Dynamic parameter estimation with EKF - Summary

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About

This document goes over the "major" findings on a term-long investigation on dynamic parameter estimation with the extended Kalman filter (EKF). In truth, the investigation didn't result in any ground-breaking discoveries and the original research question (introduced later) was not satisfactorily answered. Nonetheless, the more interesting discoveries are presented here.

If you have any questions about the summary, please reach out to the author at dzhang323@hotmail.com. I will be happy to clear up things to the best of my ability.

The codebase can be found on the aslab svn directory under:
projects/d87zhang/dynamic-param-estimation-ekf
Or in github (with a more detailed history) on:
https://github.com/d87zhang/kalman-filter-experiment

An editable google doc version of this report can be found at:
https://docs.google.com/document/d/17SDdXz34g-YytEMWhFZYZHy3b5T6V4OK46UQxBSfhlE/edit?usp=sharing

Introduction

Dynamic parameters or inertial parameters are important properties for robots when applying model-based control. A rigid-body robot's dynamic parameters can be completely characterized by specifying 10n (n being the number of links) link parameters (for each link, there is 1 parameter for mass, 3 for center of mass and 6 for moment of inertia). However, for any specific robot, not all of the set of 10n link parameters may be observable, as some parameters might only appear together as a product or as a linear combination in the equations of motion.

Yet, determining which link parameters are observable is in general difficult (there are in fact many previously proposed methods, including a recent, promising one here [1]; section A in the introduction of [1] also points to other existing work). Thus, a simple, general approach to dynamic parameter estimation is to estimate all 10n link parameters. Instead of determining each link parameter's observability, we would like to directly determine if a parameter is estimated accurately or not, or similarly if the parameter is unobservable due to not showing up independently from other parameters. Additionally, we would like to estimate and determine accuracy in real-time using the EKF (hint: the last two sentences describe the research question).

This report goes over the more important findings from estimating link parameters with the EKF. I could not find a satisfactory method for determining the accuracy of link parameter estimates or determining the relationships between parameters in the robot's dynamics. Nonetheless, the major conclusions are 1) in general link parameters are not estimated accurately but torque prediction is reasonably good 2) drops in estimated variance often happen at the same time as updates in estimates and 3) correlation estimates are inconsistent across different trajectories, noise, EKF filter tuning and initial guesses.

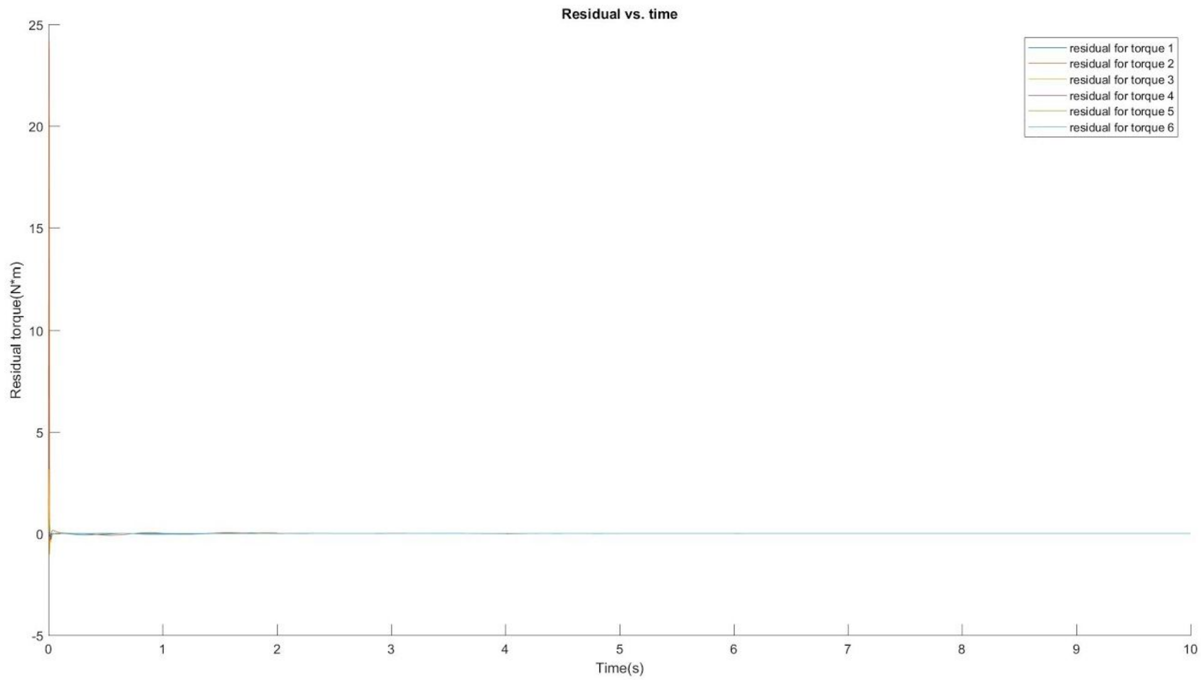
Basic experimental setup

All experiments estimate a robot's full set of 10n link parameters given potentially noisy joint torque measurements (from all of the joints) and not-noisy kinematic measurements (joint position, velocity and acceleration). A trajectory is planned for the robot and the torques to execute this trajectory are then used to generate the torque measurements. Initial guesses are randomized to be between 20% ~ 80% off (so on average 50% off) in either directions (magnitude too large or too small). When noise is injected in joint torque measurements, the noise on each torque has a normal distribution with standard deviation equal to about 1% ~ 5% of the maximum torque observed on that joint. Additionally, unless otherwise specified, Q is set to some small, non-zero value, which prevents variance from staying at a near-zero value.

If units are not given, relative error is expressed such that a 1 represents 100% error.

1. Overall estimation performance

In general, the EKF cannot accurately estimate the individual link parameters even for a relatively simple robot, likely due to unobservability in the link parameters. However, the EKF is at least able to consistently achieve a small torque residual. In the noiseless case, EKF's estimates normally converge/stabilize and continue to produce very small torque residual for the rest of the simulated trajectory. In the noisy case, EKF's estimates often do not converge.



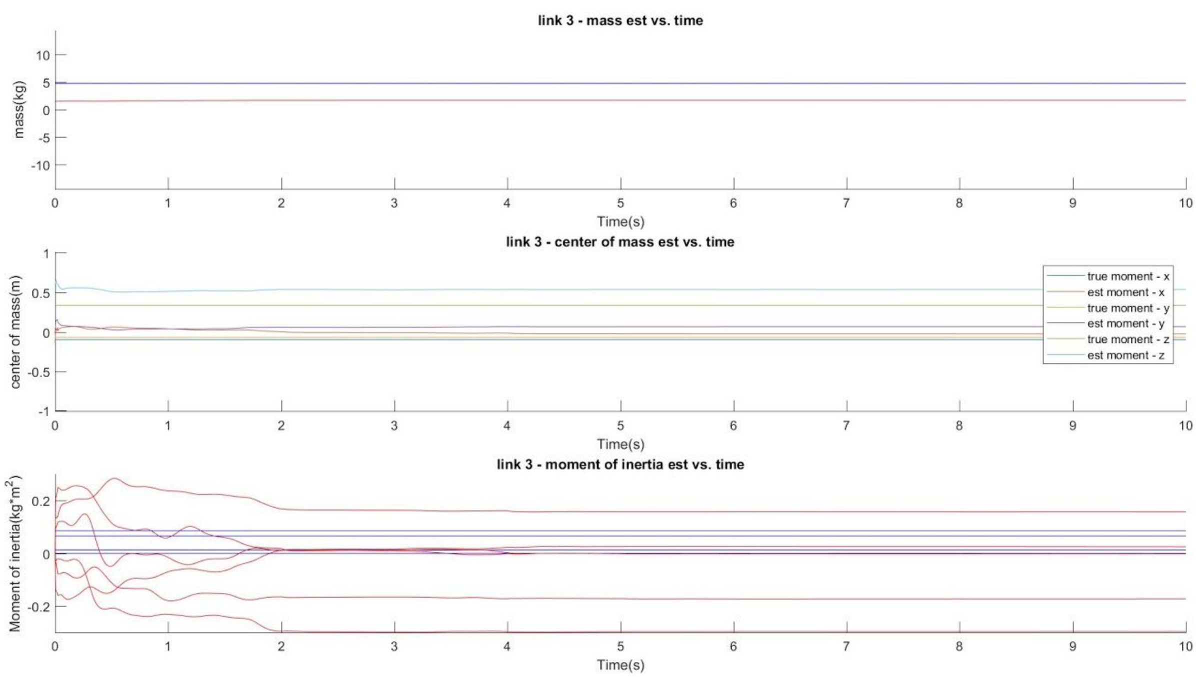
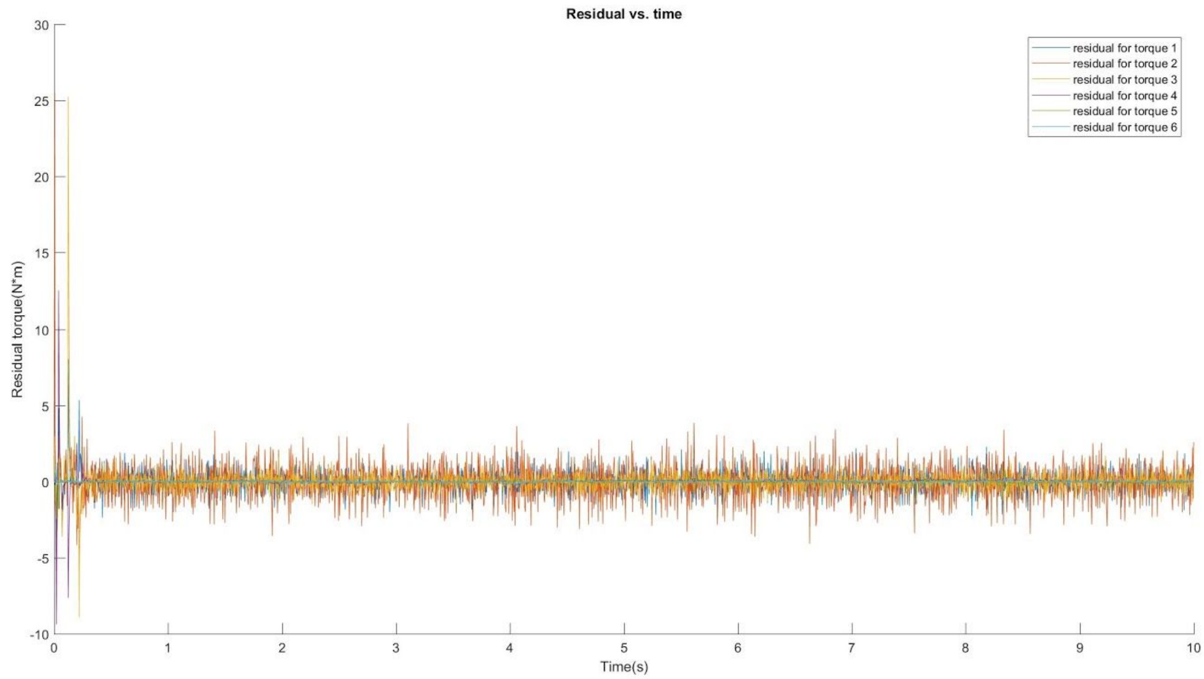


Figure 1.1: In the noiseless case, when estimating the 10n (i.e. 60) link parameters for the 6-DoF PUMA robot, the torque residual quickly converges to zero, as shown in the top figure. The parameter estimates themselves are not all accurate, but they do converge after a while. The bottom figure shows how estimates for link 3 fluctuate and converge. Fun fact, the average relative error in the final estimates is 245%!



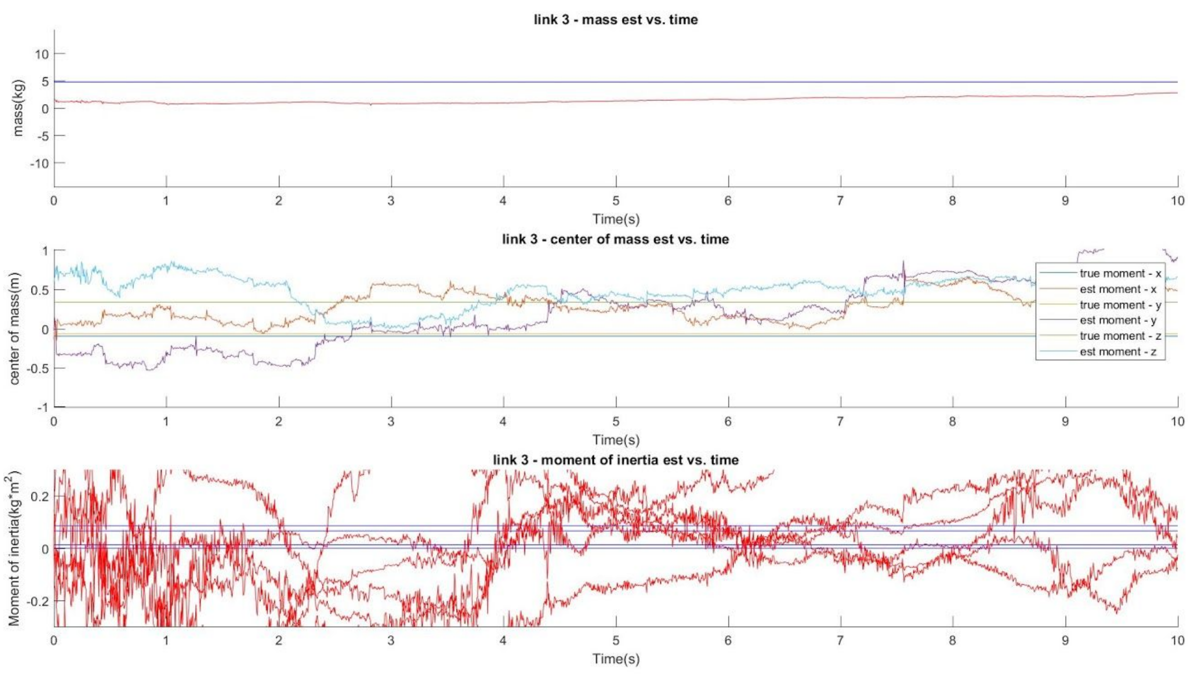
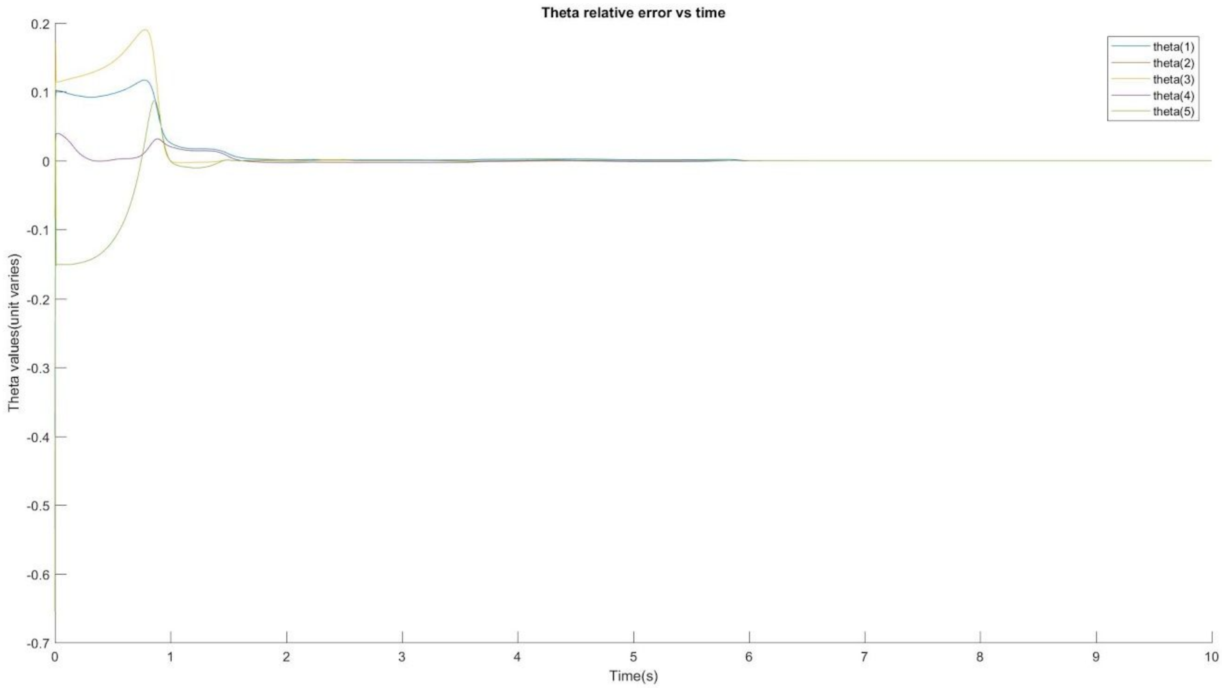


Figure 1.2: Estimating the same 60 parameters for the PUMA with noise also results in near-zero residual as illustrated in the top figure (measurement noise standard deviations were 0.03 ~ 0.5, so the residual is mostly just measurement noise). The estimates themselves however do not converge (again illustrated with link 3 estimates in the bottom figure).

In the particular case of the simple 2-DoF revolute planar arm, we have the derivation of a minimal set of observable parameter combinations from the Spong textbook [2] (see p.271; note that the minimal set of observable parameter combinations is not necessarily unique). For this robot, the estimated link parameters result in accurate values for the minimal parameter combinations set in the noiseless case meaning that the estimated parameters are guaranteed to accurately predict torque for any arbitrary trajectory (and not just the simulated one). In the noisy case, the minimal parameter set has fluctuating error, but error fluctuates around zero.



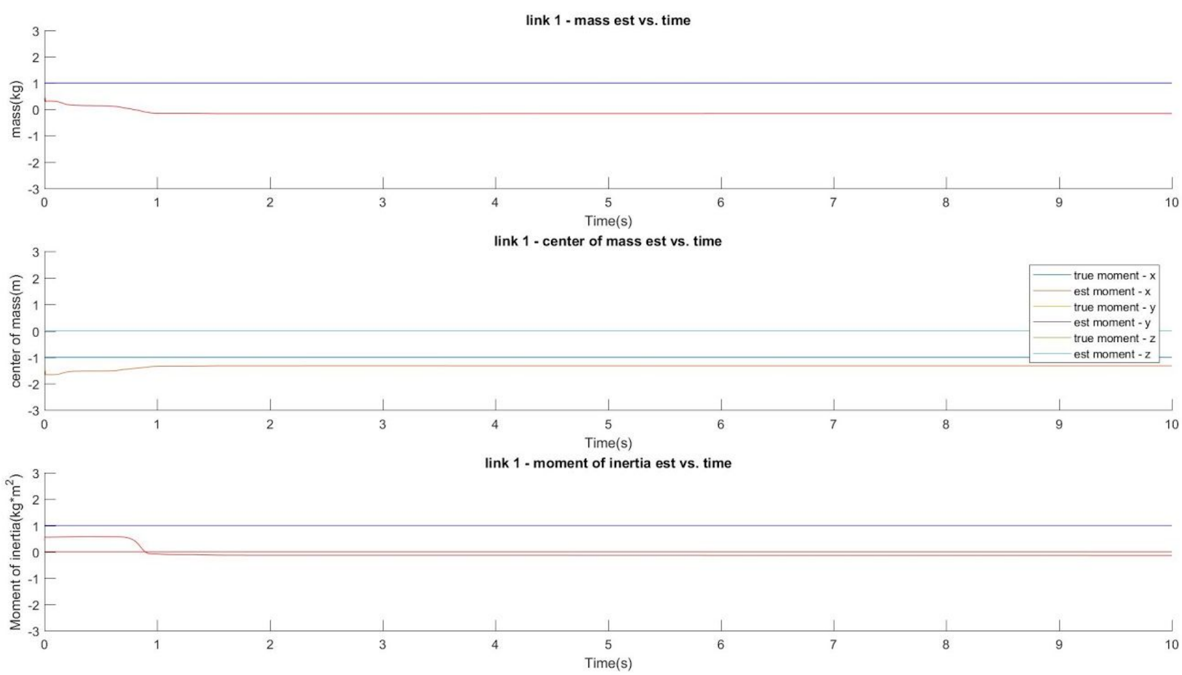


Figure 1.3: In the noiseless case for the 2-DoF arm, the relative error in the minimal parameter set converges to zero (about 1e-7 ~ 1e-8), as shown in the top figure. The parameter estimates themselves also converge quickly as shown in the bottom figure. Again, the individual estimates are not accurate, with relative error in the final estimates averaging 68%.

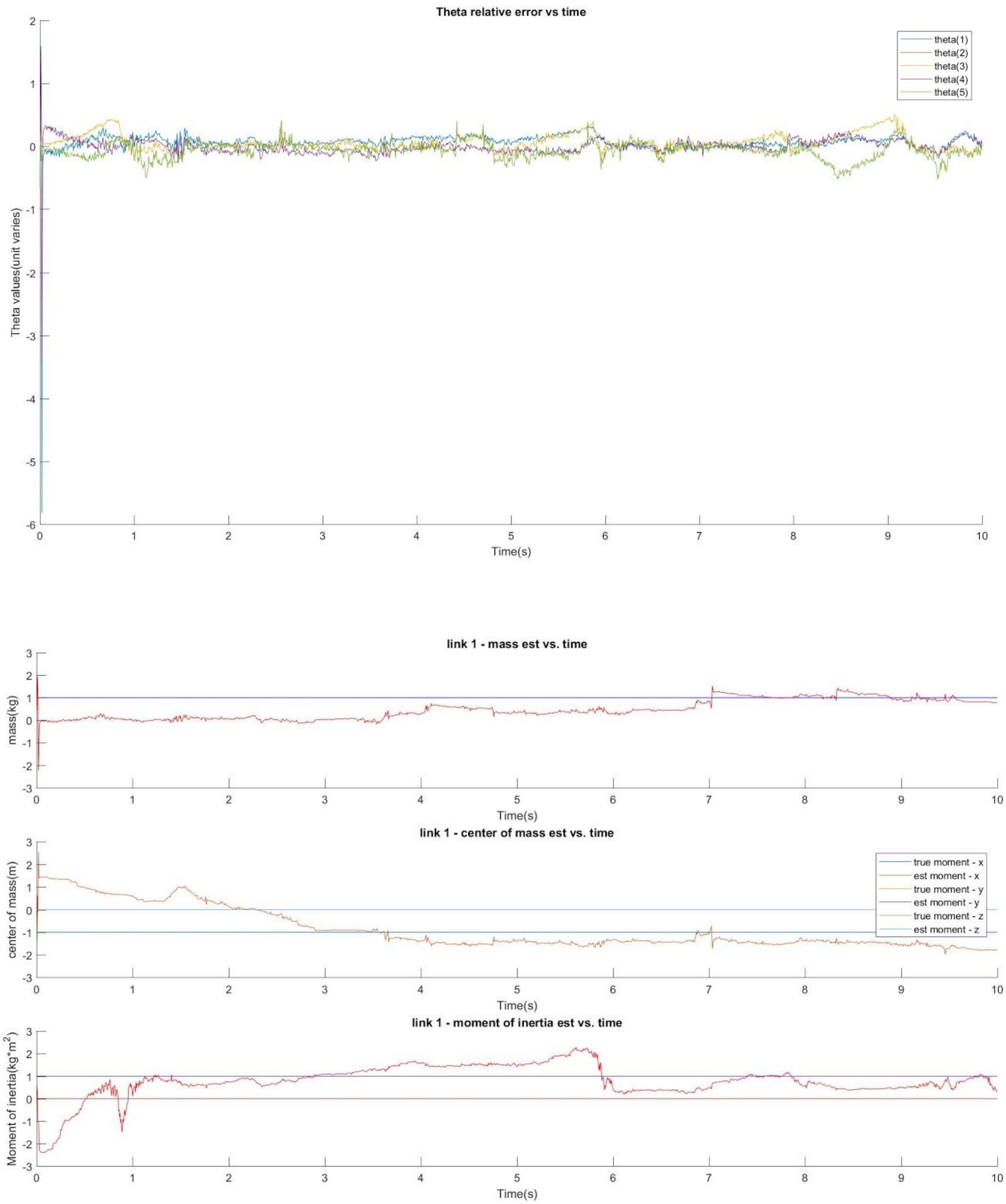


Figure 1.4: With noisy measurements, estimates again fluctuate (although less violently compared to the more complicated PUMA case) as in the bottom figure. Error in the minimal parameter set fluctuate significantly around zero as in the above figure.

In summary, EKF estimates generally produce accurate torque predictions as evidenced in the near-zero torques. In the noiseless case, the estimates converge (despite non-zero Q and variance) and continue to produce accurate torque predictions for the randomly generated, nonperiodic quintic trajectories. This suggests that the converged estimates may produce good torque predictions for any trajectory. Looking at the minimal parameter set error for the 2-DoF planar arm confirms such a possibility. On the other hand, estimates tend to not converge when measurements are noisy, although we see that torque predictions still tend to be accurate when the EKF is allowed to update its estimates. The minimal parameter set error for the 2-DoF arm fluctuating around zero also suggests that EKF is not converging on ad-hoc estimates that only predict torque accurately for certain trajectories. Overall, I would say that EKF gives reasonable estimates for the link parameters if they are just used for torque prediction/control purposes (certainly the link parameters themselves are estimated inaccurately though). Noise causes estimates to not converge, but if EKF is continuously allowed to update the parameter estimates, torque prediction should do reasonably well still.

2. Relationship between variance dips and updates in estimation

By intuition, when the EKF gains new information about the robot, two things happen simultaneously: 1) the parameter estimates are updated (or they stay still if the predicted torque is already accurate) and 2) the estimated variances for the parameters decrease. Experimental results qualitatively show that drops or dips in estimated variance and updates in estimation are indeed correlated as they often (but not always) occur together. However, it should be noted that an update in the parameter estimate doesn't necessarily mean the estimate is becoming more accurate, as dips in variance are also observed to be correlated with estimates updating in the wrong direction, especially when torque measurements are noisy.

Some examples below. Note that Q was set to a small non-zero value so variance increases when "no information is gained".

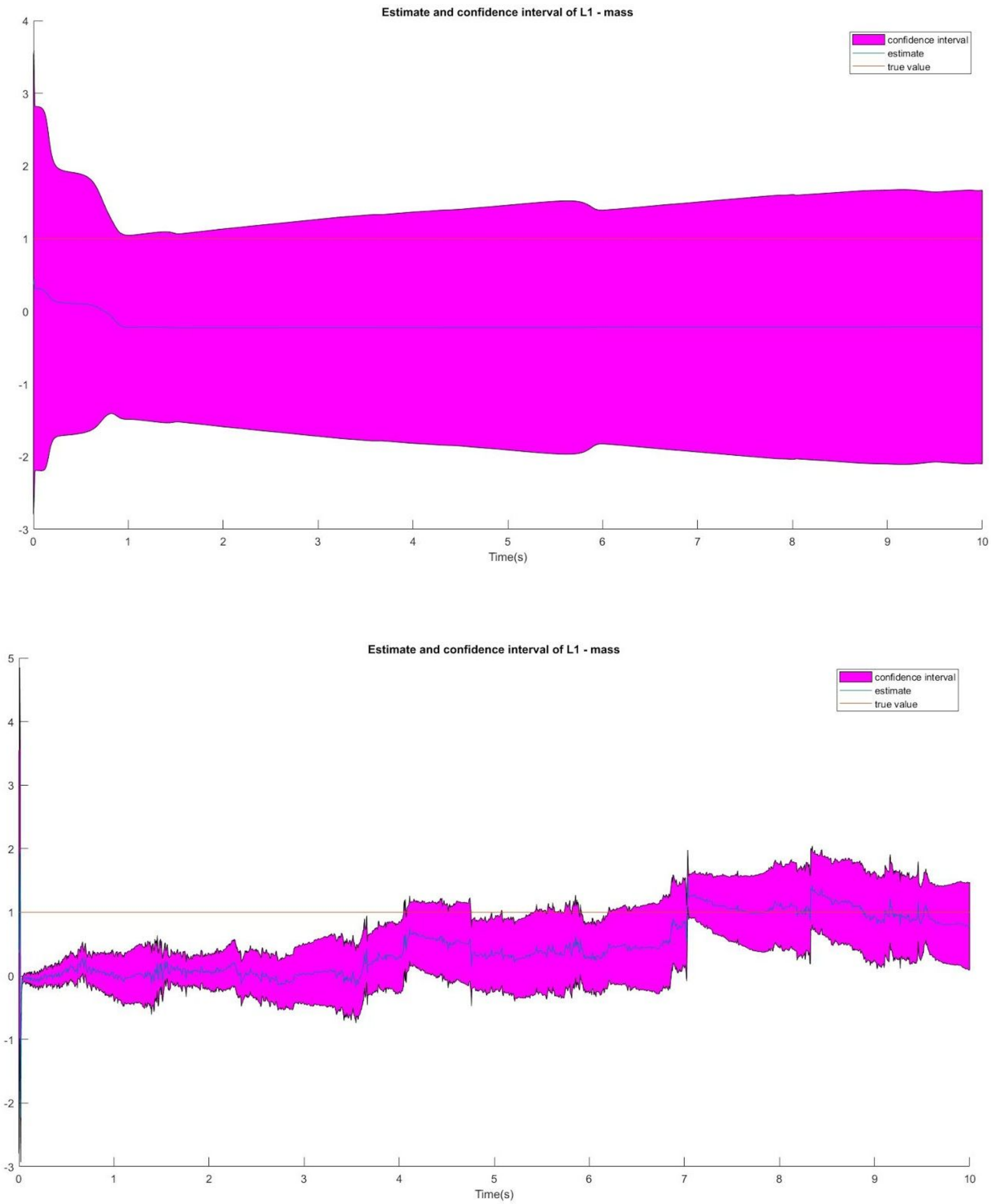


Figure 2.1: Plots of the estimates and 68% confidence intervals ($\pm$ one standard deviation) of link 1 mass in a simple 2-DoF planar arm with noiseless (top) and noisy measurements (bottom).

There is a general trend where dips in variance appear in sync with updates in estimates, especially in the noiseless case. Notice that estimates can update in the wrong direction (away from the true value). In both cases, the minimal parameter set values are estimated relatively accurately (with relative error of ~1e-7 in the noiseless case and ~0.04 in the noisy case).

A very hand-wavy mathematical explanation of this phenomenon is that a dip in variance is usually caused by a large kalman gain K_k (by looking at the variance estimate's update equation), which would contribute to a larger update to the parameter estimate.

An interesting note is that the presence of injected noise reduces the variance estimates, as can be seen by comparing the two graphs in Figure 2.1 (although they do have slightly different scales). I have seen this consistently for a while but I cannot fathom why it is the case.. It is surprising since injecting noise doesn't affect EKF's filter tuning. It only affects the residuals, which don't directly contribute to covariance updates.

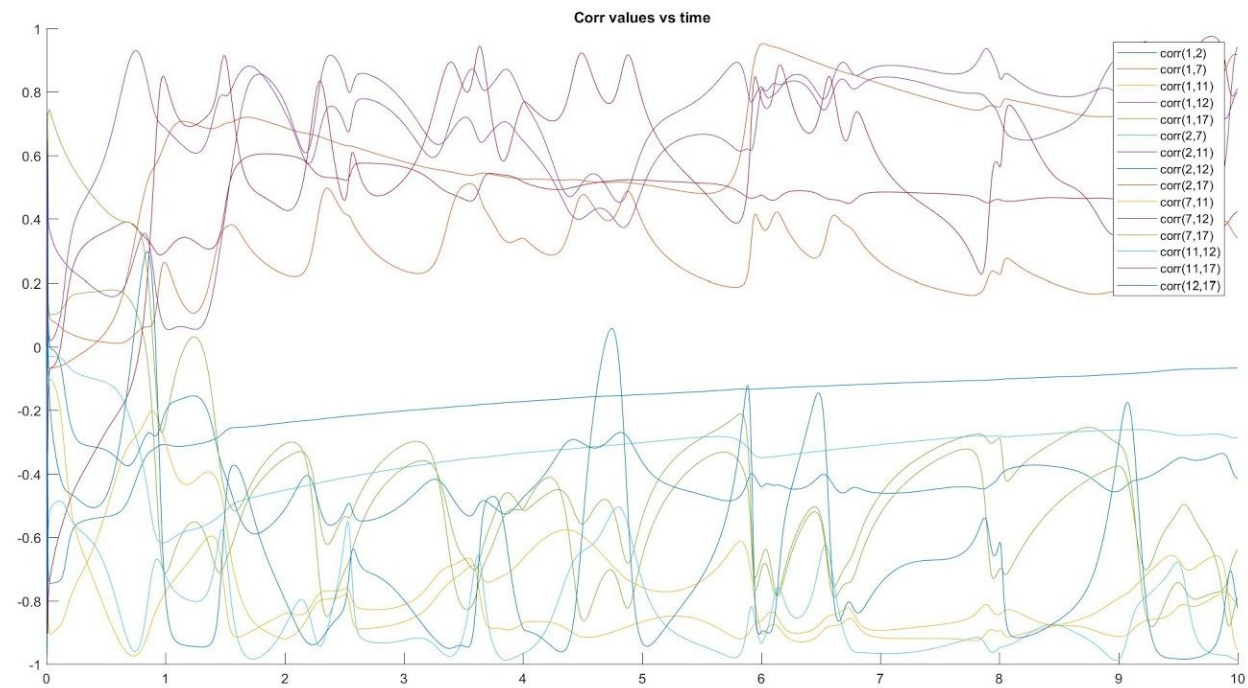
3. EKF's correlation estimates

A benefit of using the EKF is that it estimates the covariance between different parameters' estimates. Intuitively, if two parameters are only observable together as a combination, for example $(x_1 + x_2)$, then the covariance value between them is expected to be high (in magnitude) and relatively consistent over the course of a trajectory. Therefore we looked at EKF's covariance estimates to see if they can be a good indicator of dependency between parameters. Since covariance is affected by the unit of the parameters measured, the normalized Pearson correlation coefficients were observed instead. Unfortunately, experimental results show that correlation estimates do not look like promising indicators as they are very dependent on the trajectory used, on measurement noise. They are also dependent on EKF filter tuning and initial guesses, albeit to a lesser extent.

To see the unreliability of correlation estimates between different trajectories (which I think is the most insufferable of unreliabilities), it suffices to skip to section 3.3. Otherwise, we first look at correlation estimates under various cases given non-zero Q in section 3.1, then quickly see how they look given zero Q in section 3.2 and finally in section 3.3, we look at how correlation estimates vary between trajectories. Note that all examples shown in this section estimates the 2-DoF planar arm with 3n=6 relevant link parameters (so assumptions like center of mass being on the link were made).

3.1 With non-zero Q

In general, non-zero Q keeps each parameter estimate's variance from staying at zero which empirically has the effect of preventing covariance values from converging. Recall that we use random, non-periodic quintic trajectories.



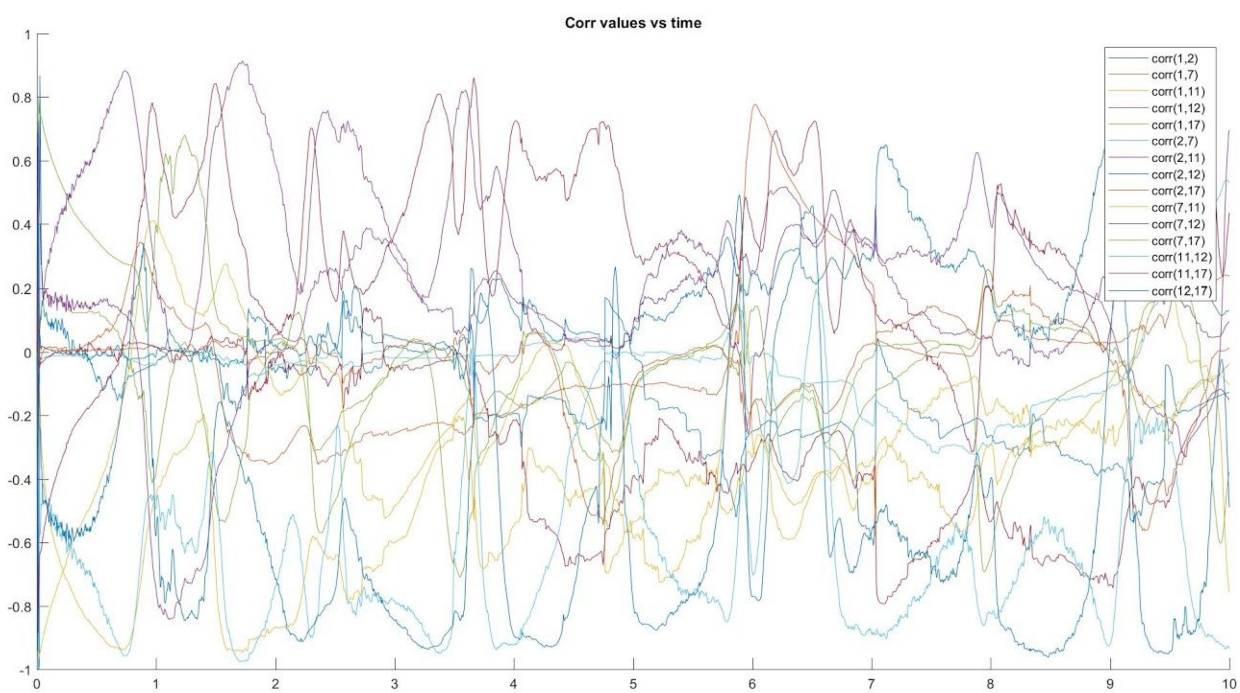


Figure 3.1: The above shows correlation estimates for a simple 2-DoF planar arm where only 3n = 6 link parameters are relevant to dynamics. The top image shows the noiseless case and the bottom image shows the noisy case.

It can be observed that between most parameter pairs, the correlation estimate varies wildly over a trajectory and all correlation estimates' value vary to some degree over time. This means looking at the correlation value is likely not a good indicator of trajectory-independent information such as dependency between parameters. However, consistency/stability is observed for some parameter pairs (like between mass 1 and center of mass length 1 and between mass 2 and moment of inertia 1 in Figure 3.1 top), at least in the noiseless case, which may be an indicator of inter-parameter dependency. But sadly the pairs of parameters showing relatively stable correlations are not very consistent across trajectories (see Figure 3.2 and later section 3.3). This implies that with non-zero Q, we can't reliably use either the value or consistency (i.e. how consistent the graphs are over time) of correlation estimates to gain information.

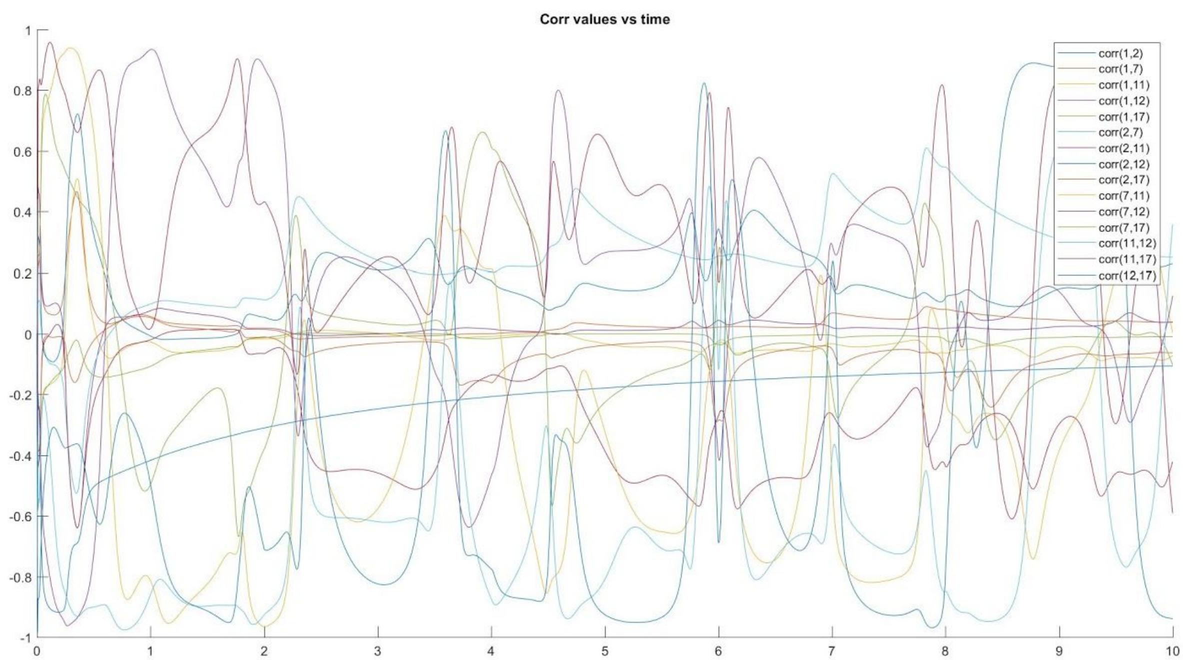


Figure 3.2: Correlation estimates under the same conditions as in Figure 3.1 (noiseless case) but with a different trajectory. Notice that the number of parameter pairs producing stable correlation estimates is much larger compared to Figure 3.1, meaning that the pairs of parameters producing stable looking correlation estimates are not entirely consistent.

Additionally, the introduction of noise has a significant effect on the correlation estimates as well. In fact, using a different random seed for generating noise results in visibly different correlation estimates as shown in Figure 3.3. However, since the noise (albeit small, being about 2% of the maximum observed torque in this case) currently has a very significant impact on estimation results in general (perhaps due to non-optimal filter tuning) this observation regarding noise should be taken with a grain of salt.

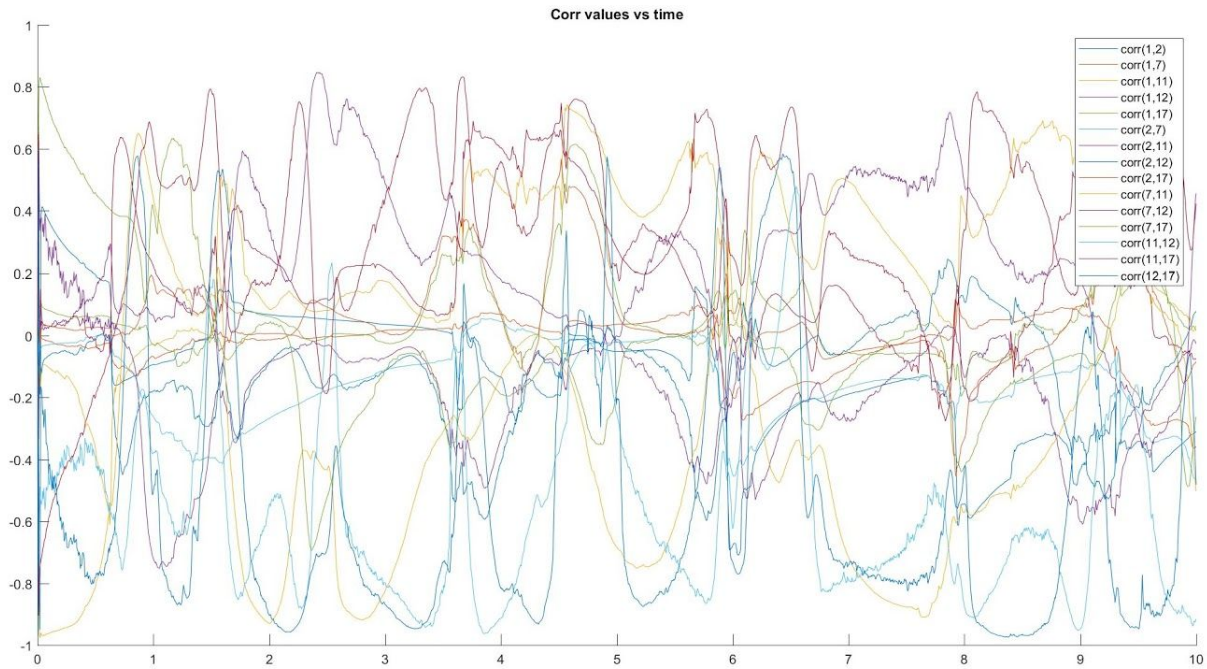


Figure 3.3: Correlation estimates under the same conditions as in Figure 3.1 (noisy case) but with a different random seed for the measurement noise.

Finally, it should be noted that filter parameter tuning and initial guesses affect correlation estimates as well. The initial guesses' part is illustrated in Figure 3.4 below. The filter parameter tuning part is not demonstrated here (I would need to redo those experiments… and this summary is getting quite long already!).

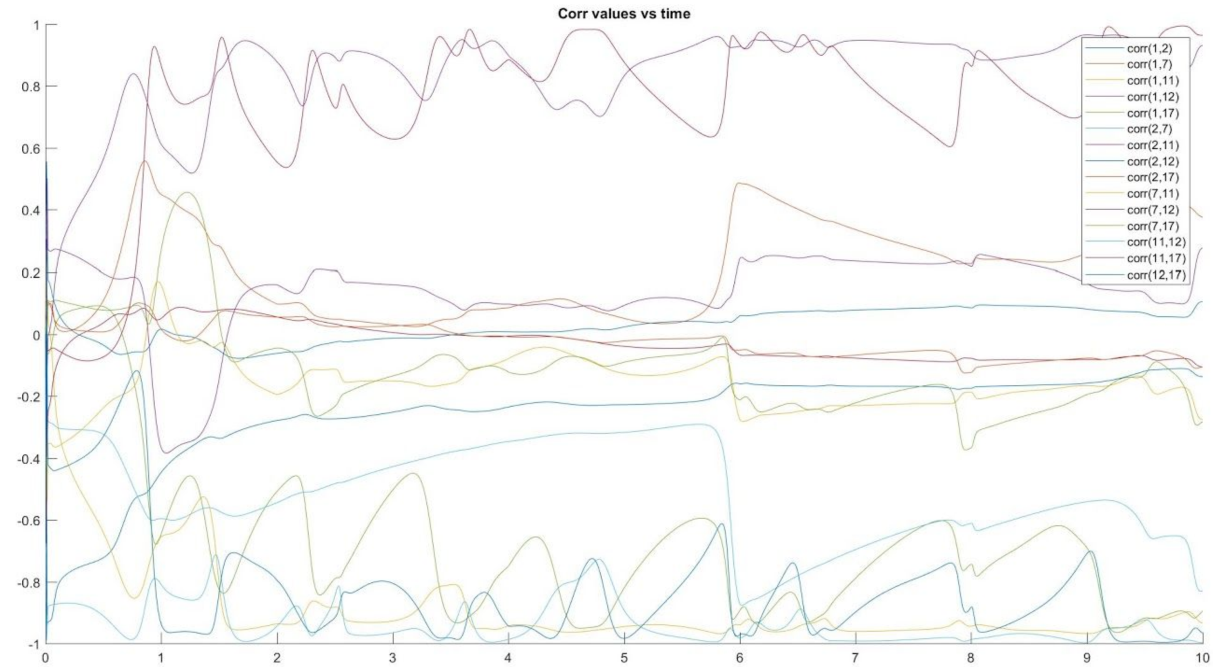
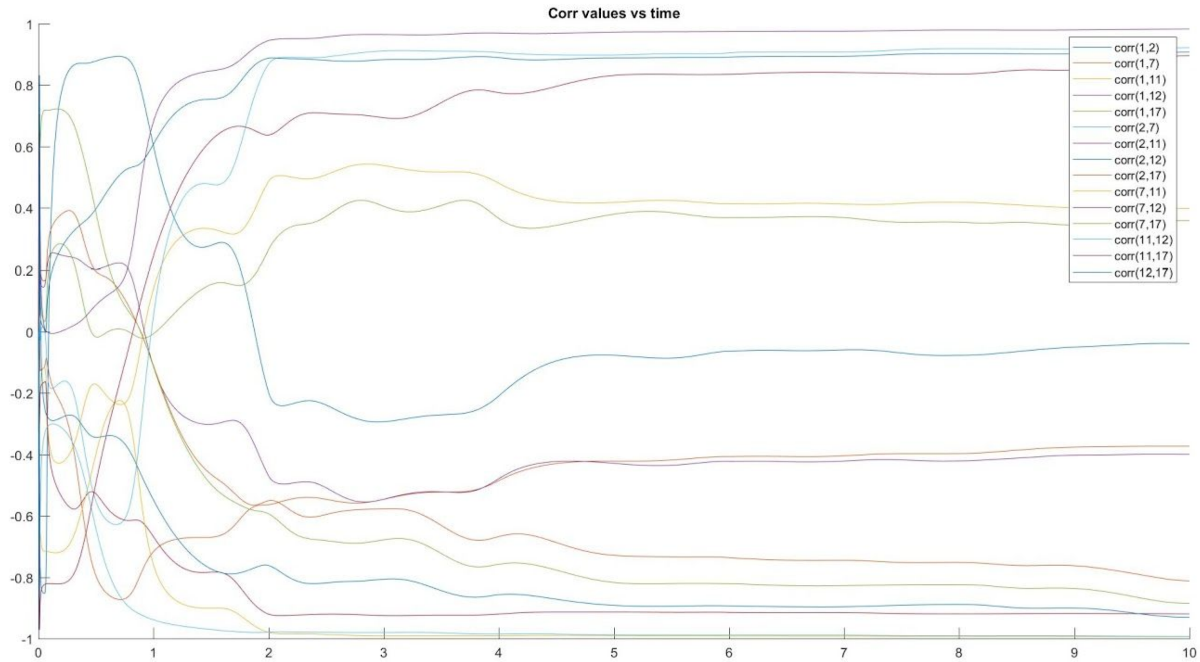


Figure 3.4: Correlation estimates under the same conditions as in Figure 3.1 (noiseless case) but with different initial guesses.

3.2 With zero Q

With zero Q, the correlation estimates converge (or at least stabilize) even with our non-periodic trajectory and noise.



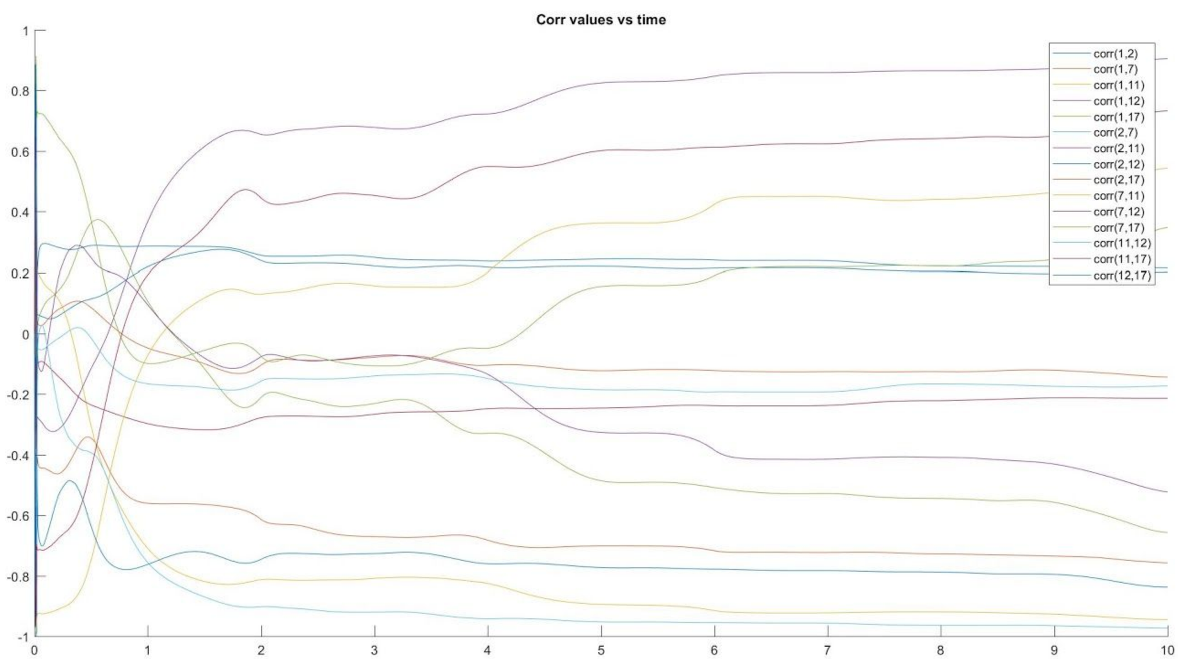


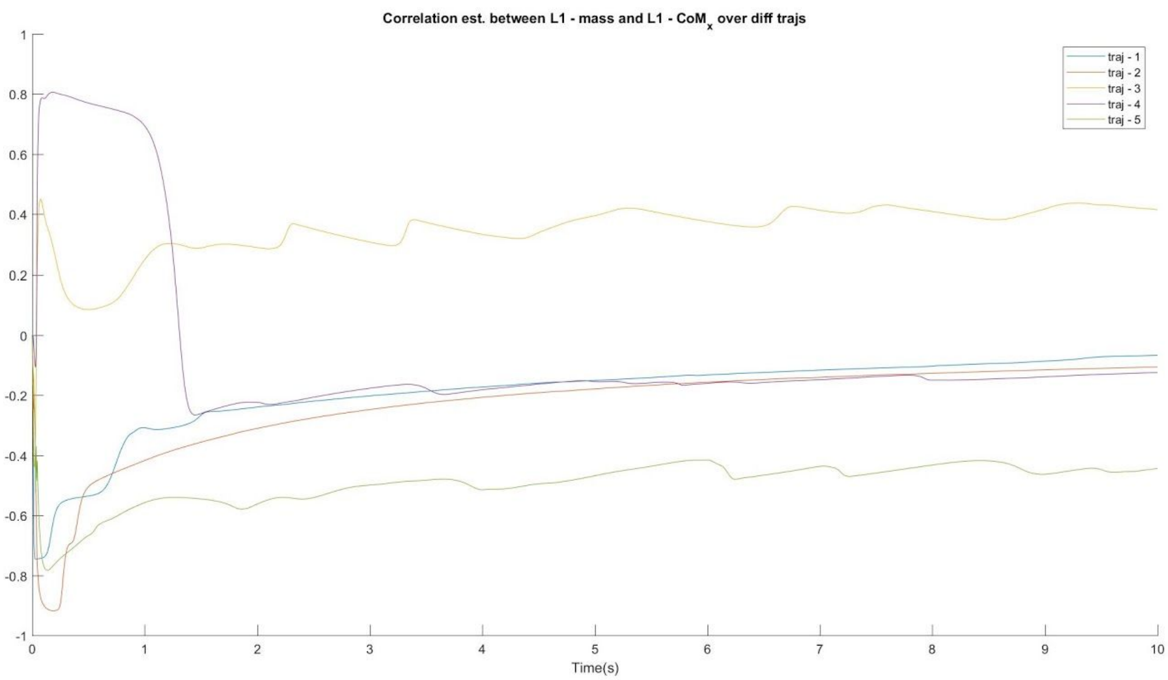
Figure 3.5: Correlation estimates under the same conditions as in Figure 3.1. The noiseless case is shown top and the noisy case shown bottom.

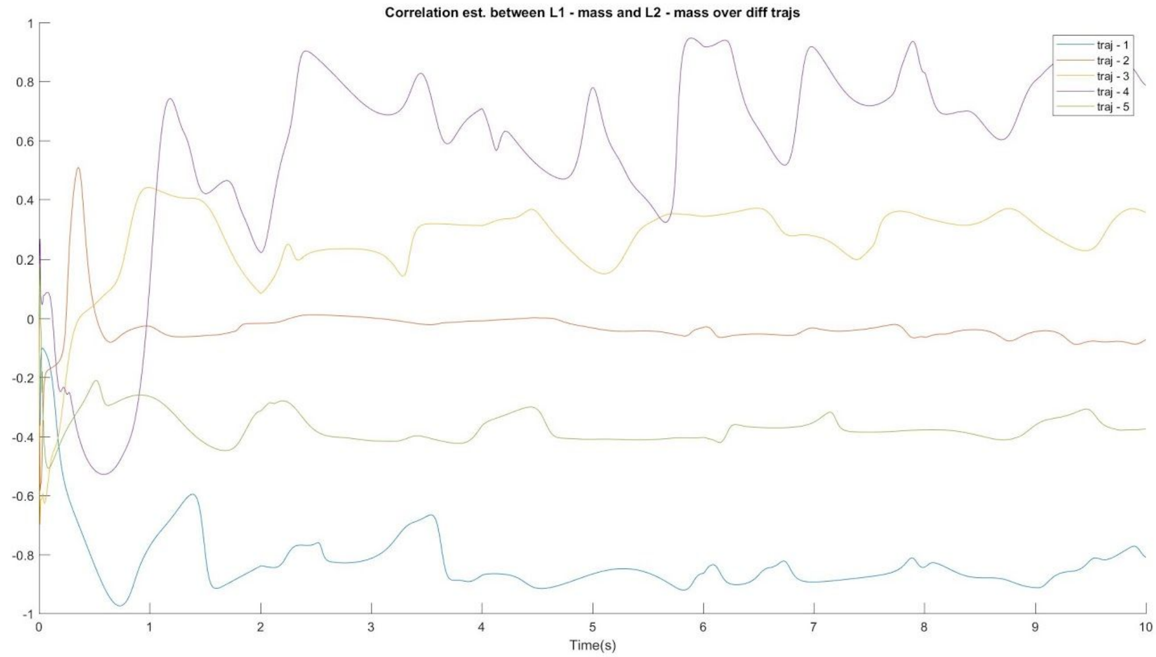
Because every parameter pair produces correlations of similar consistency (stability over time), it is harder to distinguish between parameter pairs with this metric. Regardless, we will see in the next section that correlation estimates are still very unreliable between different trajectories.

3.3 Comparing correlation estimates between different trajectories

We will see plots of correlation estimates for the same parameter pair in 5 different trajectories to get an idea of the estimates' reliability. Trajectories are randomly generated using different seeds (so they aren't related to each other) and the same 5 trajectories are used for each graph.

For brevity, we only show plots for some selected parameter pairs (with representative graphs), starting with non-zero Q:





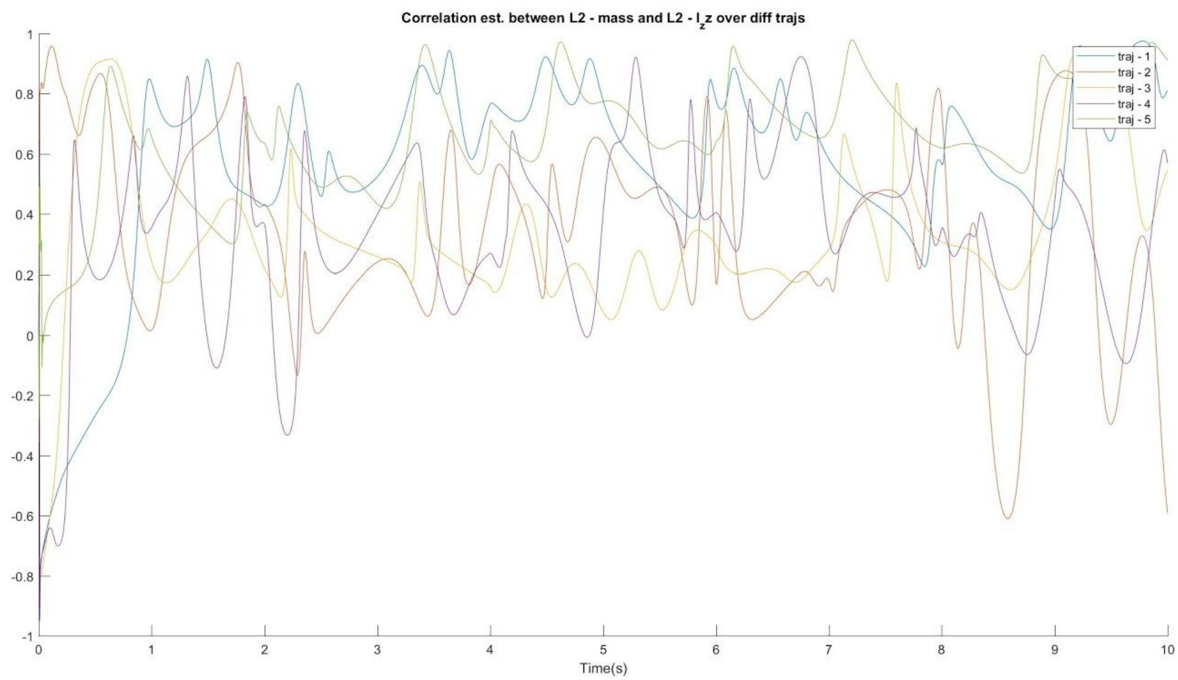
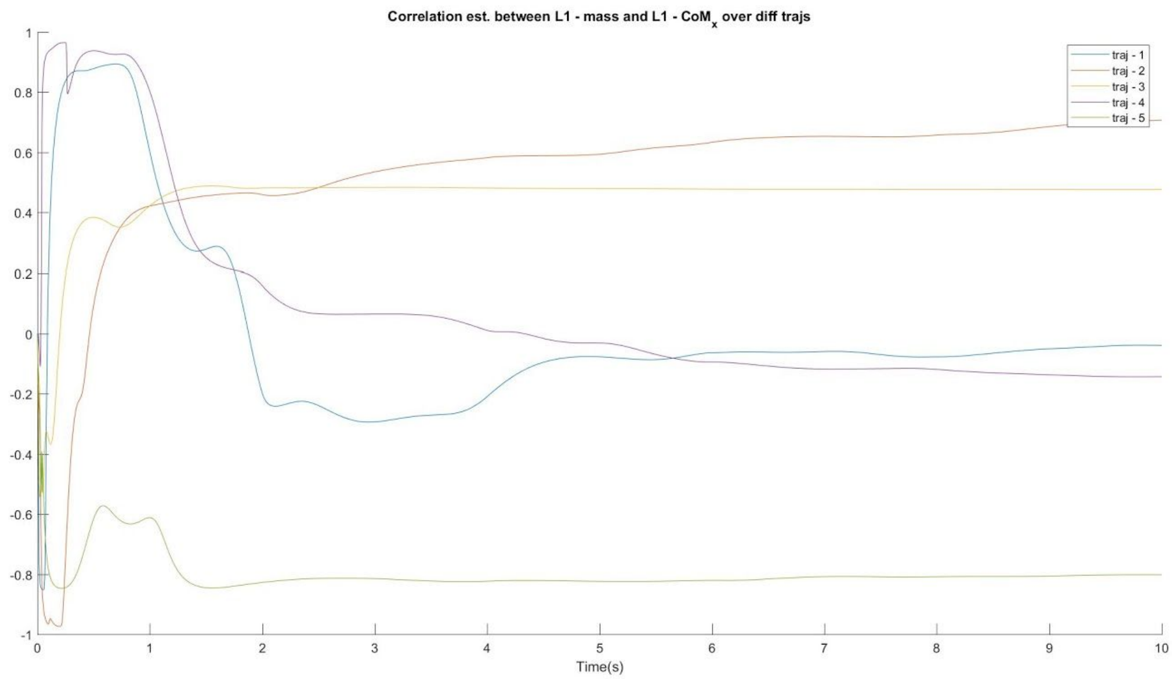


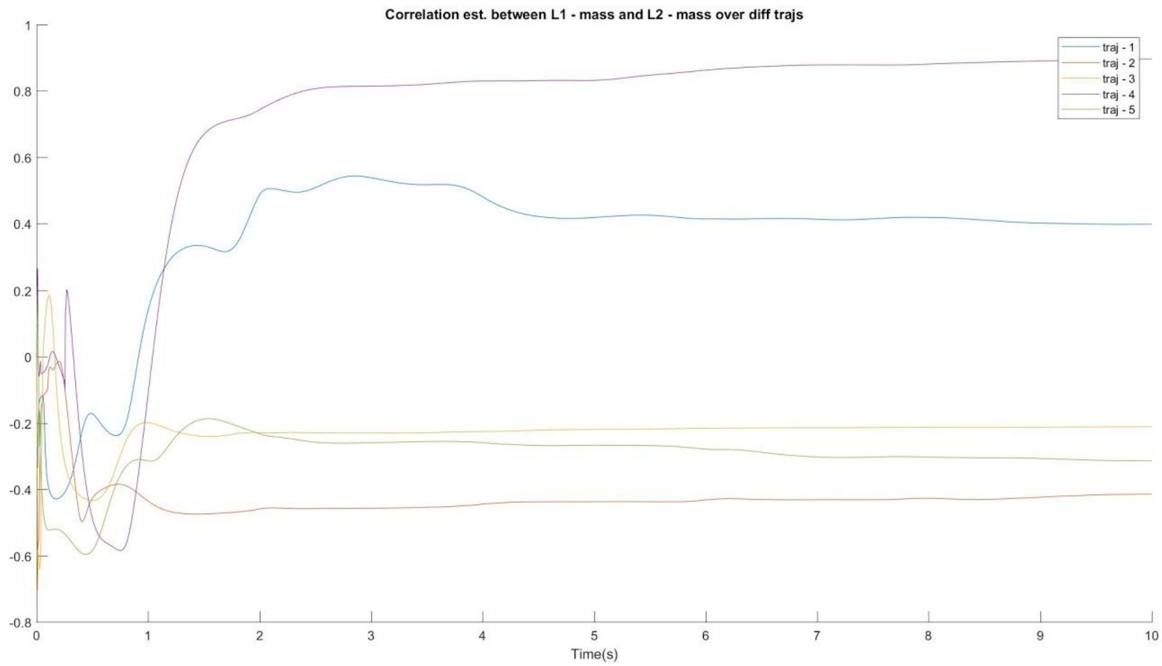
Figure 3.6: Correlation estimates given non-zero Q for the parameter pairs (from top to bottom):
1) m1 and center of mass location1 (with meter as its unit)
2) m1 and m2
3) m2 and moment of inertia2

The first thing to notice is that both the value and consistency (look at the shapes of the lines) of the correlation estimates vary significantly across trajectories. This renders these two measures inappropriate for determining trajectory-independent information.

However, it does seem like the amount of fluctuations (for instance the bottom figure in Figure 3.6 has the most fluctuations) is related to the parameter pair. For the 2-DoF planar arm with 3n=6 parameters (the one tested here), the correlation amongst link 2 parameters and between link 1 moment of inertia and any link 2 parameter seems to have relatively high amounts of fluctuations. Although, this is not a rigorous observation and I am not sure what to make of it… I thought maybe a high amount of fluctuations indicates that the parameters in the pair do not appear in the same element in minimal parameter set. However, that doesn't match the observations since all the parameters in fact appear in one element in the minimal parameter set. If this is indeed an interesting path to explore, we would still need more experiments to draw some kind of definitive conclusion.

The same parameter pairs' correlation estimates are shown below in the zero Q case:





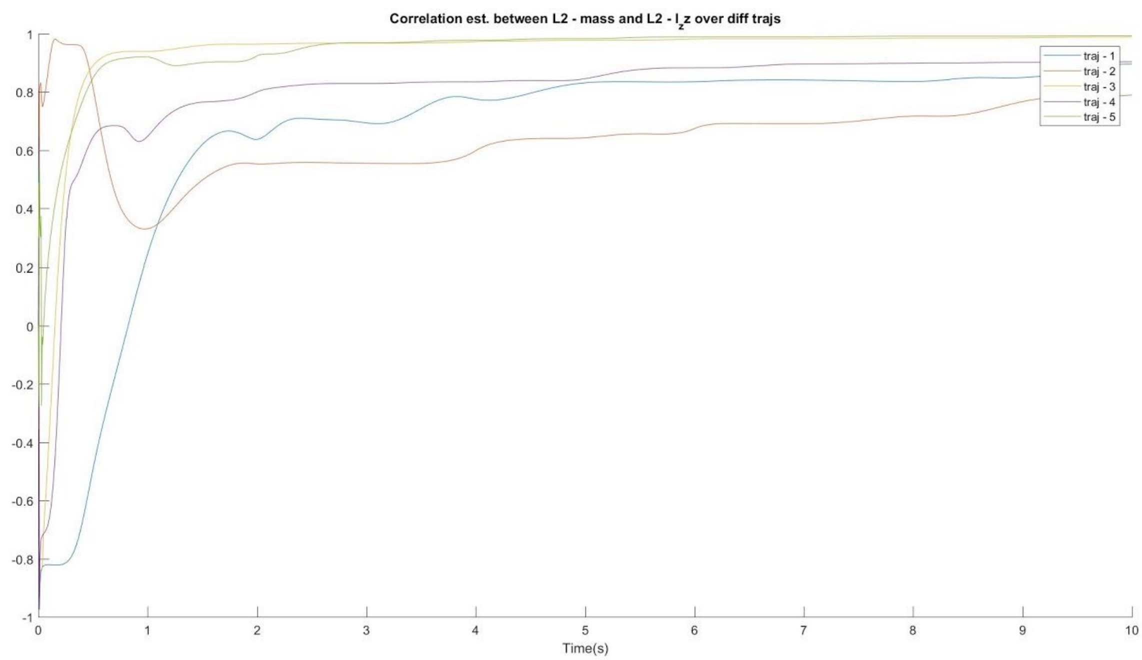


Figure 3.7: Correlation estimates given zero Q for the same parameter pairs as before (from top to bottom):
1) m1 and center of mass location1 (with meter as its unit)
2) m1 and m2
3) m2 and moment of inertia2

As mentioned before, zero Q causes correlation estimates to become consistent/converging-like in general. However, there is still significant discrepancy between the trajectories. The parameter pairs that are said to show a high amount of fluctuations tend to have closer “converged” values, although this may simply be because even though there were lots of fluctuations, the correlation estimates tended to stick closer together for these parameter pairs… for some mysterious reason.

Finally, note that noise has a very significant impact on these graphs similar to how they did in section 3.1. With zero Q, the correlation estimates with noise still look consistent but “converge” to different values (not shown here). With non-zero Q, they can be a lot messier, as exemplified below (with the same parameter pair as Figure 3.6’s top figure).

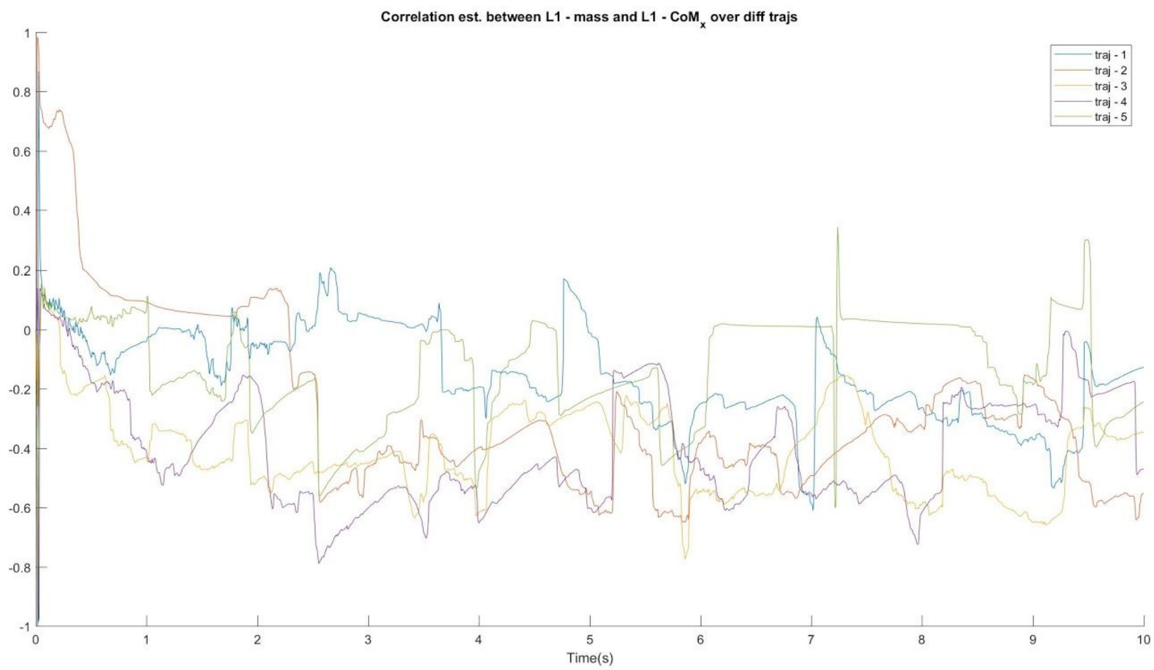


Figure 3.8: Correlation estimates given non-zero Q and noise for
- m1 and center of mass location1 (with meter as its unit)

Again, noise having a large impact might be due to non-optimal filter tuning but this unreliability is nonetheless discouraging.

Note: <project-directory>/tests/testCorrBtwnTraj.m is handy for generating the graphs in section 3.3 (this section).

All in all, EKF’s correlation estimates do not look promising as a reliable indicator of inter-parameter dependency. Even with the same robot, which yields the same dependency relationships between its parameters, a different trajectory, noise, parameter tuning or initial guesses can significantly affect the correlation estimates both in terms of their values and consistency.

4. Additional notes

Some additional notes are added here without supporting pictures, in bullet form..

- EKF's estimation results are dependent on many factors, which is a major challenge since it is hard to know if experiment results are legitimate/representative of real-life or if the filter tuning is just off or something (and it causes a lot of confusion when I forget how parameters were set for some past undocumented experiments..). Amongst more obvious factors, the estimation results are also dependent on:

    - the relative magnitudes between different parameters. For instance if one parameter is 10x larger than the other parameters versus all parameters being about the same. Messing with different true values (by randomizing them to obtain a maximum of 100x difference) has given me numerical instability issues before (i.e. Matlab complaining about close to singular matrices)...

    - the value of Q, when measurements are noisy. This is unsurprising but I want to note that Q tuning is especially impactful in the noisy case, where a high Q can lead to non-converging estimates for instance.

- Lower estimated variance doesn't necessarily mean more accurate estimates for individual link parameters. A general observation is that given multiple estimation runs with similar setups (e.g. the same setup except the initial guesses were different), a parameter may exhibit a high error and a low variance in one run while exhibiting a lower error but higher variance in another. From just eyeballing, relationship between variance and estimation error (of individual link parameters) seems random.

- Another general observation is that the EKF tends to updates multiple or all of the parameters at around the same time. So there are points in the trajectory where multiple parameters' estimates and variances simultaneously update significantly. This doesn't seem surprising, since updating some estimates likely requires updating other estimates to compensate (e.g. to maintain something like $y = x_1 + x_2$). But it is interesting to see and seeing it gives me the notion that that part of the trajectory is particularly exciting (for the current values of estimates at least).

5. Conclusion

Using EKF to estimate link parameters for simple (2-DoF and 6-DoF) manipulators has not yielded accurate estimates for the individual link parameters. However, torque predictions given the parameter estimates were reasonable with noisy measurements and excellent with noiseless measurements. Additionally, there is a general observed trend where visible dips in variance estimates often occur simultaneously with updates in the corresponding parameter estimates. Finally, EKF's correlation estimates are not consistent across trajectories, differently generated noise (even if the noise distribution characteristics are the same), EKF filter tuning and initial guesses. So these correlation estimates are likely poor indicators of general information about the robot (like the relationships between parameters in the dynamic equations). More minor but still (kind of) notable findings were listed in section 4 and are not repeated here.

As for next steps, I'm not aware of a very promising plan to try from here (or I am just very forgetful!?). There was a related research question thrown around at one point being: can we accurately estimate inertial parameters (potentially just for control) with only the forces and torque measurements at the base? Unfortunately, I did not get to doing that.. Although I believe such experiments are easy to implement from the current codebase.

Finally, I think to discover the relationships between parameter in the dynamic equations, [1] seems like a very fitting and applicable solution (although I haven't read the entire thing.. Maybe ask Vlad?).

Again, let me know if you have any questions! My contact info is in the About section.

References

[1] Patrick M. Wensing, Gunter Niemeyer and Jean-Jacques E. Slotine (2017). Observability in Inertial Parameter Identification. *CoRR, abs/1711.03896*, .
[2] Spong, Mark W., Seth Hutchinson, and M. Vidyasagar. 2006. Robot modeling and control. Hoboken, NJ: John Wiley & Sons.