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Mechanical Design and Dynamic Modeling of a Two-Wheeled Inverted Pendulum Mobile Robot

Jingtao Li, Xueshan Gao, Qiang Huang, Qinjun Du, and Xingguang Duan
Intelligent Robotics Institute, School of Aerospace Science and Engineering
Beijing Institute of Technology
5 Nandajie, Zhongguancun, Haidian, Beijing, China, 100081
jingtao.li@163.com, { qhuang, xueshan.gao, duanstar }@bit.edu.cn, qinjundu@sohu.com

Abstract - **The differential drive two-wheeled inverted pendulum mobile robot has advantage of high maneuverability on slopes and in narrow spaces. It can be used to carry human beings as well as other goods. This paper mainly discusses mechanical design and dynamic modeling of this type of mobile robot platform. The first prototype is 12 kg of weight with dimensions of 42x36x15 cm. A multi modal control system is developed which includes the human transport control system and goods transport control system mode. An embedded control system is implemented to control two motors for both modes. In human transport mode the master controller receives the data from the data conditioning module and generates the reference for PID controllers whereas in goods transport mode the master controller receives command from the teleoperator. The effectiveness of the developed mobile robot is confirmed by experiments such as climbing slope, pivot around and surpassing small obstacles.**

Index Terms - ***mobile robot, inverted pendulum, dynamic modeling, mechanical design***

I. INTRODUCTION

The human assisting robots have been used in industry and hazardous environments for a long time. There field of application may be found in radioactive and chemical installations, anti terrorism activities, fire fighting, reconnaissance operations etc [1]. Work is being done to develop robots that can assist human beings in performing normal day life tasks. The environments where human being perform task includes narrow spaces, slopes etc. The robot characteristics for such environments should be small size, light weight, low power consumer and capability to carry load.

The differential drive mobile robots with a rear balancing caster wheel have been the choice in most of the robots due to its simplicity in design and control [1-6], but have limitations of terrain adaptability. The track or leg type of mobile robots have a relatively higher adaptability to terrains as compared to the former type, but their designing and control system is complex [7]. Two questions that must be answered before designing the mechanism and control system for the robot. First where we are going to use our robot and how it will interact with the environment. Once the environment is decided the choice of appropriate sensors, and extraction of required information and use of appropriate control method for an efficient control system is to be decided.

Two-Wheeled Inverted Pendulum Mobile Robot are well suited for traversing narrow spaces, slopes etc. Its characteristics include compact size, light weight and lower energy consumption. The design of this robot shares some similarities with the other mobile robots like Joe [6], the Segway [8] and the PMP [2-3].

The Segway was recently developed and it is on the market. It is a new type of personal vehicle that has only two coaxially mounted wheels and a steering bar. Its method of propulsion is a kind of wheeled inverted pendulum that can be balanced by the driving force of each of the two wheels to stabilize the whole system including the rider. Forward and backward movement control is achieved by the rider's shifting of his center of gravity (COG), and the vehicle is steered by manual throttle mechanism. However, the total weight is approximately 38 kg, which is too heavy to carry away by user. The PMP is another type of a personal vehicle that consists only of two wheels and a standing base for the rider, without a steering bar. It is smaller and lighter as compared to Segway.

The Segway and the PMP are made for personal riding, both have only one mode: traveling control (running and steering control) based on shifting of rider's COG [3]. They can not work in semi-autonomous and the remotely control modes.

In this work, considering the security and stability of using it in narrow spaces or on slopes, a new robotic platform with improved capabilities is developed. It is designed to be a light weight, compact size, and low consumption. Moreover the robot become practically useful in daily life due to it capability of being semi-autonomous and remotely controlled. The purpose of the paper is to describe the robot design and to report experiences from the design phase.

This paper is organized as follows: Section II presents the main features and mechanical design of the robot. Dynamic system model is given in Section III. The designed hardware, high level electronics, software, sensors and actuators for the first prototype are presented in section IV. The experimental works is given in Section V. Concluding remarks and future works are listed in Section VI.

II. MECHANICAL DESIGN

The design objective is to build a new robotic platform capable of performing in narrow spaces and on slopes. The basic characteristics include small size, light weight, compactness, load transportability, easily deployment, carrying by a single person and capability of surpassing small obstacles.

The design includes the following four main features:

1. It is differentially driven system so the robot can turn around at one point, and it can be used in narrow spaces.

2. The upper part of the body can keep upright when the robot is running on a slope.

3. The layout of the robot is symmetrical.

4. Two auxiliary wheels support is provided to become more security.

In this research a mobile robot with a two-coaxial-wheeled structure, as shown in Fig. 1 is designed and developed. The design has two independent wheels attached to the rectangular frame. The symmetrical layout is designed by locating the two actuated wheels in both left and right side of the chassis and two auxiliary wheels on front and rear. The auxiliary wheels ensure the robot's safety when it is in faulty or in abnormal condition. Fig. 1 shows the robot with its three degrees of freedom (3-DOF).

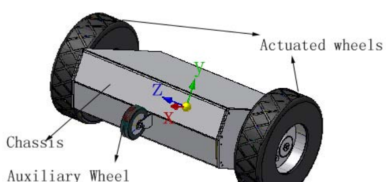
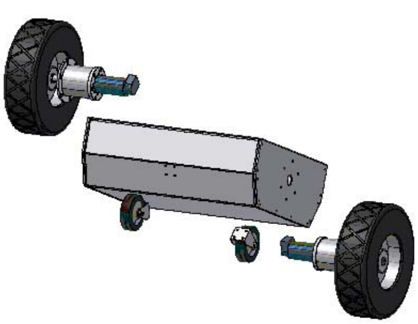
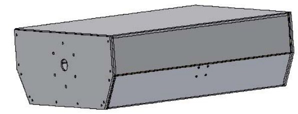


Fig. 1 Two-Wheeled Inverted Pendulum Mobile Robot

It is able to rotate around the z-axis and its movement is described by the angle and the corresponding angular velocity. The linear movement of the body along x-axis is characterized by its position and speed. Additionally, the robot can rotate around its vertical y-axis with the associated angle and angular velocity.



(a)



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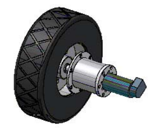
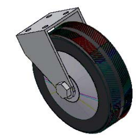
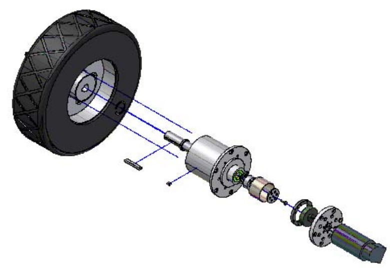




(c) (d)



(e)

Fig. 2 Mechanical Subsystems

The robot's main mechanism components are grouped into three subsystems. The dissected diagram of the robot is shown in Fig 2. Fig. 2(b), 2(c) and 2(d) are chassis, actuated wheel and castor wheel. The two actuated and caster wheels are attached to the chassis. The inside space is used to place the robot's control and power system.

The actuated wheel subsystem Fig 2 (e) is composed of wheel, axle tree, harmonic drive gear, DC-motor, sleeve etc. The sleeve is used to ensure the axletree to be inline with the harmonic dive gear and the axis of the motor and to occupy minimum space.

III. DYNAMIC MODEL

The idea of this robot is a derived from an inverted pendulum. The problem of inverted pendulum has been the subject of numerous studies in automatic controls [3]. Considering the uses in narrow space or on slope, the dynamic model of the robot must be accurate and appropriate.

Two-wheeled inverted pendulum mobile robot with a non-stable structure consists of two wheels and a chassis with rider or goods together making a pendulum.

The chassis body and wheel dynamics are analyzed separately at the beginning, but this will eventually lead to two equations of motion which completely describes the behavior of the balancing robot [6-8]. As the robot's behavior can be influenced by disturbances as well as the torque from the motor, the mathematical model will have to accommodate for such forces.

Firstly the equations of motion associated with the left and right wheels are obtained.

Fig. 3 shows the free body diagram for both wheels. Using Newton's law of motion and DC motor dynamics the left and right wheel equations are derived by equation manipulations.

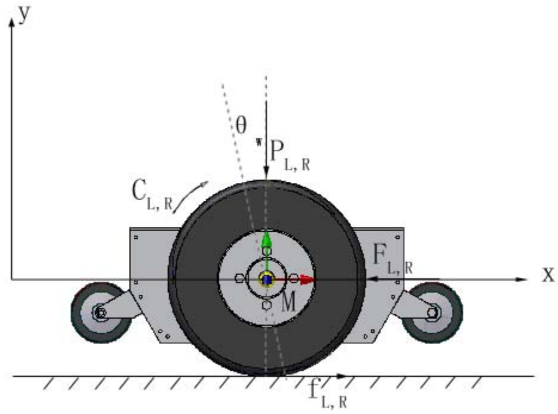


Fig. 3 Free body diagram of the wheel

M : mass of the drivable wheel
Iw : moment of inertia of the wheel
$F_{L,R}$: reaction force between the wheel and chassis
θw : rotation angle of the wheels
r : radius of the wheel
$C_{L,R}$: applied torque from the motors to the wheels
$P_{L,R}$: reaction forces between the wheel and chassis.
$f_{L,R}$: friction forces between the ground and the wheels

For the left wheel

$$M\ddot{x} = \frac{-k_m k_e}{Rr}\dot{\theta}_w + \frac{k_m}{Rr}u - \frac{I_w}{r}\ddot{\theta}_w - F_L \tag{1}$$

For the right wheel

$$M\ddot{x} = \frac{-k_m k_e}{Rr^2}\dot{x} + \frac{k_m}{Rr}u - \frac{I_w}{r^2}\ddot{x} - F_R \tag{2}$$

k_m and k_e is constant, u and R is the applied terminal voltage and resistance [7]. Because the linear motion is acting on the centre of the wheel, the angular rotation can be transformed into linear motion by simple transformation,

$$\ddot{\theta}_w r = \ddot{x} \Rightarrow \ddot{\theta}_w = \frac{\ddot{x}}{r}$$

$$\dot{\theta}_w r = \dot{x} \Rightarrow \dot{\theta}_w = \frac{\dot{x}}{r}$$

By the linear transformation, equation (1) and (2) becomes:
For the left wheel

$$M\ddot{x} = \frac{-k_m k_e}{Rr^2}\dot{x} + \frac{k_m}{Rr}u - \frac{I_w}{r^2}\ddot{x} - F_L \tag{3}$$

For the right wheel

$$M\ddot{x} = \frac{-k_m k_e}{Rr}\dot{\theta}_w + \frac{k_m}{Rr}u - \frac{I_w}{r}\ddot{\theta}_w - F_R \tag{4}$$

Adding equation (3) and (4) together yields,

$$2\left(M + \frac{I_w}{r^2}\right)\ddot{x} = \frac{-2k_m k_e}{Rr^2}\dot{x} + \frac{2k_m}{Rr}u - \left(F_L + F_R\right) \tag{5}$$

The robot's chassis can be modeled as an inverted pendulum. Fig. 4 shows the free body diagram of the chassis.

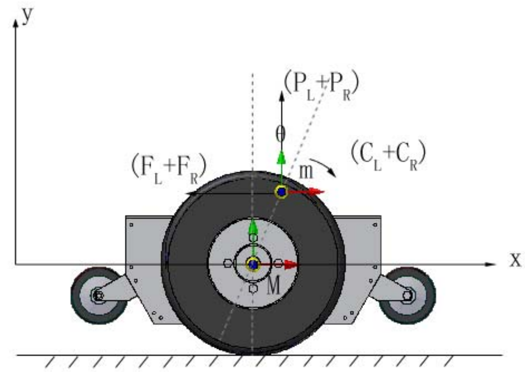


Fig. 4 Free body diagram of Chassis

θ : rotation angle of the chassis
l : distance between the centers of the wheel and chassis
m : mass of the robot's chassis
I_c : moment of inertia of the robot's chassis

Again, by using Newton's law of motion, and rearranging equations to give the non-linear equations of motion of the system,

$$(I_c + ml^2)\ddot{\theta} - \frac{-2k_m k_e}{Rr}\dot{x} + \frac{2k_m}{R}u + mgl\sin\theta = -m\ddot{x}l\sin\theta \tag{6}$$

$$\frac{2k_m}{Rr}u = \left(2M + \frac{2I_w}{r^2} + m\right)\ddot{x} + \frac{2k_m k_e}{Rr^2}\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \tag{7}$$

The above two equations can be linearised by assuming $\theta = \pi + \phi$, where ϕ represents a small angle from the vertical upward direction. This linearised model is obtained so that linear state space controllers can be used to control.

Therefore,

$$\cos\theta = -1$$
$$\sin\theta = -\phi$$
$$\left(\frac{d\theta}{dt}\right)^2 = 0$$

The linearised equations of motion are,

$$(I_c + ml^2)\ddot{\phi} - \frac{-2k_m k_e}{Rr}\dot{x} + \frac{2k_m}{R}u + mgl\phi = -m\ddot{x}l \tag{8}$$

$$\frac{2k_m}{Rr}u = \left(2M + \frac{2I_w}{r^2} + m\right)\ddot{x} + \frac{2k_m k_e}{Rr^2}\dot{x} + ml\ddot{\phi} \tag{9}$$

In order to get the state space representation of the system, equations (8) and (9) are rearranged,

$$\ddot{\phi} = \frac{ml}{I_c + ml^2}\ddot{x} + \frac{2k_m k_e}{Rr\left(I_c + ml^2\right)}\dot{x} - \frac{2k_m}{R\left(I_c + ml^2\right)}u + \frac{mgl}{I_c + ml^2}\phi \tag{10}$$

$$\ddot{x} = \frac{2k_m}{Rr\pi\left(2M + \frac{2I_w}{r^2} + m\right)}u - \frac{2k_m k_e}{Rr^2\left(2M + \frac{2I_w}{r^2} + m\right)}\dot{x} + \frac{ml}{2M + \frac{2I_w}{r^2} + m}\ddot{\phi} \tag{11}$$

By substituting equation (10) into equation (9), substituting equation (11) into equation (8) and after a series of algebraic manipulations the state space equation for the system is obtained.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2k_m k_e\left(mlr - I_c - ml^2\right)}{Rr^2\alpha} & \frac{m^2 g l^2}{\alpha} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2k_m k_e\left(\beta r - ml\right)}{Rr^2\alpha} & \frac{mgl\beta}{\alpha} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2k_m\left(I_c + ml^2 - mlr\right)}{Rr\alpha} \\ 0 \\ \frac{2k_m\left(ml - r\beta\right)}{Rr\alpha} \end{bmatrix} u \tag{12}$$

Where,

$$\beta = \left(2M + \frac{2I_w}{r^2} + m\right)$$

$$\alpha = \left[I_c\beta + 2ml^2\left(M + \frac{I_w}{r^2}\right)\right]$$

In the model above, it is assumed that the wheels of the vehicle will always stay in contact with the ground and that there is no slippage for the wheels. Effects of all the other forces are also considered negligible.

IV. First Prototype

A. *First Prototype Description*

This robot is an integration of mechanical, electrical and high level electronics and computer components. Modularity at the mechanical and control structure is the key design issues and are considered concurrently during the design process resulting in a sophisticated mobile robotic platform.

The robot offers several interesting features regarding sensors and control design. The robot is equipped with two DC drives for actuation, a rate gyro and an accelerometer for measuring the angle and angular velocity of the pendulum body respectively. Encoders are used for measuring the rotation of the wheels.

Considering all the capabilities and characteristics that the robot should have, the design requires a comprehensive process.

The robot has been successfully designed and fabricated. Fig. 1 shows picture of the first prototype. The embedded control system is used for the robot control. This control method is effective in controlling two DC drives using PID controllers.

B. *Hardware*

Fig. 5 shows the structure of integrated control system that is used in the robot and the tele-operated control station. The whole hardware system includes two parts: Control System and teleoperation control system.

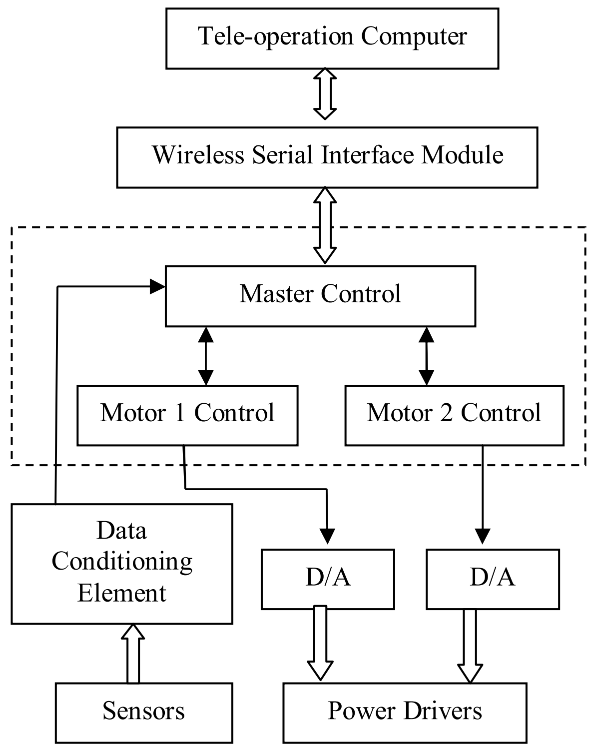


Fig. 5 Block Diagram of Control System

The micro-processor Em104-i613 was chosen for signal processing and control algorithms. The Em-i613 gives developers a low power, high performing and scalable platform with superior graphics capabilities. The Em104-i613 is perfect for applications such as military and industrial automation where uncompromised performance in a limited space is required. This processor is used for drives stabilizing control and coordinate. PC/104- I/O expansion allows features to be quickly added using any one of EMAC's many PC/104 expansion modules, a A/D and D/A module named Pm511 being used in the robot. This board is connected to PC/104 through PC/104 bus and exchanges the speed data of motors and various sensors information.

The teleoperation is done by programming RS232 bus on both PC/104 and teleoperated PC using wireless serial communication module, where the command and monitoring data are exchanged between them. The robot transmits various sensors information to the remote operator station while the remote operator issues commands to move the robot in a desired direction and speed.

C. *Sensors and Actuators*

In order to have automatic stabilizing control, the robot must be equipped with appropriate sensors and actuators. The choices of sensors and actuators, and associated algorithms, will inevitably impose constraints on achievable performance.

Two DC-drives are used to actuate the robot. In order to increase torque, a standards solution is harmonic drive gear system, which was found to effectively eliminating the back-lash problem. The drives used were a pair of Maxon RE40 motors, which produced the maximum torque 0.45 Nm each, when connected to the harmonic drive gear system. The choice of drives is done based on preliminary simulation study.

The use of a rate gyro in combination with an accelerometer is a standard configuration for inertial sensing system, which is needed in this application to estimate the angle and angular velocity of the robot body. The selected accelerometer was a TA-25 and the rate gyro was a TFG-160D, both are from To-kimec.

In order to measure the wheel angles, encoders were used. Since good angle and angular velocity information is crucial for stabilization of the system, the digital encoder of 2000 counts per revolution is used. In addition, in order to further increase the accuracy of the angular velocity measurements, the complementary encoder signals are also used with HCTL-2032, a CMOS ICs that perform the quadrature decoder and counter.

D. Software

The RT-linux real time operation system is used for robot main controller. RT-Linux 3.1 is installed with Red Hat 7.1. Much hardware i.e. USB, Keyboard can be used. Comparing with other operation system, the RT-linux has the characteristics of simplicity of structure, transplantable and small kernel etc. And it's a free OS. In the teleoperation, in order to make the tele-operating high efficiency and convenience, the software should have the better user interface. So Windows operation system has been adopted.

The primary software includes the communication protocol between PC-PC/104 and some application programs to keep balance. The communication between PC/104 and teleoperation PC includes a twelve bytes frame of data communciation. The transfer of data initiates with start byte followed by the data to select the mode. The modes include the wheel motor control mode, track data program mode, emergency break mode, communication state checking mode and shutdown mode.

The robot works in semi-autonomous mode and the remotely controlling makes the robot work practically useful. In many of real situations of hazardous duty, the robot fast deployment onto the site is possible. It requires simple and intuitive user interface. The interface for remote control, monitoring and the whole system appearance is shown in Fig.6. The teleoperation interface divided into two parts: control operating, the state information feedback.

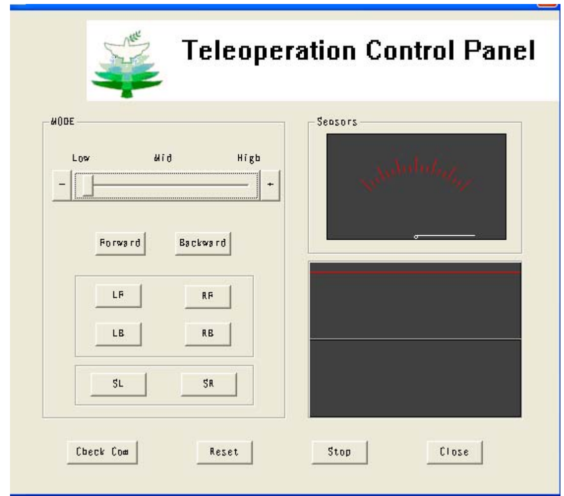


Fig. 6 Interface of Remote Tele-operation

V. Experimental Works

A. Some Basic Experiments

One basic experiment is to maintain its balance automatically. The desired angle of maximums oblique shake is 0.5°, the experiment result is reaching 2°. There would be a few reasons for this, i.e. unsuitable controller parameter, not accurate sensor data. It can be solved by repeating the experiment to get the suitable parameter and the offset of the sensor.

Another basic experiment is conducted to test the performance of the robot's speed traveling on a flat surface ground. The designed maximum speed is 7 km/h and it can be changed continuously within the speed. However, because of the slippery and the heavy power supply, the experiment result is reaching 5 km/h which is slower than expected. But this problem can be solved by changing the ratio of the harmonic dive gear, changing the radius of the drivable wheels, optimizing the wheel's surface friction characteristic and using the lighter power supply.

B. Experiments for climbing slop and traversing small obstacle

This robot is designed for different environment i.e. narrow spaces, slopes etc. So the furthermore experiments were conducted to validate its capability to climb slope and surpassing small obstacle.

1) Climbing Slope

Slop is a common obstacle type for robot negotiation and its steepness may be the most important parameter. The capability of climbing a slope was tested on a slope with angle of elevation of 15°. Fig. 7 shows the body of the robot remains upright with the horizontal when climbing a slope.

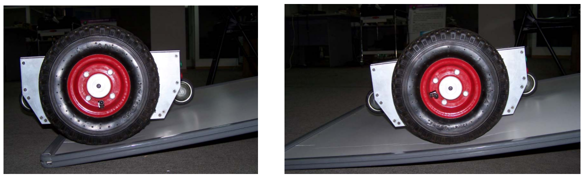


Fig. 7 Climbing Slope

2) Surpassing small obstacle

To test the stability of the robot balancing itself, some books were placed in its way when robot is moving at high speed, the robot surpass them but the change in its posture is observed, as shown in Fig. 8. This can be eliminated by the use of bigger wheels or slowing down the speed would solve this problem.

Fig. 8 Surpassing small obstacle

VI. CONCLUSION AND FUTURE WORKS

In this paper we presented a small mobile robotic platform with a differentially driven two-coaxial-wheeled structure. The overall objective is to make a robot traverse in narrow spaces and slopes. The design is light weight, compact in size, and low power consumption unit. The first prototype confirms potential of the robot to reach desired objectives by experiments. The refinement in the first prototype will be done after exploring it various capabilities such as the posture stabilization method for the robot and COG (Center of Gravity) of rider control method. Then the robot can work in semi-autonomous and the remotely controlled mode. In the very near future we will add some functional components to perform some specific tasks such as transport, reconnaissance, service etc.

ACKNOWLEDGMENT

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