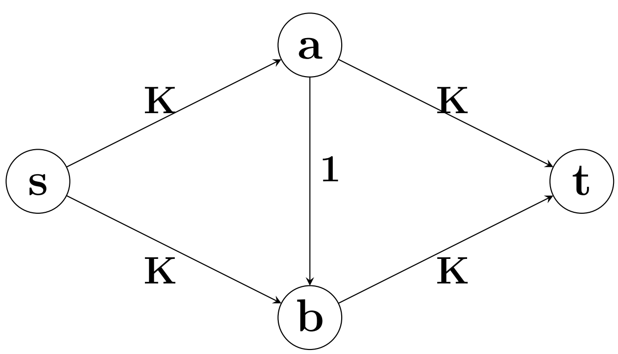
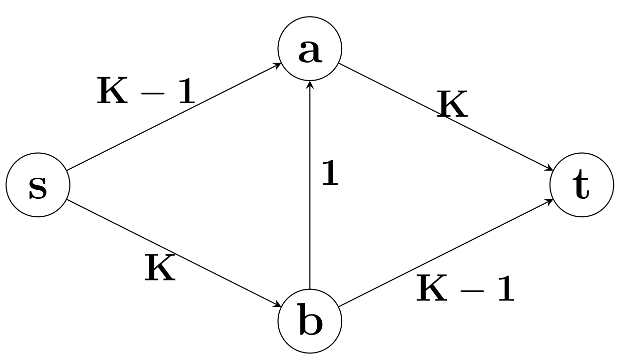
The topic of this note is covered in *Cormen*'s Section 24.2.

1 Edmonds-Karp Algorithm

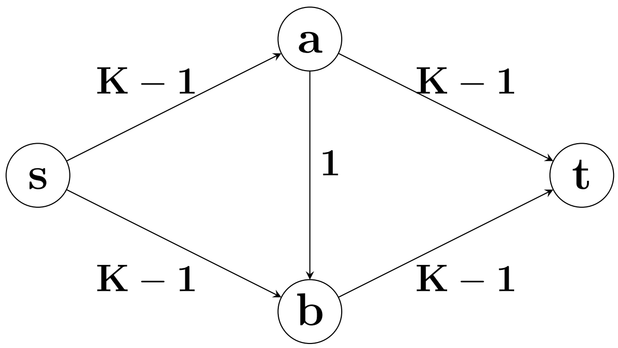
We have introduced the Ford-Fulkerson method, and argued that for a network with maximum flow M, where all capacities are integers, the method will do at most M iterations. For many networks, it will do much fewer iterations. But unfortunately, for some networks the number of iterations may be close to M, and even equal M. To see this, consider the network



where K is an integer. Assume that the Ford-Fulkerson method chooses *sabt* as augmenting path; this yields the residual network



and assuming that the method then chooses *sbat* as augmenting path, we next get the residual network



We see that if the Ford-Fulkerson method repeatedly chooses augmenting paths of the form *sabt* or *sbat*, the number of iterations will be $2K$ which equals the maximum flow value.

On the other hand, if the Ford-Fulkerson method instead as augmenting paths chooses first *sat* and next *sbt* (or vice versa), only 2 iterations are needed.

This motivates the **Edmonds-Karp** algorithm which is an instantiation of the Ford-Fulkerson method that does *not* consider *all* augmenting paths but only the **shortest**, that is those with the fewest number of edges. Such edges can be discovered by a breadth-first search; at each step there may be more than one shortest path and hence the Edmonds-Karp algorithm is not fully deterministic.

Let us consider execution of the Edmonds-Karp algorithm on a network $G = (V, E)$. Observe that for each iteration, at least one edge disappears, though other edges may (re)appear. But one can show that if an edge disappears, and then later reappears and again disappears, its distance from the source (measured as the number of edges in the shortest path) has increased by at least 2; as that distance is bounded by $|V|$, this shows that at most $O(|V| \cdot |E|)$ iterations are needed.

Since each iteration will still run in time $O(|E|)$, we infer that the **running time** of the **Edmonds-Karp** algorithm is in $\mathbf{O}(|\mathbf{V}| \cdot |\mathbf{E}|^{\mathbf{2}})$ (which may improve the bound inherited from the Ford-Fulkerson method).