CIS 575. Introduction to Algorithm Analysis
Material for January 31, 2024

Trees

The topic of this note is treated in great detail in *Cormen*'s Appendix B.5.

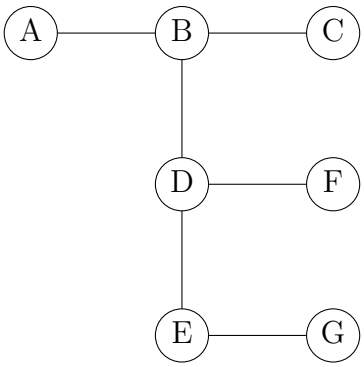
1 Trees

A **tree** is a special kind of graph, simply defined as

> a tree is an undirected graph that is connected and acyclic.

It appears obvious that this states *necessary* conditions for being a tree, as we don't want a tree to have parts not connected to each other (then it would rather be a "forest") and as we don't want branches to grow into each other so as to form a cycle.

But we may wonder: does everything connected and acyclic really deserve the honor of being called a tree? For example, let us look at a connected and acyclic graph which may not look much like a tree:



But we may pick any node and redraw the graph with that node at the top, as done in Figure 1 for A, B and G, with the resulting graphs having quite different shapes.

An in fact, relying on the unnatural but widespread convention of depicting a treet with its root up and its branches down, each such graph can be viewed as a *rooted tree*.

2 Rooted Trees

A rooted tree is a tree (as defined above), with one of the nodes being designated the **root**. We have the key property:

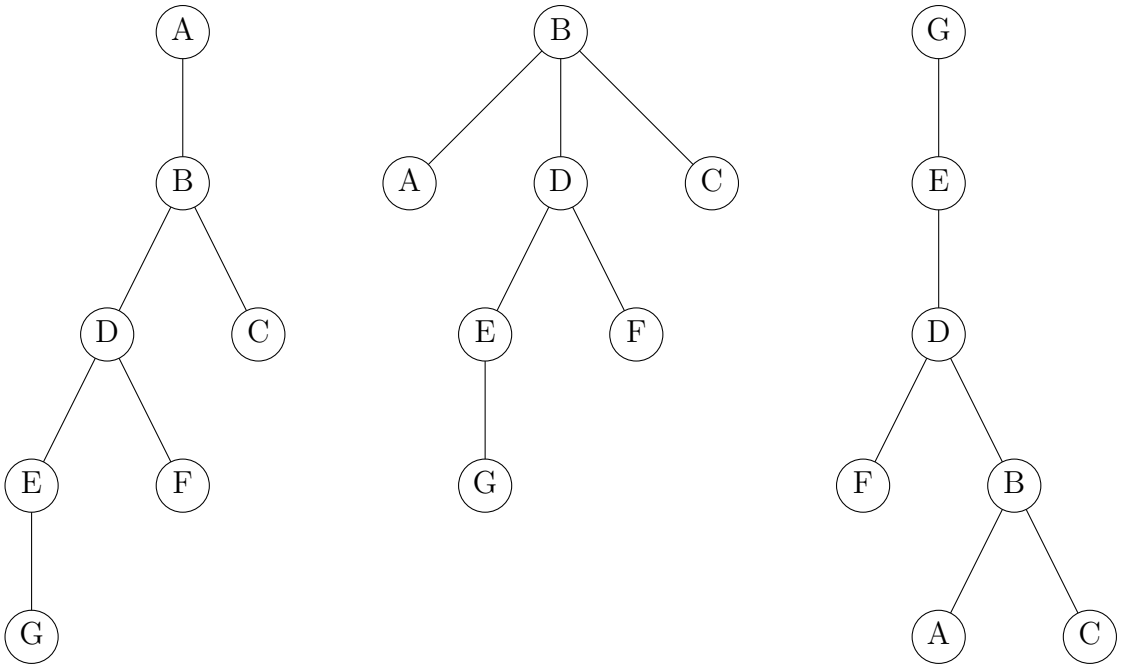


Figure 1: The same tree, viewed in 3 ways as a rooted tree.

for each node, there is exactly one path to the root from that node

where "at least one" follows from the tree being connected, and "at most one" follows from the tree having no cycles. Some terminology:

- The **parent** of a non-root node is the first node encountered on the unique path to the root.
- A node u is a **child** of w iff w is the parent of u; observe that a node may have any number of children.
- A node without children is called a **leaf**.
- The **depth** of a node is the number of edges in the path from that node to the root.
- The **height** of a tree is the maximum depth of a leaf.

For example, for the leftmost tree in Figure 1 we have: node B is the parent of nodes C and D; the leaves are C, F, and G; the depth of C is two and the depth of G is four which is also the height of the tree.

A special kind of rooted tree is a **binary** tree where each node may have at most two children: either no child at all, only a "left child", only a "right child", or both a "left child" and a "right child".