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Design and analysis of a spherical mobile robot

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ABSTRACT

A spherical mobile robot, rolling on a plane with the help of two internal rotors and working on the principle of conservation of angular momentum has recently been fabricated in our group. The robot is a classic nonholonomic system. Path planning algorithms exist in the literature for certain classes of nonholonomic systems like chained form systems, nilpotent systems and differentially flat systems. The model of this spherical mobile robot however, does not fall into any of these classes and hence these existing algorithms are rendered inapplicable to this system. The final objective is to make this robot as a testbed for feasible path planning and feedback control algorithms.

1. Introduction

Mobile robotics is one of the important branches of robotics. For mobility of the robots, different motions like rolling, walking, hopping, sliding are used. Rolling motion has certain advantages over other motions. The problem of wear and tear is less, the set of configurations is reachable by lesser number of inputs as the system is nonholonomic and the role of friction is conservative. As compared to single wheeled robots like gyroscopes [1], the spherical structure is statically stable and due to the spherical shape, the robot recovers from the collisions with unknown obstacles. Any sensor can be mounted inside the spherical shell and the robot can be effectively used. Due to these reasons and for the purpose of constructing a testbed, an autonomous spherical mobile robot has been designed and developed by our group at Systems and Control Engineering, Indian Institute of Technology Bombay, India.

A typical construction of the spherical mobile robot has a spherical shell with some internal driving unit (IDU) mounted inside the spherical shell. The robot in [1] is composed of a spherical shell and an arch shaped body. The driving unit consists of a pendulum and a controlling arch. The arched body and pendulum can control the pitch angle and the controlling arch below the pendulum controls the roll angle simultaneously with the pitch angle. In [2,3], the IDU consists of a wheel rolling inside the spherical shell which is in turn driven by a motor. The robot moves due to the disturbance of the system equilibrium due to unbalance of the inside construction. The 'Rollo', a spherical mobile robot designed and developed by Harmo et al. [4], the IDU is hanging from the rim. The rim can be rotated around the two axes of the IDU. The spherical shell rolls according to the movement of the rim. A small car like structure has been used as IDU in 'Sphericle' [5]. The car has the unicycle kinematics and is driven with the help of two stepper motors. All the robots described up till now work on the principle of change of center of mass with the help of IDU for rolling the robot. 'Rollmob', designed and developed by Ferrière et al. [6] is a spherical ball driven by an universal wheel equipped with rollers. The rotation of the roller wheel drives the sphere around the direction parallel to the wheel axis, while the sphere rotates freely around the direction perpendicular to this

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axis. The construction of the robot designed by Bhattacharya et al. [7] is driven by a set of two mutually orthogonal rotors, attached to the spherical shell of the robot from inside. Along the *Z* axis there is a single rotor and along *X* axis there are two rotors which are rotated in tandem as a single rigid connected body. When rotors are rotated, the spherical robot rolls in the opposite direction due to the conservation of angular momentum. 'GroundBot', a spherical robot developed by Rotundus is designed for extraterrestrial exploration. The center of gravity of this robot is kept close to the ground with the help of a controlled pendulum. When it is raised, the ball rolls forward. When the pendulum is moved sideways, the ball turns. Sphe-robot, a spherical mobile robot by Mukherji et al. [8] has radial spokes along which masses are placed. Radial movement of these masses create a moment about the center causing the motion of the robot. Cyclops [9] has two degrees of freedom in its locomotion system. It can pivot in place along its vertical axis and roll forward and backward along a fixed horizontal axis via a small motor and gear-head fixed inside. The review papers discussing constructional details of available spherical mobile robots are [10,11] and a concise review on the existing path planning algorithms and feedback algorithms for the system can be obtained from [12,13]. In this paper, we present a systematic study of a spherical mobile robot, designed, fabricated and analyzed in our laboratory. The organization of the paper is as follows. Section 2 describes the construction and design details of the robot. Section 3 describes the mathematical modeling of the system using quaternion. In Section 4, the properties of the robot are discussed in the quaternion space. The experimental setup along with the discussion on the experimental results is reported in Section 5. Section 6 provides concluding remarks.

2. Design

The spherical mobile robot designed in our laboratory works on the principle of conservation of angular momentum. The robot has two internal rotors similar to a few constructions mentioned above. The robot's spherical shell is made up of acrylic material having 4 mm thickness. The inner radius of the robot is 30 cm. A crucial aspect of the design is to place the internal components such that the center of mass of the robot is exactly at the geometric center of the sphere. This is very important so that the robot will not tip over on its own. The easiest way to achieve this is to place all the parts symmetrically. There are two internal rotors along two mutually orthogonal axes of the sphere. These rotors are driven by two D.C. motors MAXON EC ϕ32, Brushless, 80 W. Two batteries of the type Polyquest *PQ*08003 TWENTY LIPO PACK of capacity 800 mAh each, are used for supplying power to one motor. So in all there are four batteries. The two speed control units which control the speed of the motors receive control signals from the external controller. For symmetrically placing the components, the motor along with rotor and one speed control unit is placed on one side and the battery along with dead weight is placed on the diametrically opposite direction as shown in Fig. 2. Similarly another pair of motor assembly and battery + dead weight are placed in diametrically opposite directions. The robot is fabricated in two hemispheres as shown in Fig. 1. Each hemisphere consists of one motor assembly and one battery + dead weight assembly. The total weight of the robot is 3.4 kg. Fig. 3 shows the schematic of the robot developed. As discussed in Section 1, there are different driving mechanisms available in the literature. Some spherical robots work on the principle of change in the center of gravity while some work on the principle of conservation of angular momentum. As already stated, our spherical robot works on the principle of angular momentum. Other spherical robots working on a similar principle are reported in [7,14]. In these designs two rotors are mounted in opposite directions for mass balancing and motors moving these rotors must run in tandem which may create problems at the time of practical implementation. As against this, we have a single rotor in the *X* as well as the *Z* direction for overcoming this problem. We provide dead weights on the opposite sides to adjust the imbalance. In addition, the transparent acrylic spherical shell enables students to understand the internal mechanism while in motion. It is absolutely critical that there be no relative motion between the two hemispheres while in motion. This can be achieved by an arrangement for screwing a connecting rod along the axis of the sphere as shown in Fig. 3.

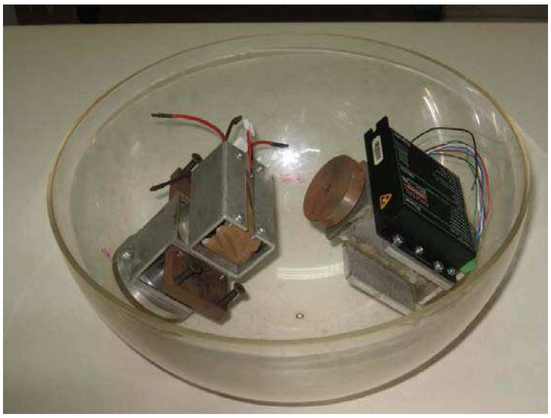

Fig. 1. Placing of components in a half shell.

Fig. 2. Construction of the spherical robot.

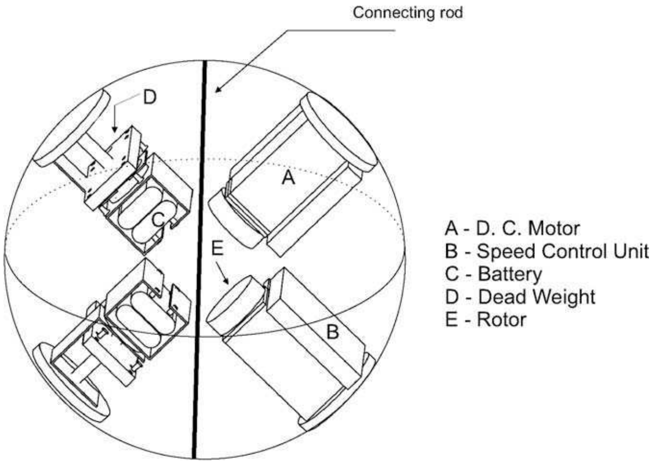

Fig. 3. Construction of the spherical robot.

3. Mathematical modeling

This section describes the development of an analytical model of the spherical rolling robot using quaternion. Consider a spherical robot rolling on a horizontal plane as shown in Fig. 4. An inertial coordinate frame is attached to the surface and denoted as $x_I y_I z_I$ with its origin at a point O. The body coordinate axes $x_b y_b z_b$ are attached to the sphere and have their origin at the center of the sphere G. The set of generalized coordinates describing the sphere consists of [15]

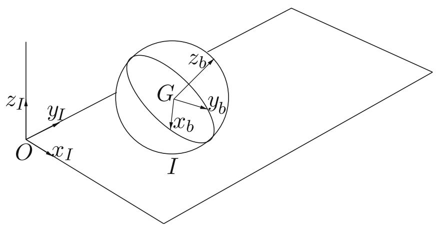

Fig. 4. Sphere rolling on a surface.

- Coordinates of the contact point I on the plane.
- Any set of variables describing the orientation of the sphere.

We use Euler parameters (instead of Euler angles) which is a set of 4 parameters to describe the orientation of the sphere. Euler parameters have the advantage of being a nonsingular two to one mapping with the rotation. In addition, Euler parameters form a unit quaternion and can be manipulated using quaternion algebra [16–19]. Using a set of Euler parameters for defining orientation, we get the set of generalized coordinates as

$$p = (x, y, e_0, e_1, e_2, e_3)^T,$$

where (x, y) are the coordinates of the contact point I and e_0, e_1, e_2 and e_3 are the Euler's parameters describing orientation forming unit quaternion such that

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1.$$

Let i_b, j_b, k_b be the unit vectors of the body frame and ω be the angular velocity of the sphere given by

$$\omega = \omega_x^b i_b + \omega_y^b j_b + \omega_z^b k_b.$$

The projection of the angular velocity vector on the body axes can be related to the rate of change of the Euler parameters w.r.t. time using the relationship given in [17,20]

$$\begin{bmatrix} 0 \\ \omega_x^b \\ \omega_y^b \\ \omega_z^b \end{bmatrix} = 2 \begin{bmatrix} e_0 & e_1 & e_2 & e_3 \\ -e_1 & e_0 & e_3 & -e_2 \\ -e_2 & -e_3 & e_0 & e_1 \\ -e_3 & e_2 & -e_1 & e_0 \end{bmatrix} \begin{bmatrix} \dot{e}_0 \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix}. \tag{1}$$

We assume the sphere to roll without slipping and no-slip constraints for a sphere of radius r are given by

$$\dot{x} + 2re_2\dot{e}_0 - 2re_3\dot{e}_1 - 2re_0\dot{e}_2 + 2re_1\dot{e}_3 = 0.$$
$$\dot{y} - 2re_1\dot{e}_0 + 2re_0\dot{e}_1 - 2re_3\dot{e}_2 + 2re_2\dot{e}_3 = 0.$$

We assume the sphere to have unit radius without any loss of generality. For a unit sphere, the no-slip constraint equations reduce to

$$\dot{x} + 2e_2\dot{e}_0 - 2e_3\dot{e}_1 - 2e_0\dot{e}_2 + 2e_1\dot{e}_3 = 0, \tag{2a}$$
$$\dot{y} - 2e_1\dot{e}_0 + 2e_0\dot{e}_1 - 2e_3\dot{e}_2 + 2e_2\dot{e}_3 = 0. \tag{2b}$$

Eqs. (1) and (3) describe the kinematics of the sphere fully giving a set of state equations as

$$Q\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{e}_0 \\ \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\omega_x^b + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\omega_y^b + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\omega_z^b, \tag{3}$$

where

$$Q = \begin{bmatrix} 1 & 0 & 2e_2 & -2e_3 & -2e_0 & 2e_1 \\ 0 & 1 & -2e_1 & 2e_0 & -2e_3 & 2e_2 \\ 0 & 0 & 2e_0 & 2e_1 & 2e_2 & 2e_3 \\ 0 & 0 & -2e_1 & 2e_0 & 2e_3 & -2e_2 \\ 0 & 0 & -2e_2 & -2e_3 & 2e_0 & 2e_1 \\ 0 & 0 & -2e_3 & 2e_2 & -2e_1 & 2e_0 \end{bmatrix}.$$

It can be shown that the matrix Q is an orthogonal matrix and hence is invertible. The internal rotors (which are actuators for the robot) are located along X and Z axis of the body frame. The robot is symmetric in construction and we therefore consider $\omega_y^b = 0$, reducing the system equations from (3) to

$$\dot{p} = X_1(p)\omega_x^b + X_2(p)\omega_z^b, \tag{4}$$

where

$$p = [x \quad y \quad e_0 \quad e_1 \quad e_2 \quad e_3]^T, \tag{5a}$$

$$X_1(p) = Q^{-1}\begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}2(e_0e_3+e_2e_1)\\(e_3^2+e_2^2-e_0^2-e_1^2)\\-\frac{1}{2}e_1\\\frac{1}{2}e_0\\\frac{1}{2}e_3\\-\frac{1}{2}e_2\end{bmatrix}, \tag{5b}$$

$$X_2(p) = Q^{-1}\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}2(e_2e_3-e_0e_1)\\-2(e_1e_3+e_0e_2)\\-\frac{1}{2}e_3\\\frac{1}{2}e_2\\-\frac{1}{2}e_1\\\frac{1}{2}e_0\end{bmatrix}. \tag{5c}$$

4. Properties of the model

4.1. Controllability

Before we proceed to path planning of the spherical robot, it is essential to check whether there exists a path that connects any arbitrary configurations of the sphere. The question can be answered using a result known as the *Chow's Theorem* [21]. In this section, we use the algorithm given in [22] to answer the question.

Consider the system described by Eq. (4).

$$\dot{p} = X_1(p)\omega_x^b + X_2(p)\omega_y^b,$$

where $p, X_1(p)$ and $X_2(p)$ are given by (5). We compute the following Lie Brackets using a Philip Hall convention [21,22]

$$X_3 = [X_1, X_2], \tag{6a}$$
$$X_4 = [X_1, X_3], \tag{6b}$$
$$X_5 = [X_2, X_3], \tag{6c}$$
$$X_6 = [X_1, X_4]. \tag{6d}$$

A distribution is formed using the above generated Lie Bracket vector fields given by

$$\Delta = [X_1, X_2, X_3, X_4, X_5, X_6],$$

where

$$X_3 = \begin{bmatrix}2e_1^2-2e_0^2+2e_3^2-2e_2^2\\4(e_2e_1-e_0e_3)\\\frac{1}{2}e_2\\\frac{1}{2}e_3\\-\frac{1}{2}e_0\\-\frac{1}{2}e_1\end{bmatrix}, \quad X_4 = \begin{bmatrix}6(e_0e_1-e_2e_3)\\6(e_1e_3+e_0e_2)\\\frac{1}{2}e_3\\-\frac{1}{2}e_2\\\frac{1}{2}e_1\\-\frac{1}{2}e_0\end{bmatrix},$$

$$X_5 = \begin{bmatrix}6(e_0e_3+e_2e_1)\\3(e_3^2+e_2^2-e_0^2-e_1^2)\\-\frac{1}{2}e_1\\\frac{1}{2}e_0\\\frac{1}{2}e_3\\-\frac{1}{2}e_2\end{bmatrix}, \quad X_6 = \begin{bmatrix}4(-e_1^2+e_0^2-e_3^2+e_2^2)\\8(-e_1e_2+e_0e_3)\\-\frac{1}{2}e_2\\-\frac{1}{2}e_3\\\frac{1}{2}e_0\\\frac{1}{2}e_1\end{bmatrix}.$$

It can be observed that all higher order brackets can be expressed in terms of X_1, X_2, X_3, X_4, X_5 giving the rank of the distribution as 5. As the Euler parameters are used, the number of the state variables is 6. We are using a system of 4 parameters to describe the orientation. But all possible orientations lie on a hyper surface defined by a level set

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1, \tag{7}$$

which is a 3 sphere. This gives the dimension of the configuration space as 5 which is equal to the rank of the distribution. So using *Chow's Theorem* the system is controllable and can be taken from any position to any arbitrary position using Lie Bracket motions described by the vector fields (6).

4.2. Conversion into a chained form

For determining the degree of nonholonomy, we construct a distribution associated with the control system (4) as

$$\Delta = \text{span}\{X_1, X_2\}.$$

We then construct filtrations associated with the distribution Δ as

$$\begin{aligned}
&E_0 = \Delta, \quad F_0 = \Delta,\\
&E_1 = E_0 + [E_0, E_0], \quad F_1 = F_0 + [F_0, F_0],\\
&E_2 = E_1 + [E_1, E_1], \quad F_2 = F_1 + [F_1, F_0],\\
&E_3 = E_2 + [E_2, E_2], \quad F_3 = F_2 + [F_2, F_0],\\
&E_4 = E_3 + [E_3, E_3], \quad F_2 = F_3 + [F_3, F_0].
\end{aligned}$$

According to [23], a feedback transformation which puts a driftless two input nonholonomic system into a chained form exists if and only if

$$\dim E_i = \dim F_i = i + 2; \quad i = 0, \dots, n-2. \tag{8}$$

In this particular case

$$\begin{aligned}
&\text{For } i = 0, \quad \dim(F_0) = 2,\\
&\text{For } i = 1, \quad \dim(F_1) = 3,\\
&\text{For } i = 2, \quad \dim(F_2) = 5 \neq i + 2,\\
&\text{For } i = 3, \quad \dim(F_3) = 5,\\
&\text{For } i = 4, \quad \dim(F_4) = 5 \neq i + 2.
\end{aligned}$$

It can be observed that the condition (8) is not satisfied and it is not possible to convert the model into a chained form.

5. Experimental setup and discussion

Rigorous experimentation has been carried out on the experimental setup shown in Fig. 5. The main controller in the system is a bluetooth enabled PC, which generates control signals according to the algorithm programmed. The control signals are transmitted to the microcontroller PIC16F877 through a bluetooth modem 'BlueSMiRF Gold'. According to the signal received from the PC, the microcontroller controls the speed of the D.C. motors using a Digital to Analog Converter (DAC). For enabling feedback, there are Hall sensors that provide information about the speed of the motors and optical encoders that give position of the rotors. During the experimentation, it was possible to control speed of the motors. We have worked on the path planning problem of the robot and the related work is reported in [24]. In this work we are mainly interested in certain maneuvers, for which the trials were carried out on the experimental setup. These maneuvers are,

- *Pivoting maneuver*:When one of the rotors is in the vertical position, the robot is expected to spin about the vertical axis. This particular maneuver is known as pivoting maneuver. The experiments were carried out with the rotor in the vertical position for this particular maneuver. It has been observed that the robot responds to the sudden change in the angular speed by spinning about the vertical axis as expected. However, the result particularly is not satisfactory for slow and gradual changes in the magnitude of the angular velocity.

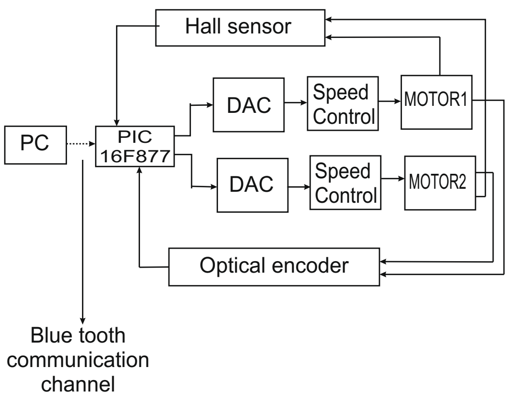

Fig. 5. Experimental setup at the laboratory.

- *Equatorial maneuver*:When one of the rotors is in the horizontal position, the robot is expected to roll along a straight line on the plane and the contact point should follow the great circle on the sphere surface, in the plane perpendicular to the axis of the rotor. According to the experiments carried out, the results of this maneuver do not match with the expected results. The probable reasons are imperfections of the sphere surface and some design related critical issues such as rotor parameters.

6. Conclusion

Design and constructional features of a spherical mobile robot rolling on a plane are presented in this paper. The kinematic model of the system is developed using quaternions for the description of the orientation of the robot. It can be observed that the model is nonsingular and valid everywhere. It is shown that the model is fully controllable and can be taken from any arbitrary configuration to any arbitrary configuration within the unit 3-sphere in the quaternion space. Further, it is not nilpotent and also can not be converted into a chained form. According to the experimentation carried out on the robot, we observe that it works for some maneuvers while for some maneuvers, the results do not match with the expected results. This is due to some imperfections in the fabrication of the robot. We plan to work on these issues. We also would like to explore the nonsingular model developed using quaternions for path planning and stabilizing controller design. These are the main avenues for our future work.

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