Speed and Balancing Control for Unicycle Robot

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***Abstract*— Unicycle mobile robot is wheeled mobile robot that can stand and move around using one wheel. It has attached a lot of researchers to conduct studies about the system, particularly in the design of the system mechanisms and the control strategies. Unlike two wheel balancing mobile robot which mechanically stable on one side, unicycle mobile robot requires additional mechanisms to keep balancing robot on all sides. By assuming that both roll dynamics and pitch dynamics are decoupled, so the balancing mechanisms can be designed separately. The reaction wheel is used for obtaining balancing on the roll angle by rotating the disc to generate momentum. While the wheeled robot is used for obtaining balancing on the pitch angle by rotating wheel to move forward or backward. A PID controller is used as balancing control which will control the rotation motor on the reaction disc and wheel based on the pitch and roll feedback from the sensor. By adding the speed controller to the pitch control, the system will compensate automatically for perfectly center of gravity on the robot. Finally, the unicycle robot will be able to balance on pitch angle and roll angle. Based on simulation result validates that robot can balance using PID controller, while based on balancing pitch experiment result, robot can achieve balancing with maximum inclination about ±23 degree on pitch angle and ±3.5 degree on roll angle with steady state error 0.1 degree.**

***Keywords*— unicycle robot, balancing, speed control, roll, pitch, reaction wheel, PID.**

I. Introduction

Balancing robot works on the principle of two-wheel inverted pendulum on a sliding carriage that has only one degree of freedom such as the Segway [1]. However, balancing robot with two wheels has some shortcomings such as: requires a lot of drive wheels, have a width physical [2], and has a large torque disturbances when the robot moves at high speed [3].

Unicycle robot solve problems that occur on two-wheeled balancing robot. By using only one wheel in direct contact to the surface, unicycle robot has several advantages compared with two wheels balancing robot like: have a higher mobility, need space smaller [2], and low torque disturbance when moving at high speed, because the robot has only one point of contact to the ground [3].

Unicycle robot works on the principle of inverted pendulum with two degrees of freedom, so it requires more complicated balancing control because the system is not linear, not stable, and underactuation. In previous studies have been produced unicycle robot [4-12], such as the unicycle robot created by Davide Falanga et al in 2013 and in Woo Han et al in 2013 using PID control, but most studies only focus on the balancing only. In this paper created a robot unicycle that can balance with additional speed control, so the robot is able to move at a certain speed. Robot consists of a wheel, robot body, and a reaction wheel. The wheel on the robot used to move the robot forward and backward so the robot is able to balance on pitch angle. Reaction wheel pendulum is used to balance the robot on roll angle, reaction wheel pendulum is a free pendulum with the reaction wheel at the top, an acceleration in the reaction wheel while spinning will generate momentum to swing the pendulum [13].

PID control is used to control the robot actuators balance separately. Pitch angle feedback is used to control the rotation of the robot wheel, while the roll angle feedback is used to control the rotation of the reaction wheel. On pitch balancing control was added with cascade speed control, so the robot speed can be controlled with desired value.

II. Modeling

A. Modeling of Unicycle Robot

To facilitate mathematical modeling on a unicycle robot, then the modeling system is divided into two part: the inverted pendulum as a model system modeling on the pitch, and the reaction wheel pendulum as a model system modeling on the roll. To determine the mathematical model of reaction wheel inverted pendulum and the pendulum robot Lagrange equation is used as follows,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_r}}\right) - \frac{\partial L}{\partial q_r} = \tau_r \tag{1}$$

L is Lagrangian function,

$$L = E_k - E_p \tag{2}$$

with,
E_k = kinetic energy
E_p = potential energy
q_r = coordinate angle on the robot
τ_r = applied torque on the robot

1) System Model Inverted Pendulum

By looking at the pitch robot robot simplified system consisting of inverted pendulum robot body and a wheel robot.

In Fig 1 below is a two-dimensional modeling inverted pendulum on a unicycle robot [5]:

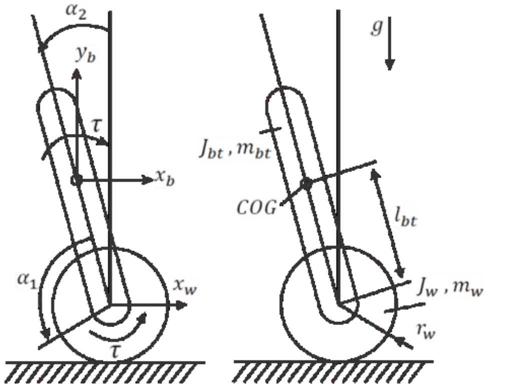

Fig. 1 Two-dimensional model of the Inverted Pendulum Robot

with:

$\alpha_1(t)$ = wheel angle of the robot body
$\alpha_2(t)$ = angle of robot body against the vertical axis
$\tau(t)$ = torque of wheel's motor
$x_w(t)$ = horizontal position of the wheel
$x_b(t)$ = horizontal position of the robot body
$y_b(t)$ = vertical position of the robot body
m_w = mass of the wheel
m_{bt} = mass of the body
J_w = the moment of inertia of wheel
J_{bt} = the moment of inertia of body
l_{bt} = the distance between the wheel's center to the center of gravity of robot body
r_{bt} = radius of wheel
g = earth's gravitational acceleration

By using the Equation (1) based on the coordinates wheel angle (α_2) and the angle of pendulum (α_1) then the dynamic equation on the pitch robot is obtained,

$$(J_w + m_w r_{bt}^2 + m_{bt} r_{bt}^2)\ddot{\alpha}_1 + m_{bt} r_{bt} l_{bt} cos\alpha_2 \ddot{\alpha}_2 - m_{bt} r_{bt} l_{bt} sin\alpha_2 \dot{\alpha}_2^2 + (\mu_f + \mu_a)\dot{\alpha}_1 - \mu_a \dot{\alpha}_2 = \tau_m \quad (3)$$

$$m_{bt} r_{bt} l_{bt} cos\alpha_2 \ddot{\alpha}_1 + (m_{bt} l_{bt}^2 + J_{bt})\ddot{\alpha}_2 - \mu_a \dot{\alpha}_1 + \mu_a \dot{\alpha}_2 - m_{bt} g l_{bt} sin\alpha_2 = -\tau_m \quad (4)$$

with,

τ_m= the torque applied to the wheel
μ_f = friction coefficient of the floor
μ_a = friction coefficient of the wheel axle

For linear model, the robot is assumed in a standing position is balanced and assumes an angle on the chassis and wheels are very small. Thus, it is assumed that $\alpha_1 = 0$ and $\alpha_2 = 0$. Linear equations pitch section can be determined,

$$(J_w + m_w r_{bt}^2 + m_{bt} r_{bt}^2)\ddot{\alpha}_1 + m_{bt} r_{bt} l_{bt} \ddot{\alpha}_2 + (\mu_f + \mu_a)\dot{\alpha}_1 - \mu_a \dot{\alpha}_2 = \tau_m \quad (5)$$

$$m_{bt} r_{bt} l_{bt} \ddot{\alpha}_1 + (m_{bt} l_{bt}^2 + J_{bt})\ddot{\alpha}_2 - \mu_a \dot{\alpha}_1 + \mu_a \dot{\alpha}_2 - m_{bt} g l_{bt} \alpha_2 = -\tau_m \quad (6)$$

From Equation (5) dan (6) can find equation for $\ddot{\alpha}_1$and $\ddot{\alpha}_2$ are,

$$\ddot{\alpha}_1 = \left[-\frac{(m_{bt} l_{bt})^2 r_{bt} g}{den}\right]\alpha_2 + \left[\frac{-(m_{bt} l_{bt}^2 + J_{bt})(\mu_f + \mu_a) - (m_{bt} r_{bt} l_{bt})\mu_a}{den}\right]\dot{\alpha}_1 + \left[\frac{-(m_{bt} l_{bt}^2 + J_{bt})\mu_a - (m_{bt} r_{bt} l_{bt})\mu_a}{den}\right]\dot{\alpha}_2 + \left[\frac{-(m_{bt} l_{bt}^2 + J_{bt}) - (m_{bt} r_{bt} l_{bt})}{den}\right]\tau_m \quad (7)$$

$$\ddot{\alpha}_2 = \left[\frac{(J_w + m_w r_{bt}^2 + m_{bt} r_{bt}^2) m_{bt} g l_{bt}}{den}\right]\alpha_2 + \left[\frac{(m_{bt} r_{bt} l_{bt})(\mu_f + \mu_a) + (J_w + m_w r_{bt}^2 + m_{bt} r_{bt}^2)\mu_a}{den}\right]\dot{\alpha}_1 + \left[\frac{-(m_{bt} r_{bt} l_{bt})\mu_a - (J_w + m_w r_{bt}^2 + m_{bt} r_{bt}^2)\mu_a}{den}\right]\dot{\alpha}_2 + \left[\frac{(m_{bt} r_{bt} l_{bt}) + (J_w + m_w r_{bt}^2 + m_{bt} r_{bt}^2)}{den}\right]\tau_m \quad (8)$$

Where,

$$den = (J_w + m_w r_{bt}^2 + m_{bt} r_{bt}^2)(m_{bt} l_{bt}^2 + J_{bt}) - (m_{bt} r_{bt} l_{bt})^2 \quad (9)$$

2) System Model Reaction Wheel Pendulum

By looking at the roll axis robot robot can be simplified as a reaction wheel pendulum system consisting of a robot body and rotating disc. In Figure 2 is a two-dimensional modeling of the reaction wheel pendulum on a unicycle robot [5].

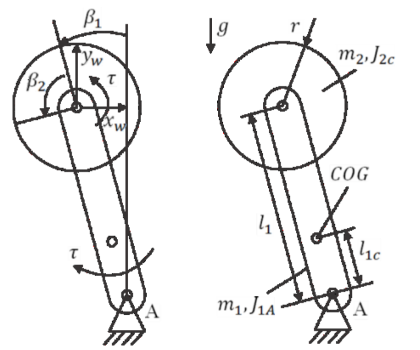

Fig. 2 Model Two Dimensional Reaction Wheel Pendulum on Robot

with,

$\beta_1(t)$ = body angle to the vertical axis
$\beta_2(t)$ = disc angle robot against robot body
$\tau(t)$ = torque of the disc motor
$x_w(t)$ = the horizontal position of the disc
$y_w(t)$ = the vertical position of the disc
m_1 = mass of the robot body
m_2 = mass of the disc
J_{1A} = moment of inertia of the body to the point A
J_p = moment of inertia robot body
J_{2c} = moment of inertia of the disc towards its center
l_1 = the distance between the center of the disc to the point A
l_{1c} = the distance between the center of mass to the point A
r = radius of the disc
g = earth's gravitational acceleration
m $= m_1 + m_2$
ml $= m_1 l_{1c} + m_2 l_1$
J $= J_p + m_1 l_{1c}^2 + m_2 l_1^2$

By using the Equation (1) to coordinate reaction disc wheel angle and roll angle pendulum, then the dynamic equation on a reaction wheel pendulum system or on the roll robot is obtained,

$$J\ddot{\beta}_1 + (\mu_{pr} + \mu_{rw})\dot{\beta}_1 - \mu_{rw}\dot{\beta}_2 + mglsin\beta_1 = -\tau_m \quad (10)$$

$$J_{2c}\ddot{\beta}_2 - \mu_{rw}\dot{\beta}_1 + \mu_{rw}\dot{\beta}_2 = \tau_m \quad (11)$$

with,

μ_{rw} = friction coefficient of the reaction wheel axle
μ_{pr} = friction coefficient of the pendulum
τ_m = torque applied to the reaction wheel

Equation (10) and (11) above is a non-linear equation on reaction wheel pendulum then linearized by robot standing balanced conditions. For that, it is assumed that $\beta_1 = 0$. Then obtained linear equations,

$$J\ddot{\beta}_1 + (\mu_{pr} + \mu_{rw})\dot{\beta}_1 + \mu_{rw}\dot{\beta}_2 + mgl\beta_1 = -\tau_m \quad (12)$$

$$J_{2c}\ddot{\beta}_2 - \mu_{rw}\dot{\beta}_1 + \mu_{rw}\dot{\beta}_2 = \tau_m \quad (13)$$

From Equation (12) and (13) can be searched equation in $\ddot{\beta}_1$ and $\ddot{\beta}_2$ that,

$$\ddot{\beta}_1 = -\frac{(\mu_{pr}+\mu_{rw})}{J}\dot{\beta}_1 + \frac{\mu_{rw}}{J}\dot{\beta}_2 - \frac{mgl\beta_1}{J} + \frac{\tau_m}{J} \quad (14)$$

$$\ddot{\beta}_2 = \frac{\mu_{rw}}{J_{2c}}\dot{\beta}_1 - \frac{\mu_{rw}}{J_{2c}}\dot{\beta}_2 - \frac{\tau_m}{J_{2c}} \quad (15)$$

III. SYSTEM DESIGN AND METHODOLOGY

Mechanical design of the robot shown in Fig 3 uses two motors, each of it is used to drive the wheel and disc. The stepper motor placed at the top of the wheel and connected to the motor shaft used timing belt mechanism. This installation method has been chosen in order to make the center of gravity of roll movement became close to the center of the robot. Stepper motors was choosed to avoid the backlash that could reduce the performance when the robot balanced. The mounting reaction wheel drive DC motors placed at the top end with the disc is located on the front side and the motor is located on the back side. In order to the center of mass on the pitch stays in the middle, then the placement of electronic components placed on the rear side of the robot.

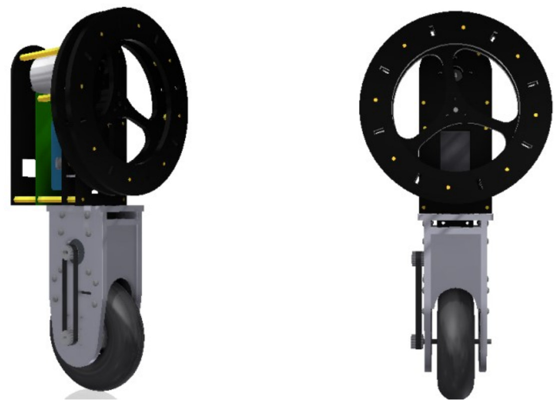

Fig. 3 Mechanical design

The electronic system block diagram of the robot is shown in Fig 4. We combine two MPU6050 IMU sensor that used to read the pitch and roll angles in interleaved mode. Rotary encoder is used to read the speed of the motor for feedback on the speed control and roll balancing control. Data from all the sensors will be processed on STM32F407VG ARM microcontroller in controlling the rotation of the reaction wheel motor and wheel stepper motor. The robot is powered by 3 Cell LiPo battery (11.1 V) 1000 mAh.

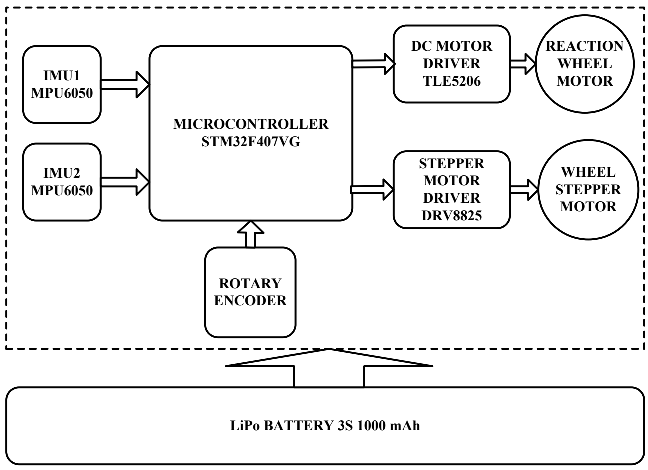

Fig. 4 Overal system

A. *Speed PID Control*

When determining the angle of offset on the balancing control pitch angle does not match to the balance center robot, the robot requires offset angles to keep its balance so that the robot will continue to move in order to achieve equilibrium angle [15]. As shown in Fig 5, by adding speed control cascade to the pitch balancing control, the robot will be able to adjust the offset value automatically based on the desired speed setpoint. By setting speed setpoint to zero, so the speed control will determine the value of the offset angle that corresponds to the condition when the robot balanced.

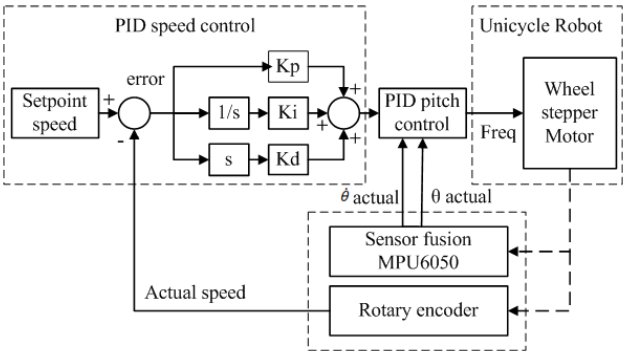

Fig. 5 Speed PID control diagram

B. Balancing Pitch PID Control

Pitch balancing control have function to control the wheel motor rotation, by sending frequency step from the PID control output. The pitch angle feedback comes from sensor sensor filter that combine the accelerometer and gyroscope data. In Fig 6 offset is the angle setpoint when robot balance, this value is not always zero, but rely on the center of mass on the robot. For derivative control section used pitch angular velocity feedback from gyro sensor, it aims to obtain a better response than did the mathematical derivation in the angle error of the robot generated from the sensor filter.

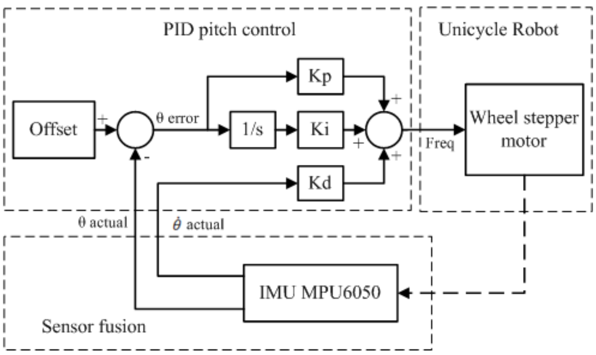

Fig. 6 Pitch balancing PID control diagram

C. Balancing Roll PID Control

Balance control section roll angle has the same method as in the balance control on pitch angle. Because the balancing control on roll angle need an acceleration, so it is necessary to add the angular velocity feedback from the disc so that resulting acceleration on the motor. Fig 7 magnitude of acceleration can be adjusted by changing the value of Ks.

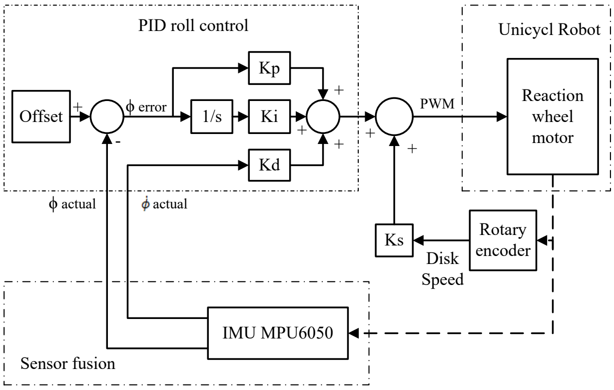

Fig. 7 Roll balancing PID control diagram

IV. RESULT

A. Balancing Response on Simulation

Simulations carried out separately for each balance control pitch and roll as shown in Fig 8 and Fig 9. Based on the simulation results obtained robot parts pitch pretty good response with overshoot of about 2% and settling time of 0.1 seconds. While the simulation section roll, longer settling time is about 0.8 seconds with overshoot about 2%.

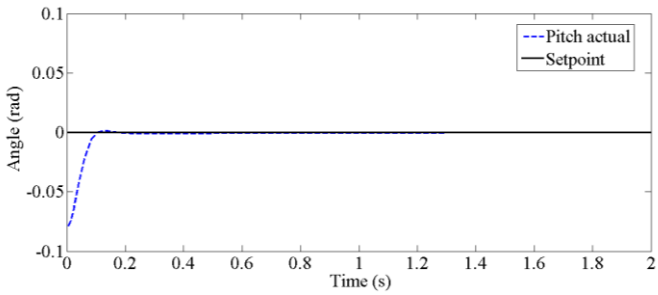

Fig. 8 Pitch balancing response on simulation

Based on the simulation results on the pitch angle with heuristic method obtained proportional value is 150, integral value is 3, while the derivative value is 4.

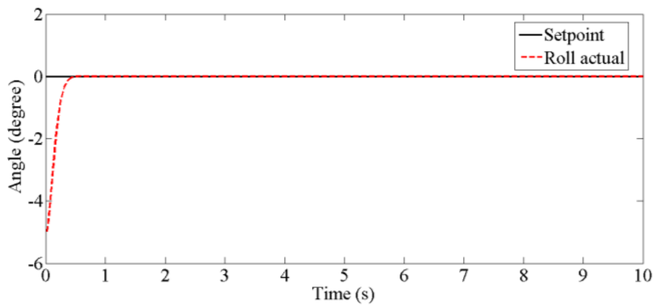

Fig. 9 Roll balancing response on simulation

While simulating the roll angle with heuristic method obtained, proportional value is 300, integral value is 3, and the derivative value is 75.

B. Speed Response on Simulation

On the simulation speed control experiment, the setpoint is set at 1 rad/s. With proportional value is 0.004, integral value is 0.018, and integral value is $4x10^{-9}$, there is overshoot about 25% with a settling time is about 1 second as shown in Fig 10. Based on experiment results, derivative control does not contribute significantly to the speed control because the response has generated nearly the same response when using PI control.

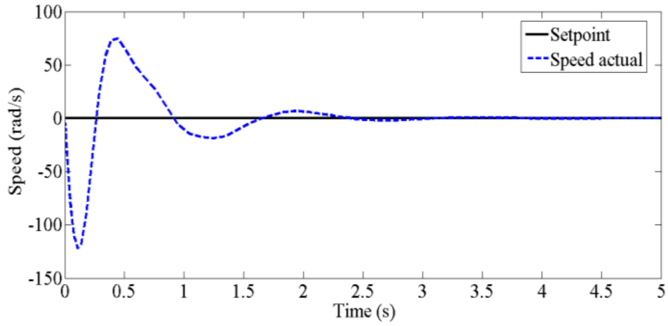

Fig. 10 Speed response on simulation

C. Balancing Response on The Robot

Experiment performed directly on the robot when both pitch and roll control are already integrated. Fig 11 shows that robot having a little oscillations when the initial balanced especially on the pitch angle. In addition to its initial angle away from the setpoint, the influence of interference between the roll and pitch oscillation also affect the amount of time the robot began balanced. There is overshoot on the pitch about 38.4% and 28.3% on the roll angle, with a settling time of 1.13 seconds on the pitch angle and 2.6 seconds on the roll angle, and steady state error is about 0.1 degrees on the both roll and pitch.

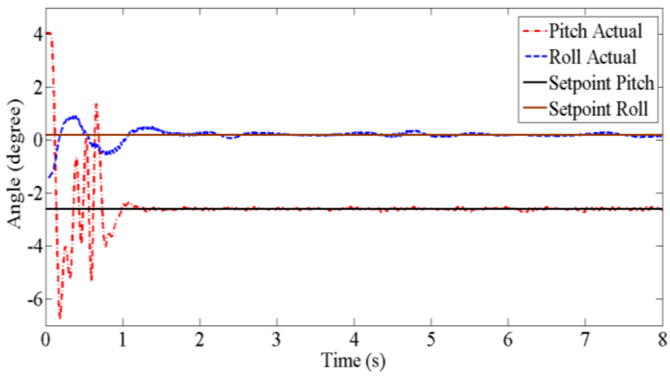

Fig. 11 Balancing response on the robot

Based on the experiment results, the robot is able to balanced with angle in the range of ± 23 degree at the pitch angle and ± 3.5 degrees on the roll angle. Recovery capability at the roll angle is strongly influenced by the amount of resources used. When the power source is enlarged using a 4 cell LiPo battery (14.8V), the angle of recovery on the roll has doubled to ± 6.2 degrees compared when using 3 cell LiPo battery. This is because the motor is able to generate greater acceleration when robot balance.

1) Pitch Balancing PID Control Tuning Method

To get the appropriate PID parameter is used heuristic method with the following procedures:

- Set all PID values at zero.
- Increase the value of Kp until the robot oscilated and capable to balance.
- Increase the value of Kd until robot oscilation could be reduced.
- If the robot was balanced but still moving, increase the value of Ki until the robot can stand still.
- When the robot has been balanced, a little disturbance is given. If there is a big oscillation then decrease value of Kp and Ki.

From the PID tuning results using heuristic methods, PID values obtained well with the value of Kp = 70, Ki = 20, and Kd = 20.

2) Roll Balancing PID Control Tuning Method

To get the appropriate PID parameter used heuristic method with the following procedures:

- Set all PID constants at zero.
- Increase the value of Kp until the robot move in oscillated condition.
- Increase the Ki value until the robot is able to balanced, with oscillations that are not too large.
- Increase Kd value to reduce oscillations in the robot.
- If the robot is still swinging, increase the value of Kd until the robot keeps standing still.
- Give a little encouragement, if the robot vibrate, decrease the Kd value until the robot does not vibrate anymore.

From the results of this tuning methods, we have obtained PID constants that respond quite well with the value of Kp = 400, Ki = 10, and Kd = 1000.

D. Balancing Response with Disturbance

When the robot is already in a balanced position given the disturbance with the oblique direction of the robot, so that the second part of the robot (pitch and roll) are both getting disturbance. Seen in Fig 12 after getting disturbance, roll angle is able to balanced within 1.2 seconds, accompanied by overshoot by 70% while the pitch angle require a faster time of about 0.8 seconds with a 42% overshoot.

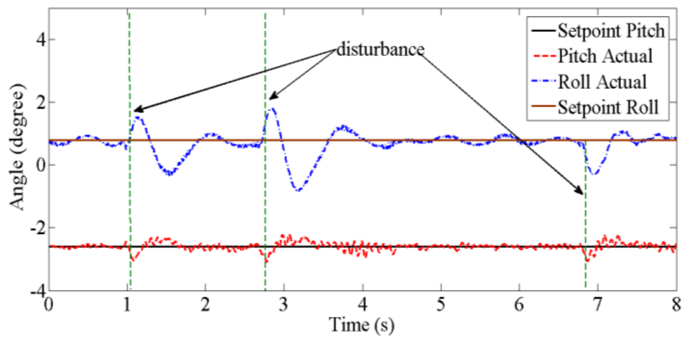

Fig. 12 Balancing response with disturbance on the robot

E. Speed Response on The Robot

This experiment shows the robot's response when the speed setpoint changes. In Fig 13 can be seen when the setpoint value is set at 0 RPM speed, there is small oscillation on the robot. This is because the offset angle on the pitch balancing control change following the change of motor rotation. When the speed setpoint value is set at 5 RPM or RPM -5, oscillation occurs in the robot looks to grow even greater. This is because when the robot walk, the robot also must maintain a balance that will cause oscillations large enough on the robot.

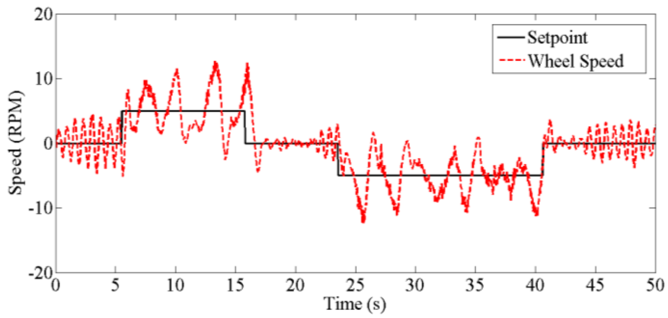

Fig. 13 Speed response on the robot

PID parameter search using the heuristic method with the following procedures:

- Set all PID constants at zero.
- Set the speed setpoint value of 2.
- Increase the value of Kp until the robot moves with a slight oscillation.
- Increase the value of Kd to oscillations in the robot can be reduced.
- Increase the Ki value until the robot is capable of running at a constant speed.
- Encourage the robot, if the robot experienced increasingly large oscillations, decrease Ki value.

From the results of tuning using heuristic methods that respond quite well PID valuess obtained with the value of Kp = 0:01, Ki = 0.001 and Kd = 0.001.

The ability of the robot to adjust its offset angle when stay balanced is tested by putting the robot on a slope with a varying angle. In this case the speed setpoint is set at a constant value of 0 RPM.

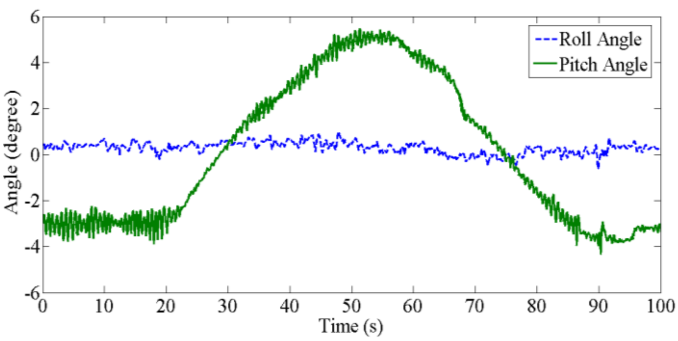

Fig. 14 Response on the varying slop angle

In Fig 14 is shown robot response when the angle of the floor changed to 30 degrees. In the second 20th, the robot's floor began to emerge slowly and robot able to follow the changes, seen from the point of oscillation robots that continuously changes with changes corner of the floor. When the floor tilt angle reaches 30 degrees, the floor is maintained in this position for 5 seconds and the robot is able to remain still in place.

V. CONCLUSIONS

Based on test results obtained pitch balance control section pretty good response with a settling time of 0.1 seconds at 1.13 seconds of simulation and testing of the robot. As for the roll has a settling time of 0.8 seconds and 2.6 seconds in simulated testing on the robot. The robot has a recovery capability of ± 23 degrees in the pitch angle and ± 3.5 degrees on the roll angle with steady state error of 0.1 degrees on both pitch and roll.

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