딥러닝 올인원

미분

8강

딥러닝호형

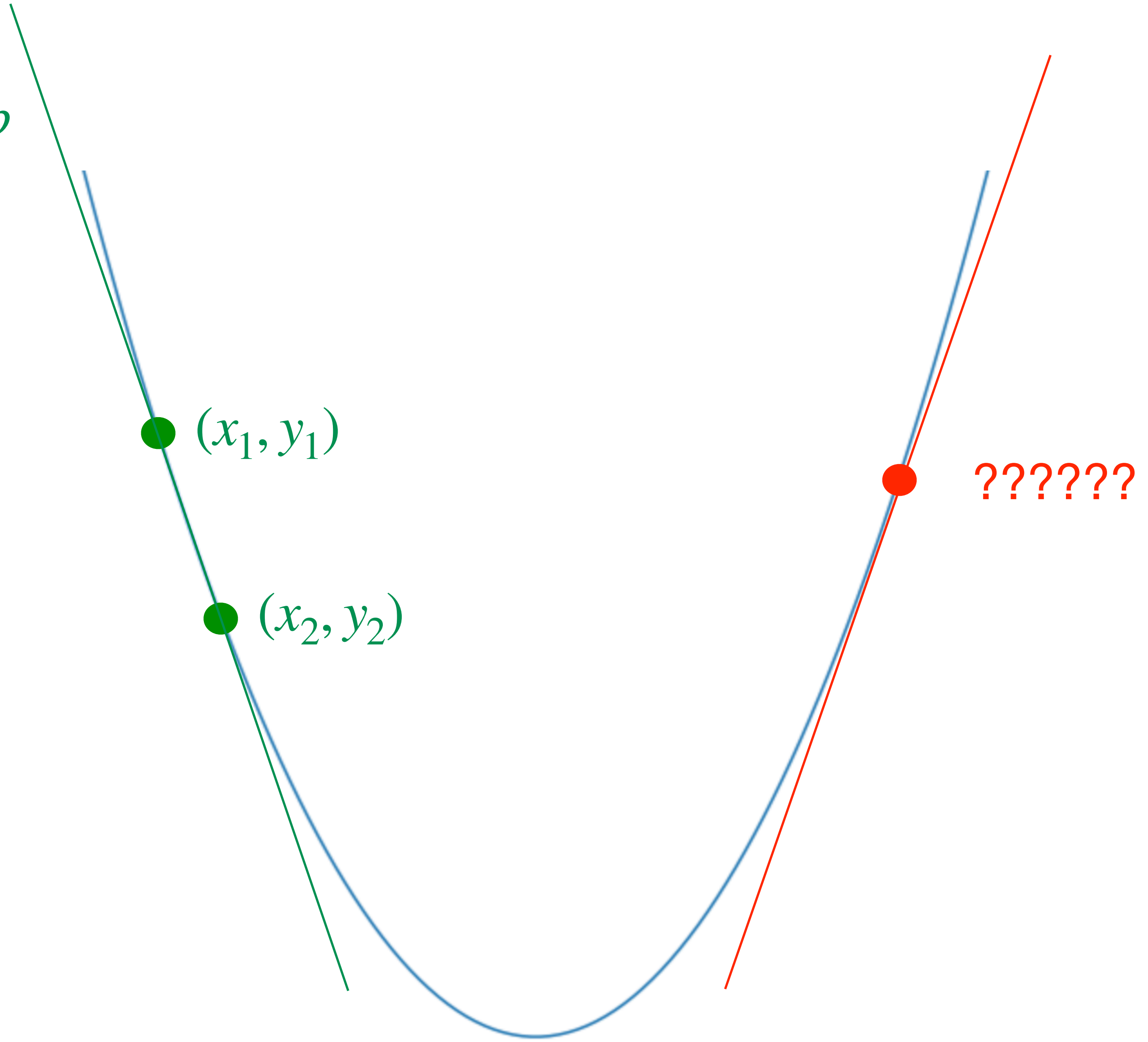
Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

기울기의 정의

$$y = ax + b$$

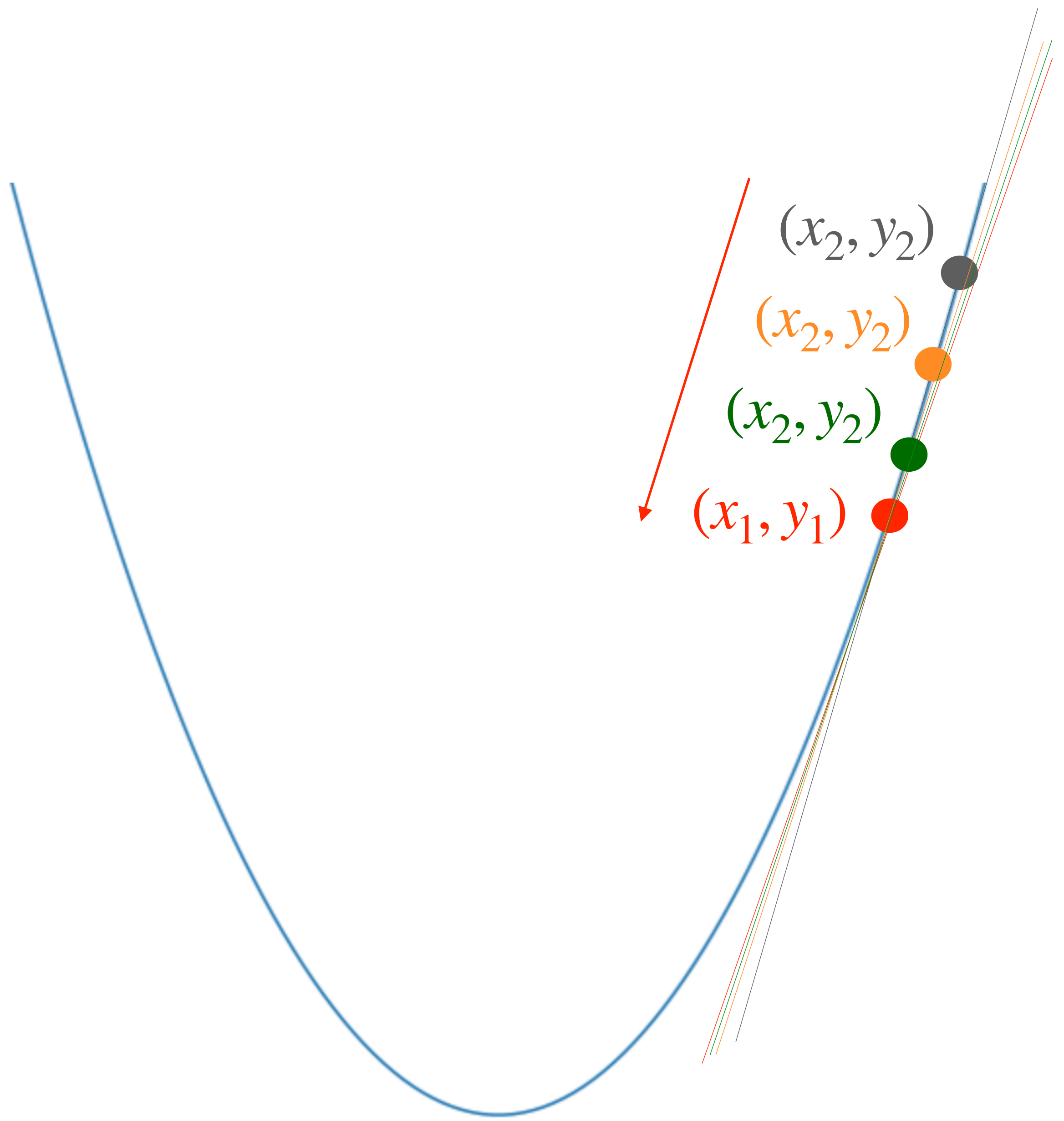
$$a = \frac{y_2 - y_1}{x_2 - x_1}$$



Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

평균 기울기 vs. 순간 기울기

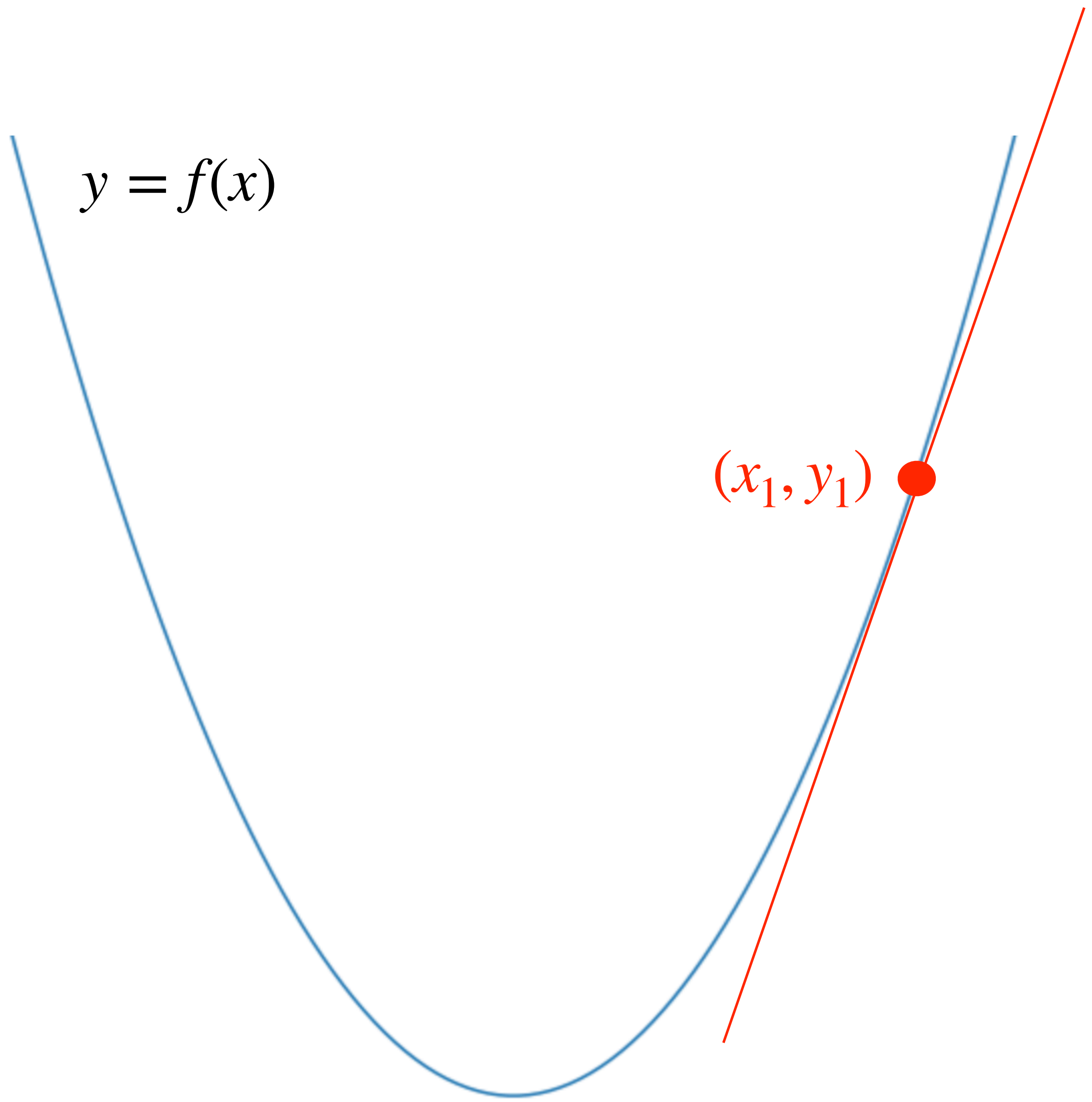


$$a = \frac{y_2 - y_1}{x_2 - x_1} \longrightarrow a = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1}$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

도함수와 미분 계수



$$a = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

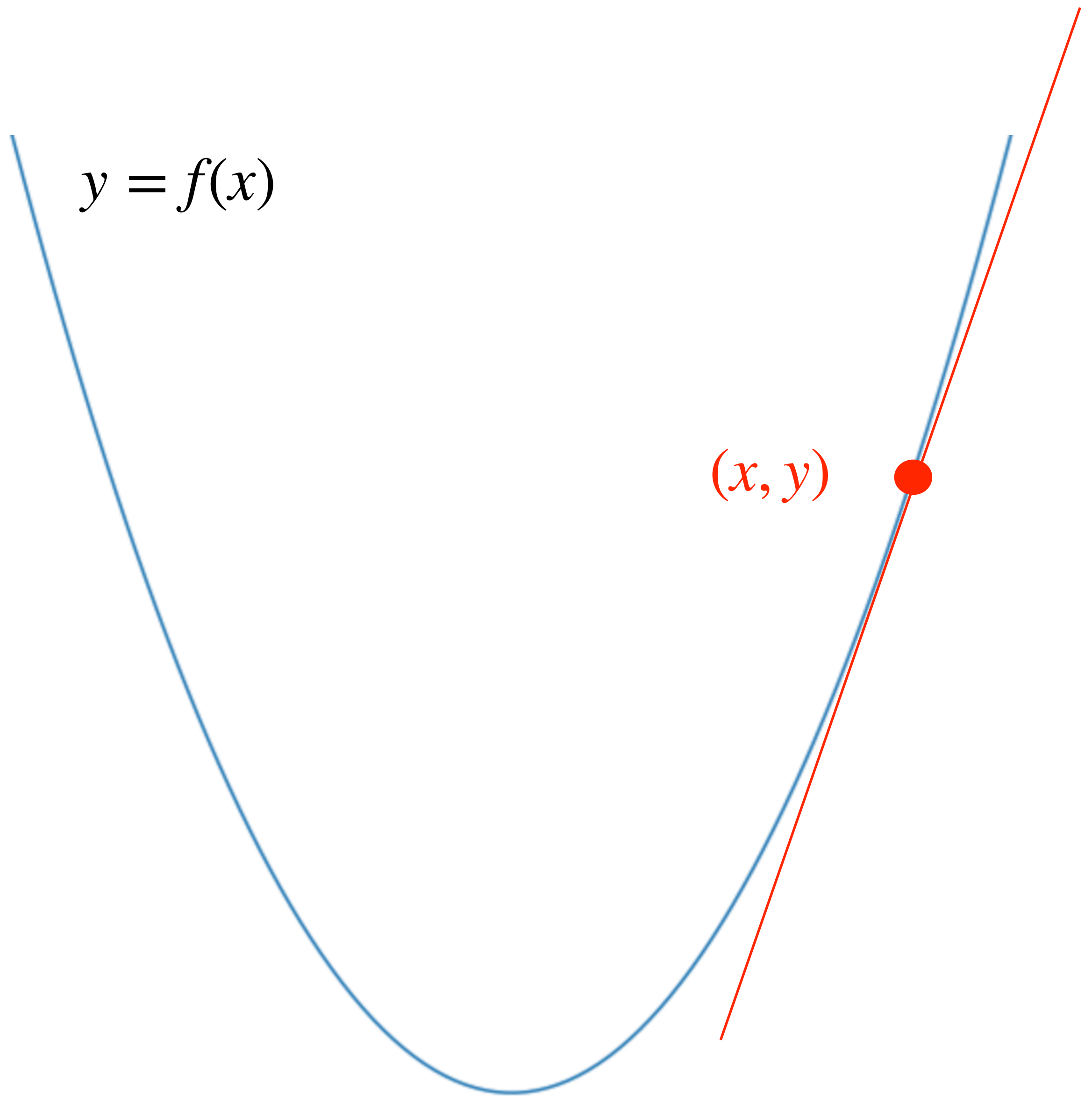
$$= \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x} \, (\Delta x = x_2 - x_1)$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

도함수와 미분 계수



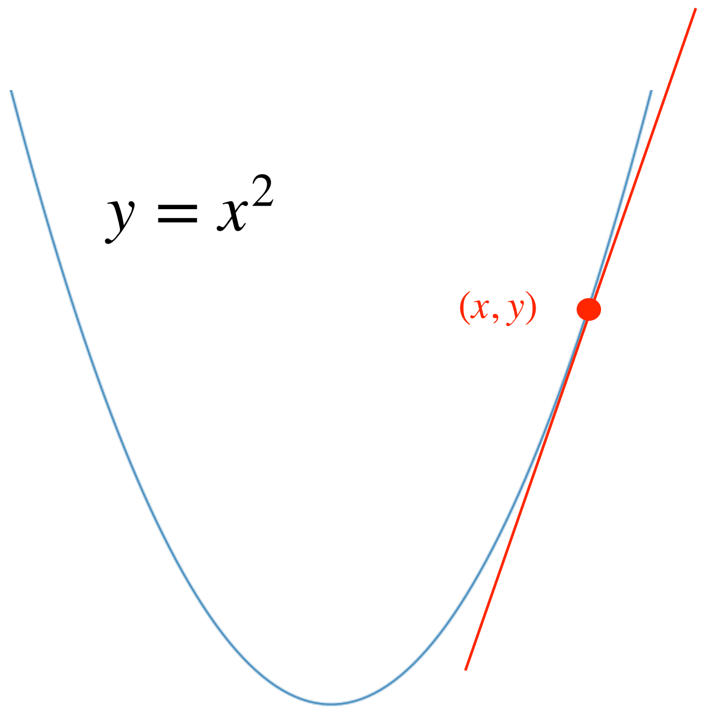
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

도함수와 미분 계수



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2}{h} = \lim_{h \to 0} (2x + h)$$

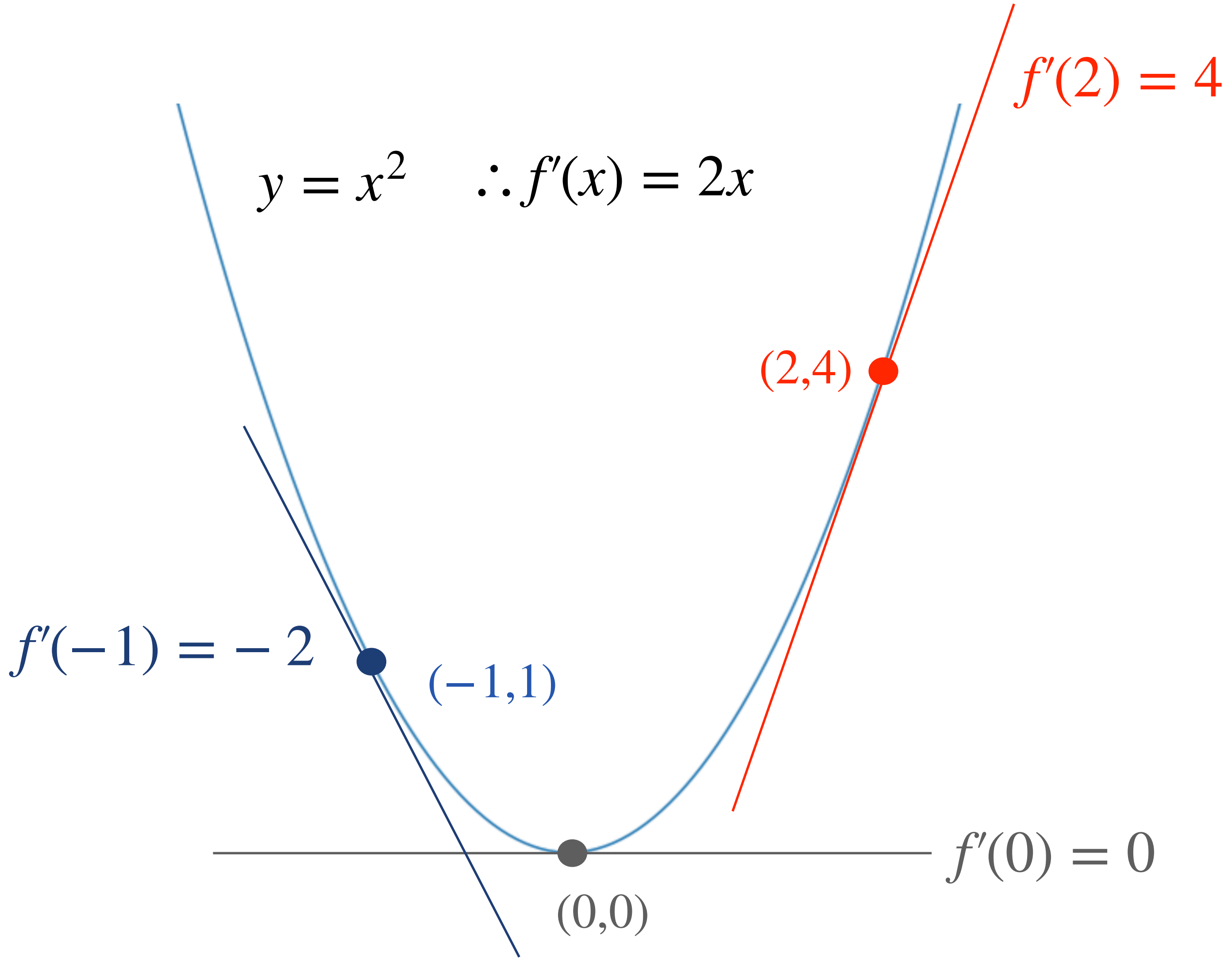
$$= 2x$$

$$\therefore f'(x) = 2x$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

평균 기울기 vs. 순간 기울기



Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

미분법

$$f(x) = x^n \rightarrow f'(x) = nx^{n-1}$$

$$f(x) = e^x \rightarrow f'(x) = e^x$$

$$f(x) = sinx \rightarrow f'(x) = cosx$$

$$f(x) = cosx \rightarrow f'(x) = -sinx$$

$$f(x) = lnx \rightarrow f'(x) = \frac{1}{x}$$

$$\vdots$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

곱의 미분법

$$\{f(x)g(x)\}' = f'(x)g(x) + f(x)g'(x)$$

$$f(x) = x^2, g(x) = e^x$$

$$\{f(x)g(x)\}' = f'(x)g(x) + f(x)g'(x) = 2xe^x + x^2e^x = (2x + x^2)e^x$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

몫의 미분법

$$\left\{\frac{f(x)}{g(x)}\right\}' = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}$$

$$f(x) = x^2, g(x) = e^x$$

$$\left\{\frac{f(x)}{g(x)}\right\}' = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2} = \frac{2xe^x - x^2e^x}{e^{2x}} = \frac{2x - x^2}{e^x}$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

합성 함수의 미분법

$$\{f(g(x))\}' = f'(g(x))g'(x)$$

$$f(x) = x^2, g(x) = sinx$$

$$\{f(g(x))\}' = \{(sinx)^2\}' = \{(sin^2x)\}' = 2sinxcosx$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

미분 표현법

$$f'(x)\,,\frac{dy}{dx}\,,\frac{df(x)}{dx}\,,\frac{\partial f(x)}{\partial x}$$

$$f(x) = x^2 \rightarrow \frac{df(x)}{dx} = 2x$$

$$f(x, y) = x^2 + 2y \rightarrow \frac{\partial f(x, y)}{\partial x} = 2x\,,\frac{\partial f(x, y)}{\partial y} = 2$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

연쇄 법칙(Chain Rule)

$$\{f(g(x))\}' = f'(g(x))g'(x)$$

$$y = f(u), u = g(x)$$

$$\{f(u)\}' = \frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

연쇄 법칙(Chain Rule)

알아두기 2.4.1 — **연쇄 법칙(chain rule).** 미분 가능한 함수들의 합성함수를 미분하는 법칙

미분 가능한 두 일변수 함수 $y = f(u)$, $u = g(x)$ 라고 하면

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

미분 가능한 두 다변수 함수 $y = f(u_1, u_2, \cdots, u_n)$, $u_i = g(x_1, x_2, \cdots, x_m)$ 라고 하면

$$\frac{\partial y}{\partial x_i} = \frac{\partial y}{\partial u_1}\frac{\partial u_1}{\partial x_i} + \frac{\partial y}{\partial u_2}\frac{\partial u_2}{\partial x_i} + \cdots + \frac{\partial y}{\partial u_n}\frac{\partial u_n}{\partial x_i}$$

예를 들어 $f(u) = u^2$, $u = 2x + 1$이면 $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 2u \cdot 2 = 4u = 8x + 4$ 이다.

Copyright 2020. 딥러닝호형 All rights reserved.

미분(Differentiation)

그레디언트(Gradient)

알아두기 **2.4.2 — 그레디언트(gradient).**

그레디언트는 함수 f를 변수 별로 한 번 미분한 것들을 모은 벡터다.

$$\nabla f(\mathbf{x}) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}, \mathbf{x} = (x_1, x_2, \ldots, x_n)$$

예를 들어 함수 $f(x_1, x_2) = x_1^2 + x_1 x_2 + 2x_2$이면 $\frac{\partial f}{\partial x_1} = 2x_1 + x_2$, $\frac{\partial f}{\partial x_2} = x_1 + 2$이므로

$\nabla f(x_1, x_2) = \begin{pmatrix} 2x_1 + x_2 \\ x_1 + 2 \end{pmatrix}$ 이다.

Copyright 2020. 딥러닝호형 All rights reserved.