



Modern Portfolio Theory of Harry Markowitz



Portfolio Weights Are Optimal...

... when they optimize an objective function while satisfying the constraints

Possible Objectives	Possible Constraints
Maximize expected return	Only positive weights
Minimize the variance	Weights sum to 1 (all capital needs to be invested)
Maximize the Sharpe Ratio	Portfolio expected return equals a target value

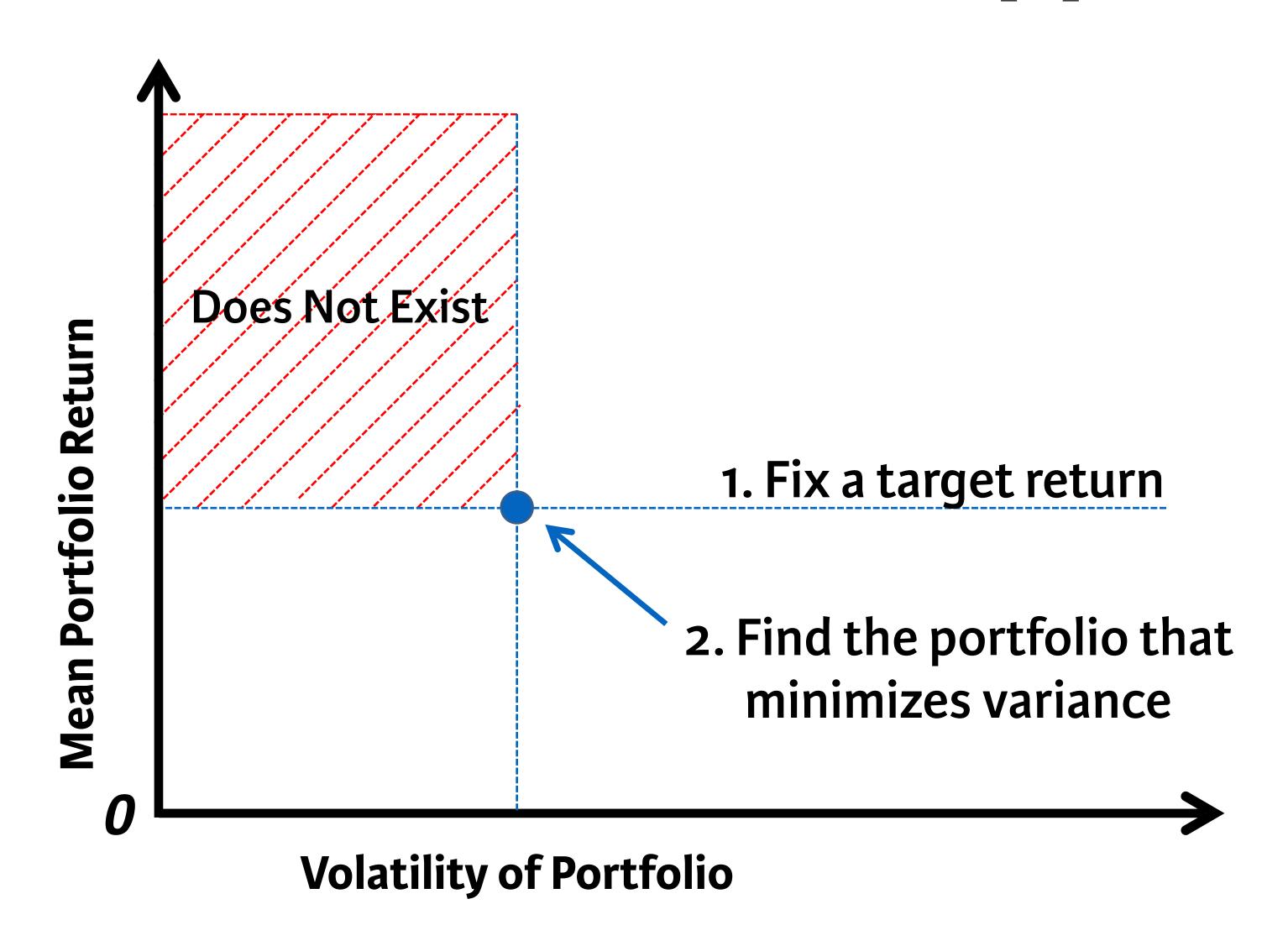


Harry Markowitz

- Nobel Prize Winner
- Recommends finding optimal portfolios by minimizing portfolio variance
 - Constraint: Expected return should be equal to a pre-specified target return



The H. Markowitz Approach







Let's practice!



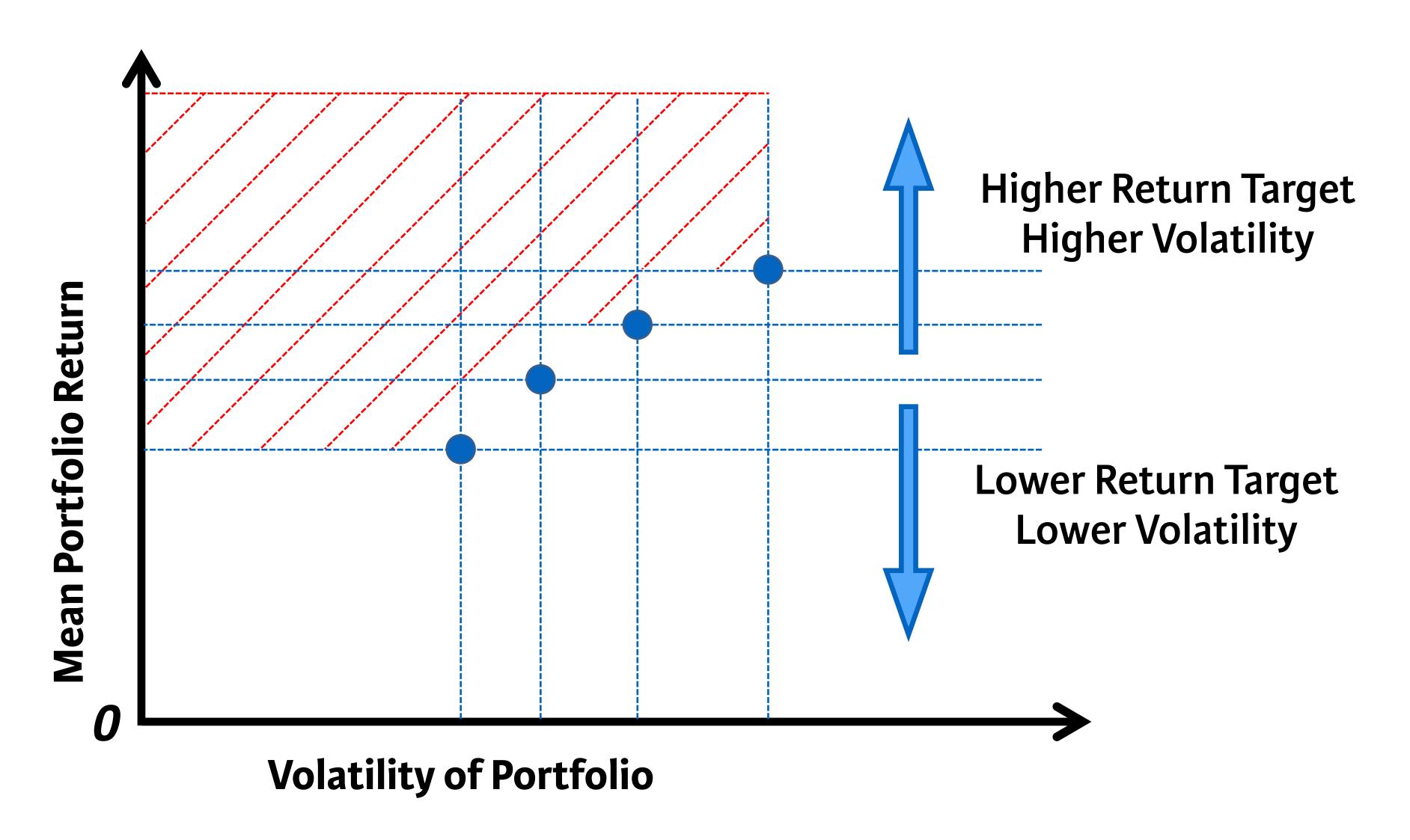


The Efficient Frontier



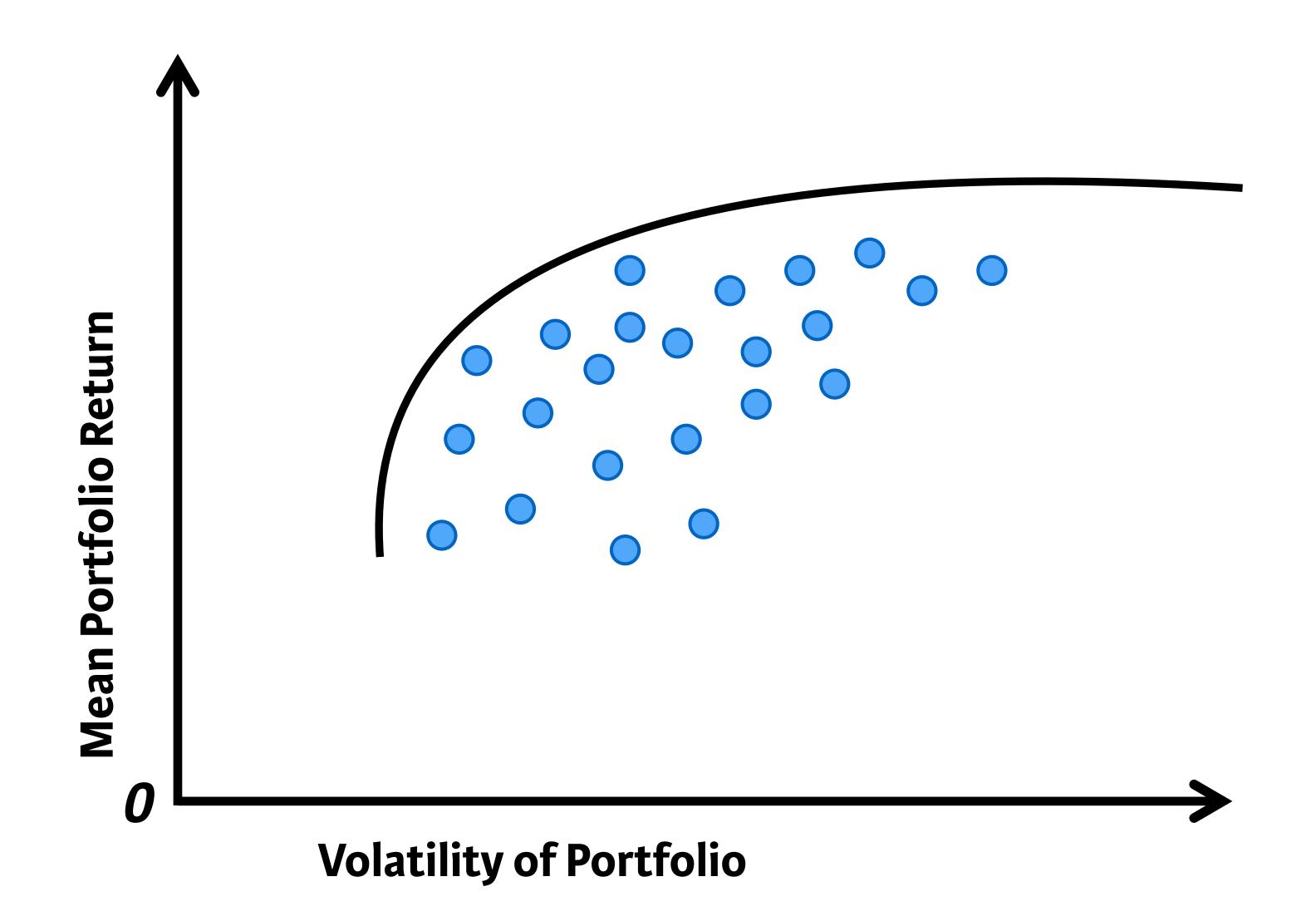


Changing Target Return



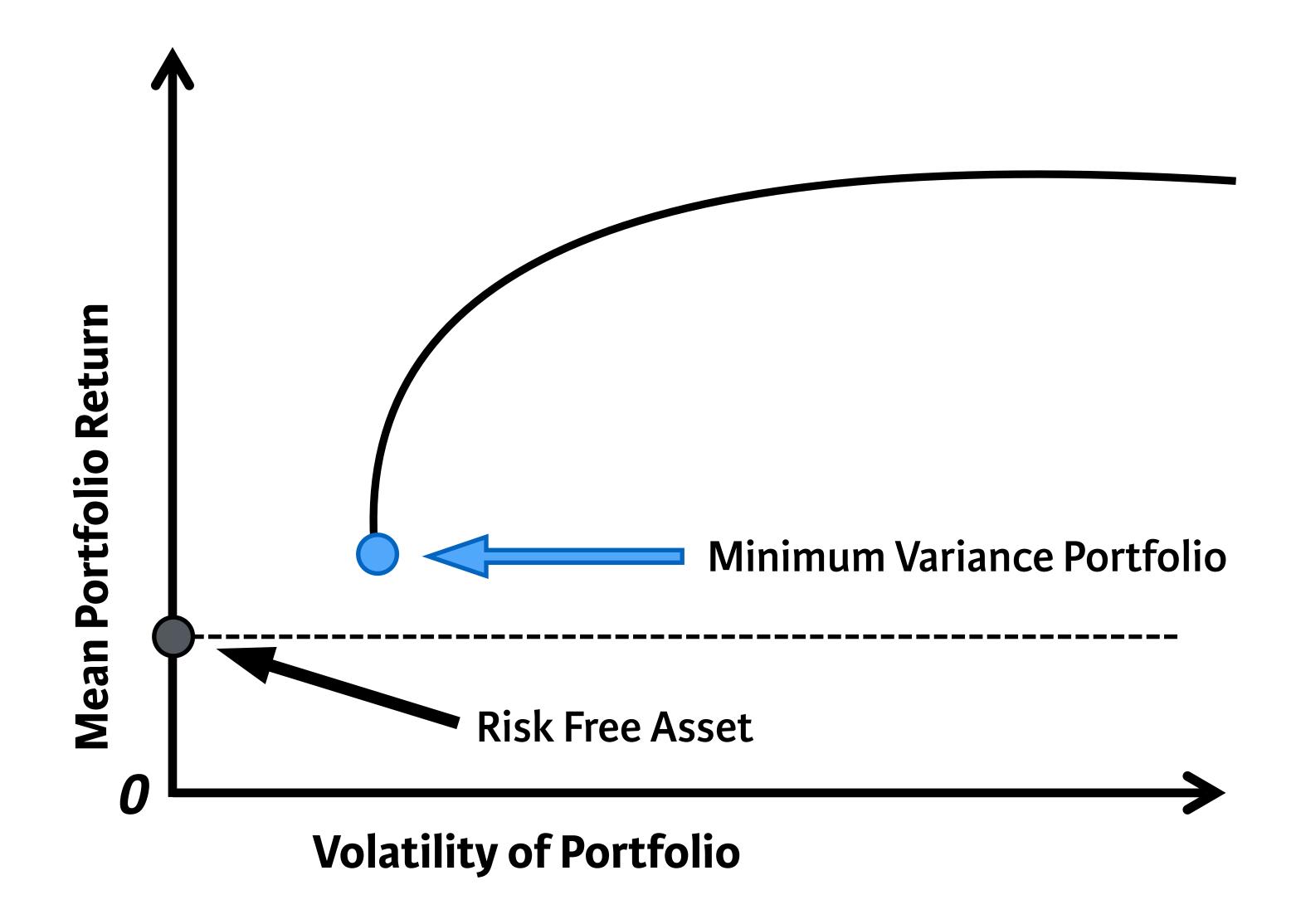


The Efficient Frontier



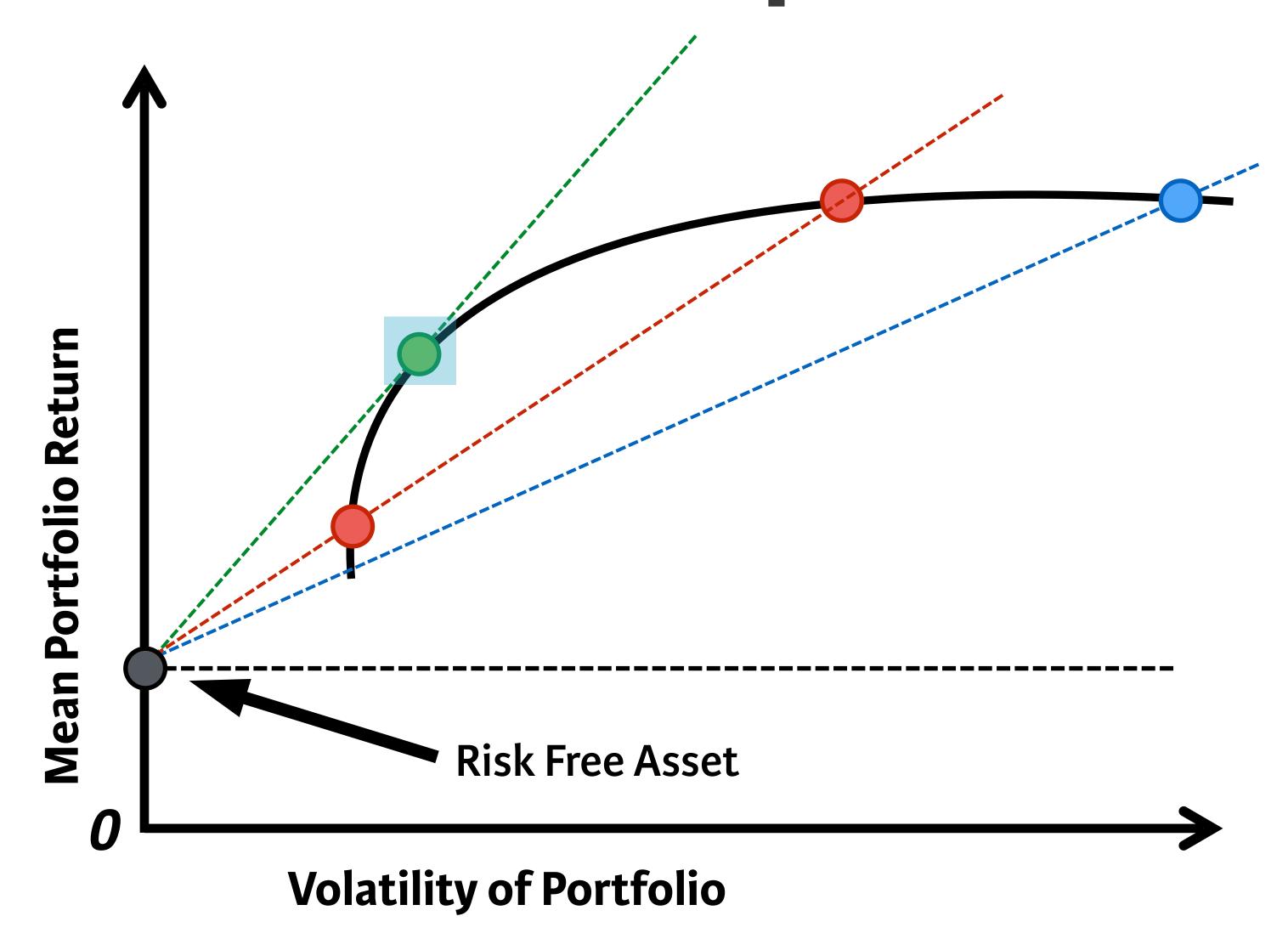


Minimum Variance Portfolio



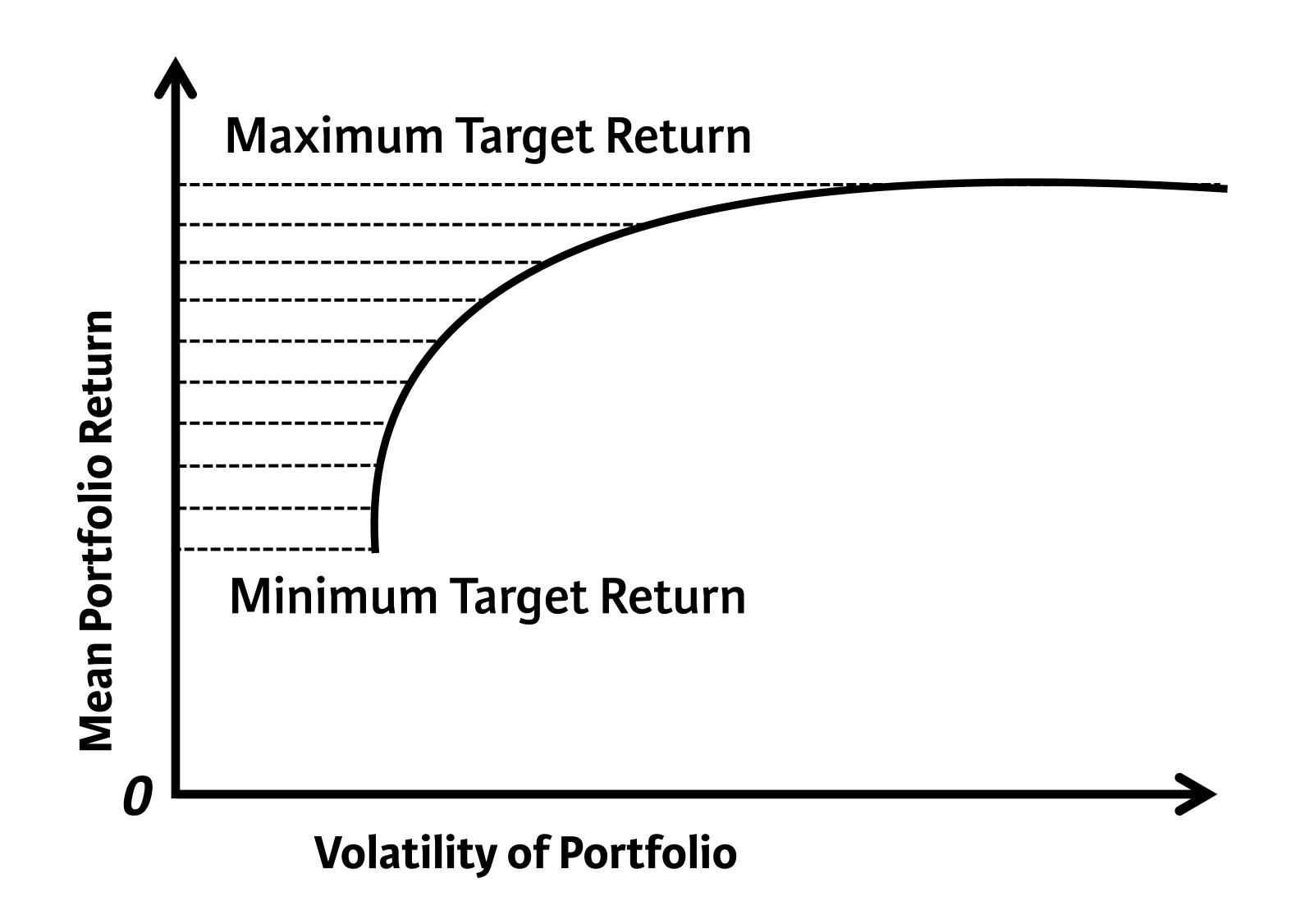


Maximum Sharpe Ratio Portfolio





Time For Practice







Let's practice!





In-Sample vs. Out-of-Sample





Bad News: Estimation Error

• Limitation to data-driven portfolio allocation:

Use in Practice

Estimated mean $\hat{\mu}$

Estimated variance $\hat{\sigma}^2$

Use In Theory

True (unknown) mean μ

True (unknown) variance σ^2

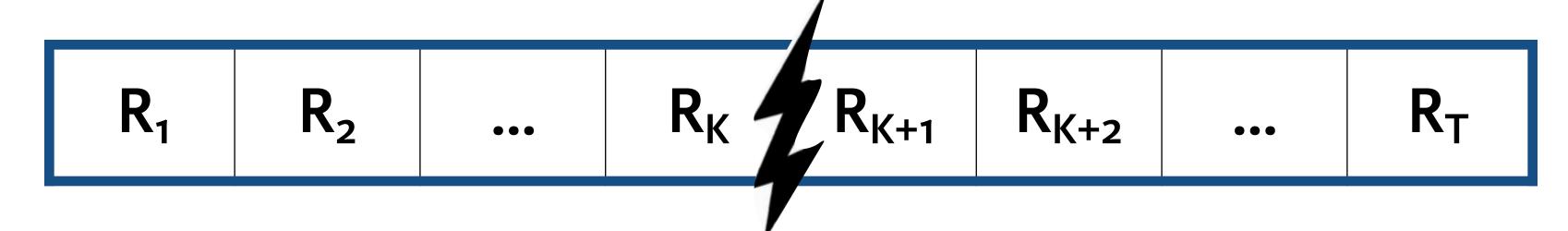
Optimized weights based on estimated mean & variance: \hat{w}

True optimal portfolio:w



Y

- Do not ignore estimation error
- Use split-sample analysis to do a realistic evaluation of portfolio performance



Estimation sample used to find the optimal weights

Out-of-Sample evaluation to give a realistic view on portfolio performance





No Look-Ahead Bias In Optimized Weights

Split-sample design matches with the investor who:

Uses at time K the returns R1, ..., Rk to compute optimal weights

Invests between time

K and time T

using optimized

weights

Time

• Function window() to do split-sample analysis in R





Let's practice!