

CIS 575. Introduction to Algorithm Analysis

Material for April 17, 2024

Depth-First Search for Directed Graphs

©2020 Torben Amtoft

The topic of this note is part of what is covered in *Cormen's* Section 20.3.

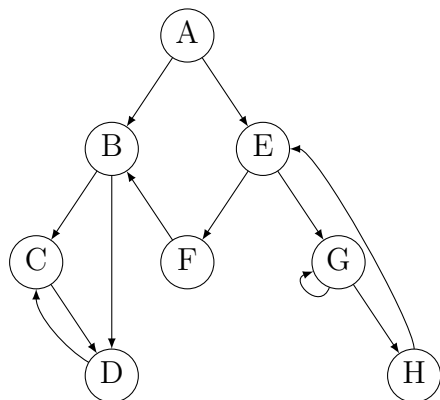
1 Depth-First Search for Directed Graphs

In the previous note, we considered depth-first search for *undirected* graphs. For **directed graphs**, much of what was written will still apply. The DFS algorithm will produce a spanning tree (rather than a spanning forest) if all nodes are reachable from the start node, as will be the case for a strongly connected graph.

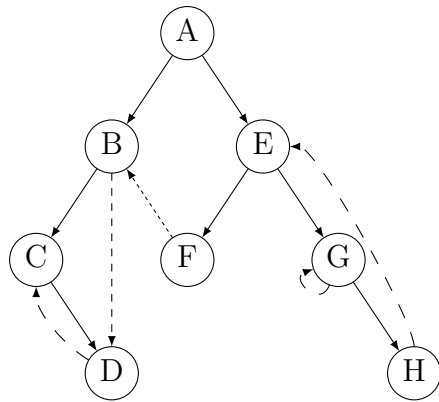
The most important difference is that for a directed graph there are several kinds of *non-tree edges*, as we shall now see. Assume that for a call $\text{DFS}(u)$, an edge from u to w is *not* included in T because w is not white.

- If w is gray, then u is a descendant of w in T , so the edge from u to w is a **back edge**. But unlike what is the case for an undirected tree, a back edge can go to the parent — and even be a self-loop!
- if w is black, there are two subcases:
 - w finished after u was discovered. Then u is a proper ancestor (but not the parent) of w in T , and the edge from u to w is a **forward edge**.
 - w finished before u was discovered. Then neither u nor w is an ancestor of the other in T , and the edge from u to w is a **cross edge**.

To illustrate this, let us look at the directed graph



If the nodes are processed in alphabetical order, the DFS algorithm produces a tree indicated by the solid lines:



- There are 3 back edges, indicated by dashed lines: from D to C, from H to E, and from G to itself.
- There is 1 forward edge, indicated by a line with shorter dashes: from B to D.
- There is 1 cross edge, indicated by a line with even shorter dashes: from F to B.

Observe that if we had chosen say B as the start node then the edge from A to B would have been a cross edge. In general, the classification of edges is highly dependent on the order in which we process the nodes.