1 Useful background for torques at the ISCO

From e.g. Zimmerman et al. (2005), we can write the maximum accretion disk temperature as

$$T_{\max} = f\left(\frac{3GM\dot{M}}{8\pi R_{in}^3\sigma}\right)^{1/4} \tag{1}$$

where M is the central mass, $\dot{M}$ is the accretion rate, R_{in} is the innermost radius of the disk and f is a parameter (O(1)) that approximates a spectral hardening modification from pure black body ($f = 1$). We can parameterize the innermost (maximum) disk temperature for a generic thin disk as

$$T_{\max} \approx 5.6\times10^5\mathrm{K}\left(\frac{\mathrm{M_{BH}}}{10^8\mathrm{M}_\odot}\right)^{-1/4}\left(\frac{\dot{\mathrm{M}}}{\dot{\mathrm{M}}_{\mathrm{Edd}}}\right)^{1/4}\left(\frac{\eta}{0.1}\right)^{-1/4}\left(\frac{\mathrm{R_{in}}}{6\mathrm{r_g}}\right)^{-3/4}\left(\frac{\mathrm{f}}{2}\right) \tag{2}$$

where M_{BH} is the black hole mass, $\dot{M}$ is the accretion rate (in units of $\dot{M}_{\mathrm{Edd}}$), the Eddington accretion rate, $\eta \sim 0.1$ is the standard accretion efficiency, and $R_{in} = 6r_g$, the ISCO for a Schwarzschild BH. For comparison, keeping all the parameters the same but changing $M_{BH} = 10M_\odot$ yields $T_{\max} \sim 3.1\times10^7$K.

If the disk is thin, we expect there to be zero-torques at R_{in}, as the material plunges in free-fall at $r < R_{in}$. The resulting zero-torque temperature (T_{ZT}) profile is given by (Zimmermann et al. 2005)

$$T_{ZT} = T_{\max}\left(\frac{r}{R_{in}}\right)^{-3/4}\left[1-\left(\frac{r}{R_{in}}\right)^{-1/2}\right]^{1/4} \tag{3}$$

and is given by the red curve in Fig. 1. However, magnetic torques (Gammie 1999; Krolik & Agol 2000) or a puffed-up disk (Narayan et al. 1997; Ashfordi & Paczynski 2003) can have a finite or quite large torque at the inner edge. In this case, the disk temperature profile looks like

$$T_{NZT} = T_{\max}\left(\frac{r}{R_{in}}\right)^{-3/4} \tag{4}$$

or the black curve in Fig. 1.

Integrating over the temperature profiles above (Zimmermann et al. 2005) find

$$L_{\mathrm{disk}}[ZT, NZT] = [1,3]\frac{GM\dot{M}}{2R_{in}}\textbf{??} \tag{5}$$

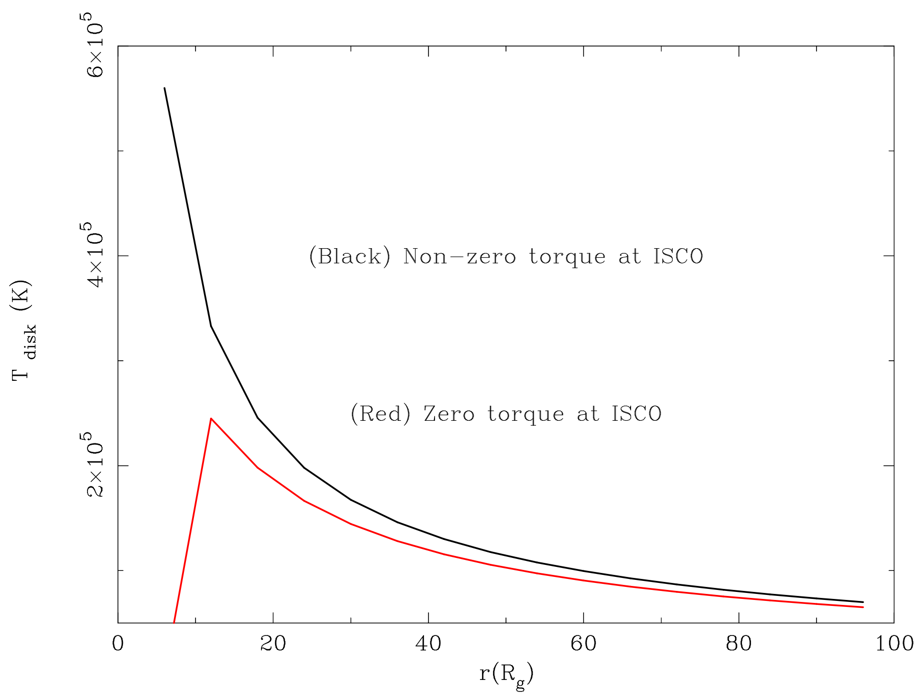

Figure 1: Comparison of the disk temperature profile using eqns.2-4, due to a change in the boundary condition at the inner edge (R_{in}). Red=standard zero torque at ISCO assumption. Black= non-zero torque at ISCO.

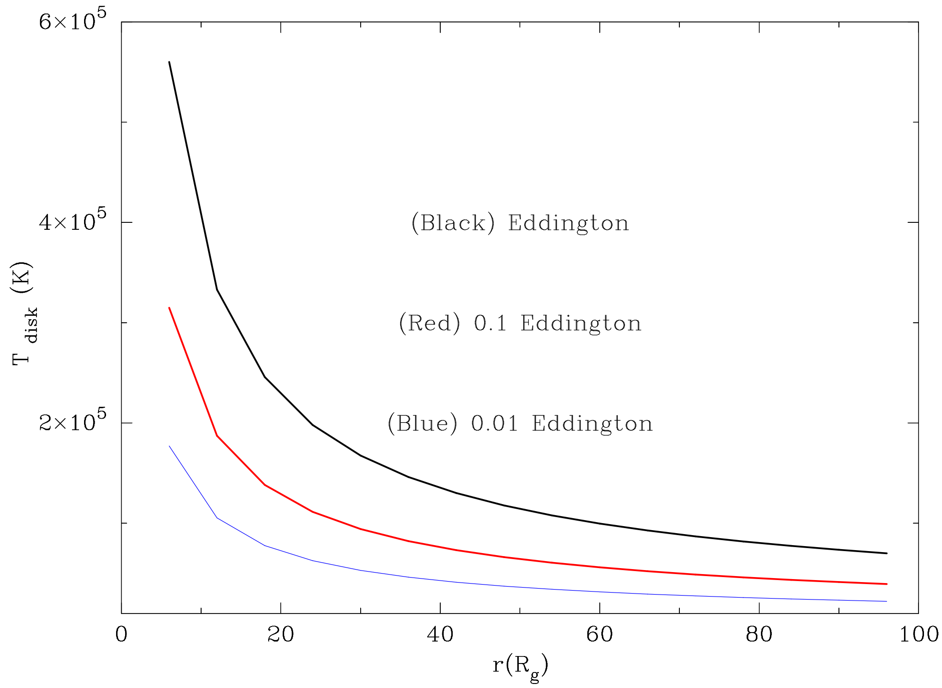

Figure 2: Comparison of the disk temperature profile using eqns.2-4, by changing the accretion rate. Black=1.0× Eddington. Red= 0.1×Eddington. Blue=0.01× Eddington.

or the disk is $\times 3$ more luminous due to this extra torquing at R_{in}.

Fig. 2 shows the effect of changing $dotM$ in eqn.(2). Black curve in Fig. 2 is the same as the black curve in Fig. 1, and the red and blue curves correspond to $\dot{M} = 0.1, 0.01\dot{M}_{\rm Edd}$ respectively.

Fig. 3 shows the effect of changing $dotM$ in eqn.(2). Black curve in Fig. 3 is the same as the black curve in Fig. 1, and the red and blue curves correspond to $R_{in} = 1.2, 9.0r_g$ respectively.

Since from eqn. (5), $L_{\rm disk} = 3GM\dot{M}/2R_{in}$, going from $R_{in} = 6r_g \rightarrow 1.2r_g$ increases disk luminosity by a factor of $\times 5$ and keeping R_{in} fixed, but changing $\dot{M}$ by an order of magnitude changes $L_{\rm disk}$ by an order of magnitude.

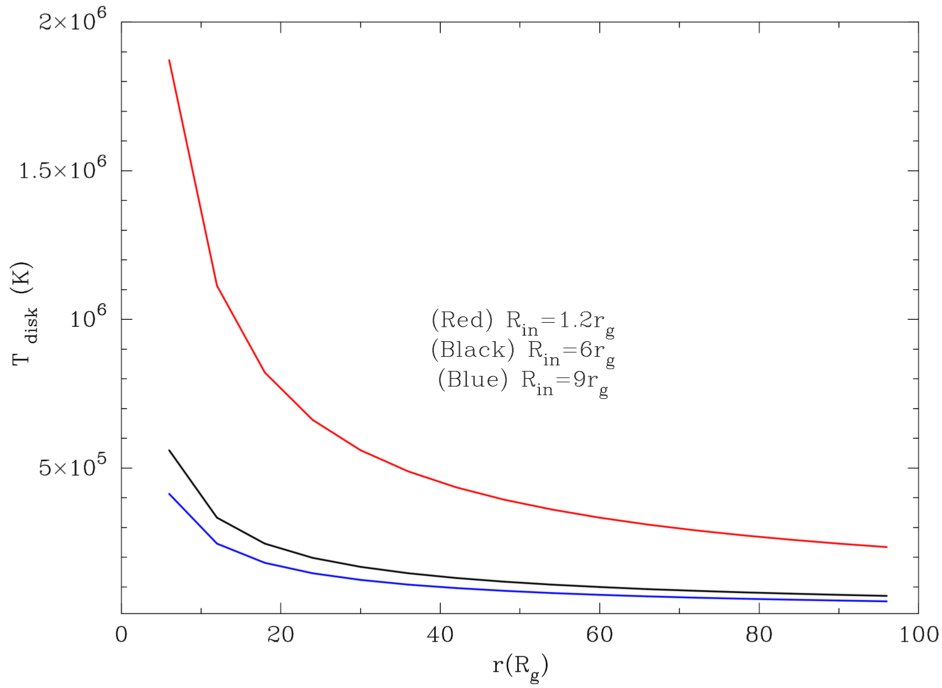

Figure 3: Comparison of the disk temperature profile using eqns.2-4, by changing the location of the disk inner edge (R_{in}). Black=1.2r_g (max. spin Kerr BH, prograde compared to gas). Red=6r_g (Schwarzschild BH, zero spin). Blue=9r_g (max. spin Kerr BH, retrograde compared to gas).

2 The heartbeat in GRS1915+105

Now if we look at the 'hearbeat' state in GRS $1915 + 105$, we have $M_{bh} = 12.4^{+2}_{-1.8} M_{\odot}$ and $d = 8.6^{+2.0}_{-1.6}$kpc. The period of the oscillation is 50s. Translating this to a $M_{bh} = 10^{8} M_{\odot}$, we get an equivalent light-crossing period of 50×10^{7}s or $\sim$ 16years. (Interesting: I wonder if e.g. PG 1302-105 is in a heartbeat state!?)

From Nielsen et al. 2011, their model (section 6 in their paper, Fig. 14, Table 3) for the heartbeat goes as:

1. a wave of excess material (from $\dot{M}$) originating between $20 - 30 r_g$ and propagates radially inward and outward.
2. Disk responds by increasing R_{in} at constant temperature.
3. Disk luminosity increases quickly. At max. R_{in} drops sharply, temp. spikes, disk becomes unstable
4. Disk ejects material. Collides with corona. Hard X-ray pulse.
5. Disk relaxes, density wave subsides.
6. Intense X-ray wind from outer disk due to X-rays
7. Short lived jet

Relevant disk timescales around a $M_{bh} = 10^{8} M_{\odot}$ at $R \sim 25 r_g$ as:

$$t_{\rm orb} \sim 2{\rm days} \left(\frac{R}{25 r_g}\right)^{3/2} \frac{r_g}{c} \tag{6}$$

$$t_{\rm th} \sim 2.5{\rm months} \left(\frac{\alpha}{0.03}\right)^{-1} \left(\frac{R}{25 r_g}\right)^{3/2} \frac{r_g}{c} \tag{7}$$

$$t_{\rm front} \sim 4{\rm yr} \left(\frac{h/R}{0.05}\right)^{-1} \left(\frac{\alpha}{0.03}\right)^{-1} \left(\frac{R}{25 r_g}\right)^{3/2} \frac{r_g}{c} \tag{8}$$

$$t_{\nu} \sim 82{\rm yr} \left(\frac{h/R}{0.05}\right)^{-2} \left(\frac{\alpha}{0.03}\right)^{-1} \left(\frac{R}{25 r_g}\right)^{3/2} \frac{r_g}{c}. \tag{9}$$

3 So is the heartbeat relevant?

From t_ν above, it seems like we'd need a slow rise (80-100years) for the first (long) part of the cycle. But that's not consistent with a $\sim$ 16yr period for the oscillation that you might expect from a simple scaling of light-crossing time ($10M_\odot \rightarrow 10^8 M_\odot$). At $10r_g$, t_ν above is 20yrs, so maybe. But why is the disk behaving like this at $10r_g$ (or $25r_g$ for that matter). Could predict observables based on the sequence above from Nielsen et al. (2011).
Nielsen et al. talk about a R_{in} getting closer to the BH accounting for the increase in $L_{\rm disk}$ but they don't explore a state-change as in Fig. 1. That should give us a different prediction and (shorter) timescales in general.