

# Fuzzy Control Solution for a Class of Tricycle Mobile Robots

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**Abstract** – The paper proposes an original fuzzy control solution to solve the tracking problem in case of a class of tricycle mobile robots with two degrees of freedom and steered front wheel. The control system structure contains two control loops to control the forward velocity and the angle between the heading direction and the  $x$ -axis. The reference trajectory is obtained on the basis of an obstacle avoidance strategy employing the artificial potential field method. The development in the linear case is done in terms of the Extended Symmetrical Optimum method and of the Iterative Feedback Tuning method resulting in two linear PI controllers. A new development method is proposed based on the transfer of results from the linear case to the fuzzy one employing Mamdani PI-fuzzy controllers. Simulation results validate the proposed fuzzy control solution.

## I INTRODUCTION

The control of nonholonomic mobile robots has received much research interest due to the implications of the nonholonomic constraints on the admissible control inputs of these systems considered particular classes in the general framework of nonsmooth or nonholonomic systems [1], [2]. The necessity of mobile robots to be capable of various and complex tasks in an autonomous and intelligent way requires the development of high performance controllers to solve the three basic navigation problems [3]: tracking control (tracking a reference trajectory, with its two versions, local and global tracking problems [4]), path following and point stabilization.

The majority of controllers developed for nonholonomic mobile robots employ kinematic or dynamic models [5], [6]. But, the dynamic models do not exploit the dynamics of the actuators, of the measuring devices and of the control equipment as part of the control system. That is the reason why a simplified dynamic model is presented, that can well characterize the considered class of tricycle mobile robots with two degrees of freedom and steered front wheel.

Current approaches to solve the tracking control problems are generally focused on nonlinear techniques [7] including the popular backstepping [2], [4], the sliding mode approach [5], linear model or passivity based approaches [8], [9], the control Lyapunov function approach [10]. Since the development of the controllers based on these approaches is rather complex, it is necessary to simplify the controller development and the further implementation. This aspect results in the aim of this paper, to offer a control solution concentrated on a development method for Mamdani PI-fuzzy controllers (PI-FCs) used as tracking controllers. The development method is based on applying firstly the Extended Symmetrical Optimum (ESO) method

[11] accompanied by the Iterative Feedback Tuning (IFT) method [12, 13] in case of the basic linear PI controllers. Then, the results are transferred to the fuzzy case in terms of the modal equivalence principle [14] having in mind the goal to improve the control system (CS) performance.

IFT is a gradient-based approach, based on input-output data recorded from the closed-loop system. The CS performance indices are specified through certain cost functions (c.f.s). Optimizing such functions usually requires iterative gradient-based minimization, but the c.f.s can be complicated functions of the plant and of the disturbances dynamics. The key feature of IFT is that the closed-loop experimental data are used to calculate the estimated gradient of the c.f. Each iteration comprises several experiments. Updated controller parameters are obtained based on input-output data collected from the system.

The control system structure used here to solve the tracking control problem contains two control loops for controlling the forward velocity and the angle between the heading direction and the  $x$ -axis. The reference inputs for these two control loops are obtained by applying the artificial potential field method used in obstacle avoidance [15] to generate the reference trajectory of the robot. In addition, calculations employing the tracking errors for the  $x$ - and  $y$ -axes and the maximum accepted errors are used.

This paper addresses the following topics. Section II is dedicated to setting the control problem employing the dynamic model used in the tracking control problem, the control system structure and a description of the artificial potential field method. Then, an overview on the IFT and ESO methods used in tuning the linear PI controllers is presented in Section III. Section IV is focused on the development method for the Mamdani PI-FCs playing the role of tracking controllers. Section V offers simulation results for a case study to validate the fuzzy control solution. Section VI draws the conclusions.

## II CONTROL PROBLEM SETTING

The dynamic model of the class of tricycle mobile robots with two degrees of freedom and steered front wheel is:

$$\begin{aligned} \dot{x} &= v \cos \theta \\ \dot{y} &= v \sin \theta \\ \dot{v} &= a_v \\ \dot{\theta} &= \omega \end{aligned} \quad , \quad (1)$$

$$\begin{aligned} T_{\Sigma 1} \dot{a}_v + a_v &= k_{p1}(u_1 + d_1) \\ T_{\Sigma 2} \dot{\omega} + \omega &= k_{p2}(u_2 + d_2) \end{aligned}$$

where (Fig. 1 (a)):  $(x, y)$  – coordinates of the centre of the rear axis of the mobile robot;  $v$  and  $a_v$  – forward velocity and acceleration, respectively;  $\theta$  – angle between the heading direction and the  $x$ -axis;  $\omega$  – angular velocity;  $u_1$  and  $u_2$  – control signals, proportional to the torques or to the generalized force variables depending on the actuator types;  $d_1$  and  $d_2$  – disturbance inputs due to the contact with the robot environment;  $k_{P1}$  and  $k_{P2}$  – gains;  $T_{\Sigma 1}$  and  $T_{\Sigma 2}$  – small time constants or time constants equivalent to the cumulative effects of the actuator dynamics, the measuring device dynamics, the control equipment dynamics and of the parasitic time constants [16].

The model structure (1) can be illustrated by means of the controlled plant (CP) structure (Fig. 1 (b)), that points out its dynamic subsystems having the transfer functions  $H_{P1}(s)$  and  $H_{P2}(s)$  of the CP:

$$H_{Pi}(s) = k_{Pi} / [s(1 + T_{\Sigma i}s)], \quad i = \overline{1, 2}, \quad (2)$$

and its kinematic subsystem (KS).

The CS structure employed in solving the tracking control problem for the considered class of mobile robots is presented in Fig. 1 (c), where: C-1 and C-2 – forward velocity controller and angle controller, respectively;  $v_r$  and  $\theta_r$  – reference inputs for the two control loops; RF-1 and

RF-2 – reference filters;  $\tilde{v}_r$  and  $\tilde{\theta}_r$  – filtered reference inputs for the two control loops;  $e_1 = \tilde{v}_r - v$  and  $e_2 = \tilde{\theta}_r - \theta$  – control errors;  $x_r$  and  $y_r$  – reference positions for  $x$  and  $y$ , respectively;  $e_x = x_r - x$  and  $e_y = y_r - y$  – tracking errors for  $x$  and  $y$ , respectively; CB-xp and CB-yp – computation blocks providing the estimates  $\hat{x}_r$  and  $\hat{y}_r$  of the derivatives  $\dot{x}_r$  and  $\dot{y}_r$ , respectively;  $\Delta x_{r,k}$  and  $\Delta y_{r,k}$  – increments of the reference positions  $x_r$  and  $y_r$ , respectively;  $k$  – index of current sampling interval.

The proposed control system structure is a cascaded one, with the two inner loops at the level of the steered front wheel to control  $v$  and  $\theta$ , and the outer loops to provide the reference inputs for the inner loops by means of the blocks CB-xp, CB-yp operating on the basis of the following algorithms:

$$\hat{x}_{r,k} = \begin{cases} \Delta x_{r,k} / h & \text{if } |e_{x,k}| \leq \epsilon_x \\ e_{x,k} / h & \text{otherwise} \end{cases}, \quad (3)$$

$$\hat{y}_{r,k} = \begin{cases} \Delta y_{r,k} / h & \text{if } |e_{y,k}| \leq \epsilon_y \\ e_{y,k} / h & \text{otherwise} \end{cases}$$

where:  $h$  – sampling period;  $\epsilon_x > 0$  and  $\epsilon_y > 0$  – maximum accepted absolute values of the tracking errors corresponding to the positions  $x$  and  $y$ , respectively. These two values must be specified by the CS specialist as function of the desired CS performance indices.

The blocks CB-xp and CB-yp are in fact estimators of derivatives. As it can be seen in (3) and Fig. 1 (c), the feedback in terms of  $x$  and  $y$  (in the outer control loops) operates only when the absolute values of the tracking errors  $e_x$  and  $e_y$  exceed the values of  $\epsilon_x$  and  $\epsilon_y$ , respectively.

To calculate the reference inputs  $v_r$  (the set-point of  $v$ ) and  $\theta_r$  (the set-point of  $\theta$ ) corresponding to the two inner control loops, there can be used the first two equations in (1), modified as follows:

$$v_r = \sqrt{(\hat{\dot{x}}_r)^2 + (\hat{\dot{y}}_r)^2}, \quad \theta_r = \tan^{-1}(\hat{\dot{y}}_r / \hat{\dot{x}}_r), \quad (4)$$

and the nonlinear blocks denoted by (4) in Fig. 1 (c) operate on the basis of (4).

The generation of the reference trajectory  $(x_r, y_r)$  for the CS in Fig. 1 (c) represents an important task, performed off-line in the initial phase. It is based on drawing a virtual potential field to ensure the obstacle avoidance. The application of the artificial potential field method [15] will be presented as follows. A local harmonic potential field  $\Psi(x, y)$  is built to fulfill the Laplace equation (5):

$$\nabla^T \cdot \nabla \Psi(x, y) = \partial^2 \Psi(x, y) / \partial x^2 + \partial^2 \Psi(x, y) / \partial y^2 = 0 \quad (5)$$

the solution to (5) giving the potential of a singular point of strength  $q$  in  $(0, 0)$ :

$$\Psi(x, y) = -0.5q \cdot \ln(x^2 + y^2), \quad (6)$$

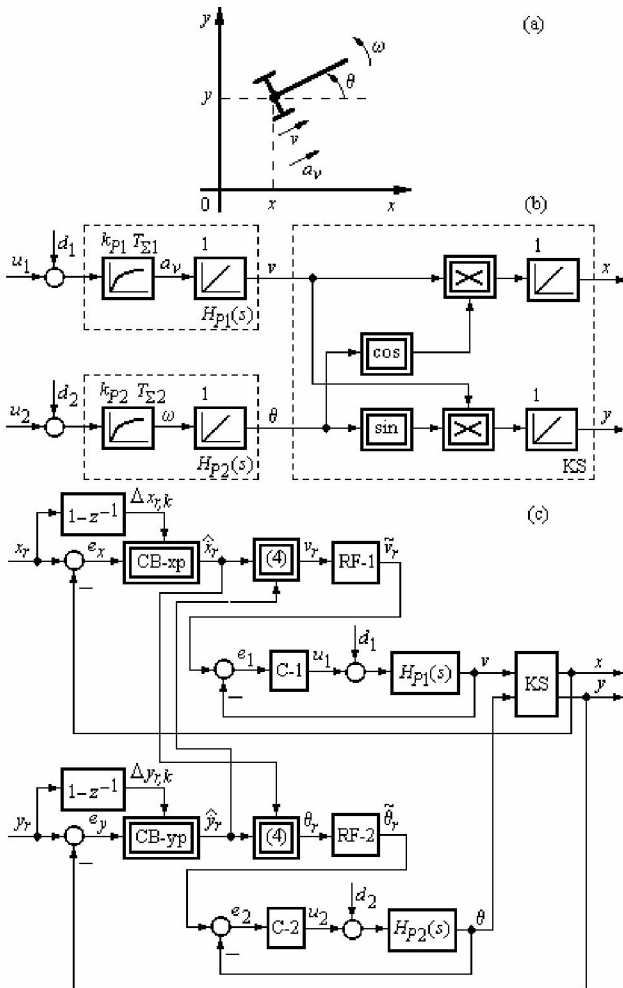


Figure 1

Mechanical variables specific to mobile robot (a), controlled plant structure (b) and control system structure (c)

and the associated gradient  $\rho(x, y) \in R^2$ :

$$\rho(x, y) = -\text{grad}\Psi(x, y) = [q/(x^2 + y^2)](x, y)^T = (\rho_x, \rho_y) \in R^2, \quad (7)$$

where the upper index  $T$  stands for matrix transposition.

A fundamental potential field configuration consists of a negative unit singular point in the goal and a positive singular point of strength  $0 < q < 1$  in the obstacle center  $q = R/(R+e)$  from the equivalent point placement method, where  $e$  is the distance between the goal point and the obstacle center, and  $R$  is the radius of the circular obstacle security zone. A collision free trajectory along the vector field lines is guaranteed when using the artificial potential field method. The obstacle avoidance aim is to control the orientation of  $\theta$  to be co-linear to the gradient  $\rho(x, y)$ . So, the reference input  $\theta_r$  at  $(x, y)$  will be:

$$\theta_r = \tan^{-1}(\rho_y / \rho_x). \quad (8)$$

The desired direction of motion representing the reference input  $v_r$  is determined by introducing the additional variable  $\gamma$ , defined as function of the angle error,  $e_\theta = \theta_r - \theta$ :

$$\gamma = \text{sgn}(v_r), \quad \gamma = \begin{cases} 1 & \text{if } -2\pi \leq e_\theta < -3\pi/2 \\ -1 & \text{if } -3\pi/2 \leq e_\theta < -\pi/2 \\ 1 & \text{if } -\pi/2 \leq e_\theta < \pi/2 \\ -1 & \text{if } \pi/2 \leq e_\theta < 3\pi/2 \\ 1 & \text{if } 3\pi/2 \leq e_\theta < 2\pi \end{cases}. \quad (9)$$

The reference inputs obtained in (7) ... (9),  $\theta_r$  and  $v_r$ , can be fed directly to the two inner control loops. But, the aim is to solve the tracking control problem and this is the reason why there is necessary to employ the CS structure in Fig. 1 (c) having the reference inputs  $x_r$  and  $y_r$  and taking the feedback in terms of the actual trajectory,  $(x, y)$ .

Finally, the reference trajectory  $(x_r, y_r)$  will be obtained by the integration the following equations of motion in terms of using adequate numerical techniques:

$$\begin{aligned} \dot{x}_r &= v_r [\rho_x / \sqrt{(\rho_x)^2 + (\rho_y)^2}] \cos \theta_r, \\ \dot{y}_r &= v_r [\rho_y / \sqrt{(\rho_x)^2 + (\rho_y)^2}] \sin \theta_r, \end{aligned} \quad (10)$$

It must be emphasized that the control problem setting presented here is different to other approaches using the reference inputs and actual posture determined by  $(x, y, \theta)$  because  $\theta$  is in fact not directly controlled, but it results with acceptable precision since the tracking at the level of  $x$  and  $y$  will be improved by fuzzy control.

### III OVERVIEW ON ITERATIVE FEEDBACK TUNING AND EXTENDED SYMMETRICAL OPTIMUM METHODS

For the considered class of plants with the transfer functions in the form (2) the use of PI controllers as the controllers C-1 and C-2 in Fig. 1 (c), having the transfer functions (11):

$$H_{ci}(s) = (k_{ci} / s)(1 + sT_{ci}), \quad i = 1, 2, \quad (11)$$

with the gains  $k_{ci}$  and the integral time constants  $T_{ci}$ , tuned in terms of Kessler's Symmetrical Optimum method [17] can ensure acceptable CS performance.

In case of these two PI controllers as one-degree-of-freedom controllers to control the plants (1) the IFT method consists of the steps A) ... E) that point out the specific design parameters [12], [13]:

A) A controller, of desired complexity, which stabilizes the system, has to be chosen. A discrete form of the controller is needed. The parameterization of the controller is such that the transfer function  $C(\mathbf{p})$  is differentiable with respect to its parameters,  $\mathbf{p}$  being the parameters vector.

B) A reference model must be chosen, which prescribes the desired output of the closed-loop system. This model is typically chosen with first- or second-order dynamics and, for the sake of simplicity and better CS performance, it can be also chosen to be without dynamics, having the transfer function equal to the unity.

C) The update law must be set by which the next set of parameters will be calculated, corresponding to a Gauss-Newton scheme of type (12):

$$\mathbf{p}^{i+1} = \mathbf{p}^i - \gamma^i (\mathbf{R}^i)^{-1} \text{est} \left[ \frac{\partial J}{\partial \mathbf{p}}(\mathbf{p}^i) \right], \quad (12)$$

where:  $i$  – index of the current iteration,  $\text{est}[x]$  – estimate (generally) of the variable  $x$ ,  $\gamma^i > 0$  determines the step size.

D) The regular matrix  $\mathbf{R}^i$  in (12) is a positive definite matrix, typically the Hessian of  $J(\mathbf{p})$ :

$$\begin{aligned} \mathbf{R}^i &= \frac{1}{N} \sum_{k=1}^N \left( \text{est} \left[ \frac{\partial y}{\partial \mathbf{p}}(k, \mathbf{p}^i) \right] \text{est} \left[ \frac{\partial y}{\partial \mathbf{p}}(k, \mathbf{p}^i) \right]^T + \right. \\ &\quad \left. + \lambda \cdot \text{est} \left[ \frac{\partial u}{\partial \mathbf{p}}(k, \mathbf{p}^i) \right] \text{est} \left[ \frac{\partial u}{\partial \mathbf{p}}(k, \mathbf{p}^i) \right]^T \right) \end{aligned} \quad (13)$$

Choosing the identity matrix for  $\mathbf{R}^i$  ensures the negative direction of the gradient, but it is recommended to calculate  $\mathbf{R}^i$  by a quasi-Newton method or as the Hessian of the c.f.

E) The general expression of the c.f.  $J$  is defined in (14):

$$\begin{aligned} \mathbf{p}^* &= \arg \min_{\mathbf{p} \in SD} J(\mathbf{p}), \quad J(\mathbf{p}) = \frac{1}{2N} \sum_{k=1}^N \{ [L_y(q^{-1}) \cdot \\ &\quad \cdot \delta y(k, \mathbf{p})]^2 + \lambda [L_u(q^{-1}) u(k, \mathbf{p})]^2 \} \end{aligned} \quad (14)$$

where:  $N$  – length of each experiment,  $L_y, L_u$  – weighting filters, introduced to emphasize certain frequency regions,  $\lambda$  – weighting constant,  $\delta y$  – output error, the difference between the actual output ( $y$ ) and the desired output ( $y_d$ ):

$$\delta y = y - y_d. \quad (15)$$

IFT algorithms are involved to solve the optimization problem (14), where several constraints (for example, one constraint with  $SD$  – stability domain) can be imposed

regarding the plant or the closed-loop CS. These techniques are iterative, meaning that in case of one-degree-of-freedom controllers as the PI controllers considered here two experiments are required, a normal one and a special one. The latter is also referred to as the gradient experiment, having as reference input the control error of the first experiment. Then, the input-output data recorded from the two experiments are used to calculate the estimated gradients of the output and control signal required in calculating the estimated gradient of the c.f.  $J(\mathbf{p})$ .

The IFT algorithm considered here (with the aim to obtain the next set of parameters), not presented for the sake of simplicity, usually consists of eight steps [12], [13].

A simple and efficient way to tune the parameters of the PI controllers (11) controlling the plants (2) is represented by the ESO method [11], and the PI tuning conditions are:

$$k_{ci} = 1/(\sqrt{\beta_i^3 T_{\Sigma i}^2 k_{pi}}), \quad T_{ci} = \beta_i T_{\Sigma i}, \quad i = \overline{1,2}, \quad (16)$$

where  $\beta_i$ ,  $i = \overline{1,2}$ , represent design parameters, only one for each controller.

By the choice of the design parameters  $\beta_i$  in the recommended domain  $1 < \beta_i < 20$ , the CS performance indices  $\{\sigma_1 - \text{overshoot}, \hat{t}_s = t_s/T_{\Sigma i} - \text{settling time}, \hat{t}_1 = t_1/T_{\Sigma i} - \text{first settling time}, \varphi_r - \text{phase margin}\}$  can be accordingly modified. A compromise to these indices can be reached by using the diagrams illustrated in Fig. 2.

The CS performance indices can be corrected by suppressing the action of the zero in the open-loop transfer functions. Other versions are also available suppressing some complex conjugated poles. This can be accomplished by adding the reference filters RF-1 and RF-2 with the transfer functions  $H_{RF1}(s)$  and  $H_{RF2}(s)$ , respectively [11]:

$$H_{RF1}(s) = 1/(1 + \beta_i T_{\Sigma i} s), \quad i = \overline{1,2}. \quad (17)$$

Connecting the IFT method to the ESO method, the application of the IFT algorithms is relatively simple because the presence of only two design parameters in case of the ESO method results in:

$$\mathbf{p} = [\beta_1 \quad \beta_2]^T. \quad (18)$$

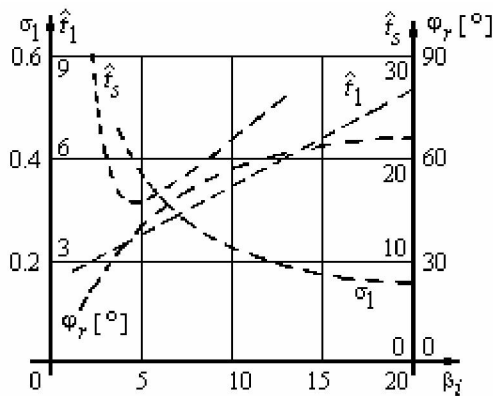


Figure 2  
Control system performance indices versus design parameters  $\beta_i$

#### IV DEVELOPMENT METHOD FOR MAMDANI PI-FUZZY CONTROLLERS

To develop the PI-FCs, the continuous-time PI controllers (11) are discretized resulting in the discrete-time equations of the quasi-continuous digital PI controllers expressed in their incremental versions (19):

$$\begin{aligned} \Delta u_{i,k} &= K_{pi} \Delta e_{i,k} + K_{fi} e_{i,k} = \\ &= K_{pi} (\Delta e_{i,k} + \lambda_i \cdot e_{i,k}), \quad i = \overline{1,2}, \end{aligned} \quad (19)$$

where the parameters  $\{K_{pi}, K_{fi}, \lambda_i\}$  depend on  $\{k_{ci}, T_{ci}\}$ , exemplified in (20) in case of Tustin's method:

$$\begin{aligned} K_{pi} &= k_{ci} T_{ci} [1 - (h/2T_{ci})], \quad K_{fi} = k_{ci} h, \\ \lambda_i &= K_{fi} / K_{pi} = 2h/(2T_{ci} - h), \quad i = \overline{1,2}. \end{aligned} \quad (20)$$

Both Mamdani PI-FCs playing the roles of the controllers C-1 and C-2 in Fig. 1 (c) have the unified structure presented in Fig. 3. This structure is based on adding the dynamics to the basic fuzzy controllers without dynamics  $FC_i$ ,  $i = \overline{1,2}$ , by the numerical differentiation of the control error  $e_{i,k}$  as the increment of control error,  $\Delta e_{i,k} = e_{i,k} - e_{i,k-1}$ , and the numerical integration of the increment of control signal,  $\Delta u_{i,k} = u_{i,k} - u_{i,k-1}$ ,  $i = \overline{1,2}$ .

The fuzzification in the basic fuzzy controllers without dynamics  $FC_i$ ,  $i = \overline{1,2}$ , can be solved in the initial phase as follows (Fig. 4):

- for the input variables,  $e_{i,k}$  and  $\Delta e_{i,k}$ , five linguistic terms with regularly distributed triangular type membership functions having an overlap of 1 are chosen,
- for the output variable  $\Delta u_{i,k}$ , seven linguistic terms with non-regularly distributed singleton type membership functions are chosen,

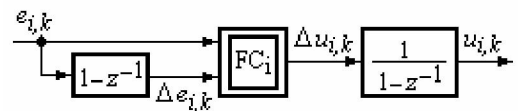


Figure 3  
Structure of Mamdani PI-FCs

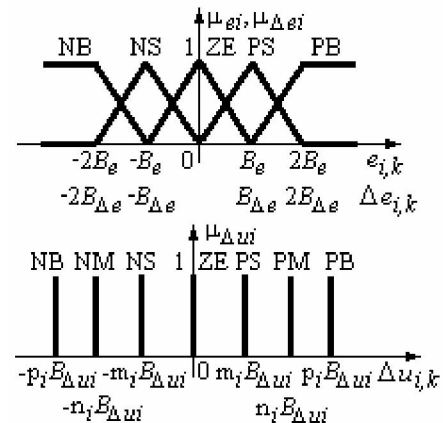


Figure 4  
Accepted initial membership functions of  $FC_i$

- other shapes of membership functions can contribute to CS performance enhancement.

The (strictly positive) parameters of the Mamdani PI-FCs are  $\{B_{ei}, B_{\Delta ei}, B_{\Delta ui}, m_i, n_i, p_i\}$ . The parameters  $B_{ei}$ ,  $B_{\Delta ei}$  and  $B_{\Delta ui}$  are in correlation with the shapes of the input and output membership functions. The parameters  $m_i$ ,  $n_i$  and  $p_i$ ,  $m_i < n_i < p_i$ , have been added to the standard version of PI-FCs to improve the CS performance by modifying the input-output static map of the blocks  $FC_i$ . The inference engine of the blocks  $FC_i$  employs the Mamdani's MAX-MIN compositional rule of inference assisted by complete rule bases expressed as decision tables presented in Table 1, and the centre of gravity method is used for defuzzification. The development method for the two Mamdani PI-FCs as part of the accepted control system structure consists of the following development steps regarding both the linear and the fuzzy part:

Step 1. Express the last two equations in the simplified dynamic model (1) of the mobile robot in terms of the mathematical models of the subsystems of the controlled plant with the transfer functions in (2),  $H_{P1}(s)$  and  $H_{P2}(s)$ , and compute the parameters  $k_{P1}$ ,  $k_{P2}$ ,  $T_{\Sigma 1}$  and  $T_{\Sigma 2}$ .

Step 2. Choose the values of the design parameters  $\beta_1$  and  $\beta_2$  as function of the desired / imposed CS performance indices by using the diagrams in Fig. 2 and the maximum accepted absolute values  $\varepsilon_x$  and  $\varepsilon_y$  of the tracking errors corresponding to the positions  $x$  and  $y$ , respectively.

Step 3. Apply one of the IFT algorithms specific to the IFT method according to Section III, resulting in the optimal values of the design parameters,  $\beta_1^*$  and  $\beta_2^*$ .

Step 4. Tune the parameters of the basic continuous-time PI controllers  $\{k_{c1}, T_{c1}\}$  (for C-1) and  $\{k_{c2}, T_{c2}\}$  (for C-2) in terms of (16) with  $\beta_1 = \beta_1^*$  and  $\beta_2 = \beta_2^*$ .

Step 5. Add to the CS structure the reference filters RF-1 and RF-2 (for example, the versions in (17)).

Step 6. Choose a sufficiently small sampling period,  $h$ , accepted by quasi-continuous digital control, take into account the presence of zero-order hold blocks, and discretize the two continuous-time PI controllers and compute the parameters of the two quasi-continuous digital PI controllers,  $\{K_{P1}, K_{I1}, \lambda_1\}$  and  $\{K_{P2}, K_{I2}, \lambda_2\}$ , by (19).

Step 7. Choose the values of the parameters  $m_i$ ,  $n_i$ ,  $p_i$  and  $B_{ei}$  of the PI-FCs and apply (21), resulted from the modal equivalence principle, to obtain the values of the parameters  $B_{\Delta ei}$  and  $B_{\Delta ui}$ :

$$B_{\Delta ei} = \lambda_i B_{ei}, \quad B_{\Delta ui} = K_{fi} B_{ei}, \quad i = \overline{1, 2}. \quad (21)$$

## V CASE STUDY. DIGITAL SIMULATION RESULTS

The case study considers the model (1) characterizing sufficiently well the accepted class of mobile robots [16].

Table 1  
Decision table of  $FC_i$

$\Delta e_{i,k} \backslash e_{i,k}$	NB	NS	ZE	PS	PB
PB	ZE	PS	PM	PB	PB
PS	NS	ZE	PS	PM	PB
ZE	NM	NS	ZE	PS	PM
NS	NB	NM	NS	ZE	PS
NB	NB	NB	NM	NS	ZE

With this respect, the following values of the parameters have been accepted to validate the proposed fuzzy control solution:  $k_{P1} = k_{P2} = 1$  and  $T_{\Sigma 1} = T_{\Sigma 2} = 1$  s.

There is considered one experiment, with the obstacles placed in the points (1, 4), (10, 8) and (2, 10) having the potentials 0.3, 0.3 and 0.5, respectively, and the initial position of the robot is in (5, 2). The goal, representing the desired final position, is placed in the point (6, 11) with the potential -1. For the considered obstacles and robot positions the application of the artificial potential method (Section II) leads to the vector field and to the reference trajectory  $(x_r, y_r)$  presented in Fig. 5.

The control system structure is that presented in Fig. 3. The Mamdani PI-FCs playing the roles of C-1 and C-2 are developed in terms of the method presented in the previous Section.

For the accepted case study, in the conditions of  $\beta_1 = \beta_2 = 4$  and  $h = 0.1$  s, applying one IFT algorithm for the c.f. (22):

$$J = \frac{1}{2N} \sum_{k=1}^N (\delta y^2(k)), \quad (22)$$

results in the optimal values  $\beta_1^* = 3.82$  and  $\beta_2^* = 4.23$ .

The parameters of the quasi-continuous digital PI controllers obtain the values  $K_{P1} = K_{P2} = 0.4938$ ,  $K_{I1} = K_{I2} = 0.0125$ ,  $\lambda_1 = \lambda_2 = 0.0253$ . Choosing  $B_{ei} = 0.3$ ,  $B_{e2} = 0.45$ ,  $m_1 = m_2 = 0.8$ ,  $n_1 = n_2 = 1.2$  and  $p_1 = p_2 = 1.4$ , the values of the other parameters of the Mamdani PI-FCs will be tuned as  $B_{\Delta e1} = 0.0076$ ,  $B_{\Delta u1} = 0.0037$ ,  $B_{\Delta e2} = 0.0114$ ,  $B_{\Delta u2} = 0.0056$ .

The following disturbance inputs,  $d_1$  and  $d_2$ , were used:

$$\begin{aligned} d_i = & \delta_i \sigma(t - 110) - \delta_i \sigma(t - 220) - \\ & - \delta_i \sigma(t - 440) + \delta_i \sigma(t - 550) + \\ & + \delta_i \sigma(t - 770) - \delta_i \sigma(t - 880), \\ & t \in [0, t_f], \quad i = \overline{1, 2}, \end{aligned} \quad (23)$$

where:  $\sigma$  – unit step signal,  $\delta_1 = 0.003$ ,  $\delta_2 = 0.07$ ,  $t_f = 1000$  s. These variations are acceptable to model the contact of the robot with its environment.

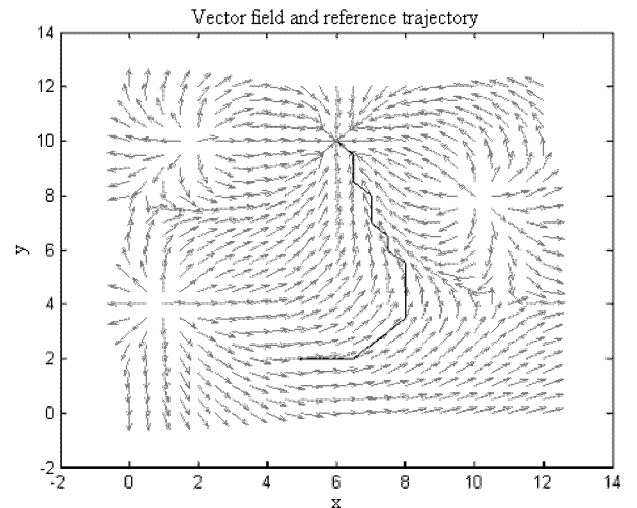


Figure 5  
Vector field and reference trajectory

In the conditions of the disturbance inputs (23) and of the reference inputs computed off-line in terms of by using (8) ... (10) and illustrated in Fig. 5, part of the digital simulation results for the control system with Mamdani PI-FCs is presented in Fig. 6.

## VI CONCLUSION

The paper proposes a new fuzzy control solution represented by a development method for Mamdani PI-fuzzy controllers as part of a relatively simple control system structure, to solve the tracking problem for a class of tricycle mobile robots with two degrees of freedom and steered front wheel. The fuzzy controllers can be implemented as low cost tracking controllers in a cascaded control system structure with two control loops acting at the level of the steered front wheel, using the off-line generation of the reference inputs by means of the artificial potential field method for obstacle avoidance.

The fuzzy control solution has been validated with good results on a simplified dynamic model dedicated to the considered class of mobile robots through one digitally simulated experiment.

The presence of the double integrator on both control channels (of  $x$  and  $y$ ) ensures a favorable behavior with respect to a ramp variation of the reference input and the rejection of the effects of constant disturbances as well. The control system behavior can be improved by the development of optimal PI-FCs or of other advanced Takagi-Sugeno fuzzy controllers [18], [19].

It has been proved here that fuzzy control can cope with tracking control involving relatively small initial tracking errors, not arbitrary ones as in the global tracking problem. It has also potential in simpler or more complex control problems such as path following and point stabilization.

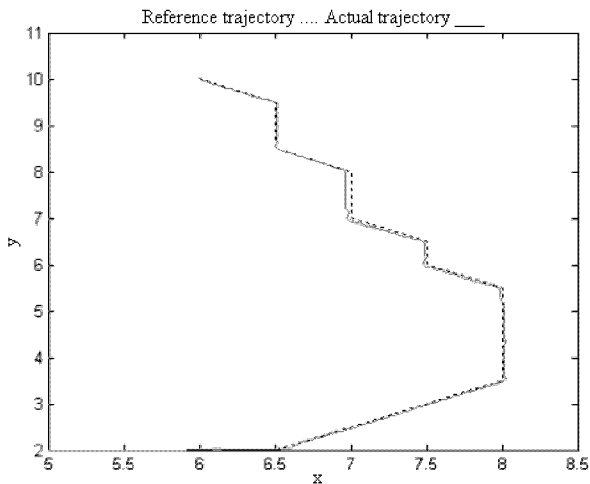


Figure 6  
Reference and actual trajectory

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