Black Hole Binaries

From X-ray binaries to black hole binaries

X-ray binaries (XRBs) are referred to as the binary, X-ray emitting systems.

The compact, X-ray emitting source may be a WD, a NS or a black hole; in this last case, the system is called black hole binary (BHB).

Accretion is still the main driver of X-ray emission.

⇓

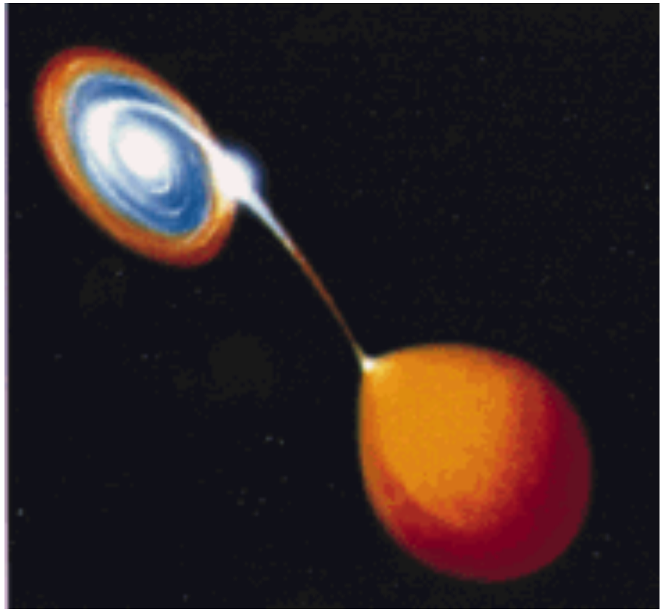
Most X-ray properties in common with systems with an accreting NS

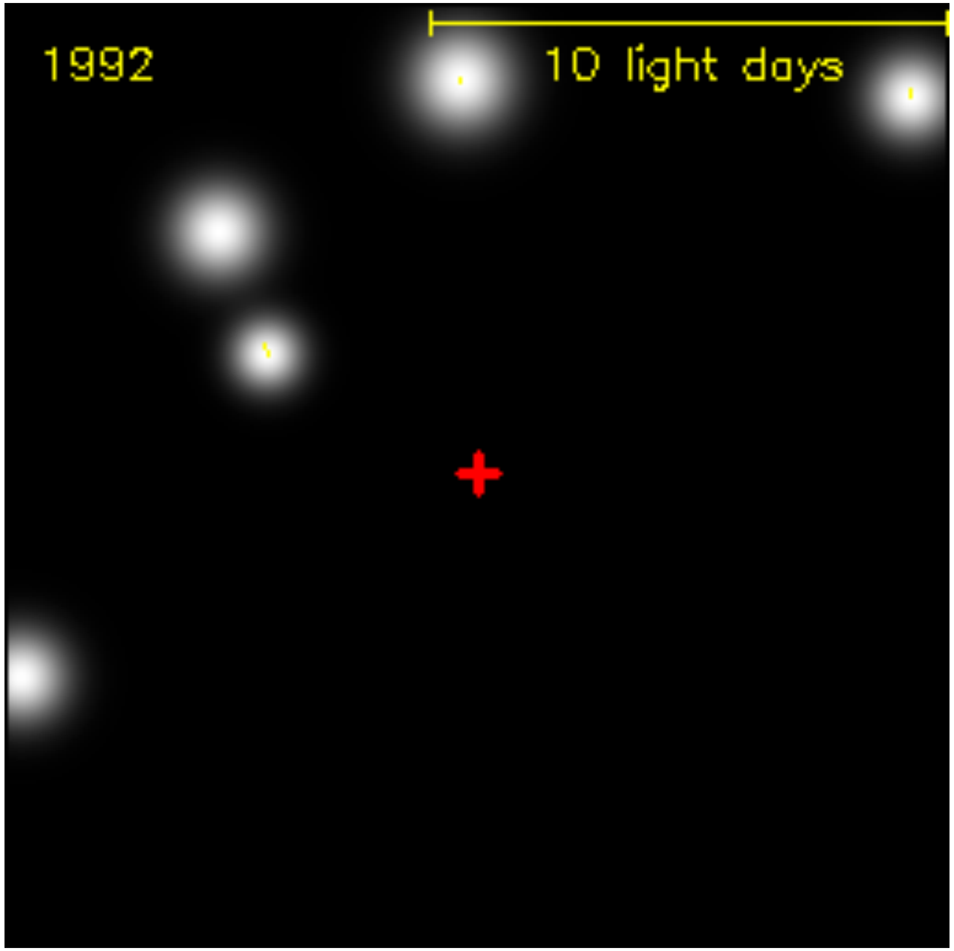
Black Hole Candidates (BHCs) among soft X-ray transients

How can we infer the presence of a Black Hole (it is *black*!)

One way is via gravitational effects on nearby stars (but only for the Galactic Center, SgrA*).

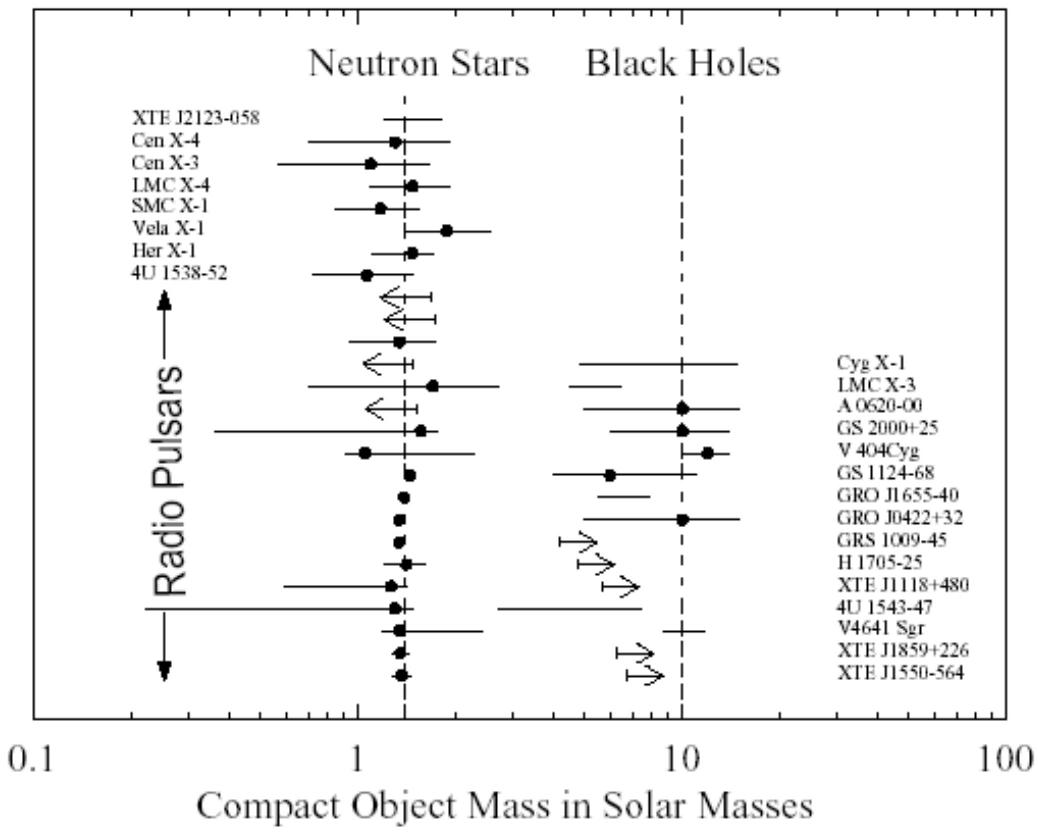
Another way via its "hot" accretion disc and consequent high-energy emission





Confirmed BHBs: statistics (I)

• About 20 BHBs + further 20 BHCs (candidates), where the presence of the black hole is suggested by e.g. the mass function and the X-ray spectral properties (soft thermal + hard power-law emission)



Confirmed BHBs: statistics (II)

Only 3 persistently X-ray bright BHBs, all being HMXBs

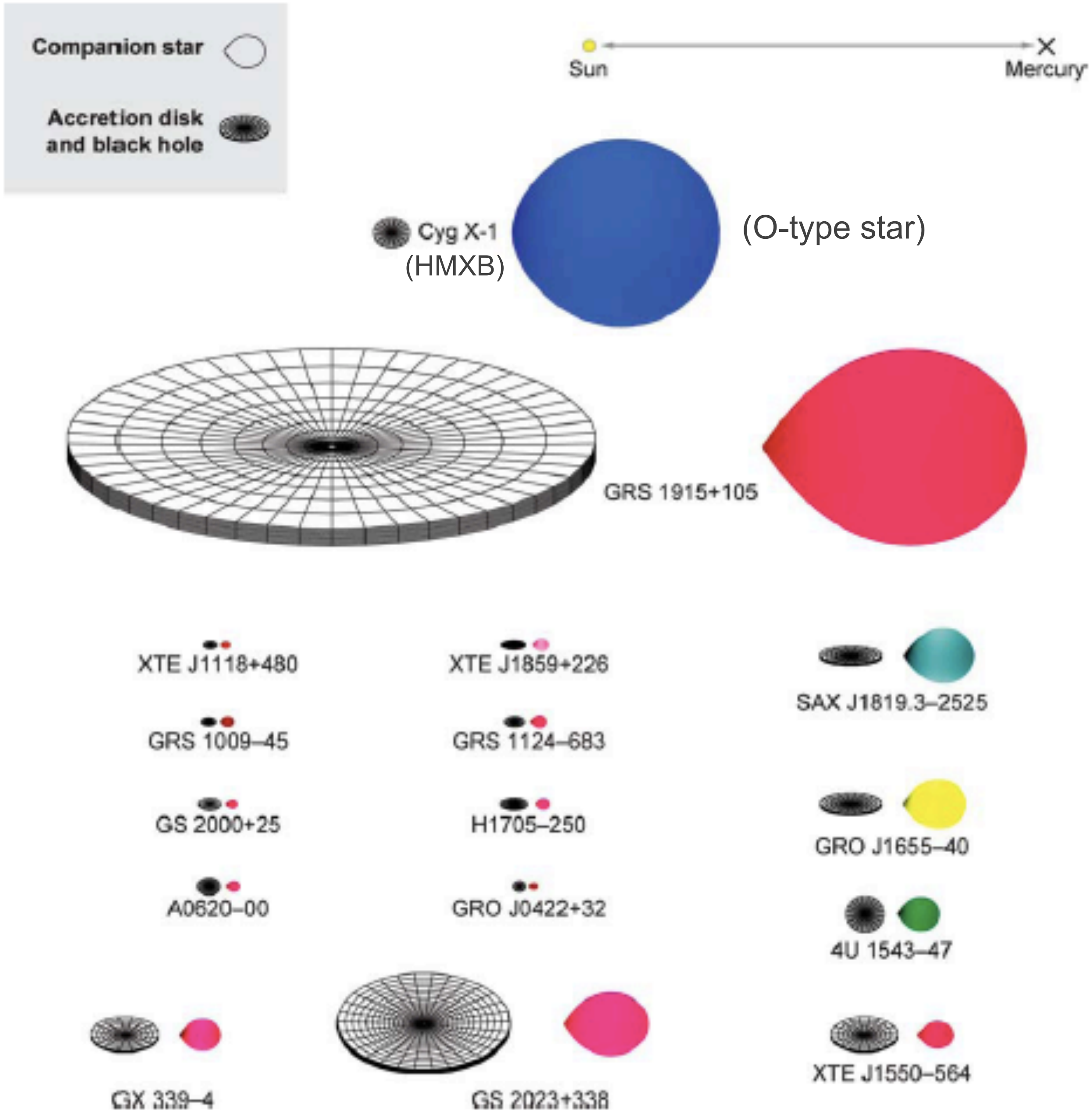
All BHBs have recurrent outbursts

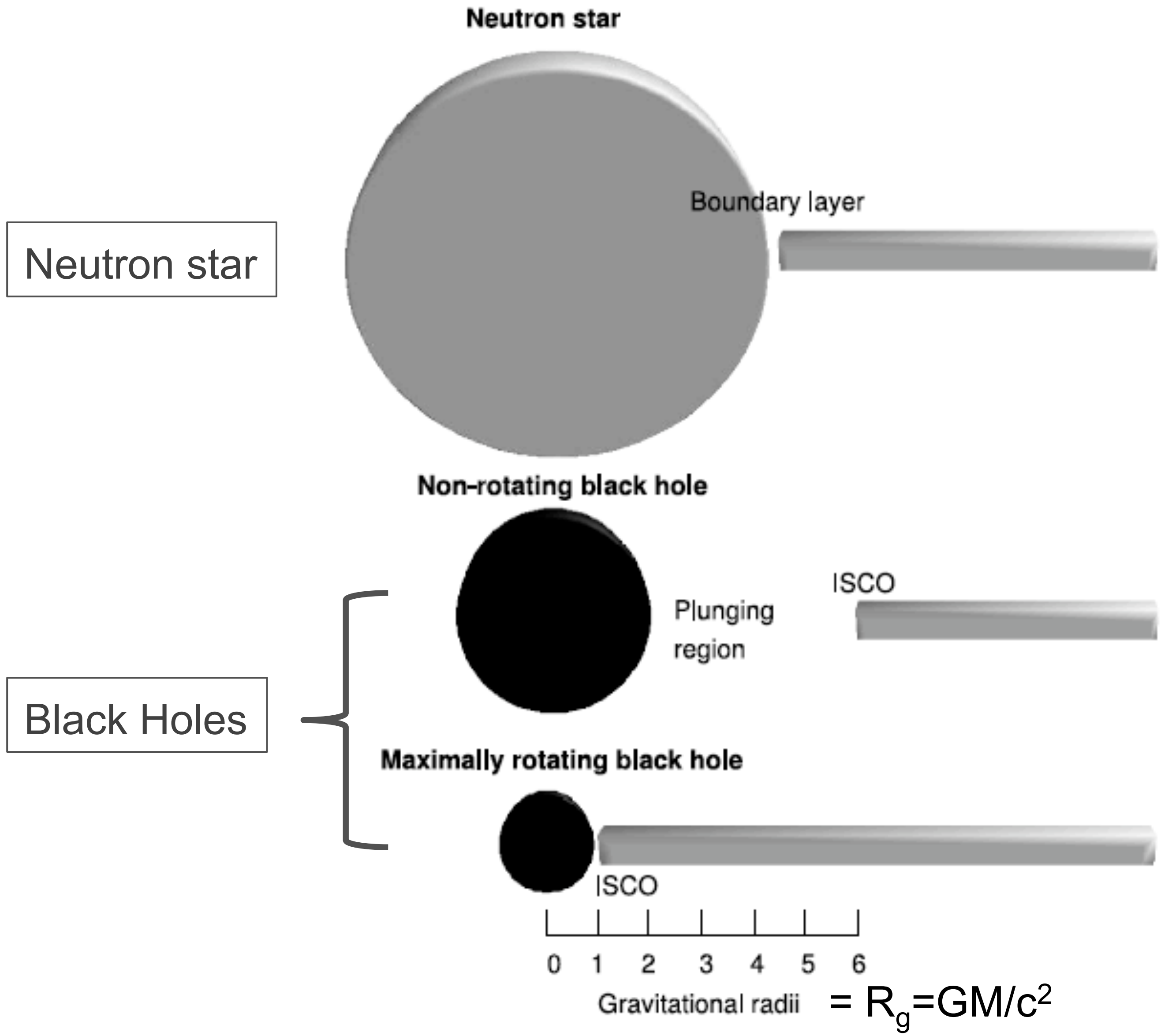
Table 16.1 Black-hole binaries confirmed with the mass function

Source name		Year[a]	Type[b]	$f(M)$ ($M_\odot$)	M_x ($M_\odot$)	X-ray[c] spectrum
Cyg X-1			H, P	0.244±0.005	6.9–13.2	S + PL
LMC X-3			H, P	2.3 ±0.3	5.9–9.2	S + PL
LMC X-1			H, P	0.14±0.05	4.0–10.0	S + PL
J0422 + 32	V518 Per	'92	L, T	1.19±0.02	3.2–13.2	PL
0620–003	V616 Mon	'17, '75	L, T	2.72±0.06	3.3–12.9	S + PL
1009–45	MM Vel	'93	L, T	3.17±0.12	6.3–8.0	S + PL
J1118 + 480	KV Uma	2000	L, T	6.1 ±0.3	6.5–7.2	PL
1124–684	GU Mus	'91	L, T	3.01±0.15	6.5–8.2	S + PL
1354–645	BW Cir	'71,'87,'97	L, T	5.75±0.3	>7.8	S + PL
1543–475	IL Lup	'71,'83,'92	L, T	0.25±0.01	7.4–11.4	S + PL
J1550–564	V381 Nor	'98	L, T	6.86±0.71	8.4–10.8	S + PL
J1650–500		2001	L, T	2.73±0.56	4–7.3	PL
J1655–40	V1033 Sco	'94	L, T	2.73±0.09	6.0–6.6	S + PL
1659–487	GX 339–4	(P~460d)	L, T	5.8±0.5	>5.8	S + PL
1705–250	V2107 Oph	'77	L, T	4.86±0.13	5.6–8.3	S + PL
J1819.3–2525	V4641 Sgr	'99	L, T	3.13±0.13	6.8–7.4	S + PL
J1859 + 226	V406 Vul	'99	L, T	7.4±1.1	7.6–12	S + PL
1915 + 105	V1487 Aql	'92—	L, T	9.5±3.0	10–18	S + PL
2000 + 251	QZ Vul	'88	L, T	5.01±0.12	7.1–7.8	S + PL
2023 + 338	V404 Cyg	'38,'56,'89	L, T	6.08±0.06	10.1–13.4	PL

[a] The year of outburst, including earlier records as optical novae.
[b] H: high-mass binary. L: low-mass binary: P: persistent. T: transient.
[c] X-ray spectral type near the maximum luminosity. S: soft thermal. PL: power-law.

Situation in 2007 (2008)

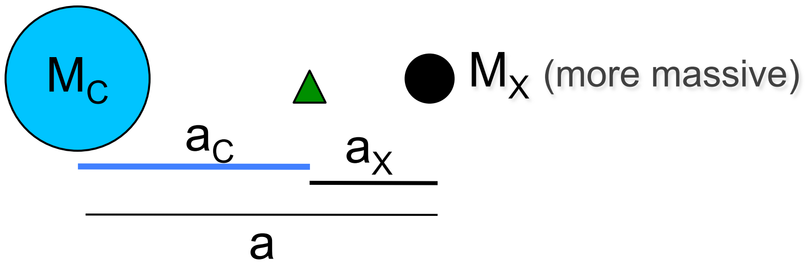

Companion star
Accretion disk and black hole
Sun
Mercury
Cyg X-1
(HMXB)
(O-type star)
GRS 1915+105
XTE J1118+480
XTE J1859+226
SAX J1819.3–2525
GRS 1009–45
GRS 1124–683
GS 2000+25
H1705–250
GRO J1655–40
A0620–00
GRO J0422+32
4U 1543–47
GX 339–4
GS 2023+338
XTE J1550–564


Neutron star
Neutron star
Boundary layer
Black Holes
Non-rotating black hole
Plunging region
ISCO
Maximally rotating black hole
ISCO
0 1 2 3 4 5 6
Gravitational radii = $R_g = GM/c^2$

Mass Function (I)

$$a = a_C + a_X$$
$$M_C a_C = M_X a_X$$

Center of mass



$$v_{C,proj} = v_{C,true}\sin(i) = \left(\frac{2\pi}{P_{orb}}\right) a_C \sin(i)$$

v=projected orbital velocity
i=inclination angle between the orbital plane and the plane of the sky

$$G\frac{(M_C + M_X)}{a^3} = \left(\frac{2\pi}{P_{orb}}\right)^2$$

III Kepler's law

Mass Function (II)

$$\boxed{a = a_C + a_X = a_C + \frac{M_C}{M_X} a_C = a_C \frac{M_C + M_X}{M_X}}$$

$$\Rightarrow \quad \boxed{G \frac{(M_C + M_X) M_X^3}{a_C^3 (M_C + M_X)^3} = G \frac{M_X^3}{a_C^3 (M_C + M_X)^2} = \left(\frac{2\pi}{P_{\mathrm{orb}}} \right)^2}$$

$$\boxed{G \frac{M_X^3 \ \sin(i)^3}{a_C^3 (M_C + M_X)^2} = \frac{4\pi^2}{P_{\mathrm{orb}}^2} \sin(i)^3 \rightarrow \frac{(M_X \sin(i))^3}{(M_C + M_X)^2} = \frac{4\pi^2}{G P_{\mathrm{orb}}^2} (a_C \sin(i))^3}$$

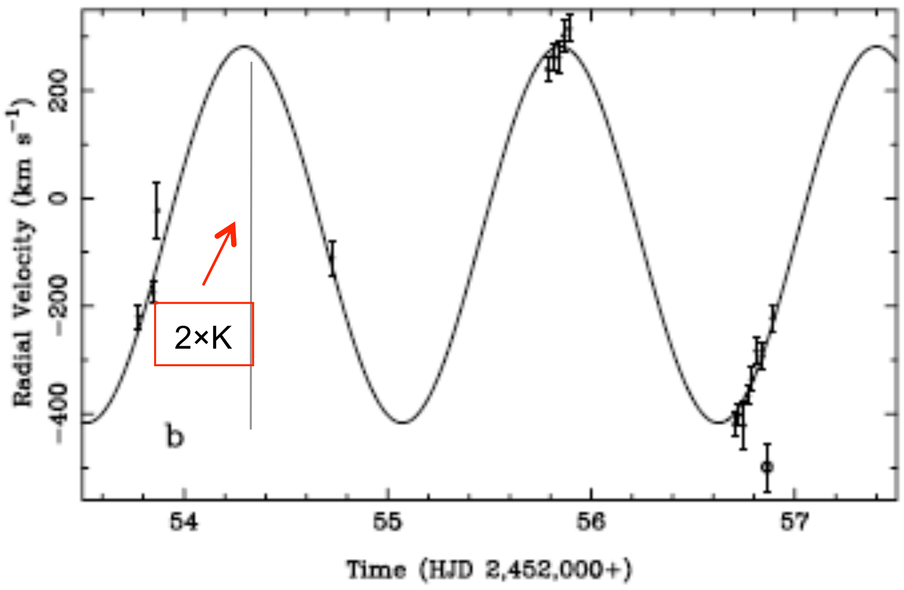
$$\Rightarrow \quad \boxed{f(M) = \frac{(M_X \sin(i))^3}{(M_C + M_X)^2} = \mathrm{v}_C^3 \frac{P_{\mathrm{orb}}}{2\pi G}}$$

Mass function=f(M)

Mass Function (III)

$$f(M) = \frac{M_X^3 (\sin i)^3}{(M_X + M_c)^2} = \frac{PK^3}{2\pi G}$$

X: compact source (accretor)
C: companion source
i: inclination of the binary orbit
P: orbital period
K: amplitude of the Doppler curve which provides the line-of-sight component of the radial velocity=K_C=$V_{C,proj}$



P and **K** are optical measurable quantities from the radial velocity curve (using the Doppler frequency shift of the spectral lines as a function of time/orbital phase)

Application of the III Kepler's law

The mass function sets a lower limit to the real mass of the compact object

Mass Function (IV)

$$f(M) = \frac{M_X^3 (\sin i)^3}{(M_X + M_c)^2} = \frac{PK^3}{2\pi G}$$

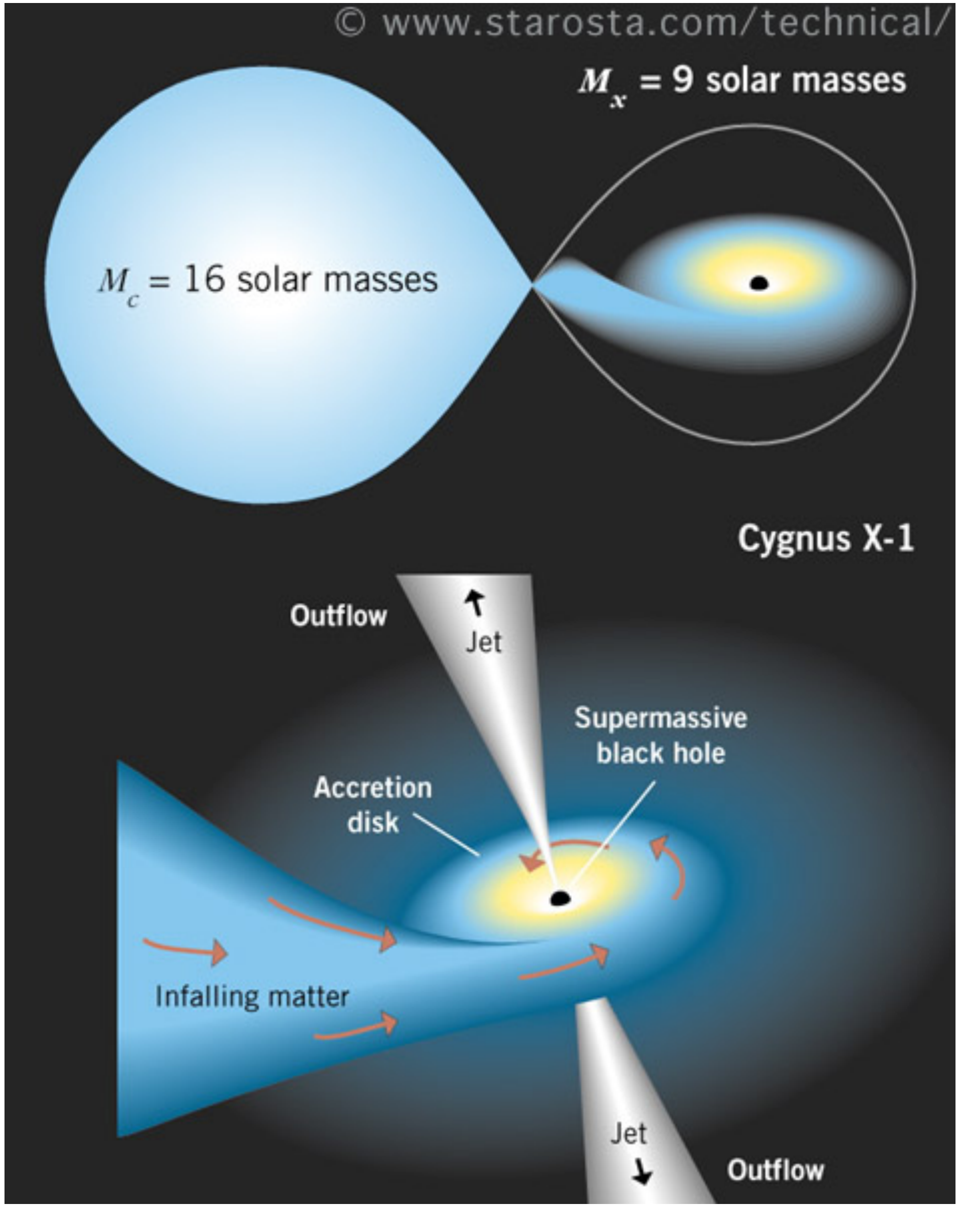
$$q = \frac{M_C}{M_X} \text{ mass ratio} \rightarrow f = \frac{M_X^3 (\sin i)^3}{(qM_X + M_X)^2} = \frac{M_X (\sin i)^3}{(q+1)^2}$$

$$\Rightarrow M_X = f \underbrace{\boxed{\frac{(q+1)^2}{(\sin i)^3}}}_{>1} \rightarrow M_X > f(M)$$

➔ If f(M)>3 $M_\odot$, then the compact object should reasonably be a black hole

LMXBs: weak companions
HMXBs: optical light from the companion dominates

Easier to observe the companions once bursts are over



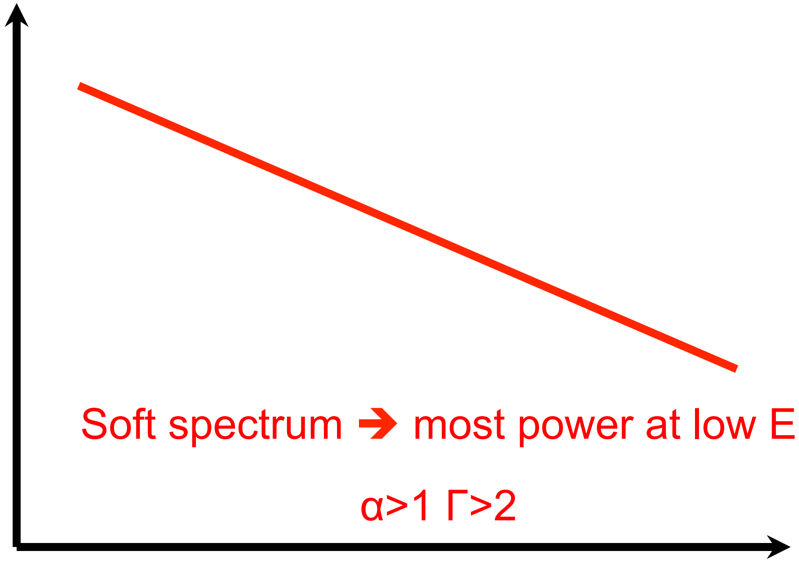
The binary system Cyg X-1

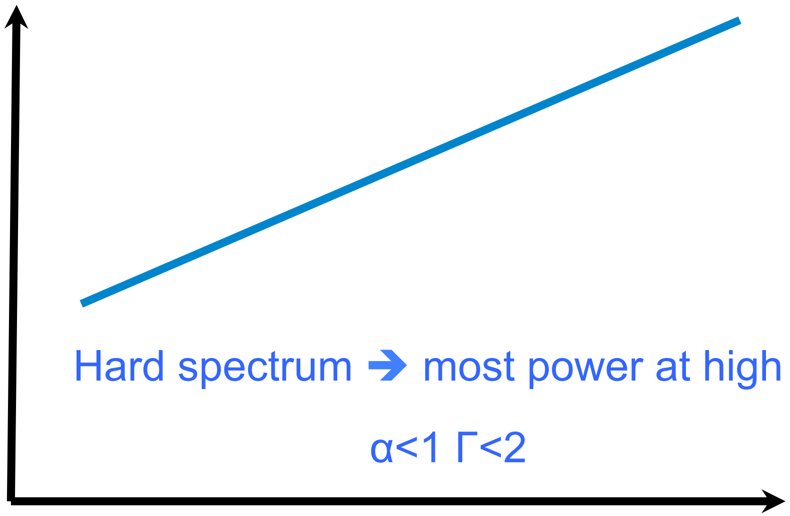
“Internal” structure of BHBs

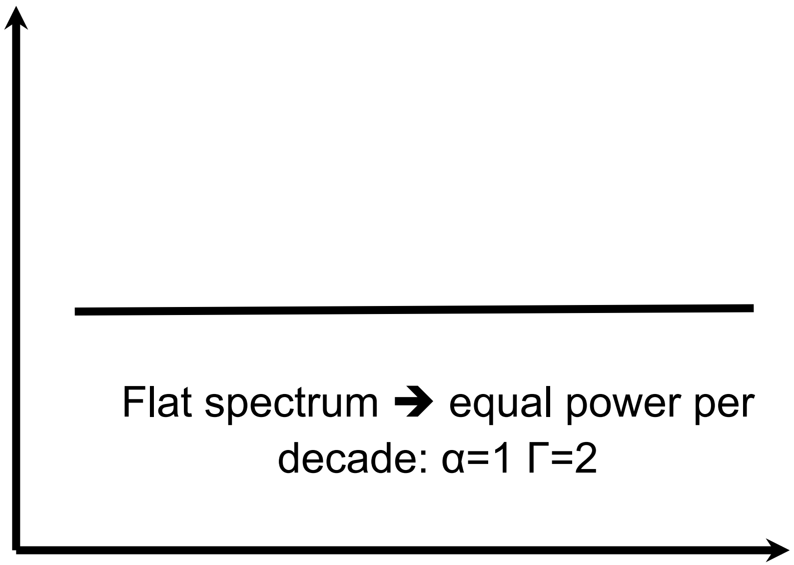
Spectra (basics)

- Plot $\nu f(\nu)$ as this peaks at energy where power output of source peaks.
- $N(E)=AE^{-\Gamma}$
- $F(E)=EN(E)= AE^{-\Gamma+1}=AE^{-\alpha}$

  $\alpha=\Gamma-1$

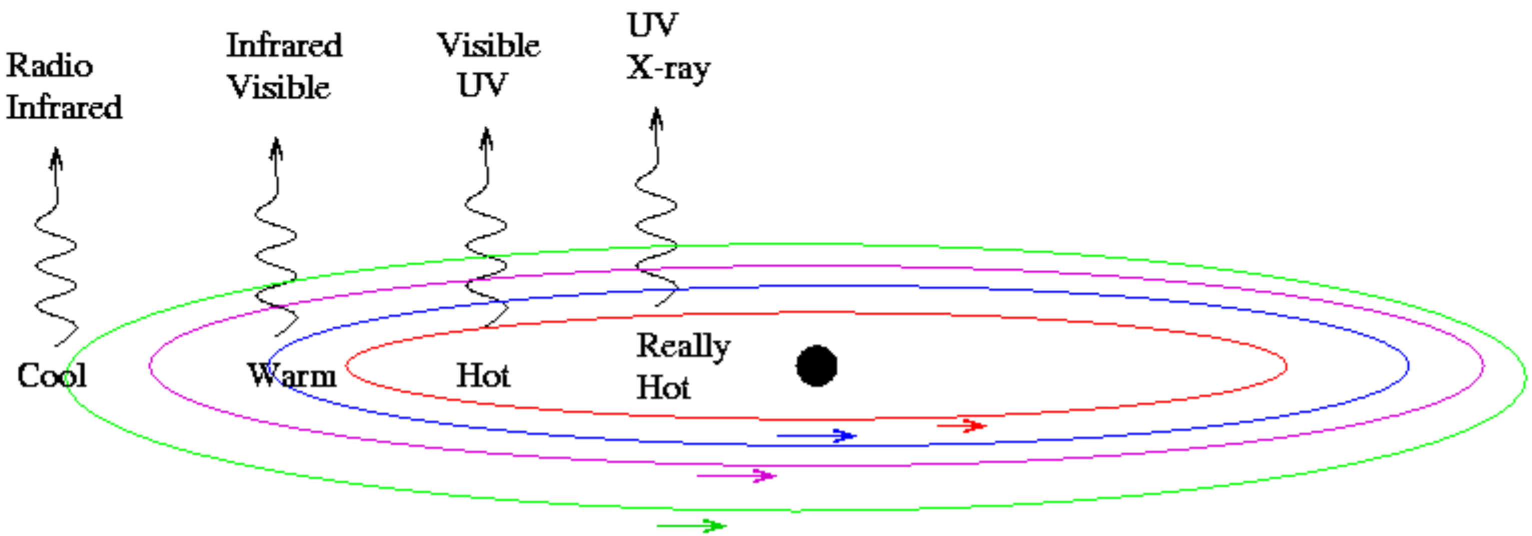






$dL= F(E)\, dE = EF(E)\, dE/E = EF(E)\, d\log E$

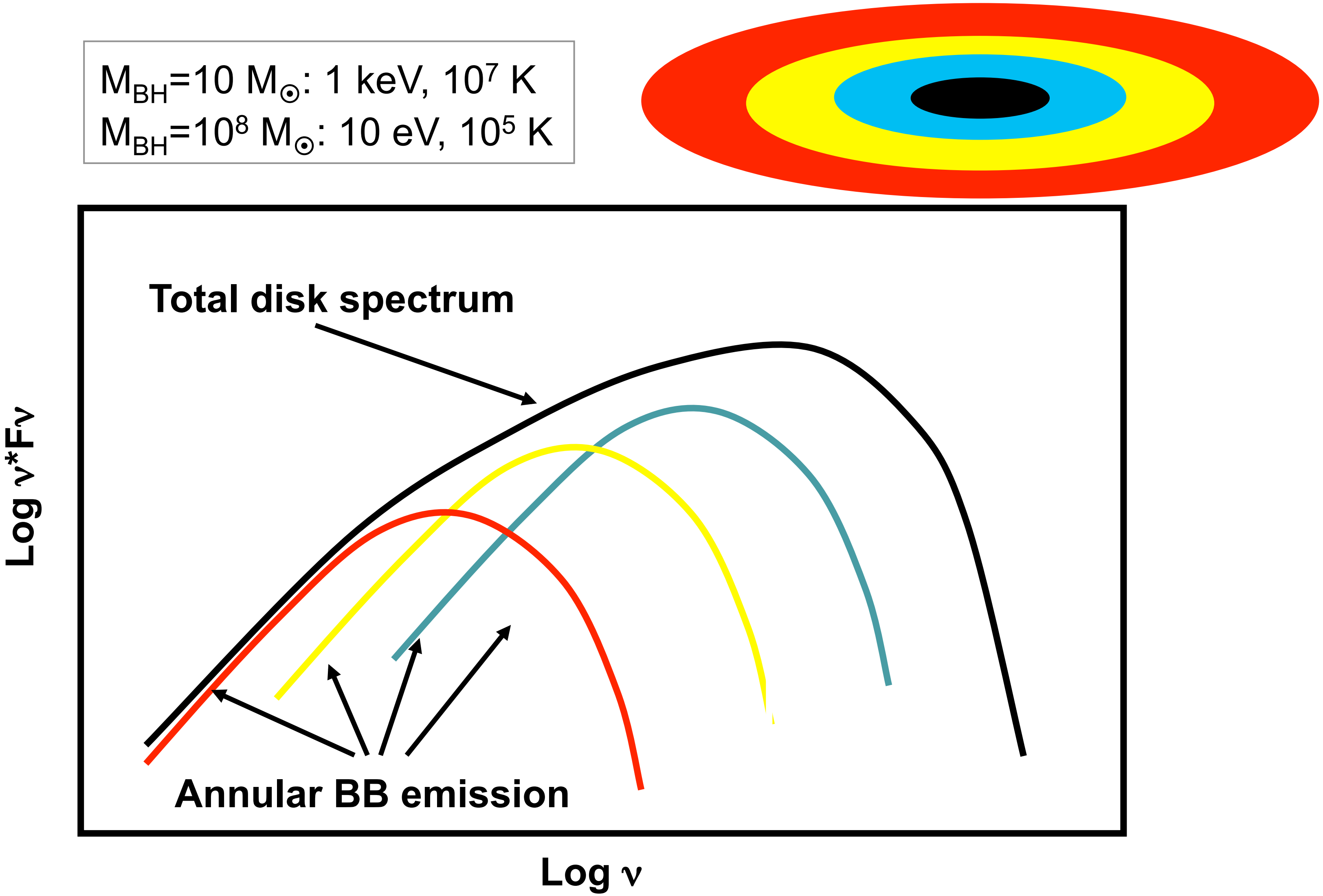
The accretion disc



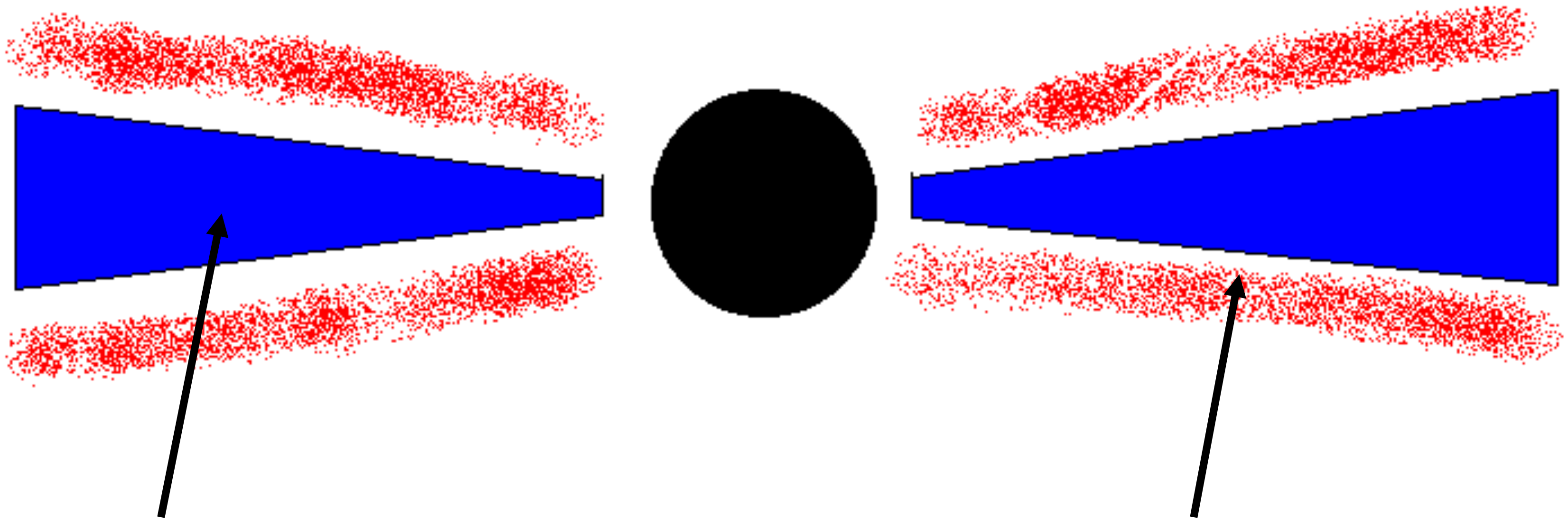
• Friction between adjacent layers which converts gravitational potential energy of the accreting matter into radiation
• Gas in differential rotation, viscosity transports angular momentum outward, while matter is driven inward
• Multi-color blackbody (MCD) emission=BB from layer at different temperatures

The accretion disc spectrum: multi-color blackbody

$M_{BH}=10\ M_{\odot}$: 1 keV, 10^7 K
$M_{BH}=10^8\ M_{\odot}$: 10 eV, 10^5 K



Accretion disc + corona modeling

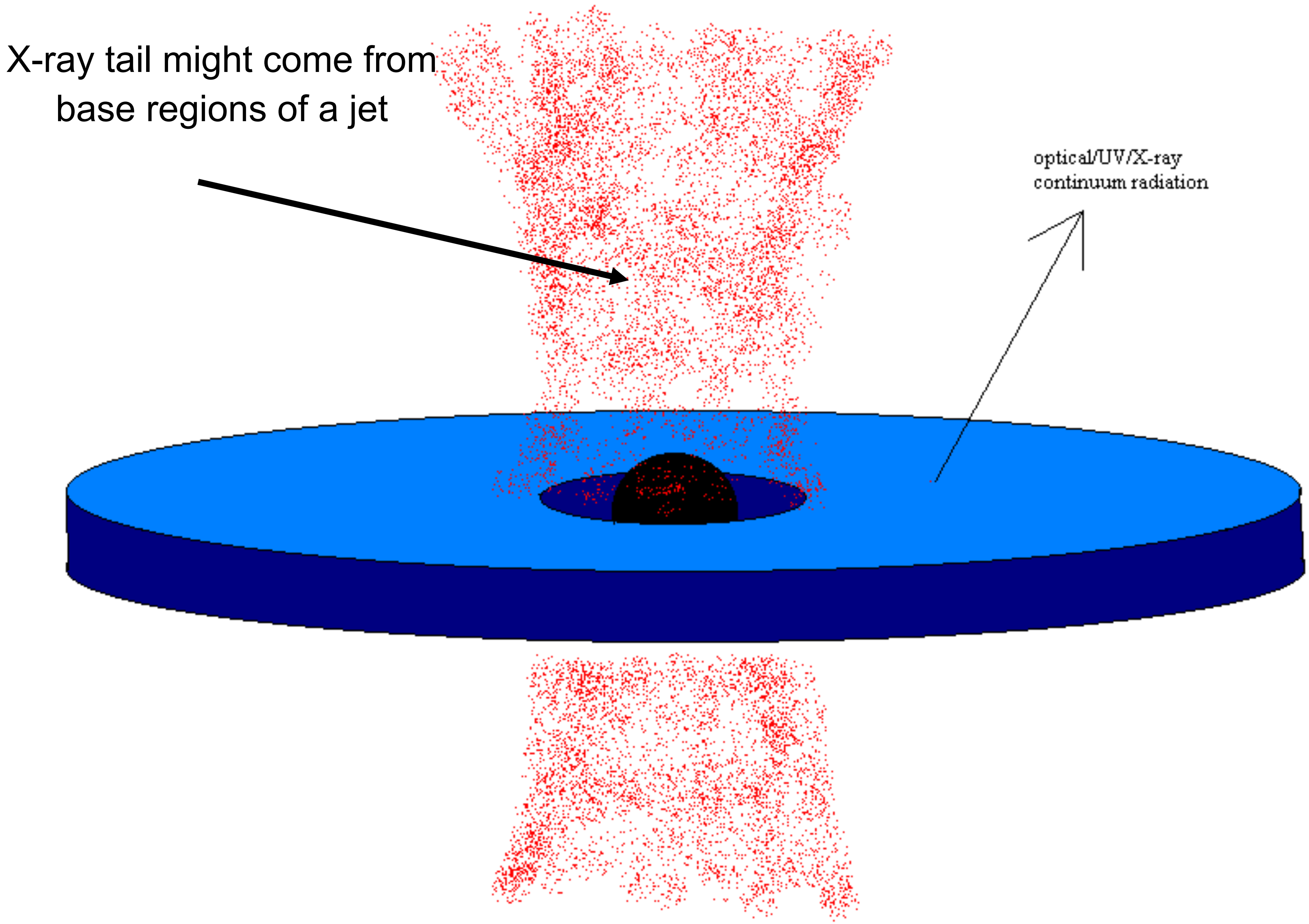


Optically-thick part of the accretion disk emits thermal spectrum… black body radiation with

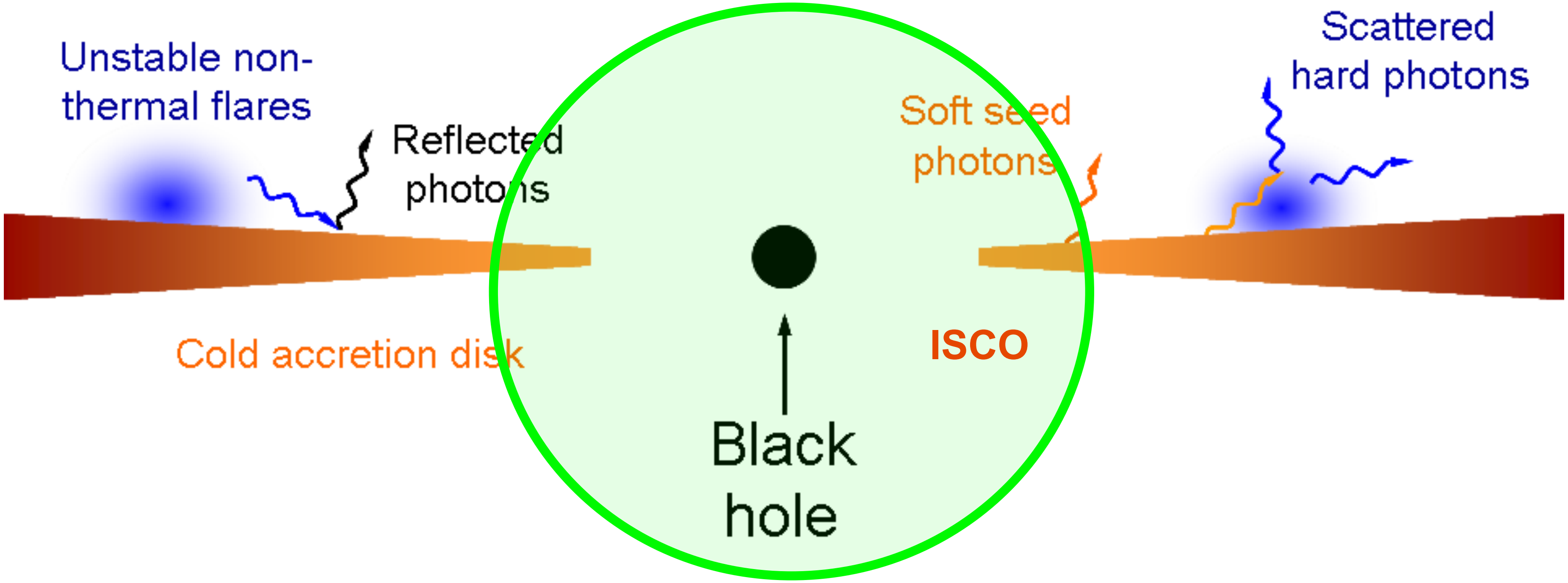
$$T = \left(\frac{3GM\dot{M}}{8\pi r^3 \sigma_{\rm SB}} \right)^{1/4}$$

X-ray "tail" probably comes from a hot corona that sandwiches the disk… inverse Compton scattering of thermal disk emission by electrons with T~10^9K

Similarities with accretion onto super-massive black holes in AGN


X-ray tail might come from
base regions of a jet
optical/UV/X-ray
continuum radiation

Overview of the central region of GBHCs/AGN

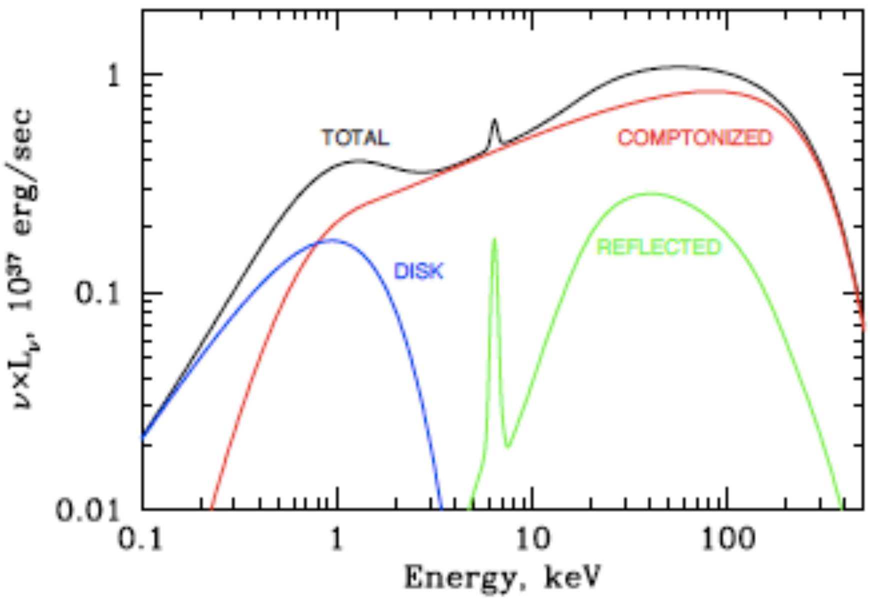


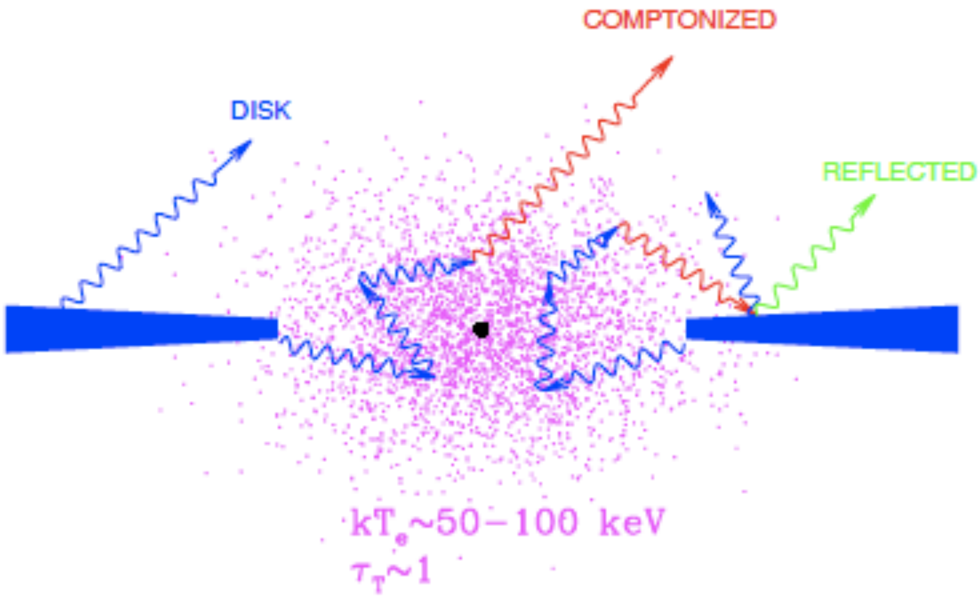
Spectrum:

- Direct Continuum (PL + MCD)
- Reflection (Comptonized PL)
- Reprocessed (via photoionization) (emission lines; Fe Ka @ ~6.4 keV or ionized)

Timing:

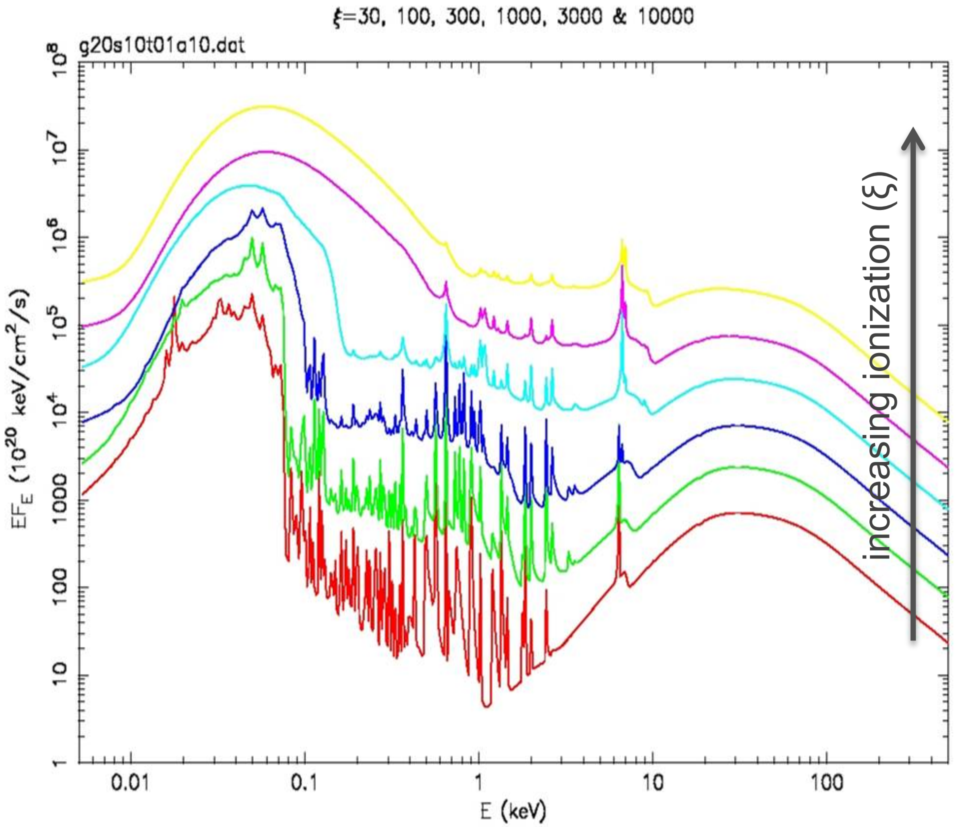
- Broad-band/Intrinsic Variability
- Local Variability (QPOs)

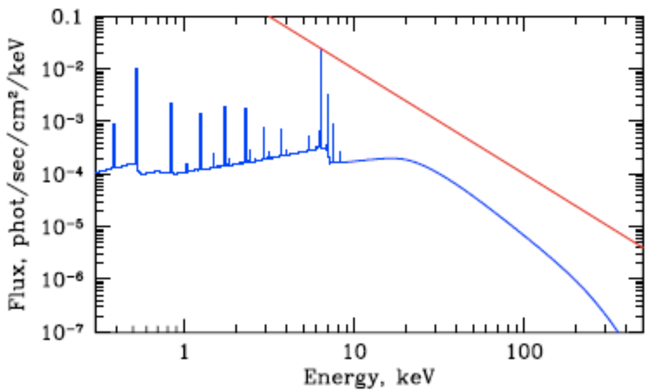

1
0.1
0.01
0.1
1
10
100
Energy, keV
$\nu \times L_\nu$, 10^{37} erg/sec
TOTAL
COMPTONIZED
DISK
REFLECTED

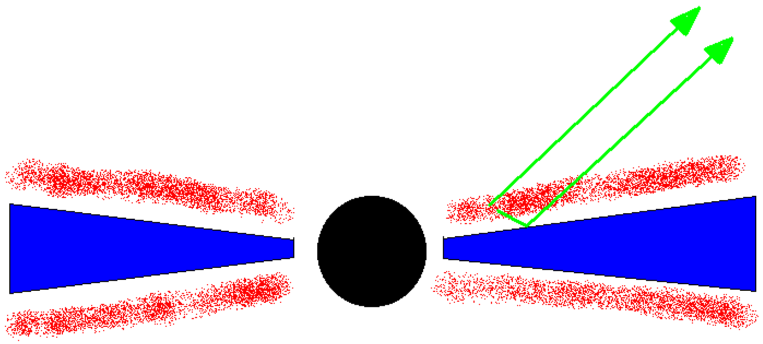

COMPTONIZED
DISK
REFLECTED
$kT_e \sim 50-100$ keV
$\tau_T \sim 1$

X-ray reflection

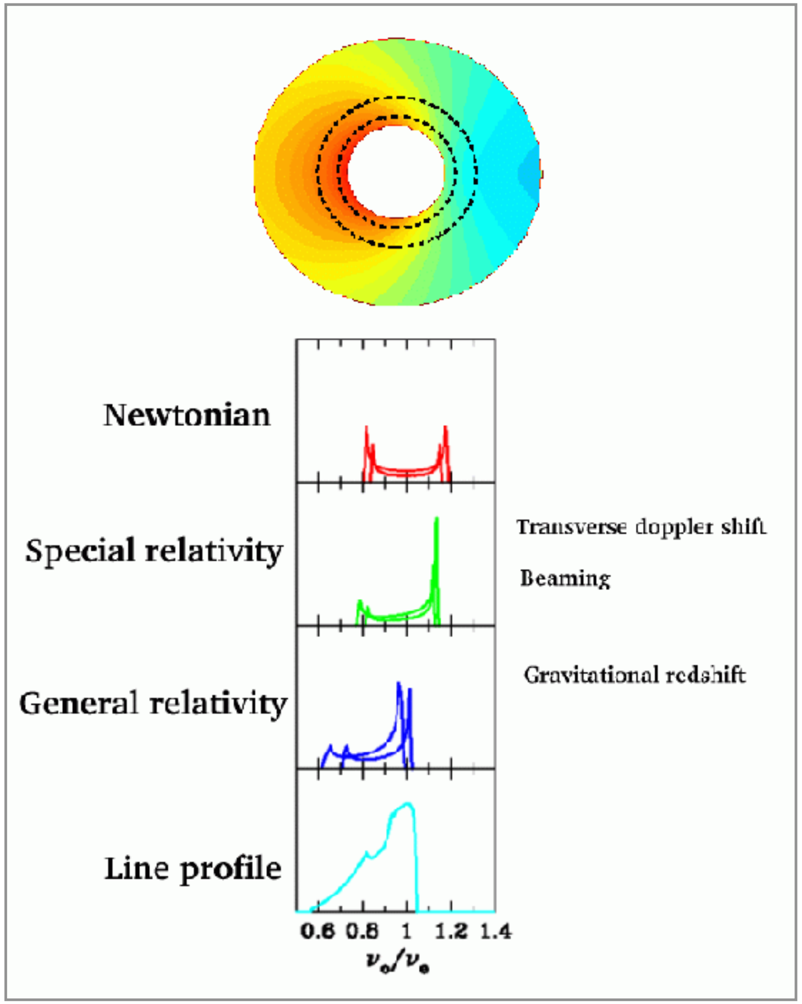
Important consequence of corona: underlying disk is irradiated by intense X-ray source… results in a characteristic spectrum being "reflected" from the disk surface layers

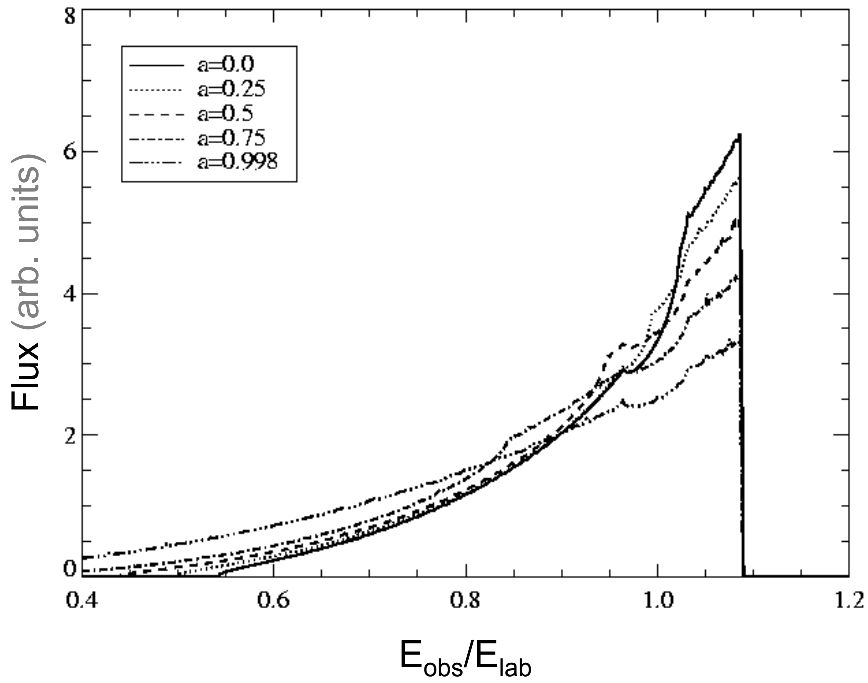






Relativistic effects





X-ray spectral states

X-ray spectral states of BHBs

Very High State (VHS) Steep PL State

- **PL emission dominant** ($\Gamma \approx 2.5$) from thermal Comptonization, up to high energie (≈MeV)
- High-frequency QPOs
- At high Mdot, from thin to thick (**slim**) **disc** (Mdot>>L_{Edd}/c^2)

High State (HS)

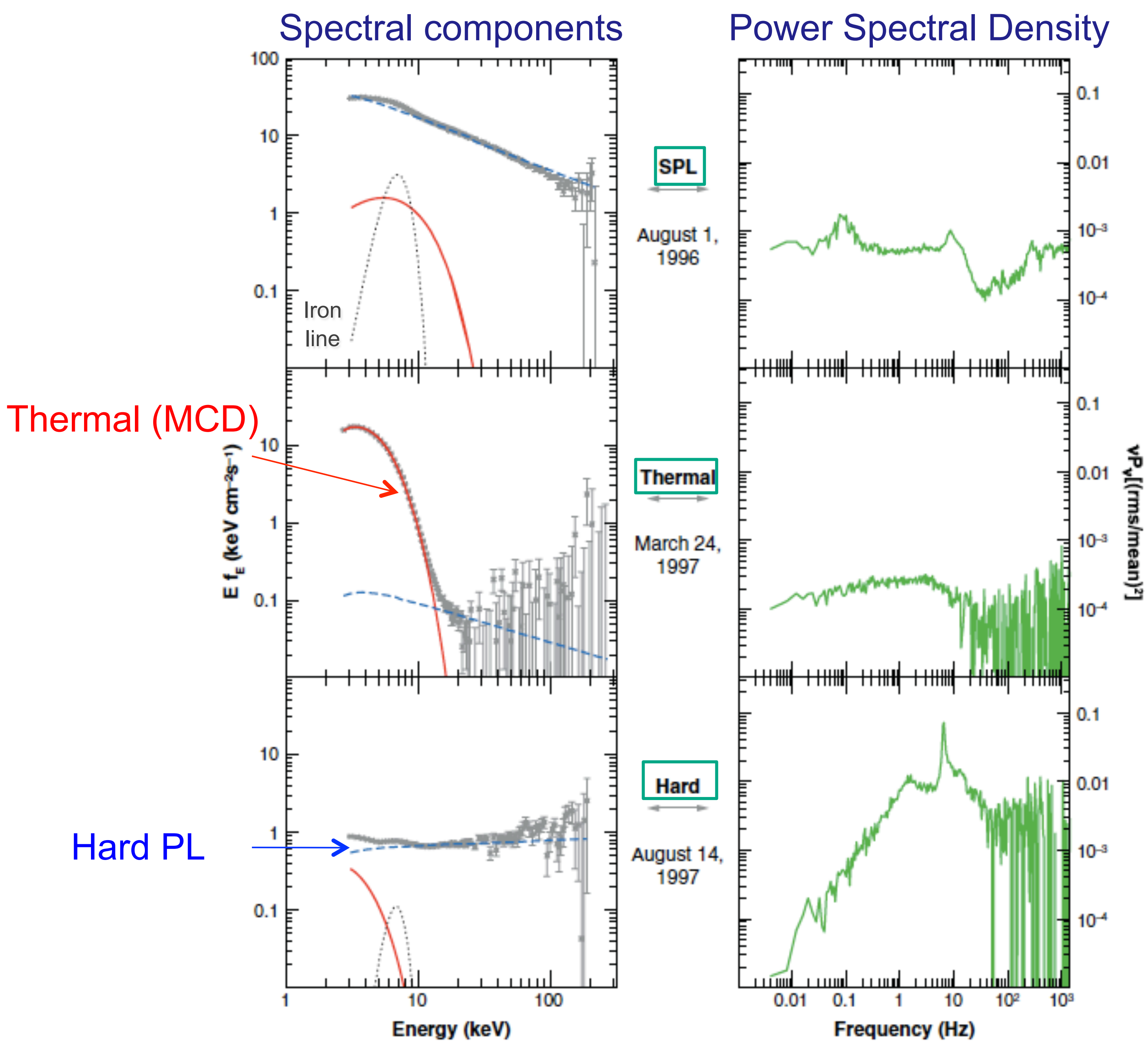
- **Mostly thermal emission from inner accretion disc** (MCD)
- Faint $\Gamma \approx 2.5$ PL emission due to non-thermal Comptonization
- Absent/very weak QPOs

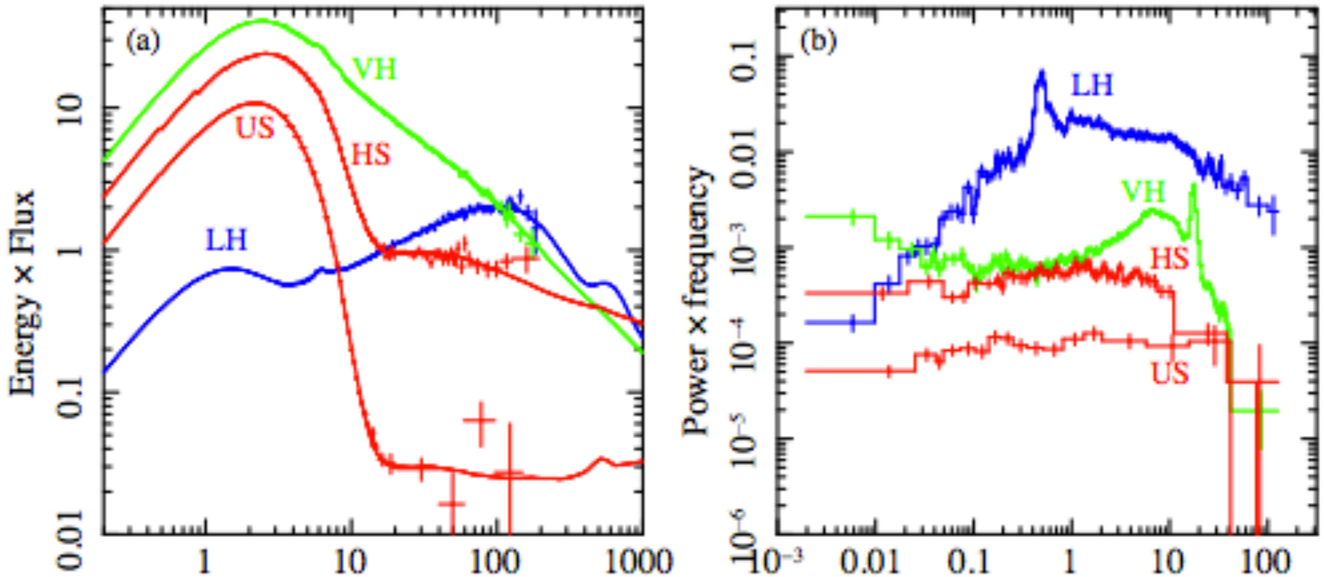
Low State (LS)

- **Hard PL dominant** ($\Gamma \approx 1.4-2.1$, *thermal Comptonization*) + high-E cut-off
- $T_e \approx 100$ keV, $\tau_e \approx 1$
- Presence of a *steady jet*
- QPOs present/absent

INTERPLAY DISC-CORONA

- In the soft/high state, the disc is very close to the last marginally stable Keplerian orbit ➔ no "room" for the corona to Comptonize soft X-ray photons
- In the hard/low state, the accretion disc is probably truncated at larger radii
- Thermal emission is less variable than PL emission


Spectral components
Power Spectral Density
Thermal (MCD)
Hard PL
Iron line
SPL
August 1, 1996
Thermal
March 24, 1997
Hard
August 14, 1997
E f_E (keV cm^-2 s^-1)
Energy (keV)
νP_ν[(rms/mean)^2]
Frequency (Hz)



Done 2010; see also Gierlinski & Done 2003

- HS (high/soft): disc dominated − looks like a disc but small tail to high energies (L prop. T^4)
- Very high/intermediate states − at least know something about a disc
- LH (low/hard) state − looks really different, not at all like a disc!

Summary of the main spectral states

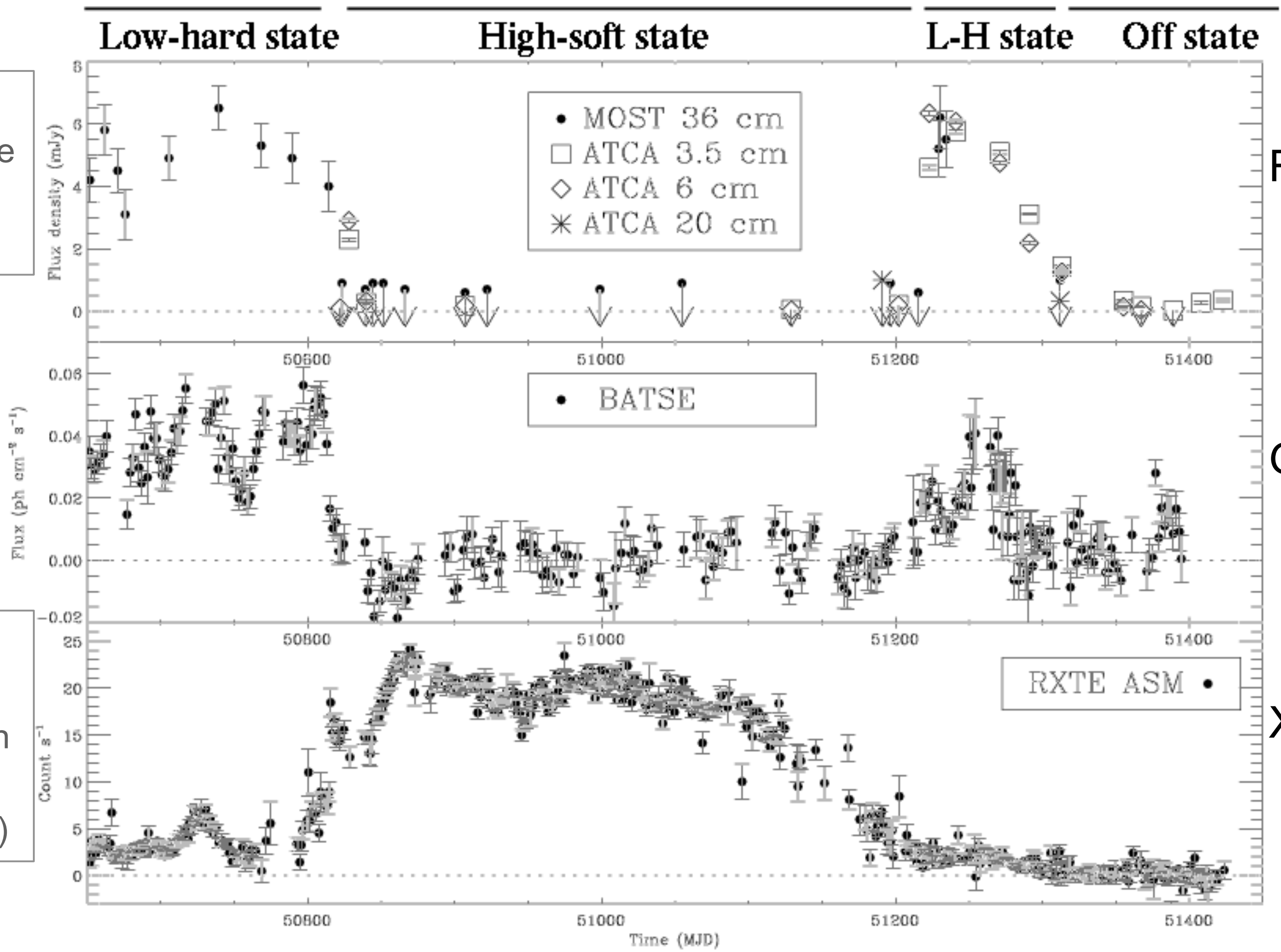
Table 2 Outburst states of black holes: nomenclature and definitions

New State Name (Old State Name)		Definition of X-Ray State[a]
Thermal (High/Soft)	HS	Disk fraction $f^b > 75\%$ QPOs absent or very weak: $a^c_{max} < 0.005$ Power continuum level $r^d < 0.075^e$
Hard (Low/Hard)	LH	Disk fraction $f^b < 20\%$ (i.e., Power-law fraction > 80%) $1.4^f < \Gamma < 2.1$ Power continuum level $r^d > 0.1$
Steep Power Law (SPL) (Very high)	VH	Presence of power-law component with $\Gamma > 2.4$ Power continuum level $r^d < 0.15$ Either $f^b < 0.8$ and 0.1–30 Hz QPOs present with $a^c > 0.01$ or disk fraction $f^b < 50\%$ with no QPOs

- HS = thermal, high disk fraction
- LH = hard emission, low disk fraction
- VHS = steep power law: sort of intermediate state with both components

Strong radio in the LH state (jet)

Strong X-ray emission In the HS stare (disc)



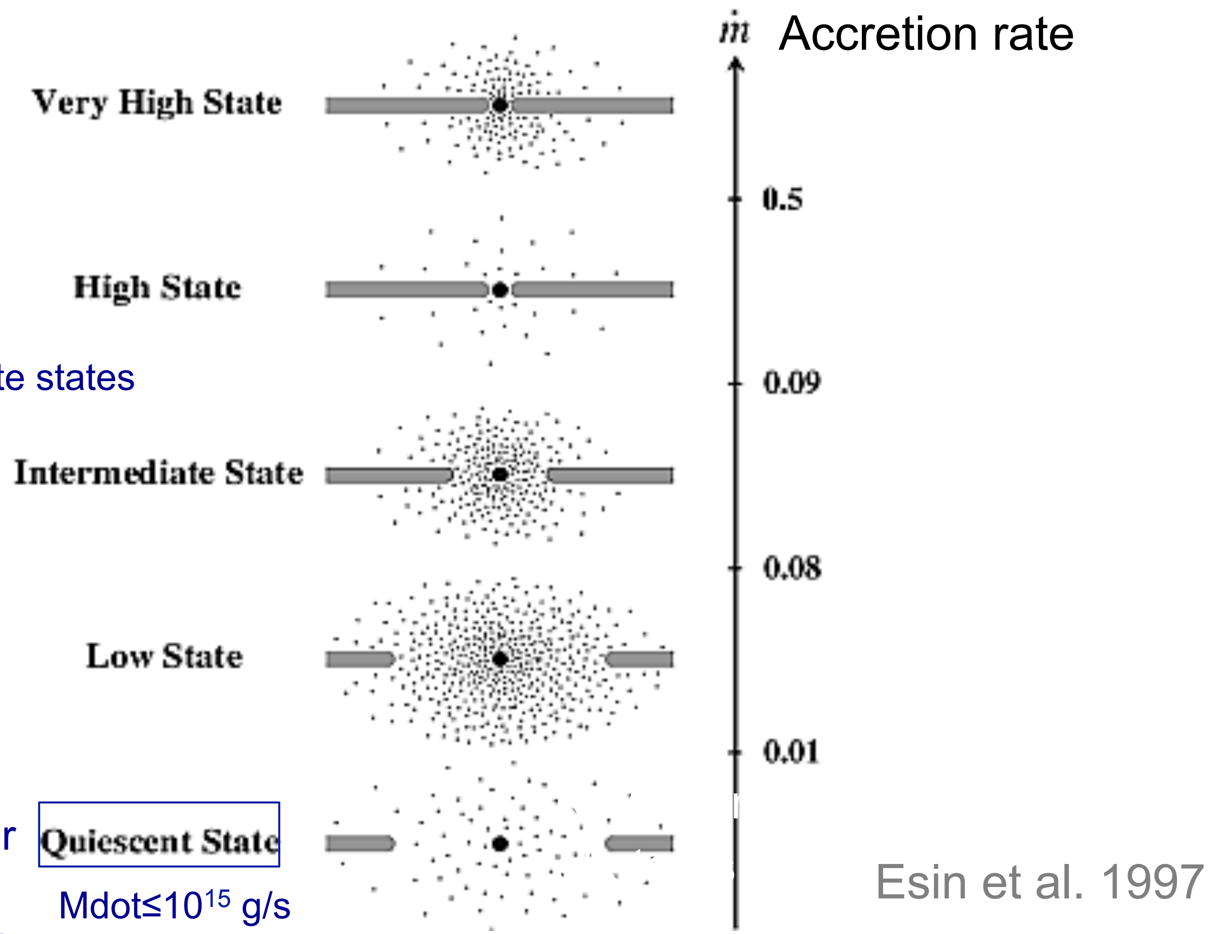
Radio/mm

Gamma-ray

X-ray

GX339-4

Spectral states



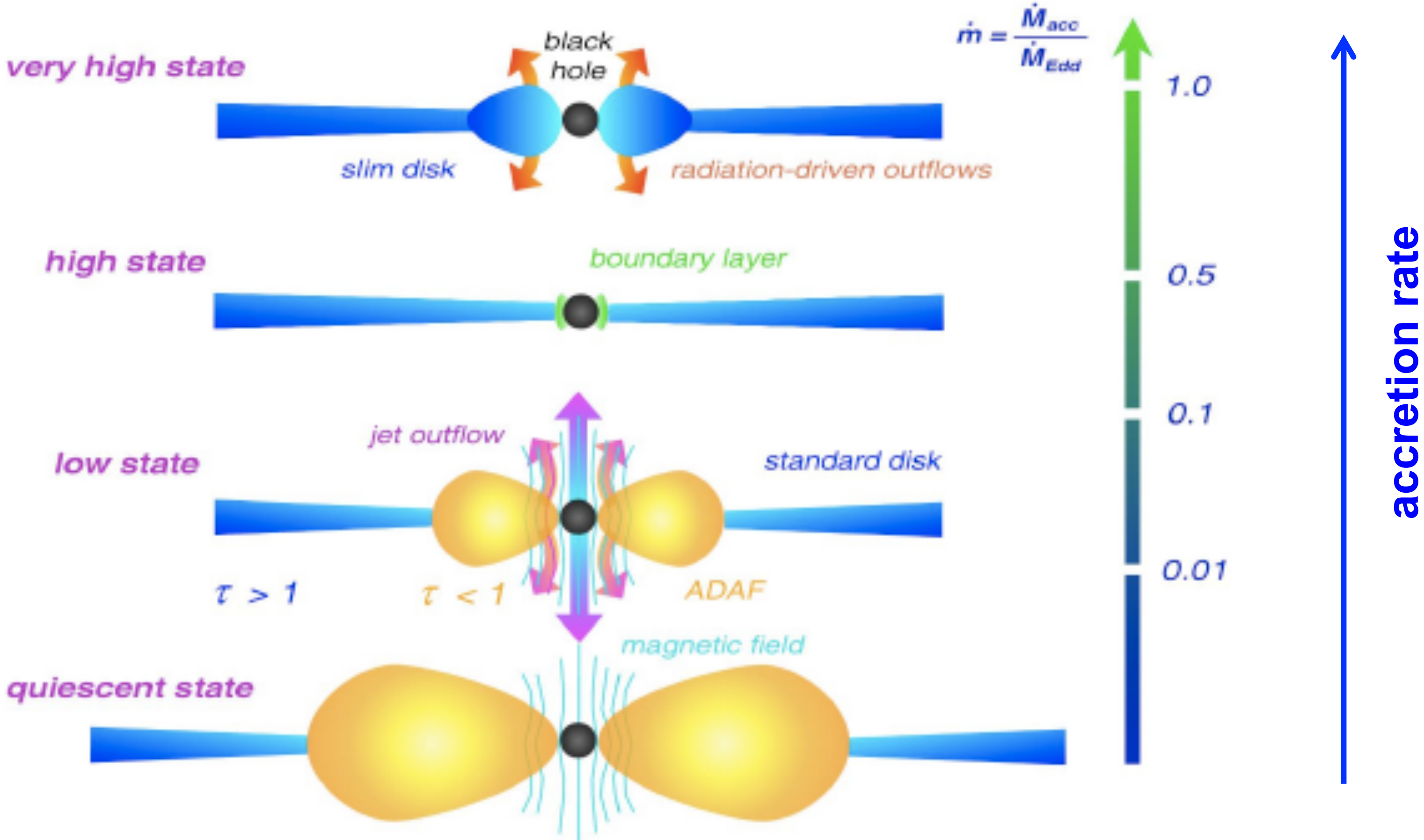
Presence of intermediate states

Quiescent state:
$\Gamma \approx 1.5$-2.1 (+BB in NS)
$L_x \approx 10^{30.5\text{-}33.5}$ erg/s
Adequate conditions for studies of the secondary star (limited heating)

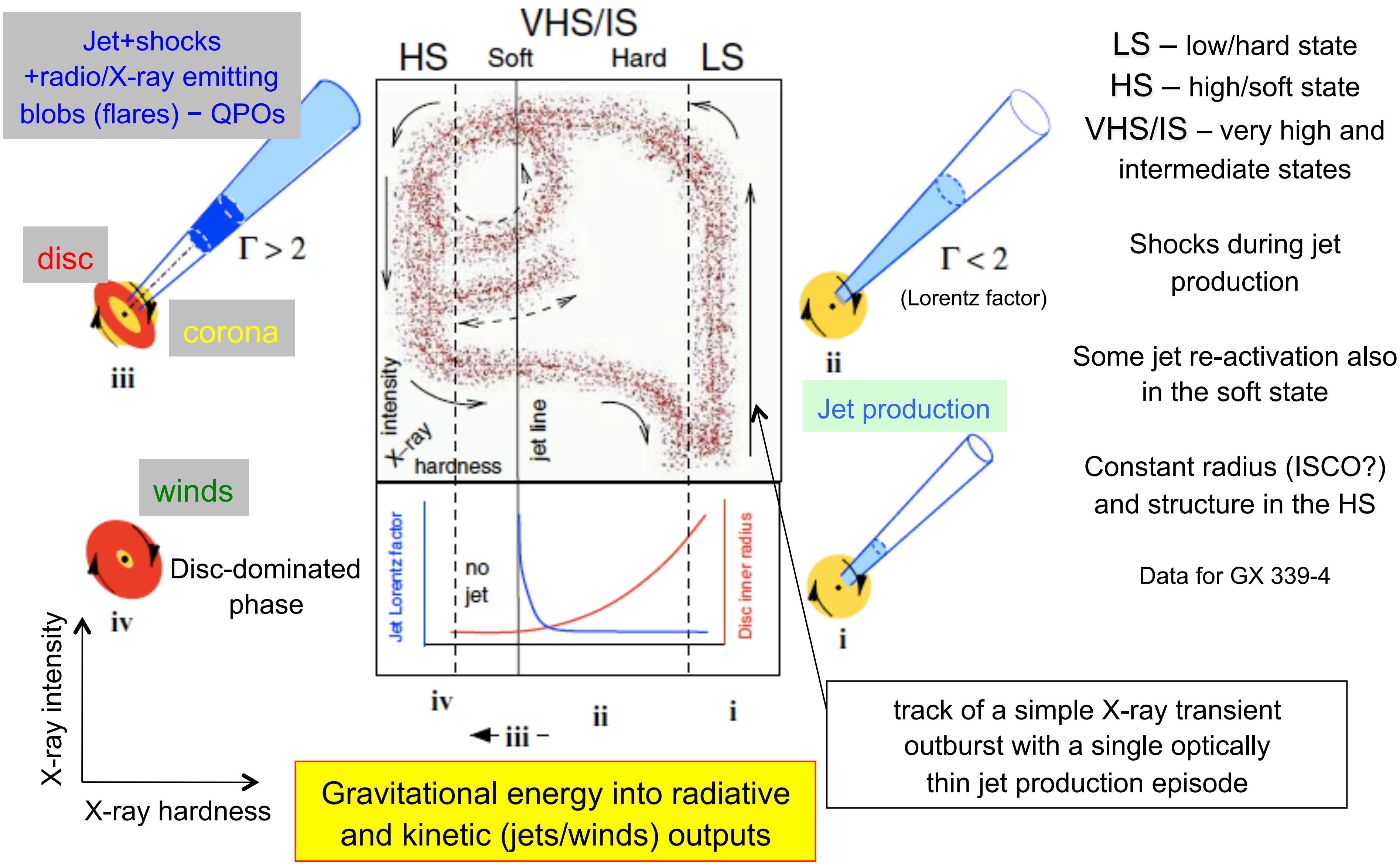
Mdot$\leq 10^{15}$ g/s

Spectral states mainly due to different disc/corona geometry and their interplay

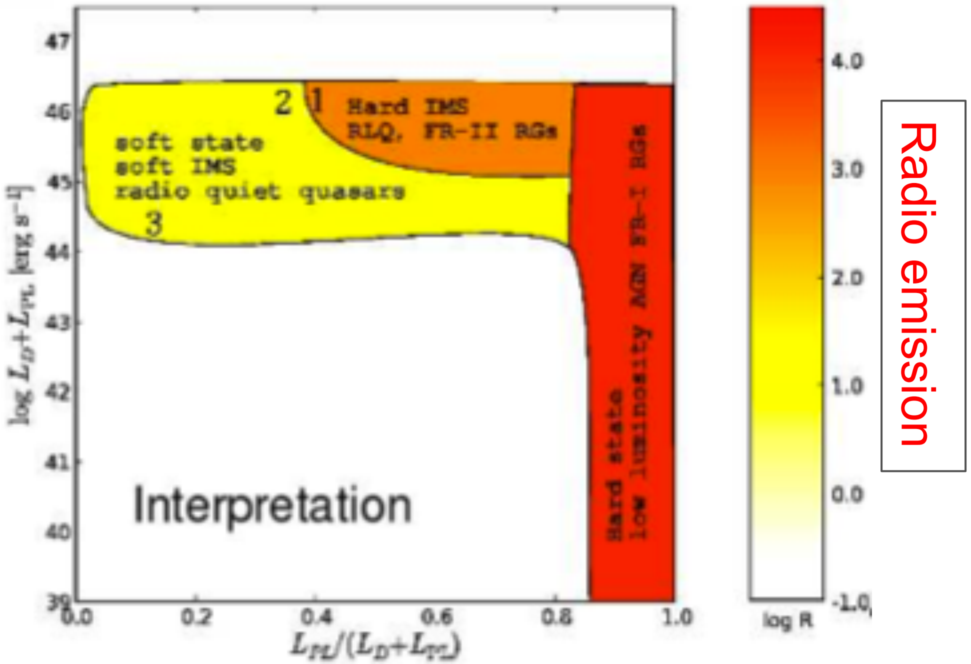
Inefficient radiation (ADAF?) in quiescent state (outer disc as merely a store of matter)


very high state
black hole
slim disk
radiation-driven outflows
high state
boundary layer
low state
jet outflow
standard disk
$\tau > 1$
$\tau < 1$
ADAF
magnetic field
quiescent state
$\dot{m} = \frac{\dot{M}_{acc}}{\dot{M}_{Edd}}$
1.0
0.5
0.1
0.01
accretion rate

Discs and jets (disc-jet coupling)



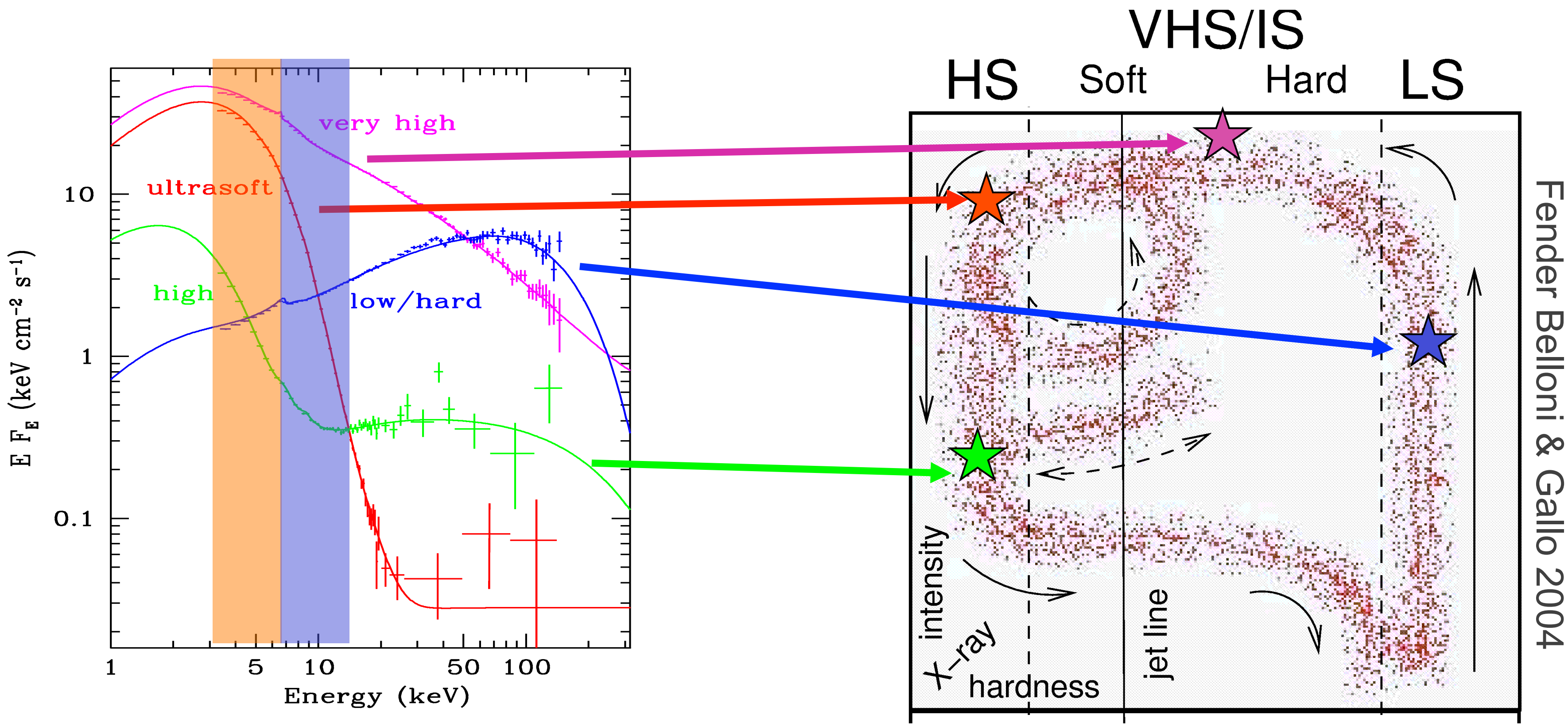
(from Fender et al. 2004; Remillard and McClintock 2007)



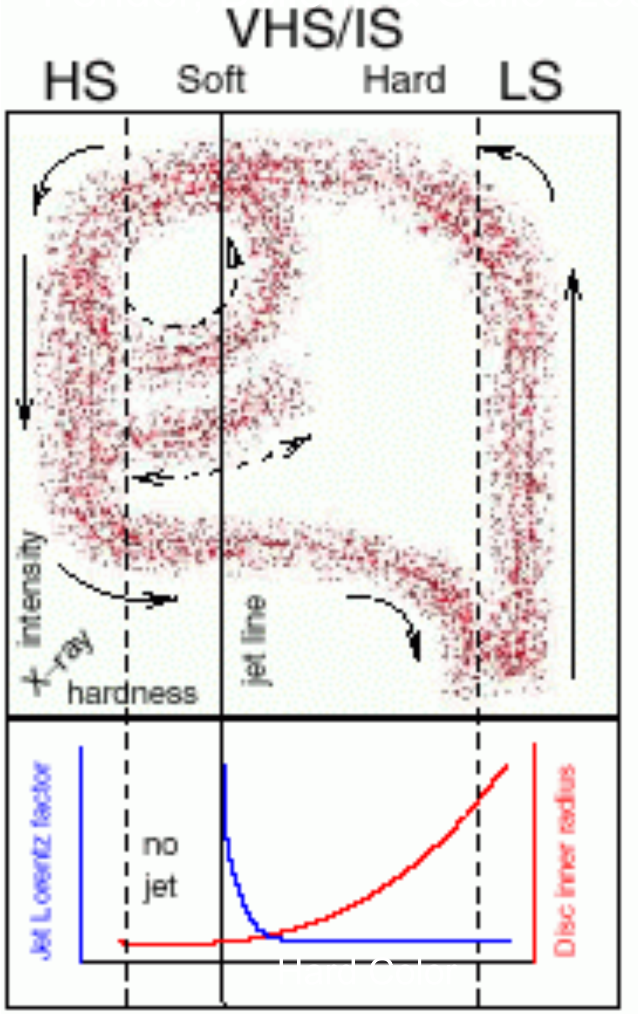
soft state
soft IMS
radio quiet quasars
Hard IMS
RLQ, FR-II RGs
Hard state
low luminosity AGN FR-I RGs
1
2
3
Interpretation
log L_D+L_{PL} [erg s^{-1}]
$L_{PL}/(L_D+L_{PL})$
log R
Radio emission

Hardness – intensity diagram

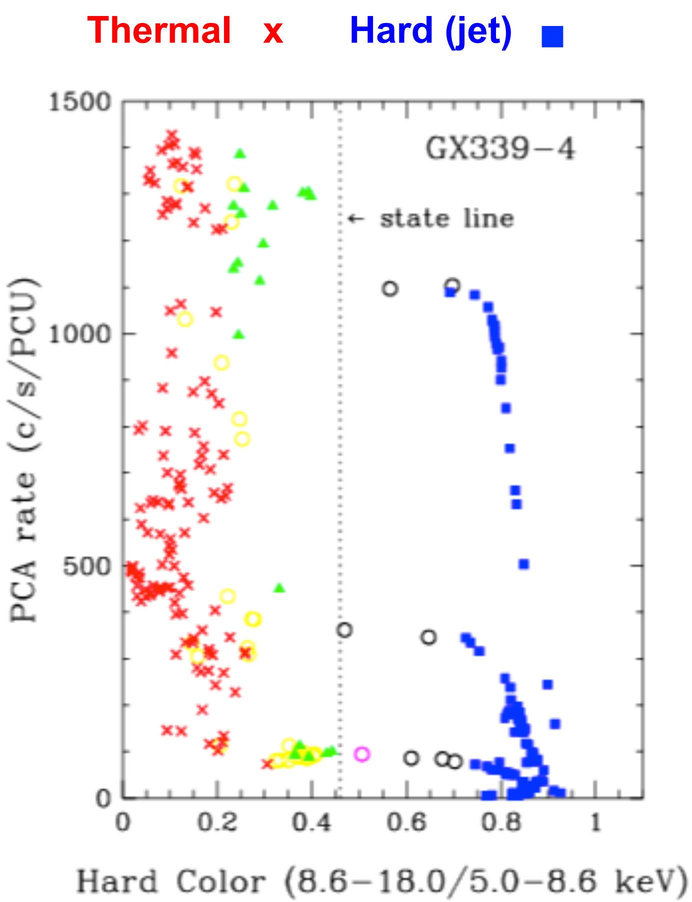
- Outburst starts hard, source stays hard as source brightens
- Then source softens to intermediate/very high state/steep power law state ➔ major hard-soft state transition
- Then the source is in the disc-dominated state, then hardens to make transition back to low/hard state – hysteresis as generally at lower L



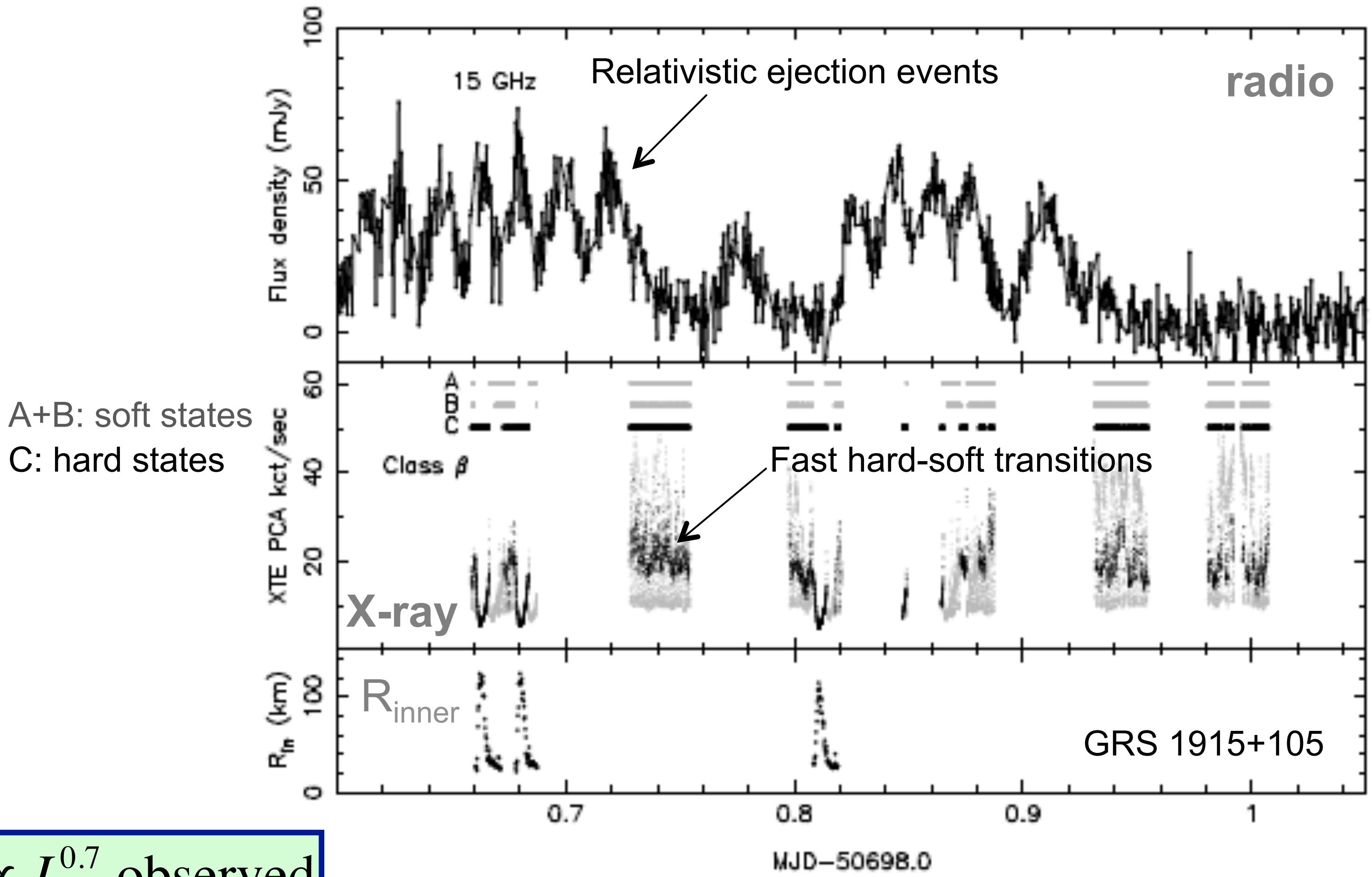
Unified model for jets in BH binaries



Remillard 2005



Support from observations



$L_R \propto L_X^{0.7}$ observed

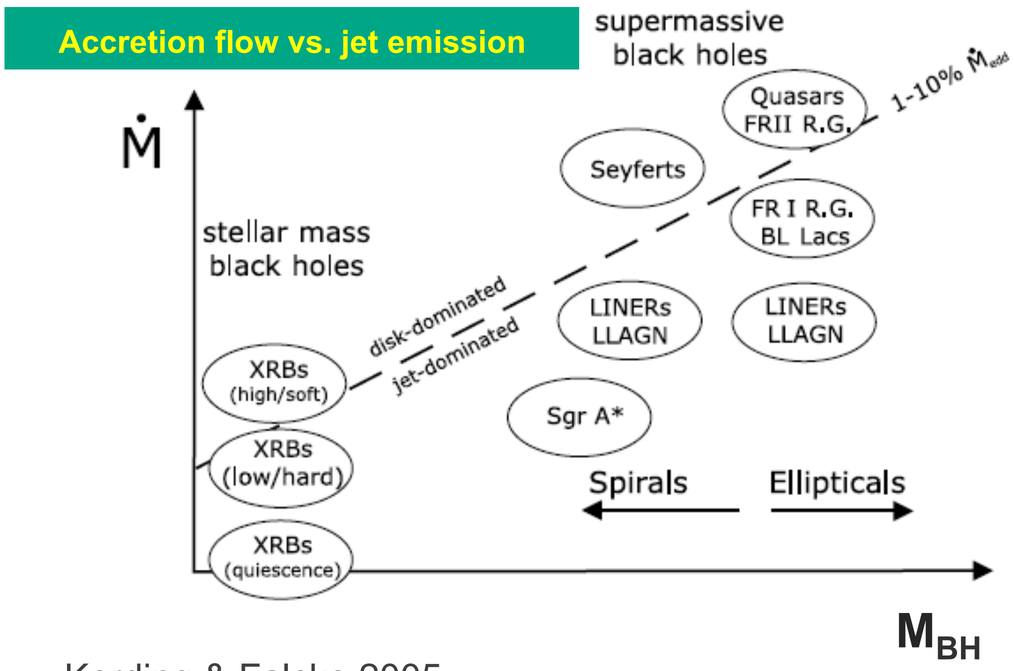
$L_R \propto P_{jet}^{1.4}$ theory

$\rightarrow\ \mathrm{P_{jet}} \propto L_X^{0.5}$

Major ejection events during state transitions

In BHB low states, jets may dominate

Unified model for BHB and low-power AGN



Kording & Falcke 2005

Weakly accreting systems: the jet is in equipartition without loosing a significant fraction of its energy via radiation (synchrotron phase)

At increasing accretion rates, SSC and EC play a role and the jet can be quenched if there is material around the BH that scatters the photons from the accretion disc

$$L_{\rm synch} \approx \left(q_j \frac{\dot{M}}{\dot{M}_{\rm Edd}} \right)^2 M$$

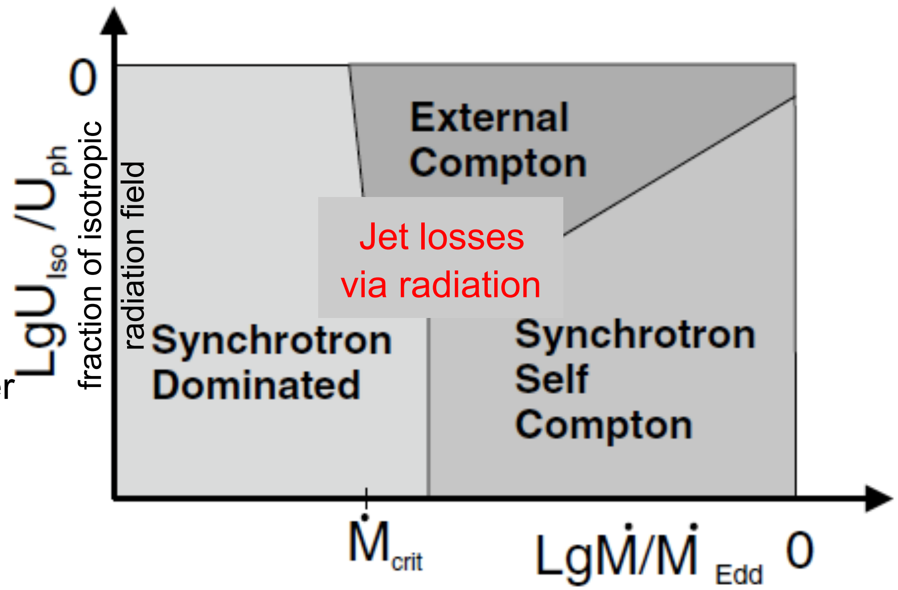
$$L_{\rm SSC} \approx \left(q_j \frac{\dot{M}}{\dot{M}_{\rm Edd}} \right)^3 M$$

M=BH mass

$$L_{\rm EC} \approx \left(q_j \frac{\dot{M}}{\dot{M}_{\rm Edd}} \right)^2 M U_{\rm ph}$$

q_j=percentage of accretion power injected into the magnetic field in the jet

U_{ph}=energy density of external photon field



A unified scheme for accretion

I) The accretion flow and disc form a coupled jet-disc system, with both components always present

II) Below a certain critical accretion rate (≈0.01−0.1 Edd. accretion rate), the inner part of the accretion flow becomes radiatively inefficient (low emission, ADAF)

III) Below this critical Mdot, or for face-on orientation (relativistic beaming), the jet emission (mostly radio/IR) dominates the emission from the accretion flow (X-rays, with maybe some contribution from the jet via synchrotron)

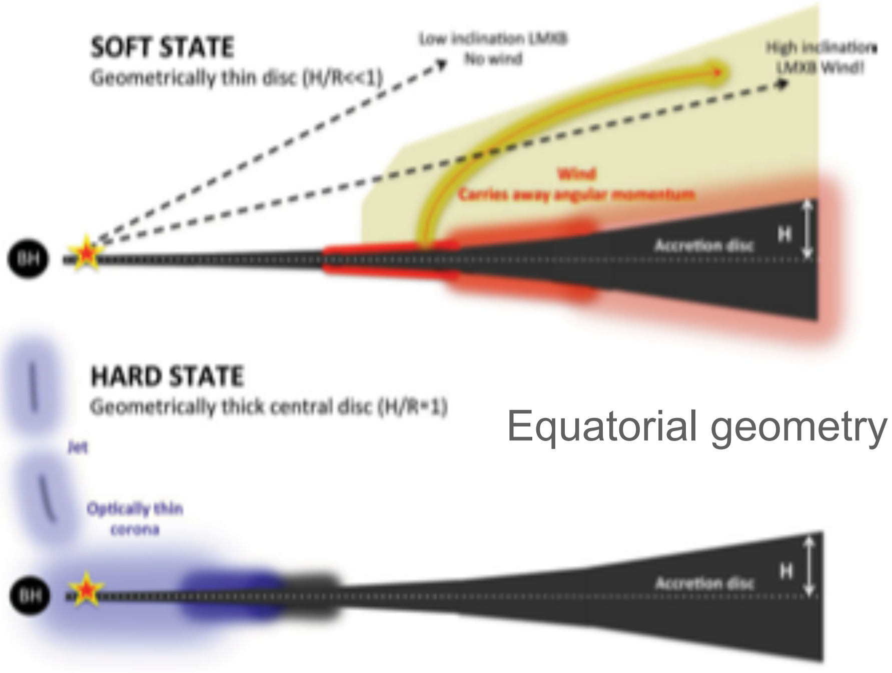
➔ jet-dominated accretion flow

➔ Near-Eddington, black holes are *disc-dominated* (with production of winds), while at sub-Eddington rates, black holes are *jet-dominated* (not thermally dominated)

➔ low-state binaries ≅ BL Lacs, FRIs, LINERS

One prediction is that the region of the onset of particle acceleration in the jet is at ≈100−1000 R_G

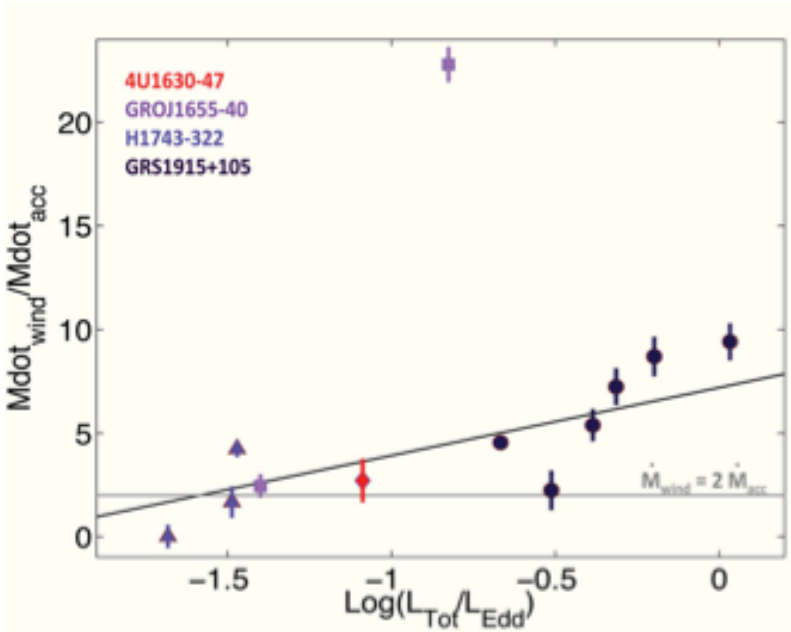
Winds in high/soft state



Equatorial geometry

$$\dot{M} = 4\pi R^2 n m_p v_{out} \frac{L_X}{\xi} \frac{\Omega}{4\pi}$$

V_{out}: wind outflow
ξ: ionization parameter (from abs. lines)
Ω: solid angle subtended by the wind
Thermal pressure to launch, then radiative and magnetic pressure

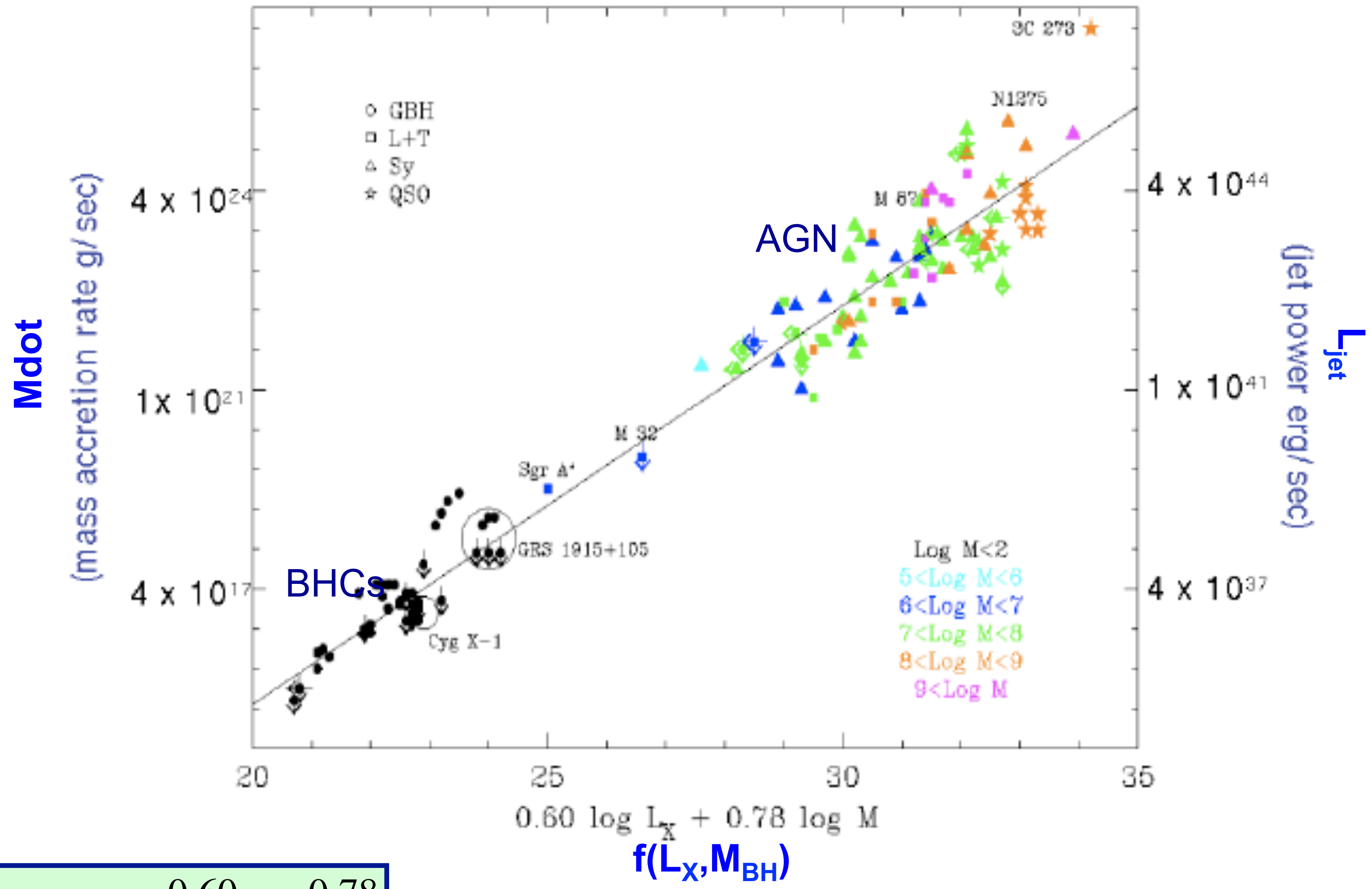


The mass outflow rate carried out by these winds may be higher than the inner accretion rate
➔ Responsible for the quenching of the jet?

Ponti et al. 2012

AGN/BHCs "fundamental plane"

A “fundamental” plane for AGN and BHBs

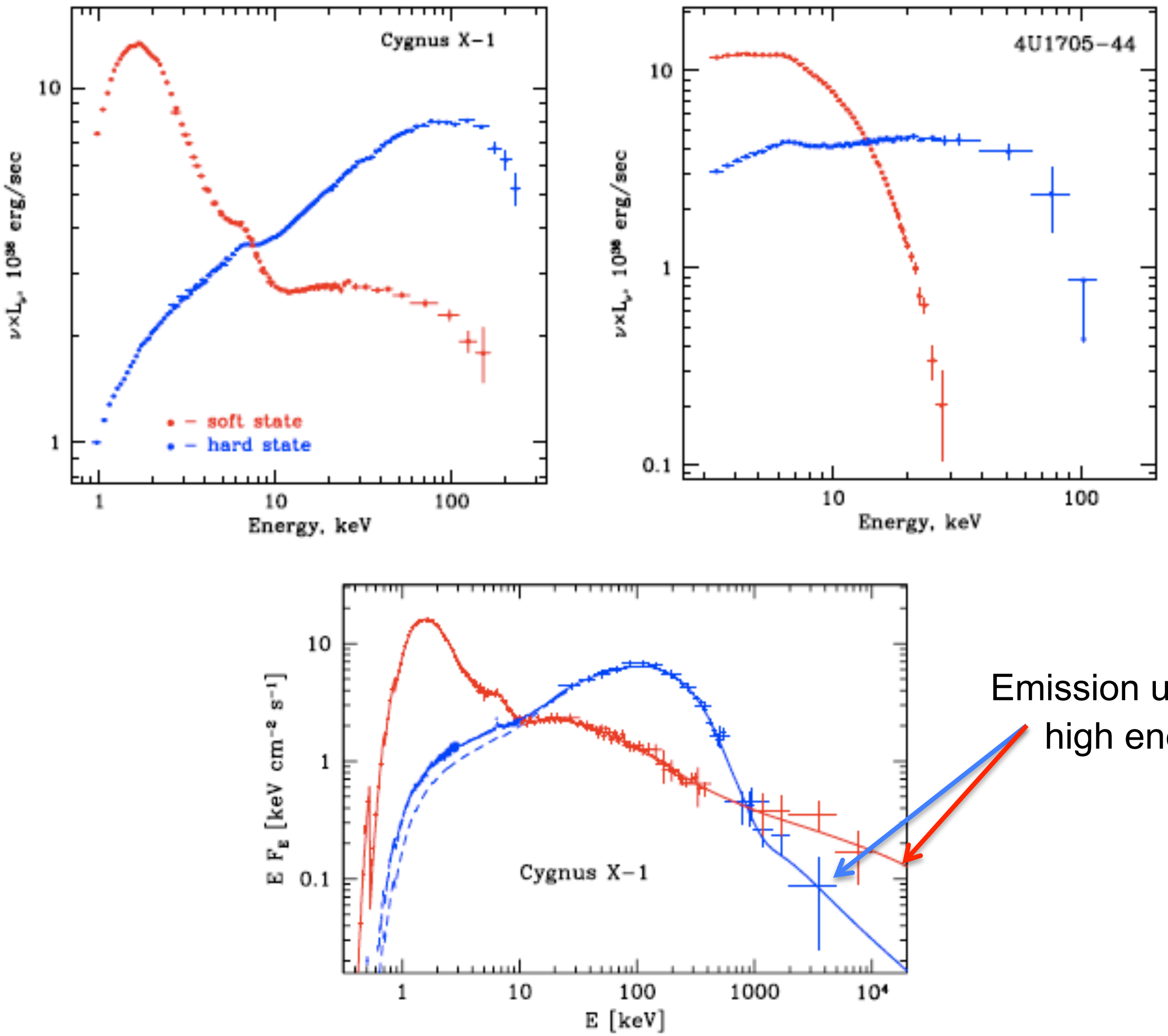


$$\boxed{L_{\rm radio} \propto L_X^{0.60} M^{0.78}}$$

From Merloni et al. 2003 – Here L_{jet} instead of L_{radio}

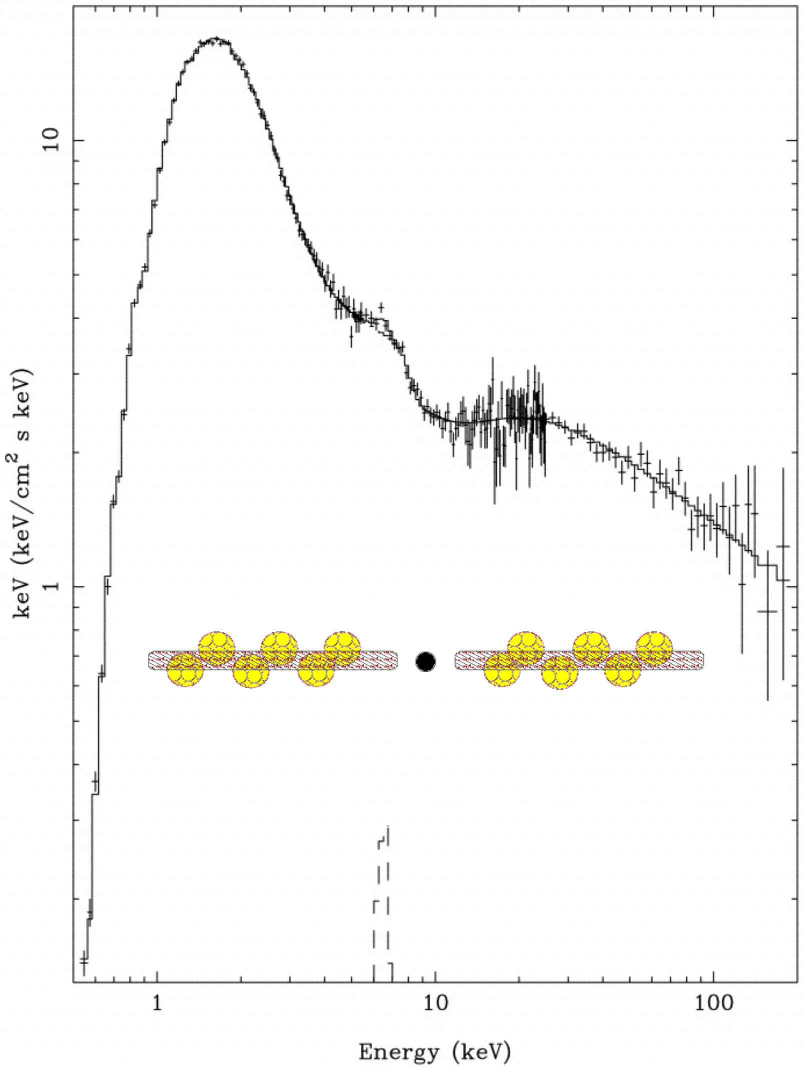
Some “real” X-ray spectra

Soft vs. hard state in BHBs

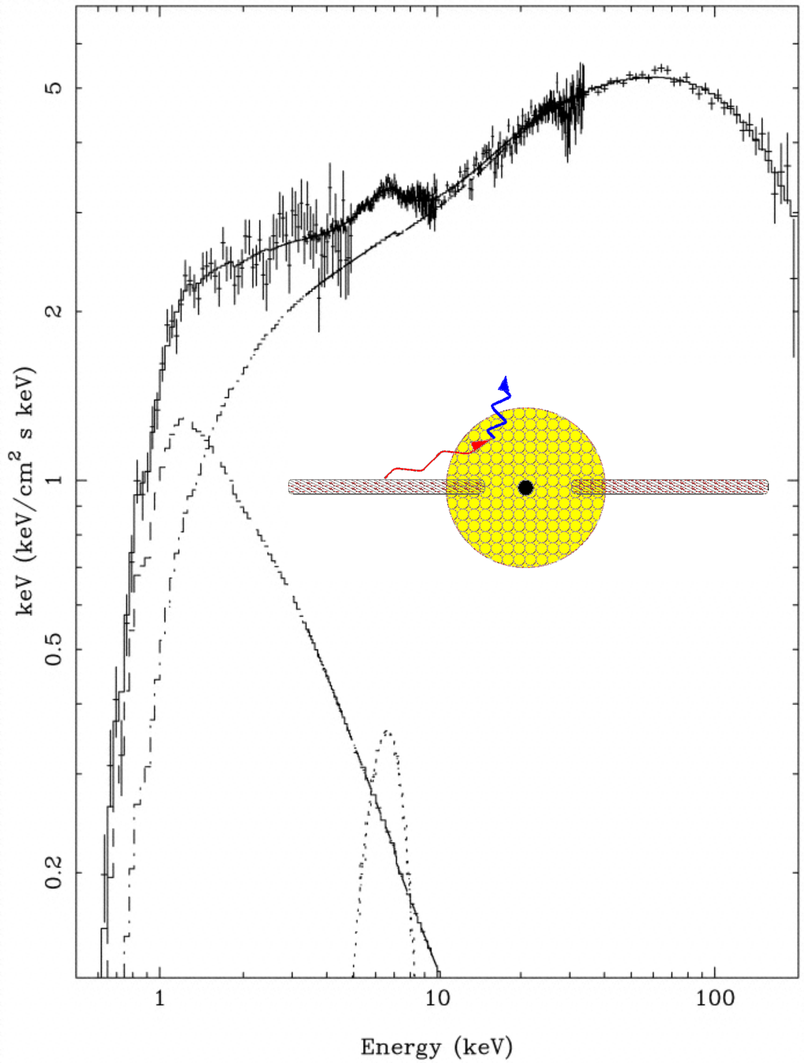


Emission up to very high energies

BeppoSAX



Soft (high) state
thermal disk emission + hard tail



Hard (low) state
hard X-ray spectrum, little thermal disk

Quasi periodic oscillations in BHBs

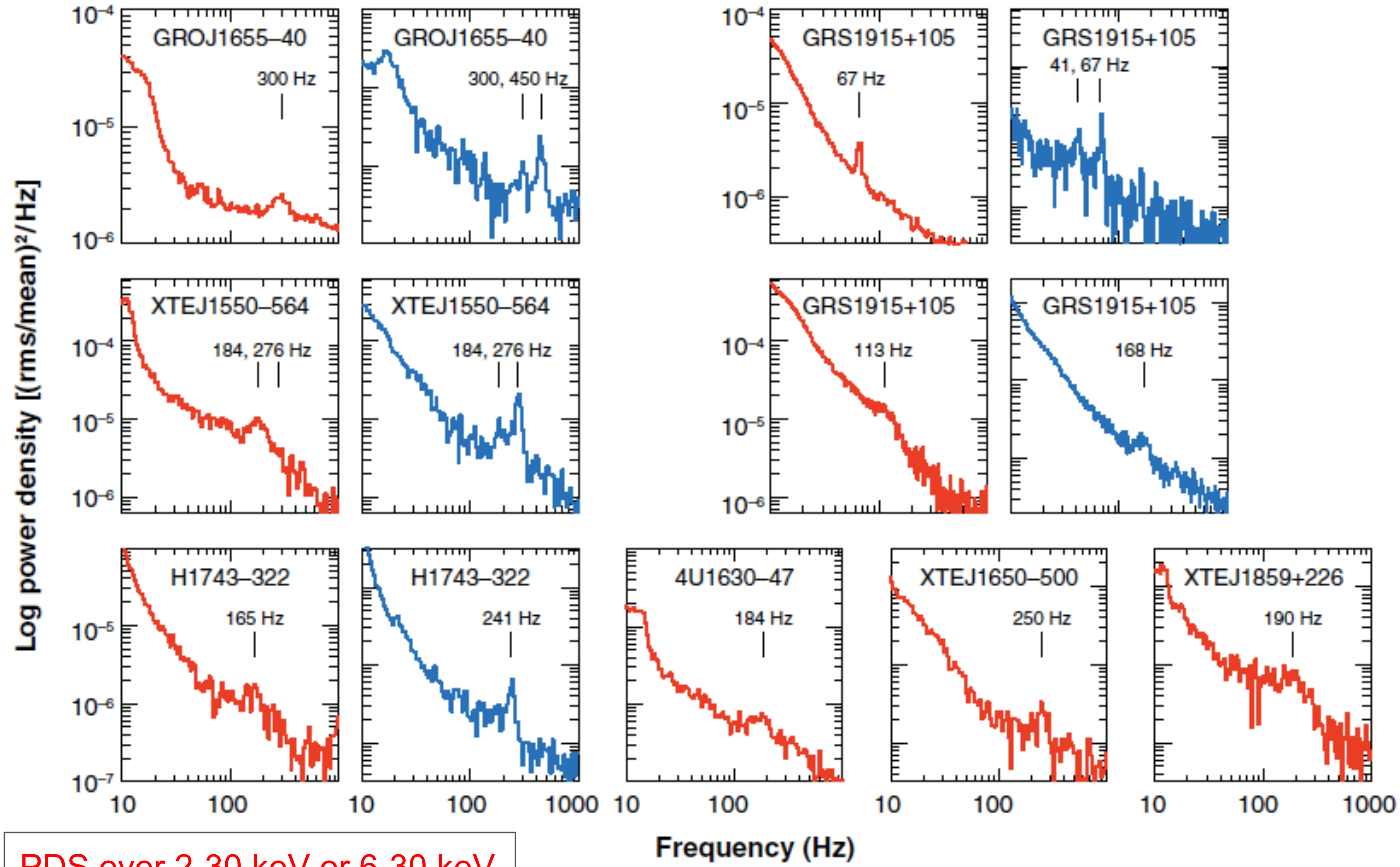
Main properties of QPOs in BHBs

Low-frequency QPOs (LFQPO: 0.1−30 Hz): tied to the flow of matter in the AD
$\nu_{LFQPO} << \nu_{Kepl,disc}$ ➔ $R \approx 100\ R_g$
Typically they are stable and persistent

High-frequency QPOs (HFQPO: 40−450 Hz): tied to $R \approx R_{ISCO}$
stable, do not shift in frequency (vs. NS)
3:2 pairing in frequency (resonance)
➔models explains such QPOs if the BH is rapidly spinning (Kerr BH) ➔ all sources with such QPOs have jets
Transient and subtly variable

Possible models: global disk oscillations, radial oscillations associated with spiral shocks, oscillations in the region separating the cool disk from the hot corona

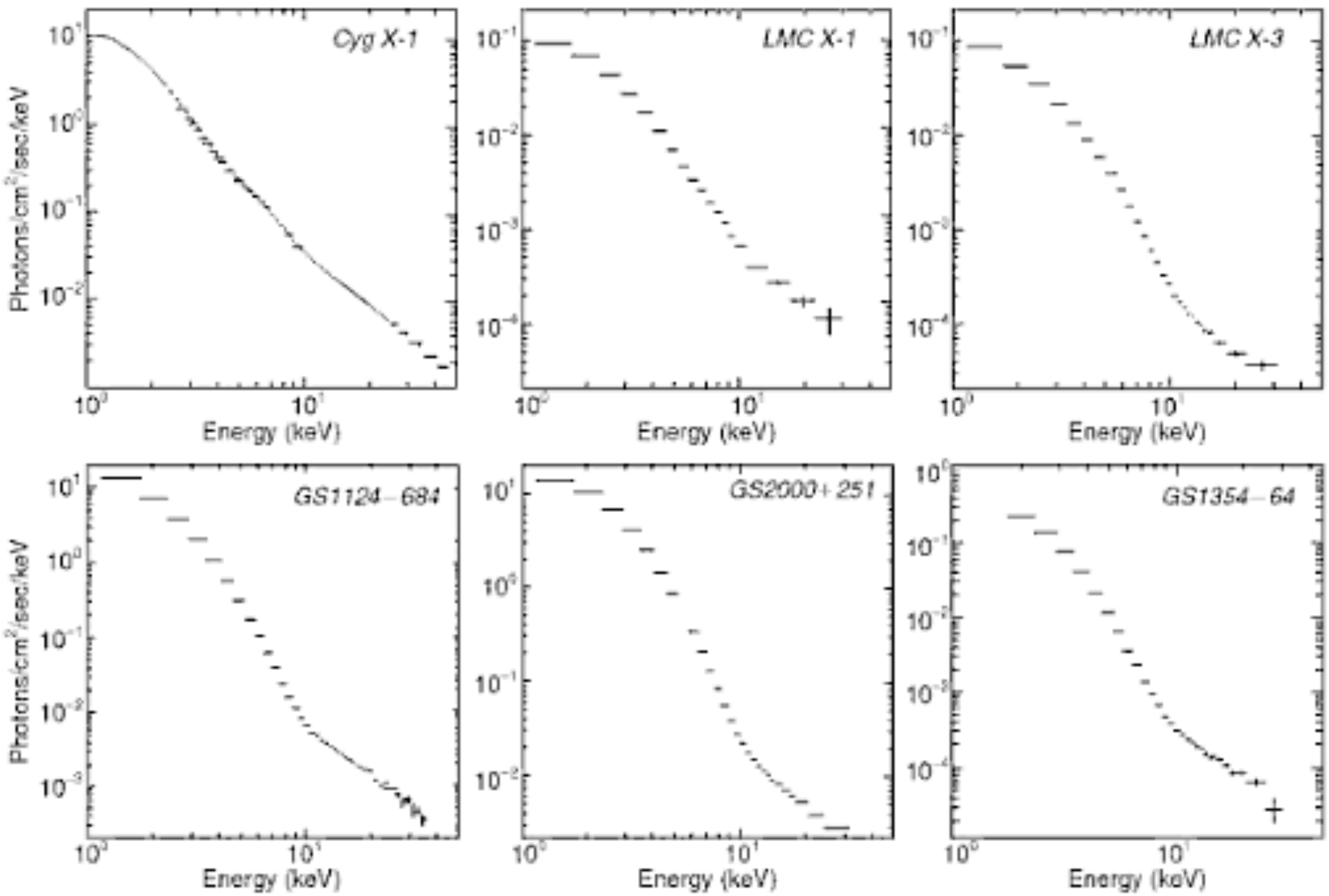
QPOs in BHBs and BHCs



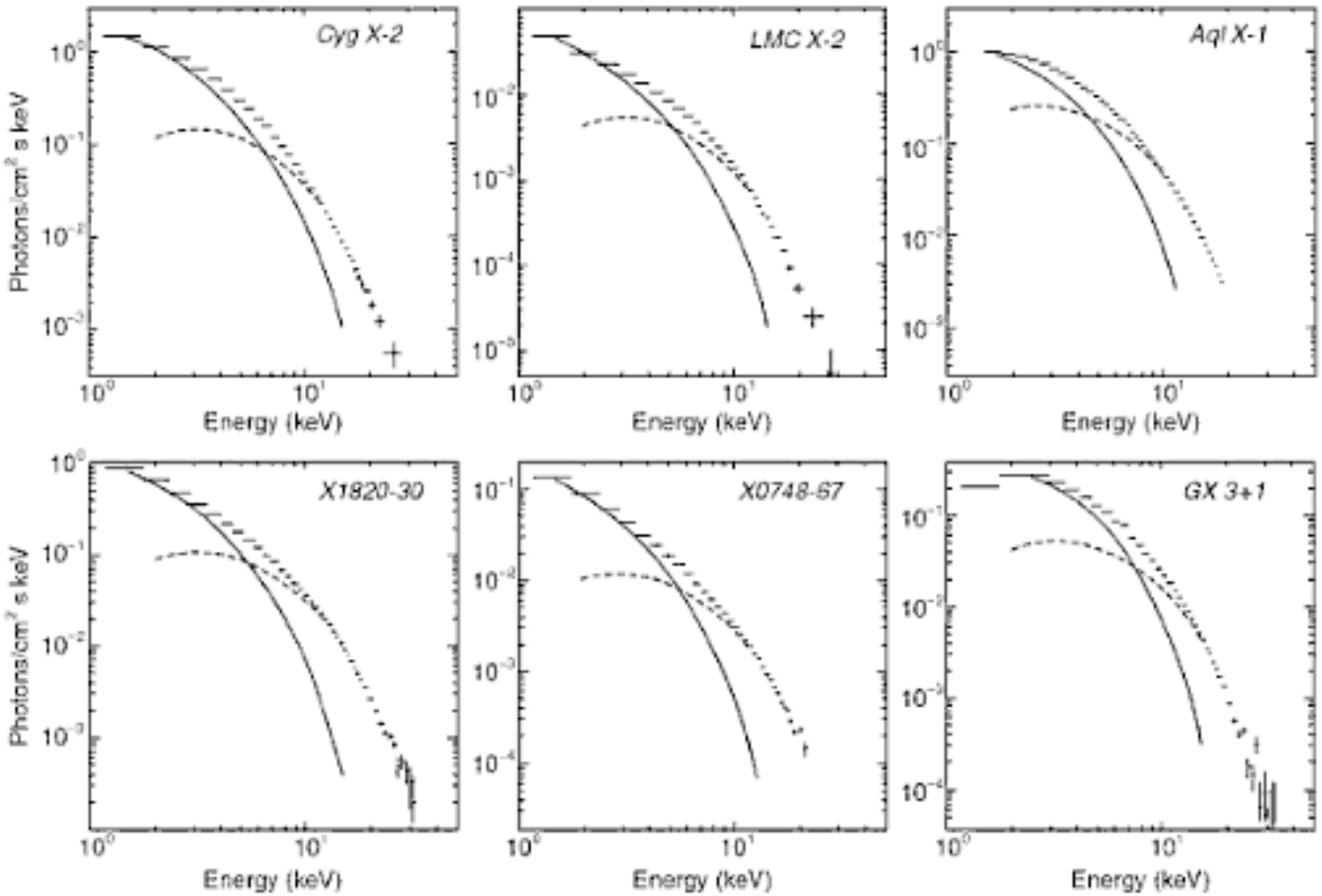
PDS over 2-30 keV or 6-30 keV
PDS over 13-30 keV

BHBs vs. NS in the same state

High-state spectra of BHBs (thermal + PL)



High-state spectra of NS (MCD+BB component)

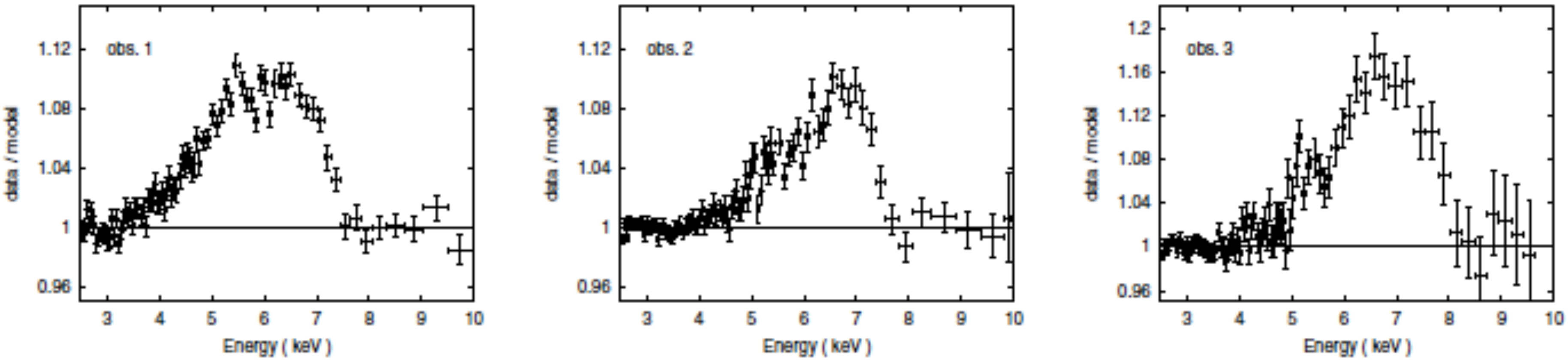


- NS have an additional ≈2 keV blackbody emission probably linked to the cooling of the NS surface (BH have no surface!) If not observed, possible obscuration by the corona
- No Type I bursts in BHBs
- kT_{in} prop $M_X^{-1/4}$ ➔ NS slightly “hotter” than BH

Broad (relativistic) iron lines

Broad (relativistic) iron Kα lines in BHCs

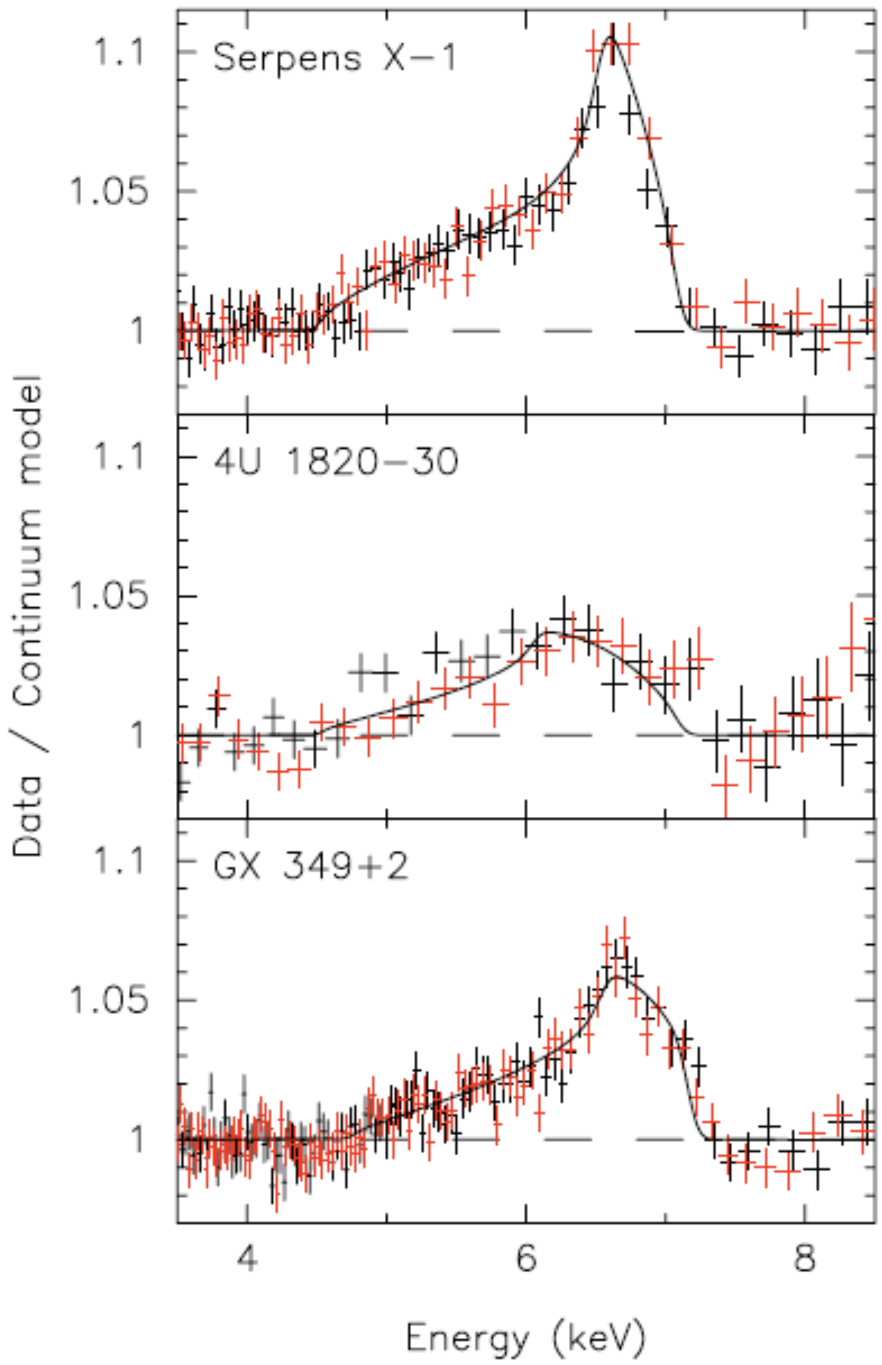
XTE J1650-500 (BH candidate) during outburst



Miniutti et al. (2004)

and in NS LMXBs

LMXBs with *Suzaku* observations

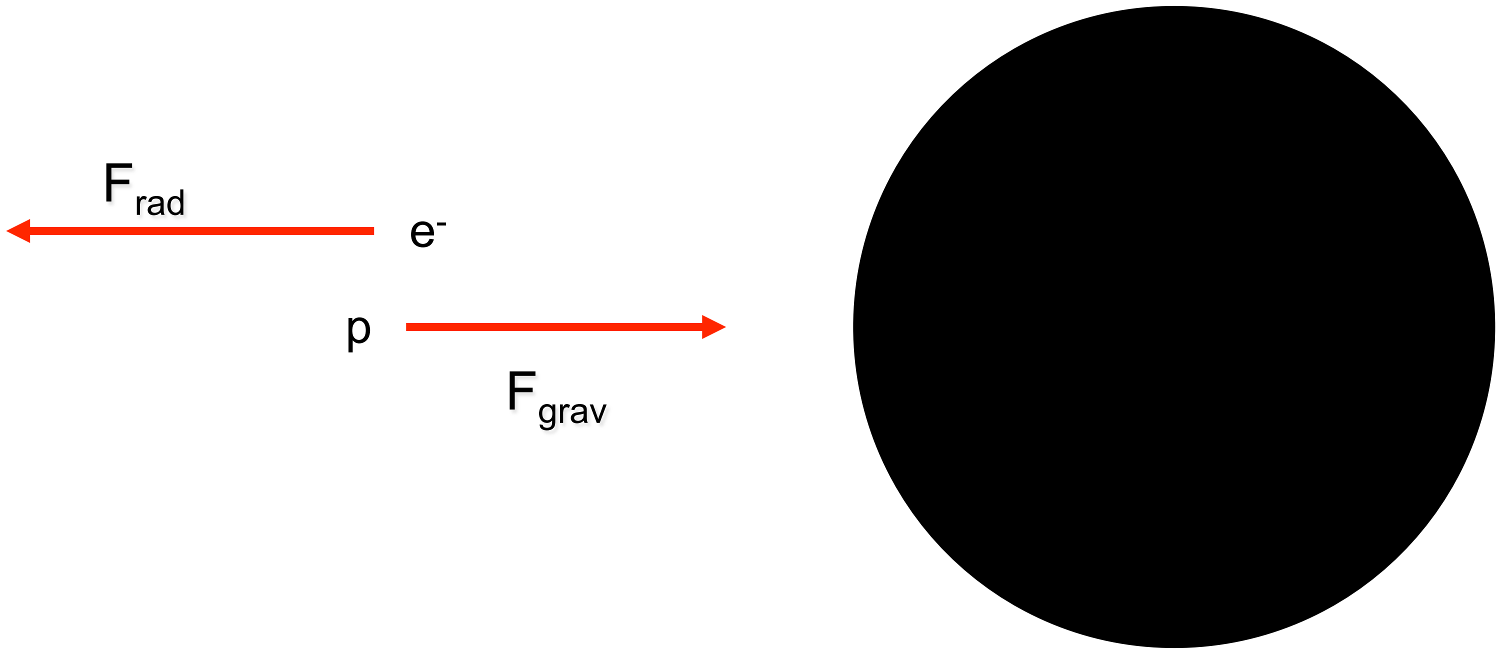


Cackett et al. (2008)

Eddington Luminosity

How luminous can an accreting black hole be?

ASSUMPTION: spherically symmetric steady-state radial infall of ionized H



$$P_{\mathrm{rad}} = \frac{F_{\mathrm{rad}}}{\mathrm{c}} = \frac{L}{4\pi R^2 c}$$

Outward momentum flux = pressure

$$F_{\mathrm{rad}} = \frac{L\sigma_T}{4\pi R^2 c}$$

Outward radiation force on a single electron

$$F_{\mathrm{grav}} = \frac{G(m_p + m_e)M}{\mathrm{R}^2} \approx \frac{G m_p M}{R^2}$$

Gravitational energy acting on a proton

Eddington luminosity (limit)

max luminosity allowed under the defined assumptions

$$F_{\mathrm{rad}} \leq F_{\mathrm{grav}} \rightarrow \frac{L\sigma_T}{4\pi R^2 c} \leq \frac{GMm_p}{R^2} \rightarrow L \leq \frac{4\pi G c m_p}{\sigma_T} M$$

$$\rightarrow L_{\mathrm{Edd}} = 1.26 \times 10^{38} \left(\frac{M}{M_{\mathrm{sun}}} \right) \mathrm{erg/s}$$

Accretion luminosity and accretion rate

Accretion luminosity: luminosity released by gravitational energy due to gas falling onto a star/compact object with mass M and radius R

$$L = \frac{GM\dot{M}}{\mathrm{R}} = \eta \dot{M} c^2$$

η=efficiency of the conversion of gravitational energy into radiation

$$L_{\mathrm{Edd}} = \frac{4\pi G c m_p M}{\sigma_{\mathrm{T}}} = \frac{GM\dot{M}_{\mathrm{Edd}}}{\mathrm{R}} \rightarrow \dot{M}_{\mathrm{Edd}} = \frac{4\pi c m_p}{\sigma_{\mathrm{T}}} R$$

$$\rightarrow \dot{M}_{\mathrm{Edd}} \approx 1.5 \times 10^{-8} \left(\frac{R}{10\ \mathrm{km}} \right) M_{\mathrm{sun}}/\mathrm{yr}$$

[cgs system]

$m_p = 1.7\times10^{-24}$ g

$M_\odot = 2\times10^{33}$ g

$\sigma_T = 6.65\times10^{-25}$ cm^2

$G = 6.67\times10^{-8}$ cm^3/g/s^2

All calculations

$$\dot{M}_{\mathrm{Edd}} = \frac{4\pi c m_p}{\sigma_{\mathrm{T}}} R = \frac{12.56\ 3\ 10^{10}\,(\mathrm{cm/s})\ 1.7\ 10^{-24}\,(\mathrm{g})}{6.65\ 10^{-25}\,(\mathrm{cm}^2)} R =$$

$$= 9.63\ 10^{11}\ \mathrm{R}\ (\mathrm{g/cm\ s}) = 9.63\ 10^{11} \left(\frac{R}{10\ \mathrm{km}}\right) 10^{6}\,(\mathrm{cm})\ \frac{3600 \times 24 \times 365\ (\mathrm{s})}{2\ 10^{33}\ (\mathrm{g})}\ [M_{\mathrm{sun}}/\mathrm{yr}] =$$

$$= \dot{M}_{\mathrm{Edd}} \approx 1.5 \times 10^{-8} \left(\frac{R}{10\ \mathrm{km}}\right) [M_{\mathrm{sun}}/\mathrm{yr}]$$

Accretion rate in a super-massive black hole (I)

$$L_{\mathrm{Edd}} = \eta \dot{M} c^2 = 1.3\times 10^{38}\left(\frac{M}{M_{\mathrm{sun}}}\right)\ \mathrm{erg/s}$$

$$\eta_{0.1} = \eta / 0.1$$

$$\mathrm{erg{=}g\ cm^2/sec^2}$$

$$\frac{\mathrm{g}}{\mathrm{s}} = \frac{\mathrm{M_{sun}}}{2\times 10^{33}} \times \frac{365\times 24\times 3600}{\mathrm{yr}} = 1.58\times 10^{-26}\ \frac{\mathrm{M_{sun}}}{\mathrm{yr}}$$

$$\rightarrow \dot{M}_{\mathrm{Edd}} = \frac{L_{\mathrm{Edd}}}{\eta c^2} = \frac{1.3\times 10^{38}(M / M_{\mathrm{sun}})}{0.1\times \eta_{0.1}\times 9\times 10^{20}}\left[\frac{\mathrm{erg/s}}{\mathrm{cm^2/s^2}}\right] =$$

$$= \frac{1.44\times 10^{18}}{\eta_{0.1}}\left(\frac{\mathrm{M}}{\mathrm{M_{sun}}}\right)[\mathrm{g/s}] = \frac{1.44\times 10^{18}}{\eta_{0.1}}\times 1.58\times 10^{-26}\left[\frac{\mathrm{M_{sun}}}{\mathrm{yr}}\right] =$$

$$\approx \frac{2.2\times 10^{-8}}{\eta_{0.1}}\left[\frac{\mathrm{M_{sun}}}{\mathrm{yr}}\right] = 2.2\ M_8/\eta_{0.1}\ [\mathrm{M_{sun}/yr}]$$

$$\rightarrow \dot{M}_{\mathrm{Edd}} = 2.2 M_8/\eta_{0.1} \qquad [\mathrm{M_{sun}/yr}]$$

M_8= M in units of $10^8\ M_\odot$
$\eta_{0.1}=\eta/0.1$

Accretion rate in a super-massive black hole (II)

Eddington time

Eddington time: time taken by a body to radiate its entire mass at the Eddington rate

$$\dot{M}_{\rm Edd} = \frac{M}{t_{\rm Edd}} \rightarrow t_{\rm Edd} = \frac{M}{\dot{M}_{\rm Edd}} = \frac{M}{2.2 M_8 / \eta_{0.1}} = \frac{10^8}{2.2}\eta_{0.1} = 4.5 \times 10^7 \eta_{0.1}\ {\rm yr}$$

Rate of increase in mass for a black hole

$$\frac{dM}{dt} = \dot{M}_{\mathrm{Edd}} = \frac{M}{t_{\mathrm{Edd}}} \rightarrow \int_{M_0}^{M} \frac{1}{M'}\,dM = \frac{1}{t_{\mathrm{Edd}}} \int_{t_0}^{t} dt' = \frac{1}{t_{\mathrm{Edd}}}(t - t_0) \overset{t_0=0}{=} \frac{t}{t_{\mathrm{Edd}}}$$

$$\rightarrow \ln(\mathrm{M}) - \ln(\mathrm{M}_0) = \frac{t}{t_{\mathrm{Edd}}} \rightarrow M = M_0\, e^{(t/t_{\mathrm{Edd}})}$$

T(R) in accretion discs (I)

$$L = \frac{1}{2}\frac{GM\dot{M}}{R}$$

Half of the E_{grav} is radiated, half goes into heating of the gas

$$L = 2\pi R^2 \sigma_{SB} T^4$$

πR^2=area of the disc (×2: both surfaces)
$\sigma_{SB}T^4$: flux passing through the surface

Stefan-Boltzmann law

$$T = \left(\frac{GM\dot{M}}{4\pi\sigma_{SB}R^3}\right)^{1/4} \rightarrow \left[\frac{3GM\dot{M}}{8\pi\sigma_{SB}R^3}\left(1-\left(\frac{R_{in}}{R}\right)^{1/2}\right)\right]^{1/4}$$

Proper treatment

$$R_{in} \approx R_S = 2GM/c^2$$

$$R >> R_{in} \rightarrow 1-(R_{in}/R)^{1/2} \approx 1$$

$$R^{-3/4} = (R/R_S)^{-3/4} \times (2GM/c^2)^{-3/4}$$

$$T(R) = \left(\frac{3c^6}{64\pi\sigma_{SB}G^2}\right)^{1/4} \mathrm{M}^{-1/2}\ \dot{M}^{1/4}\left(\frac{R}{R_S}\right)^{-3/4}$$

see also Done 2010, arXiv:1008:2287

All calculations

$$T = \left(\frac{3GM\dot{M}}{8\pi\sigma_{SB}}\right)^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} \left(\frac{2GM}{c^2}\right)^{-3/4} =$$

$$= \left(\frac{3G}{8\pi\sigma_{SB}}\right)^{1/4} M^{1/4}\,\dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} \left(\frac{c^2}{2G}\right)^{3/4} M^{-3/4} =$$

$$= \left(\frac{3GG^{-3}c^6}{8\pi\sigma_{SB}2^3}\right)^{1/4} M^{-1/2}\,\dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} =$$

$$= \left(\frac{3G^2c^6}{8\pi\sigma_{SB}2^3}\right)^{1/4} M^{-1/2}\,\dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4} =$$

$$= \left(\frac{3c^6}{64\pi\sigma_{SB}G^2}\right)^{1/4} M^{-1/2}\,\dot{M}^{1/4} \left(\frac{R}{R_S}\right)^{-3/4}$$

T(R) in accretion discs (II)

$$\dot{M}_{\rm Edd} = 2.2 M_8/\eta_{0.1} \qquad [M_{\rm sun}/{\rm yr}]$$

+

$$T = {\rm cons} \times M^{-1/2}\,\dot{M}^{1/4}\left(\frac{R}{R_S}\right)^{-3/4}$$

η=0.1

$M^{-1/2}$

$M_{dot}^{1/4}=(M_{dot,Edd} \times M_{dot}/M_{dot,Edd})^{1/4}$

$$T(R) = {\rm cost} \times (10^8 M_8 M_{\rm sun})^{-1/2}(2.2 M_8)^{1/4}\left(\frac{\dot{M}}{\dot{M}_{\rm Edd}}\right)^{1/4}\left(\frac{R}{R_S}\right)^{-3/4}$$

$$T(R) \approx 2.1 \times 10^5\ M_8^{-1/4}\ (\dot{M}/\dot{M}_{\rm Edd})^{1/4}\ ({\rm R/R_S})^{-3/4}\ {\rm K}$$

$$\propto M^{-1/4}\ (\dot{M}/\dot{M}_{\rm Edd})^{1/4}$$

$T_{BB}(R) \approx M^{-1/4}$: thermal (disk) emission mostly emitting in UV for AGN (*big blue bump*), in soft X-rays in BHBs

T(R) in accretion discs (III)

- Disc annuli not blackbody – too hot, so little true opacity. Compton scattering important.
- Modified blackbody (Shakura & Sunyaev 1973)
- Describe by colour temperature fcol
- Relativistic smearing effects on the spectra at each radius