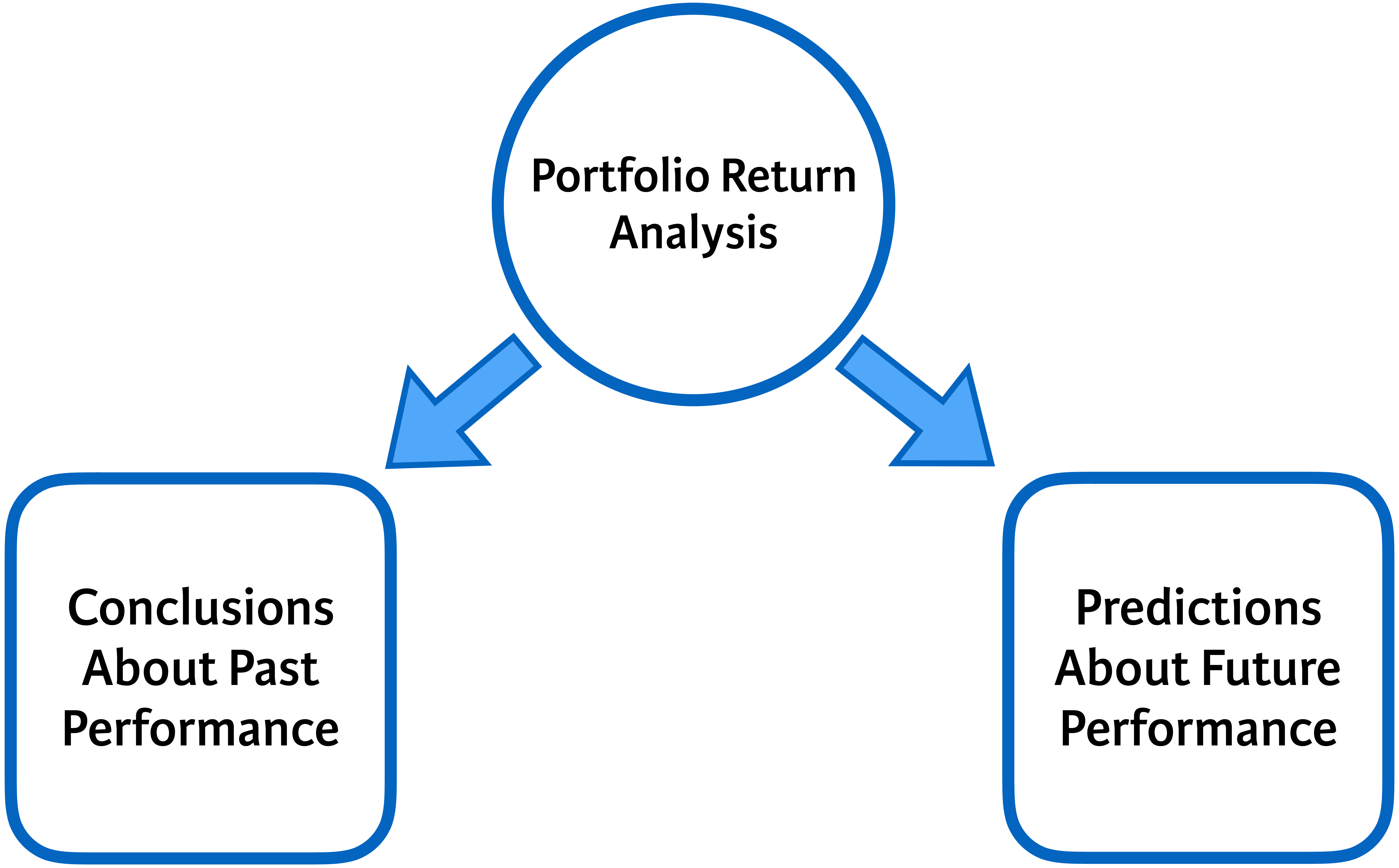
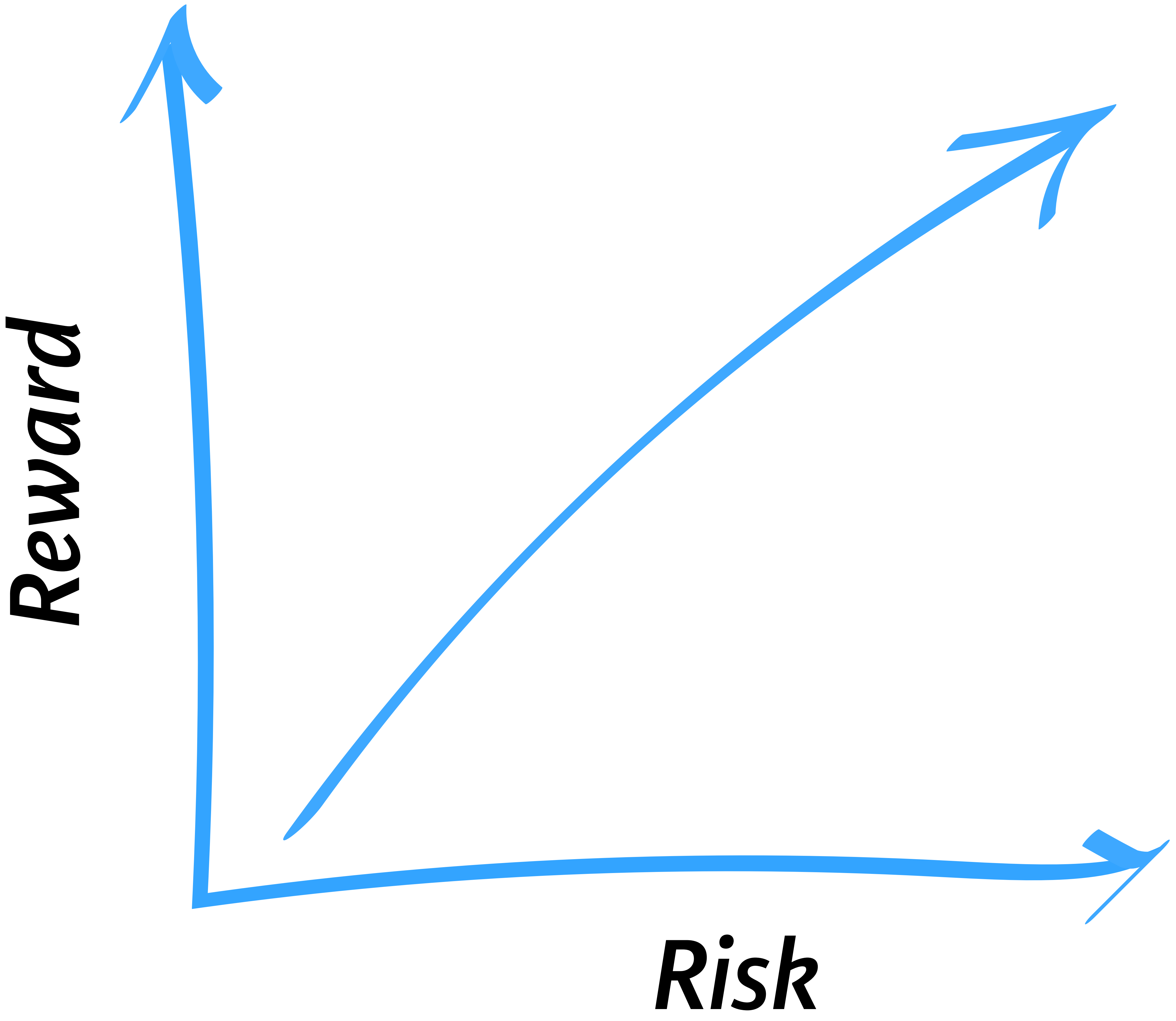
Dimensions of Portfolio Analysis

Interpretation of Portfolio Returns



Risk vs. Reward



Need For Performance Measure

Portfolio Returns

↓

Performance & Risk Measures

Reward → portfolio mean return

Risk → portfolio volatility

↓

Interpretation

Arithmetic Mean Return

- Assume a sample of *T* portfolio return observations:

$$R_1, R_2, \ldots, R_T$$

- Reward Measurement: Arithmetic mean return is given:

$$\hat{\mu} = \frac{R_1, R_2, \ldots, R_T}{T}$$

- It shows how large the portfolio return is on average

Risk: Portfolio Volatility

- De-meaned return

$$R_i - \hat{\mu}$$

- Variance of the portfolio

$$\hat{\sigma}^2 = \frac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + \cdots + (R_T - \hat{\mu})^2}{T - 1}$$

- Portfolio Volatility:

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

No Linear Compensation In Return

- Mismatch between average return and effective return

final value=
*initial value * (1 +0.5)*(1-0.5)= 0.75 * initial value*

↓

Average Return = (0.5 - 0.5) / 2 = 0

Geometric Mean Return

- Formula for *Geometric Mean* for a sample of **T** portfolio return observations $R_1, R_2, ..., R_T$:

$$\text{Geometric mean} = [(1+R_1)*(1+R_2)*...(1+R_T)]^{\frac{1}{T}} - 1$$

- Example: +50% & -50% return

$$\begin{aligned}\text{Geometric mean} &= (1+0.50)*(1-0.50)]^{\frac{1}{2}} - 1 \\ &= 0.75^{\frac{1}{2}} - 1 \\ &= -13.4\%\end{aligned}$$

Application to the S&P 500

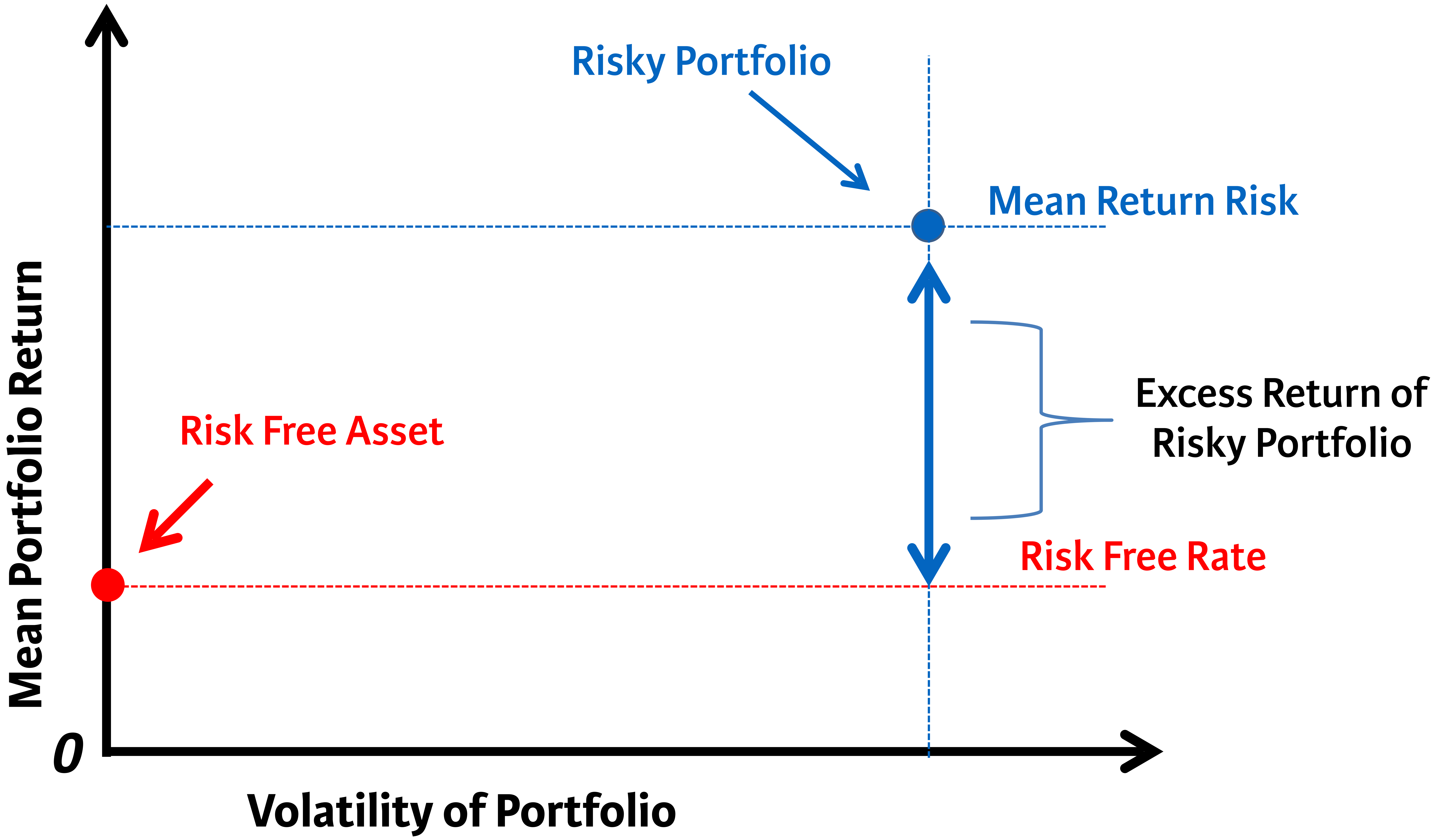
Let's practice!

The (Annualized) Sharpe Ratio

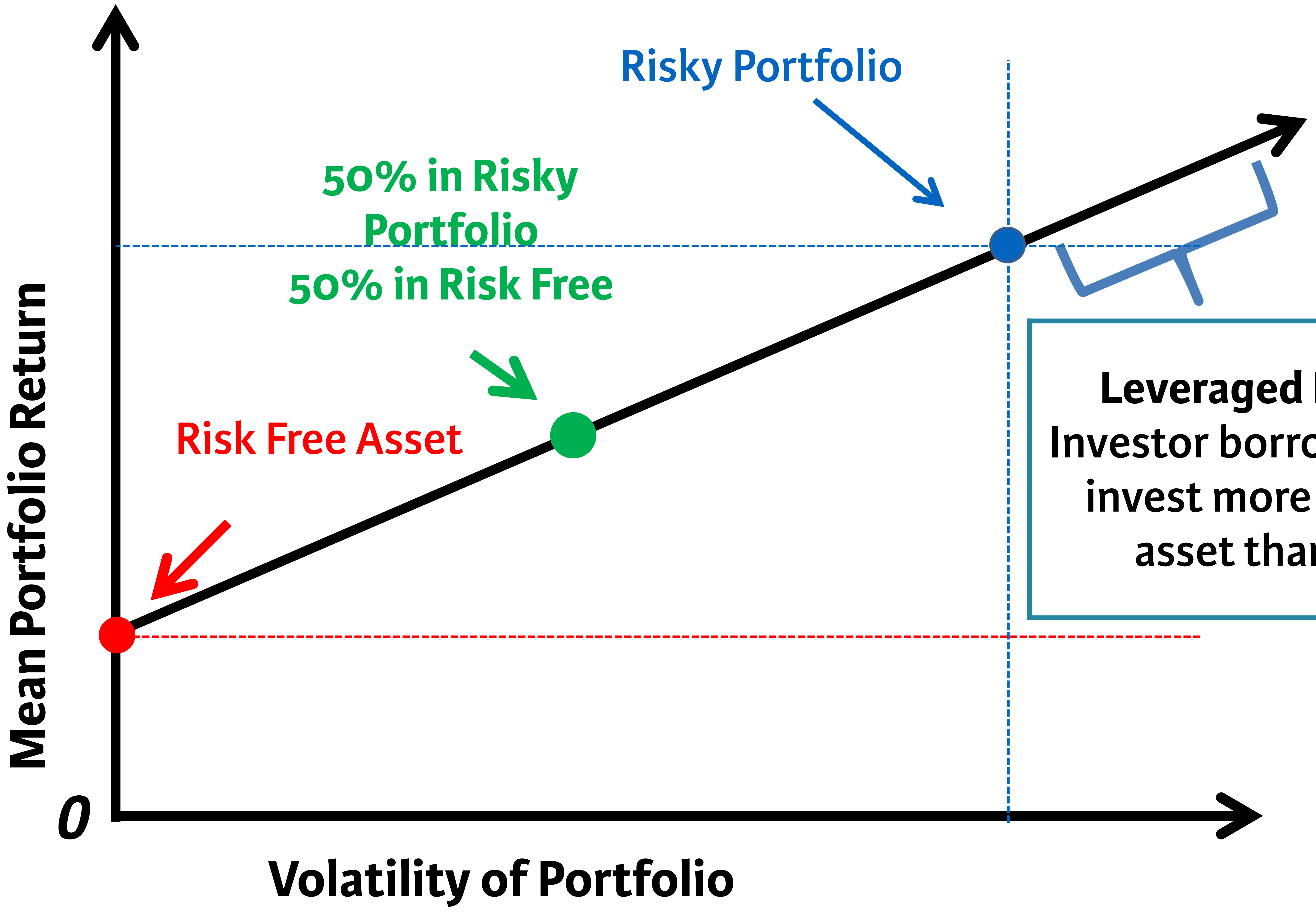
Benchmarking Performance

Risky Portfolio **E.g:** portfolio invested in stocks, bonds, real estate, and commodities	Risk Free Asset **E.g:** US Treasury Bill
Reward: measured by mean portfolio return	**Reward**: measured by risk free rate
Risk: measured by volatility of the portfolio returns	**Risk**: No risk, volatility = 0 return = risk free rate

Risk-Return Trade-Off

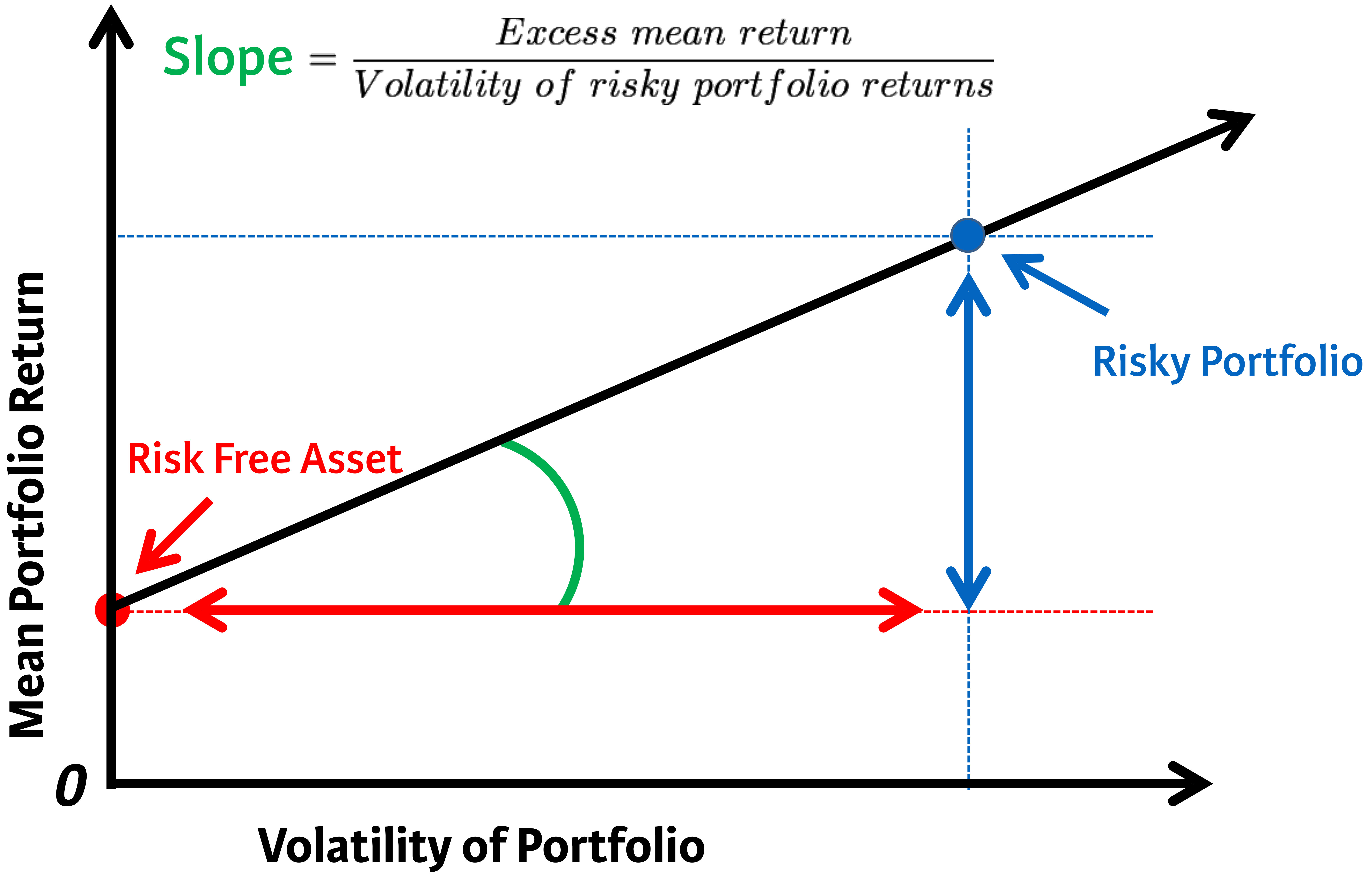


Capital Allocation Line



Leveraged Portfolios: Investor borrows capital to invest more in the risky asset than she has

The Sharpe Ratio



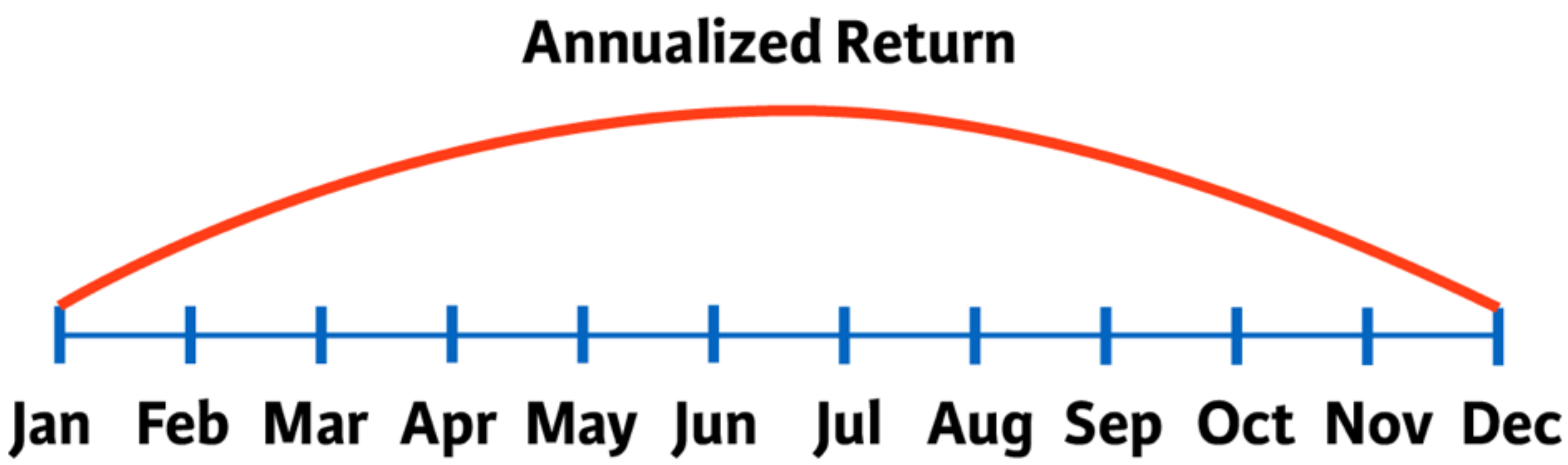
Performance Statistics In Action

```
> library(PerformanceAnalytics)
> sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)

> (mean(sample_returns) - 0.004)/StdDev(sample_returns)
```

returns	-0.02, 0 , 0 , 0.06, 0.02, 0.03, -0.01, 0.04
arithmetic mean	0.015
geometric mean	0.01468148
volatility	0.02725541
sharpe ratio	0.4035897

Annualize Monthly Performance



Arithmetic mean: monthly mean * 12

Geometric mean, when $\mathbf{R_i}$ are monthly returns:

$$[(1+R_1)*(1+R_2)*\ldots*(1+R_T)]^{\frac{12}{T}}-1$$

Volatility: monthly volatility * sqrt(12)

Performance Statistics In Action

```
> library(PerformanceAnalytics)
> sample_returns <- c( -0.02, 0.00, 0.00, 0.06, 0.02, 0.03, -0.01, 0.04)

> Return.annualized(sample_returns, scale = 12) /
Std.Dev.annualized(sample_returns, scale = 12)
```

	monthly	FACTOR	annualized
arithmetic mean	0.015	12	0.18
geometric mean	0.01468148	$[0.98 * \ldots * 1.04]^{\frac{12}{8}} - 1$	0.1911235
volatility	0.02725541	sqrt(12)	0.0944155
sharpe ratio	0.4035897	sqrt(12)	1.398076

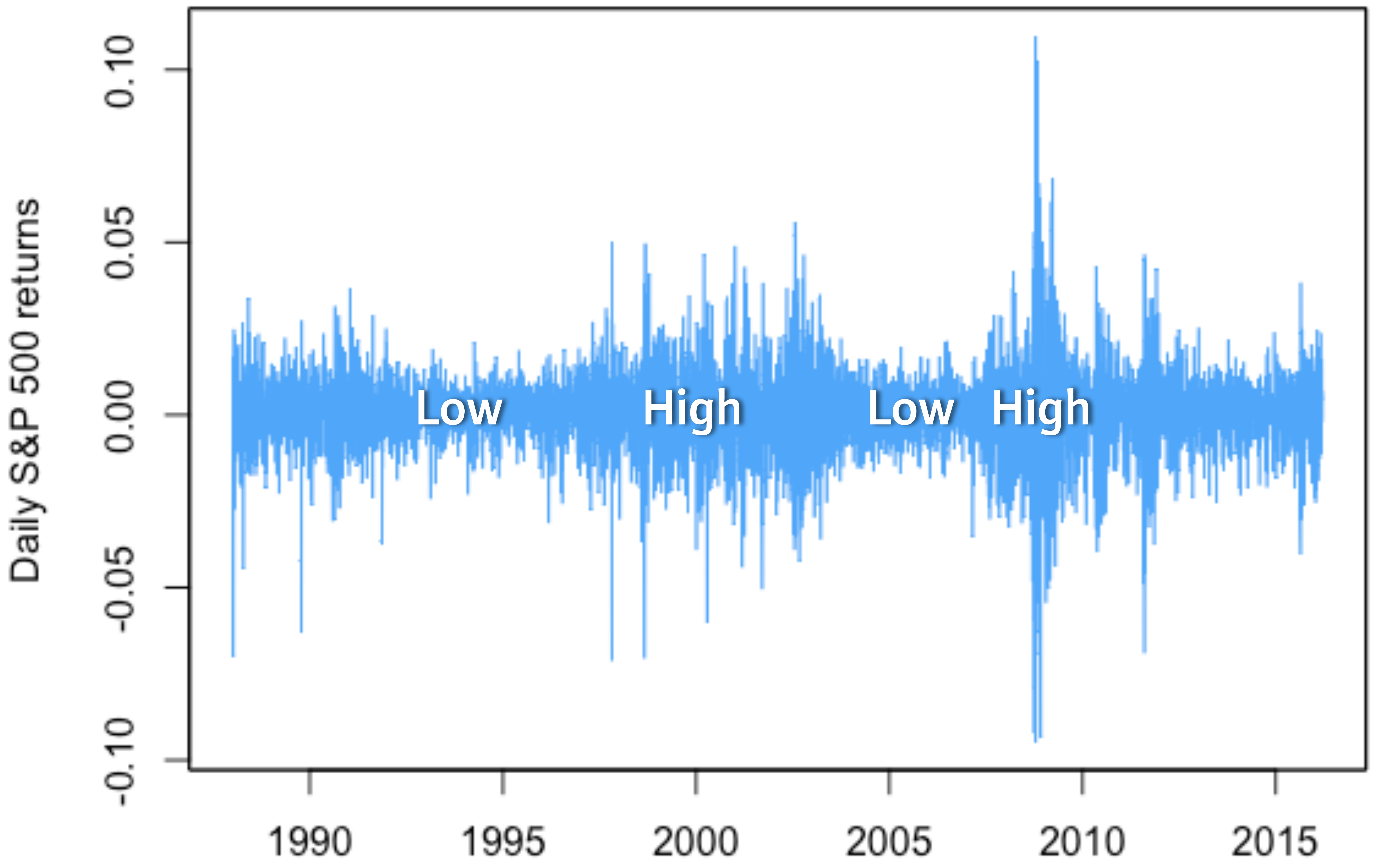
Let’s practice!

Time-Variation In Portfolio Performance

Bulls & Bears

- Business cycle, news, and swings in the market psychology affect the market

Clusters of High & Low Volatility



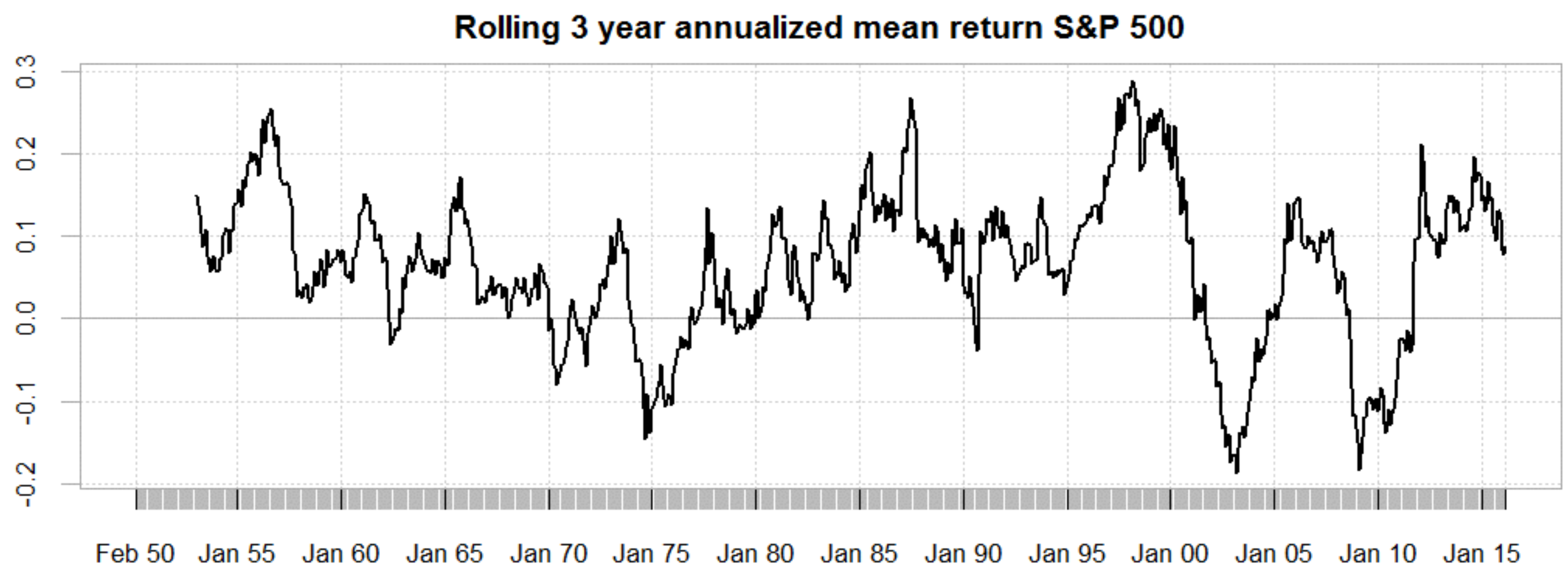
Internet Bubble Financial Crisis

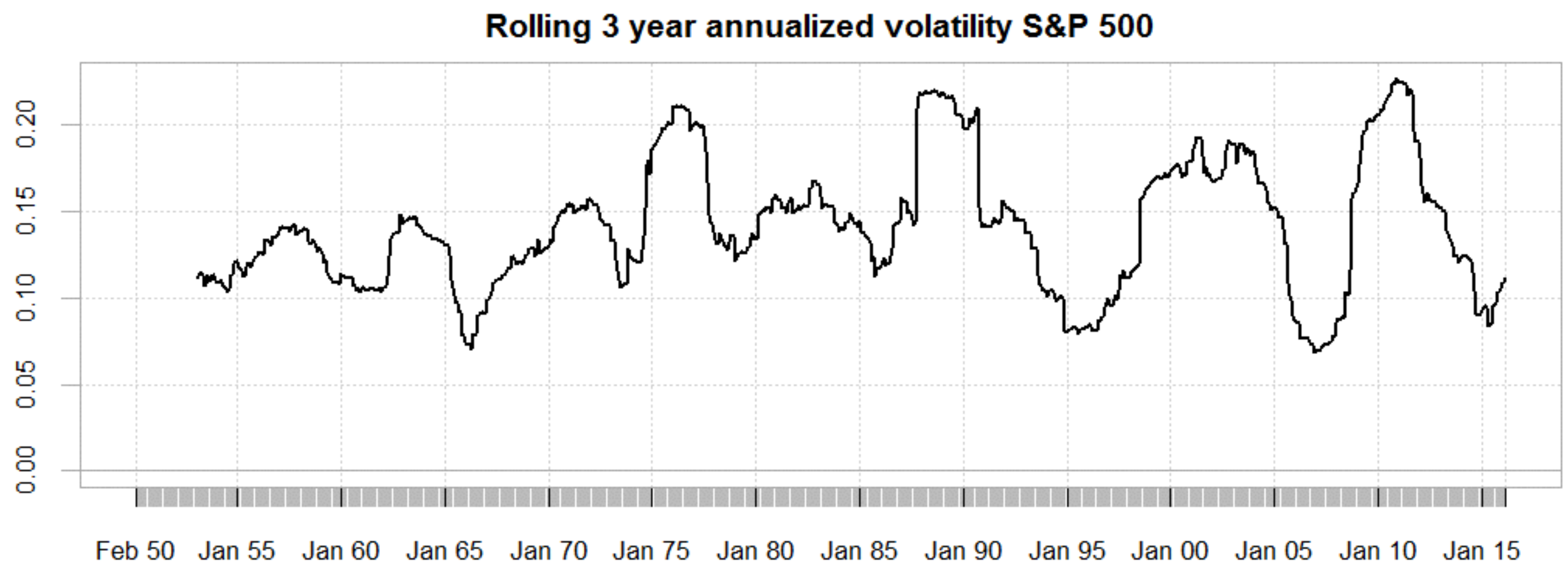
Rolling Estimation Samples

- Rolling samples of K observations:
    - Discard the most distant and include the most recent

R_{t-k+1}	R_{t-k+2}	R_{t-k+3}	...	R_t	R_{t+1}	R_{t+2}	R_{t+3}

Rolling Performance Calculation





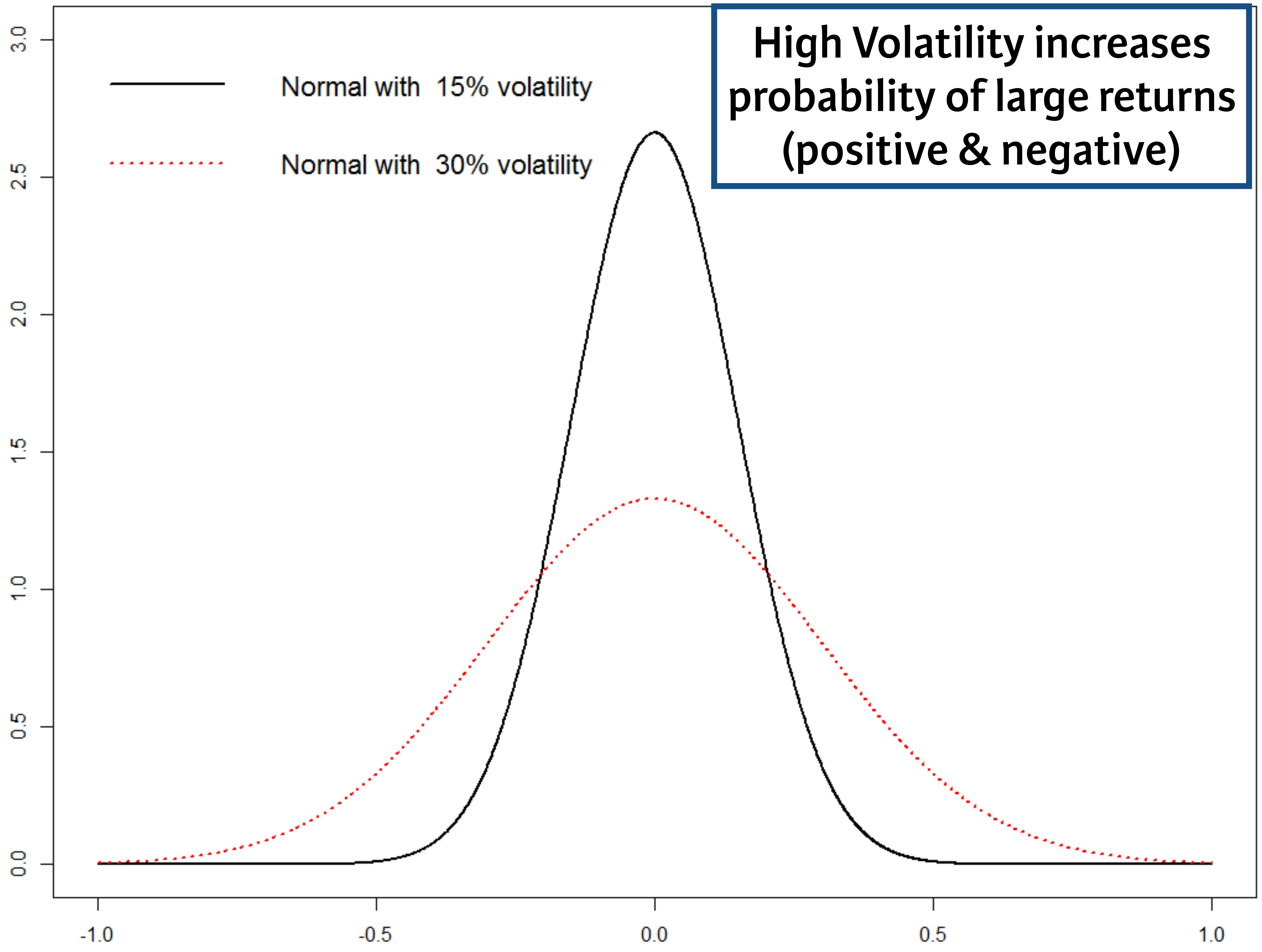
Choosing Window Length

- Need to balance noise (long samples) with recency (shorter samples)
- Longer sub-periods smooth highs and lows
- Shorter sub-periods provide more information on recent observations

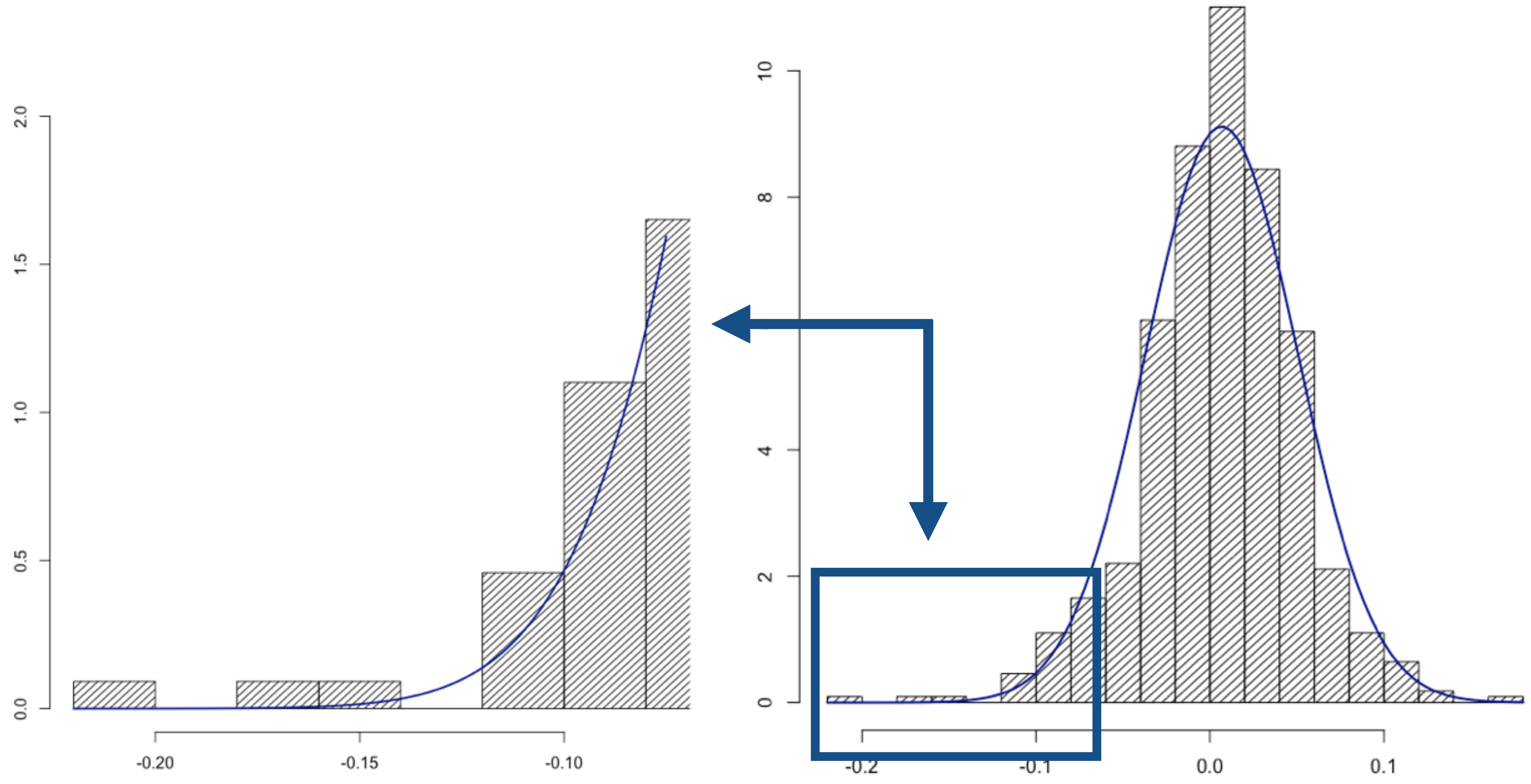
Let's practice!

Non-Normality of the Return Distribution

Volatility Describes "Normal" Risk



Non-Normality of Return



Portfolio Return Semi-Deviation

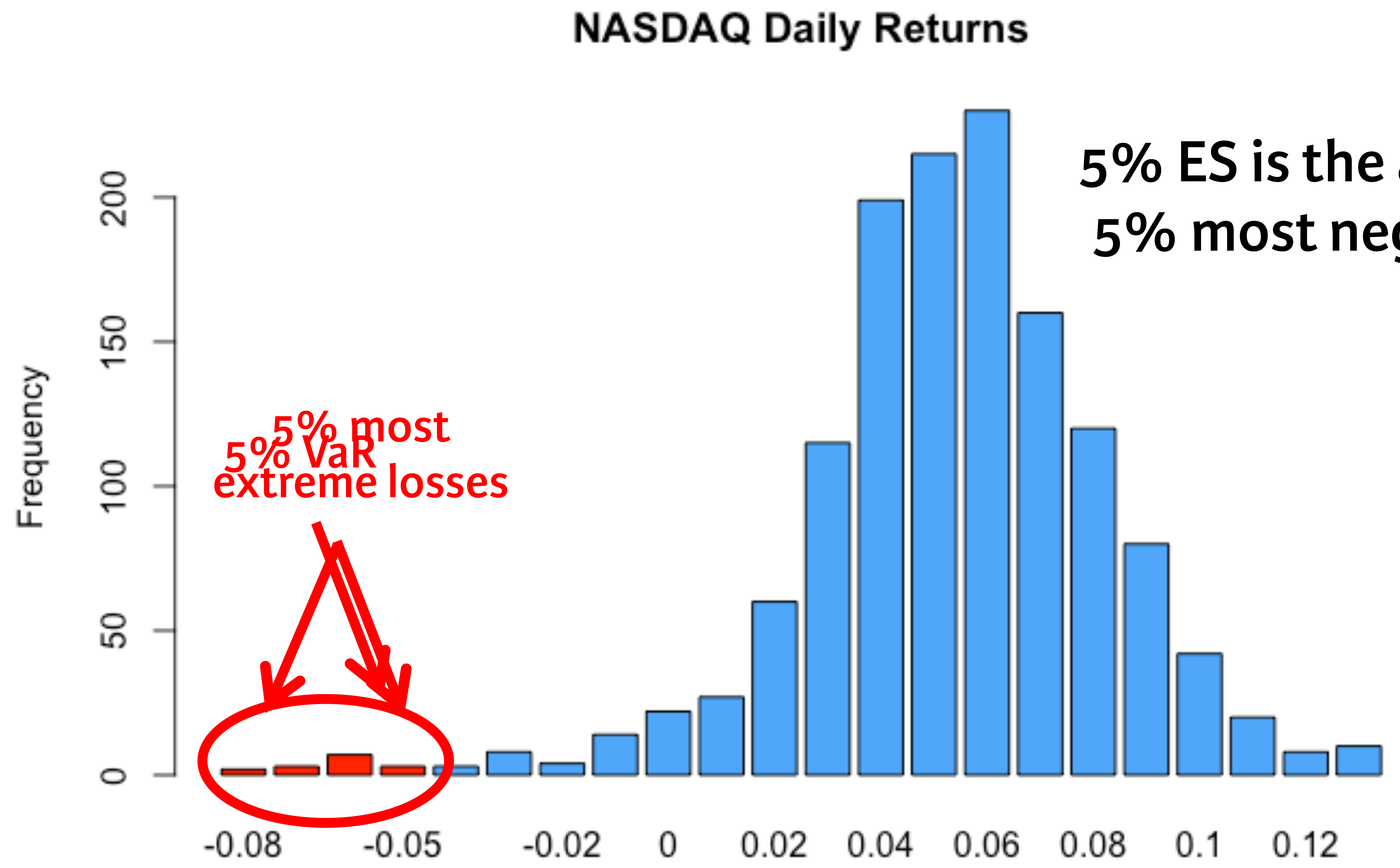
- **Standard Deviation** of Portfolio Returns:
  - Take the *full sample* of returns

$$SD = \sqrt{\frac{(R_1 - \mu)^2 + (R_2 - \mu)^2 + ... + (R_T - \mu)^2}{T - 1}}$$

- **Semi-Deviation** of Portfolio Returns:
  - Take the *subset* of returns below the mean

$$SemiDev = \sqrt{\frac{(Z_1 - \mu)^2 + (Z_2 - \mu)^2 + ... + (Z_n - \mu)^2}{n}}$$

Value-at-Risk & Expected Shortfall



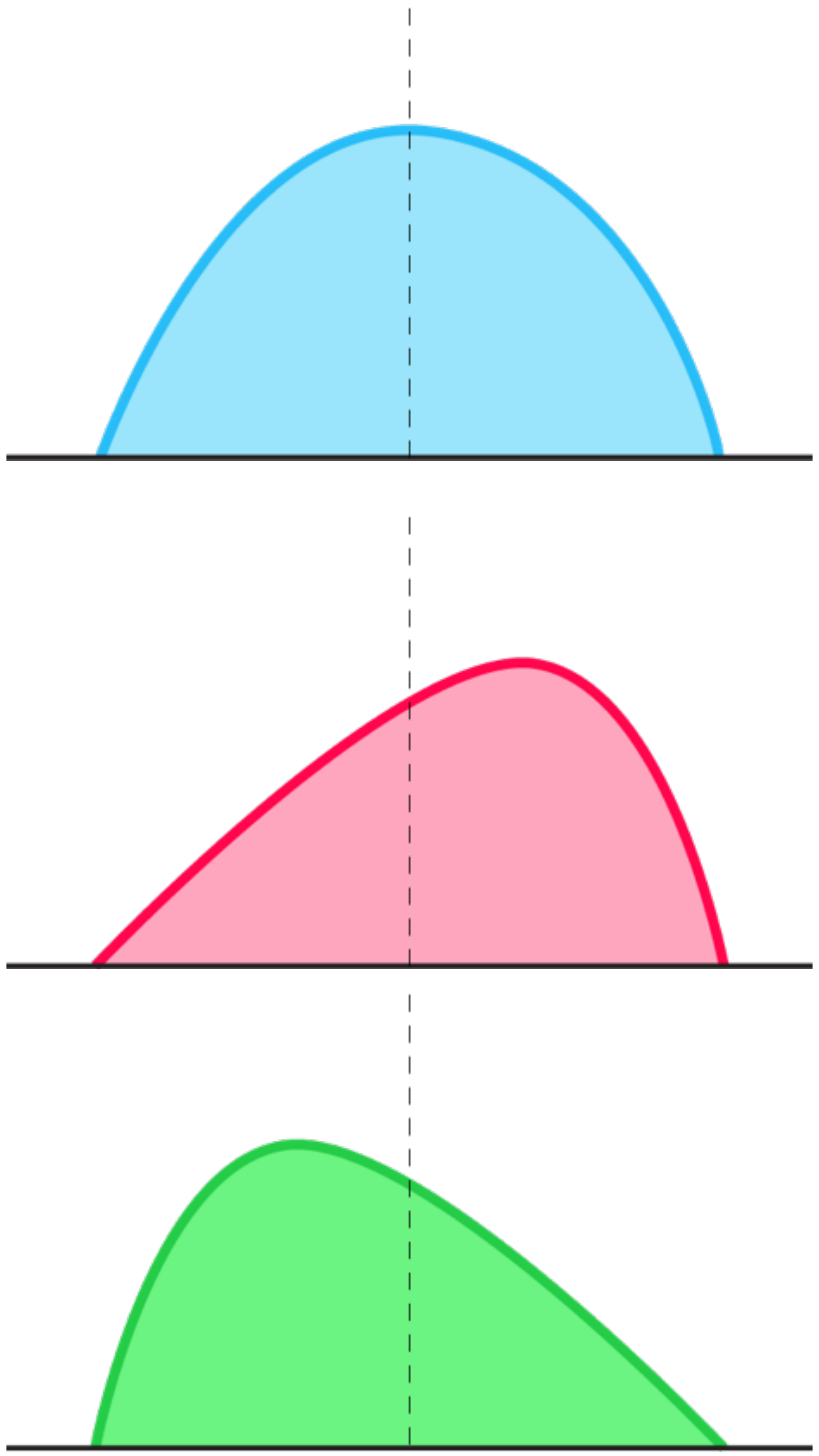
5% ES is the average of the 5% most negative returns

Shape of the Distribution

- Is it symmetric?
  - Check the skewness
- Are the tails fatter than those of the normal distribution?
  - Check the excess kurtosis

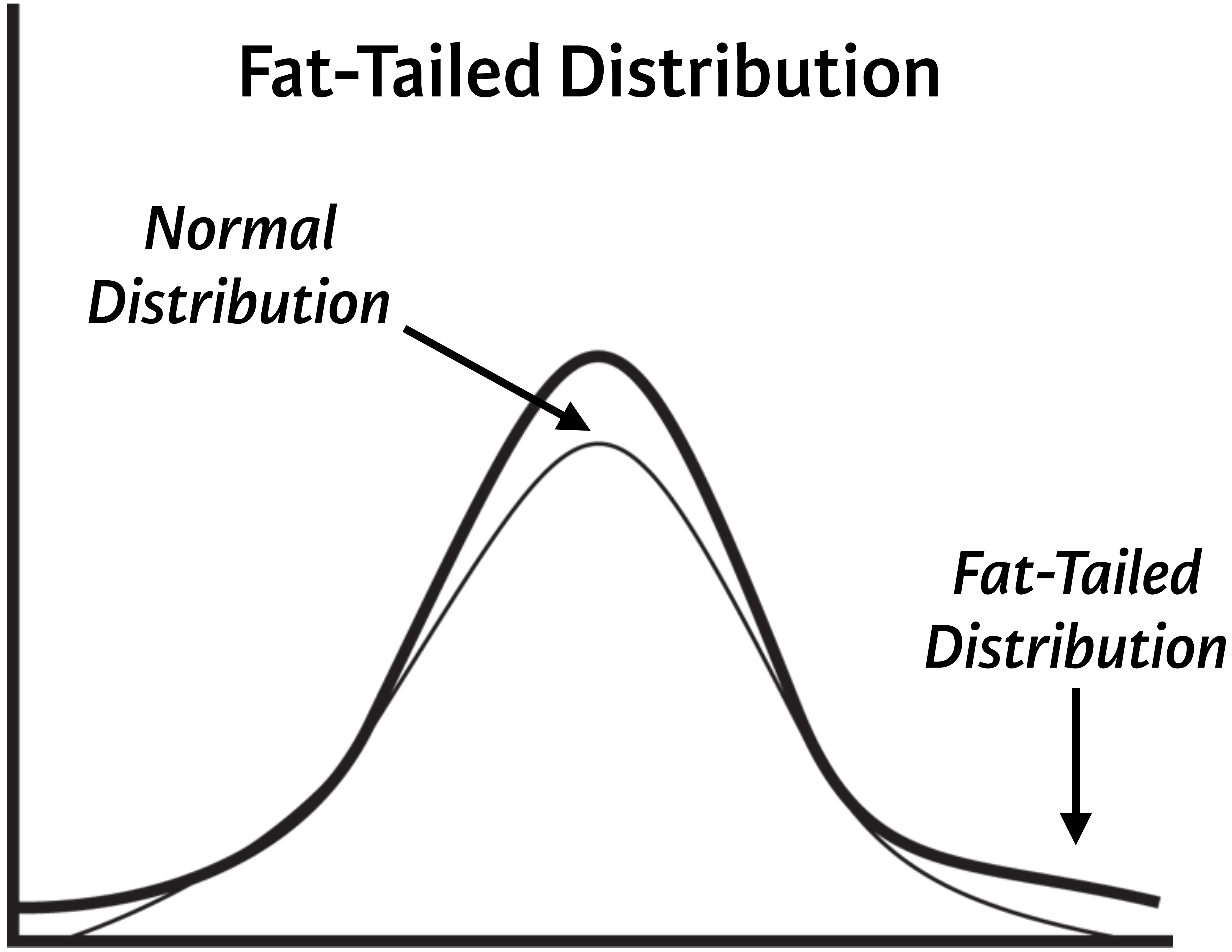
Skewness

- Zero Skewness
  - Distribution is symmetric
- Negative Skewness
  - Large negative returns occur more often than large positive returns
- Positive Skewness
  - Large positive returns occur more often than large negative returns



Kurtosis

- The distribution is fat-tailed when the excess kurtosis > 0



Let’s practice!