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The Kinematics Model of a Two-wheeled Self-balancing Autonomous Mobile Robot and Its Simulation

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***Abstract*—The researches on two-wheeled self-balancing robots have gained momentum in recent decade around the world. This paper describes the kinematics model of a two-wheeled self-balancing autonomous mobile robot. After mechatronics system design of the robot is completed, the analysis of the whole kinematics model can be divided into two wheels and a body. Then the velocity decompositions of robot wheel and body are analyzed respectively. After the left and right wheel kinematic model is established based on above velocity decomposition method, and the kinematic model of self-balancing robot's body is also calculated by this method. The whole kinematics model of the two-wheeled self-balancing robot system is then established. The effectiveness of the kinematics model is testified by the simulation analysis on ADAMS and experimental validation.**

Keywords- Kinematics model; Two-wheeled self-balancing; robot; Kinematics simulation

I. INTRODUCTION

As two-wheeled self-balancing autonomous mobile robots are characterized by the ability to balance on its two wheels and spin on the spot, many researches have been focused on it in recent decades with many valuable fruits [1-5]. The uniqueness of the inverted pendulum system for the two-wheeled self-balancing autonomous robot has drawn interest from many researches [1-2]. As the robot is mechanically unstable, it is necessary to keep the system in equilibrium [3]. Googol developed a self-balancing robot for education [4]. A two–wheeled robot called JOE [5] is also based on the inverted pendulum. A control system, made up of two decoupled state-space controllers, pilots the motors to keep the system in equilibrium. A similar and commercially available system, SEGWAY [6], which is a man-machine vehicle [7], has been invented by Dean Kamen.

However, existing kinematics model approaches to the two-wheeled self-balancing autonomous robots always ignore the working environments, such as a slope with a sloping angle.

This paper is successful in overcoming the current researches' insufficiency and achieving its aims to balance a two-wheeled autonomous robot based on the inverted pendulum model. After the left and right wheel kinematic model is established based on velocity decomposition method, and the kinematic model of self-balancing robot's body is then calculated. The whole kinematics model of the two-wheeled self-balancing robot system is then established. The model is correct by ADAMS simulation analysis and experimental validation.

II. KINEMATICS MODEL OF TWO-WHEELED SELF-BALANCING AUTONOMOUS MOBILE ROBOT

There are space postures changes (including the position and posture problems) due to the motion in the movement of self-balancing mobile robot. It is the foundation to set up the motion state equations and robot trajectory planning.

Robots can move on the flat and complex terrain, such as on the slope, we must establish a reasonable and accurate kinematic model to adapt changeable environment. The mobile robot is a robot platform, which can achieve self-regulation pose. Then the kinematics model for its implementation platform laid the foundation for self-control. This requires the mobile robot platform can achieve the path tracking in its movement, and be able to estimate its own position, direction, speed, acceleration, and then to determine a driving wheel torque according to the desired moving speed. The establishment of mobility kinematic model of the platform is very necessary.

A. Mechatronics Systems Design

The robotic systems include mechanical systems, control systems and sensing systems. The two-wheeled self-balancing autonomous robot is composed of a chassis carrying two dc servo motors coupled to a harmonious gearbox for each wheel, the NI cRIO board with NI Compact module 9014, 9401, 9205, 9411, and 9263 used to implement the controller, the necessary sensors, such as (digital inclinometer) CXTA02 and digital rate gyroscope) miniAHRS to measure the vehicle's states for control unit, as well as a vertical steel bar. The batteries are bolted onto the aluminum bar. The wheels of the self-balancing autonomous robot are directly coupled to the output shaft of the harmonious gearboxes by key. Figure. 1 shows the prototype of self-balancing autonomous robot with its three degrees of freedoms (3-DOFs). It is able to rotate around the axis (pitch), a movement described by the angle and the corresponding angular velocity. Additionally, the vehicle can

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rotate around its vertical axis (yaw) with the associated angle and angular velocity. These six state-space variables fully describe the dynamics of the 3-DOF system. In Figure 1, there are two big wheels and two small wheels. The two small wheels are used to assistant in our debugging stage. When the two-wheeled self-balancing mobile robot is moving on the road, these two small wheels do not touch on the ground.

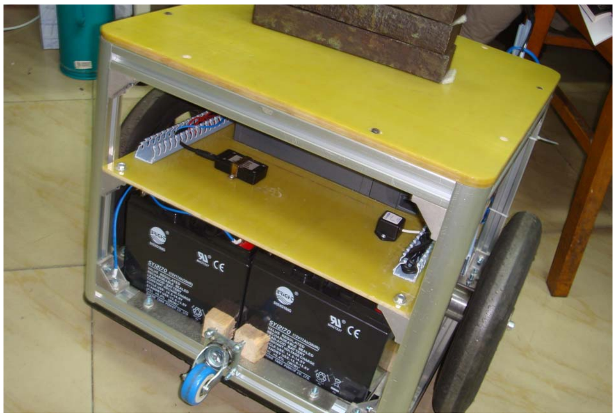

Figure 1. Prototype of the two-wheeled self-balancing autonomous robot.

B. Establishment of Coordinates System and Modeling Assumptions

First of all, it is necessary to establish a coordinate system to kinematics analysis. The mobile platform in three-dimensional plane of the coordinate system is shown in Figure 2 and 3. It is necessary to establish a mathematical model of control object with quantitative and accurate analysis and design. The vertical axis through the midpoint of two wheels is Y-axis positive direction, and the moving direction of robot is X-axis positive direction, and two wheel's axis is Z-axis positive direction.

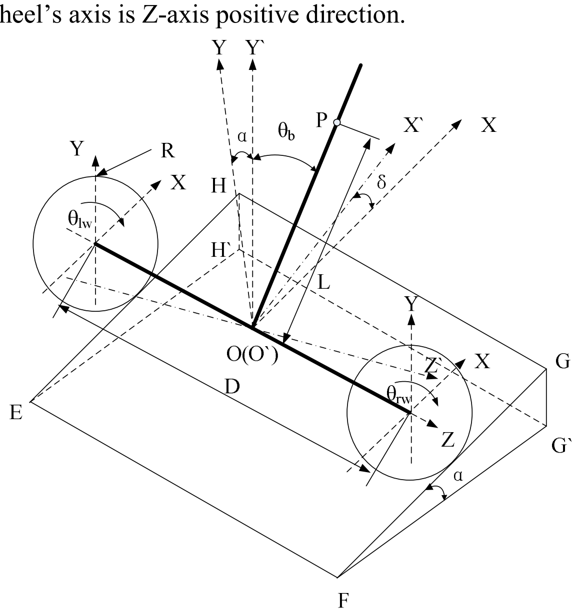

Figure 2. Establishment of coordinate and structure figure of two-wheeled self-balancing mobile robot

In Figure 2,

D: the distance between two wheels (m);

X direction: the initial moment direction;

Z direction: the initial moment of the wheel axis;

Y direction: vertically upward;

δ : yaw angle between the current moment of robots and the initial moment direction (X direction);

α : sloping angle of sloping road (rad);

$\dot{\theta}_{rw}$: the angular speed of right wheel (rad/s);

$\dot{\theta}_{lw}$: the angular speed of left wheel (rad/s);

R : wheel radius (m);

θ_b : the angle between mass center and O`Y` axis (rad).

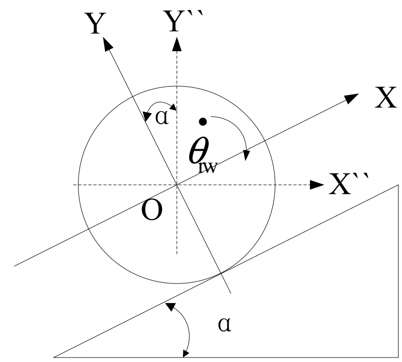

Figure 3. Two-wheeled self-balancing mobile robot with slope.

In order to achieve the control of motion state, two-wheeled self-balancing robot kinematics model must be set up. As shown in Figure 2, there are two coordinate systems OXYZ and O`X`Y`Z` system, which coordinate system OXYZ in OXZ plane and parallel to the flat road plane EFGH, OY-axis perpendicular to the flat road plane EFGH, which coordinate system O`X`Y`Z` in O`X`Z` plane and parallel to the sloping road plane EFG`H`, O`Y`-axis perpendicular to the sloping road plane EFG`H`.

As the complexity of self-balancing mobile robot, it is difficult to establish an accurate model. In order to facilitate kinematic model, planning and control self-balancing mobile robot pose, the model must be simplified to meet the requirements of approximate model.

The self-balancing mobile robot's body can be seen as a rigid body. The two wheels can also be seen as rigid bodies, and it does not take wheels' lateral sliding into account, that is, only rolling movement is considered. There is no movement in Z-axis direction, and no rotation in X-axis direction, so rotation force can be ignored. Two wheels can be seen with the same geometry; assuming that the robot is moving in the horizontal plane and the wheels do not slip, that is, speed of points of contact between wheel and ground is zero.

C. Kinematics Model of Wheels

We use velocity decompositions to solve kinematics model of robot. The velocity decompositions of right wheel and body are shown in Figure 4. The Figure 5 is A-A section view of Figure 4.

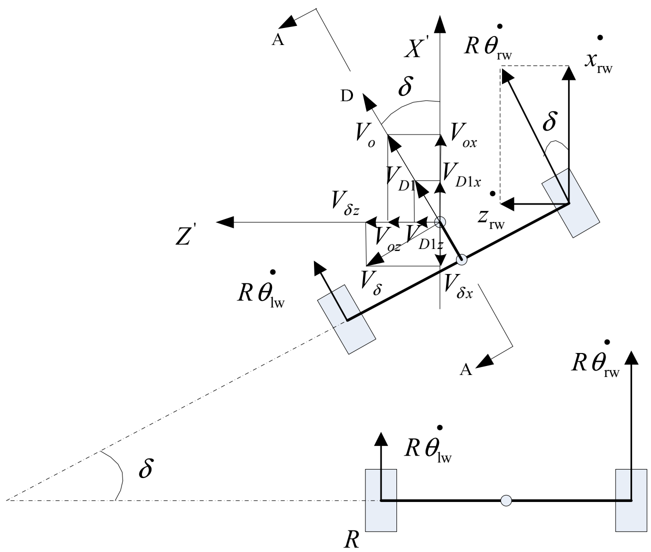

Figure 4. Velocity decomposes of robot

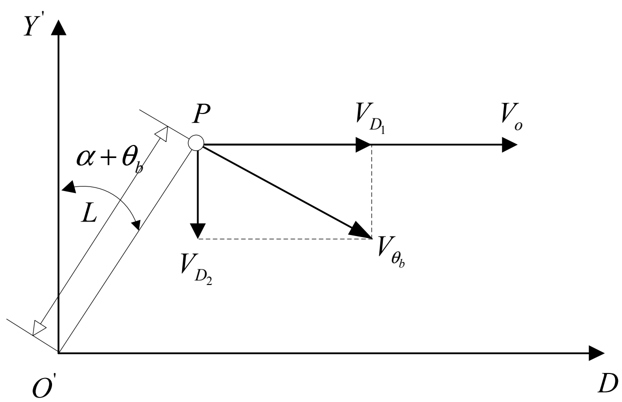

Figure 5. A-A section view of Figure 4.

To the right side of the wheel as research object, the velocity decomposition of the robot is shown in Figure 4, right wheel movement analysis at the moment t and the moment t+Δt.

In Figure 4 and 5,

$R\dot{\theta}_{rw}$: moving speed of right wheel (m/s).

$\dot{x}_{rw}$: moving speed of right wheel in the direction of X axis (m/s).

$\dot{z}_{rw}$ moving speed of right wheel in the direction of Z axis (m/s).

δ : yaw angle between the current moment of robots and the initial moment direction (X direction) (rad).

The right wheel speed is available from the analysis of Figure 4, the right wheel speed is:

$$\begin{cases} \dot{x}_{rw} = R\dot{\theta}_{rw}\cos\delta \\ \dot{y}_{rw} = 0 \\ \dot{z}_{rw} = R\dot{\theta}_{rw}\sin\delta \end{cases} \tag{1}$$

Where,

$\dot{\theta}_{rw}$: the angular speed of right wheel (rad/s);

R: wheel radius (m);

δ: robot yaw angle (rad);

By the same method, left wheel speed is:

$$\begin{cases} \dot{x}_{lw} = R\dot{\theta}_{lw}\cos\delta \\ \dot{y}_{lw} = 0 \\ \dot{z}_{lw} = R\dot{\theta}_{lw}\sin\delta \end{cases} \tag{2}$$

$\dot{\theta}_{lw}$: the angular speed of left wheel (rad/s).

D. Kinematics Model of Body

In Figure 4 and Figure 5,

V_{θ_b} : the speed of mass center of rotation around two-wheel OX axis (rad/s);

L : the distance between body mass center and the two-wheel axle(m);

V_{D2} : V_{θ_b} projection in the O\`Y\`axis (m/s);

V_{D1} : V_{θ_b} projection in the car track tangent direction (OD axis) (m/s);

V_o : velocity of robot chassis center (m/s);

V_δ : The speed of mass center of rotation around O\`Y\` axis (m/s).

V_{D1}, V_o, and V_δ are decomposed along X-axis and Z axis in to V_{D1x}, V_{ox}, and $V_{\delta x}$, V_{D1z}, V_{oz}, and $V_{\delta z}$. And V_{D2} is decomposed along Y-axis V_y.

From the above analysis, the robot mass center velocity in X, Y, Z-axis velocity components are as follows:

X direction:

$$\begin{cases} V_{D1x} = V_{D1}\cos\delta \\ V_{D1} = V_{\theta b}\cos(\alpha+\theta_b) \\ V_{\theta b} = L(\alpha+\dot{\theta}_b) \end{cases} \tag{3}$$

$$\begin{cases} V_{ox} = V_o\cos\delta \\ V_o = \dfrac{R\dot{\theta}_{l\omega}+R\dot{\theta}_{r\omega}}{2} \end{cases} \tag{4}$$

$$\begin{cases} V_{\delta x} = -V_\delta \sin\delta \\ V_\delta = \dot{\delta} L \sin(\alpha+\theta_b) \\ \dot{\delta} = \dfrac{R\dot{\theta}_{l\omega} - R\dot{\theta}_{r\omega}}{D} \end{cases} \tag{5}$$

Then,

$$V_x = V_{D1x} + V_{ox} + V_{\delta x} =$$

$$L(\dot{\alpha+\theta_b})\cos(\alpha+\theta_b)\cos\delta + \frac{R\left(\dot{\theta}_{l\omega}+\dot{\theta}_{r\omega}\right)}{2}\cos\delta$$

$$-\frac{R\left(\dot{\theta}_{l\omega}-\dot{\theta}_{r\omega}\right)}{D} L\sin(\alpha+\theta_b)\sin\delta \tag{6}$$

Y direction:

$$V_y = V_{D2} = -V_{\theta b}\sin(\alpha+\theta_b) = -L(\dot{\alpha+\theta_b})\sin(\alpha+\theta_b) \tag{7}$$

Z direction:

$$\begin{cases} V_{D1z} = V_{D1}\sin\delta \\ V_{D1} = V_{\theta b}\cos(\alpha+\theta_b) \\ V_{\theta b} = L(\dot{\alpha+\theta_b}) \end{cases} \tag{8}$$

$$\begin{cases} V_{oz} = V_o \sin\delta \\ V_o = \dfrac{R\dot{\theta}_{l\omega} + R\dot{\theta}_{r\omega}}{2} \end{cases} \tag{9}$$

$$\begin{cases} V_{\delta z} = V_\delta \cos\delta \\ V_\delta = \dot{\delta}\cdot L\sin(\alpha+\theta_b) \\ \dot{\delta} = \dfrac{R\dot{\theta}_{l\omega} - R\dot{\theta}_{r\omega}}{D} \end{cases} \tag{10}$$

$$\therefore V_z = V_{D1z} + V_{oz} + V_{\delta z} =$$

$$L(\dot{\alpha+\theta_b})\cos(\alpha+\theta_b)\sin\delta + \frac{R\left(\dot{\theta}_{l\omega}+\dot{\theta}_{r\omega}\right)}{2}\sin\delta$$

$$+\frac{R\left(\dot{\theta}_{l\omega}-\dot{\theta}_{r\omega}\right)}{D} L\sin(\alpha+\theta_b)\cos\delta \tag{11}$$

E. *Kinematics Model of Robot*

In order to study robot kinematics model, the following variables are substituted:

The relative circumgyration angle of left wheel (rad):

$$\phi = \theta_{\mathrm{lw}} - \theta_b \tag{12}$$

The relative circumgyration angle of right wheel (rad):

$$\varphi = \theta_{\mathrm{rw}} - \theta_b \tag{13}$$

Then the track of robot chassis center:

$$p = \frac{p_{lw}+p_{rw}}{2} = \frac{R\theta_{lw}+R\theta_{rw}}{2} = \frac{R}{2}(\phi+\varphi) + R(\alpha+\theta_b) \tag{14}$$

Robot yaw angle:

$$\delta = \frac{p_{lw}-p_{rw}}{D} = \frac{R\theta_{lw}-R\theta_{rw}}{D} = \frac{R}{D}(\phi-\varphi) \tag{15}$$

In Equation (14) and (15),

p_{lw} : track of left wheel,

p_{rw} : track of right wheel.

With the robot kinematics model, it can be seen that the robot can be achieved along the arbitrary trajectory of robot by controlling the robot speed of two wheels.

III. APPLICATION

A. *Basic Data of Kinematic Analysis*

The basic data of self-balancing mobile robot shown in Figure 1 are:

1) wheel radius R = 0.145 m;

2) the distance between body mass center and two-wheel axle L = 0.031 m;

3) two wheels centroid axial distance D = 0.5294 m;

Get this kinematic equations from equation (14) and (15), as follows:

$$\mathrm{P} = 0.0725\left(\theta_{lw}+\theta_{rw}-2\alpha-2\theta_b\right)+0.145\theta_b \tag{16}$$

$$\delta = 0.000136947\left(\theta_{lw}-\theta_{rw}\right) \tag{17}$$

B. *Kinematics Simulation*

ADAMS is the most widely used multibody kinematics and dynamics analysis software in the world. Adams helps engineers to study the kinematics and dynamics of moving parts, how loads and forces are distributed throughout mechanical systems, and to improve and optimize the performance of their products. The simulation model is built up in ADAMS, as shown in Figure 6.

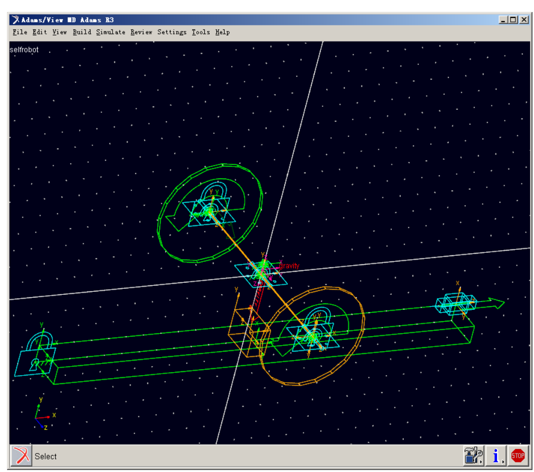

Figure 6. Kinematics simulation based on ADAMS

We do some disturbance in 2.6 second. And some results of simulation, such as position, velocity, acceleration, and angular velocity of robot chassis center are shown in Figure 7-9.

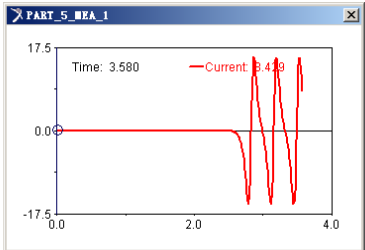

Figure 7. The position of robot chassis center.

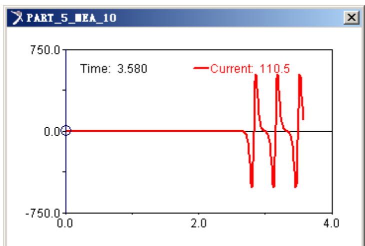

Figure 8. The velocity of robot chassis center

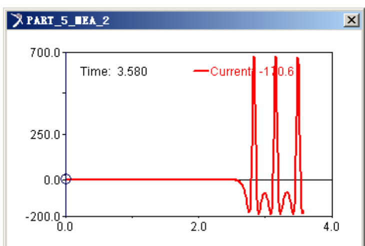

Figure 9. The acceleration of robot chassis center

IV. CONCLUSIONS

This paper is successful in kinematics model of a two-wheeled self-balancing autonomous robot based on the inverted pendulum model. After the left and right wheel kinematic model is established based on velocity decomposition method, and the kinematic model of self-balancing robot's body is then calculated. The whole kinematics model of the two-wheeled self-balancing robot system is then established. The model is correct by ADAMS simulation analysis and validation.

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