딥러닝 올인원

XOR 문제
5강

딥러닝호형

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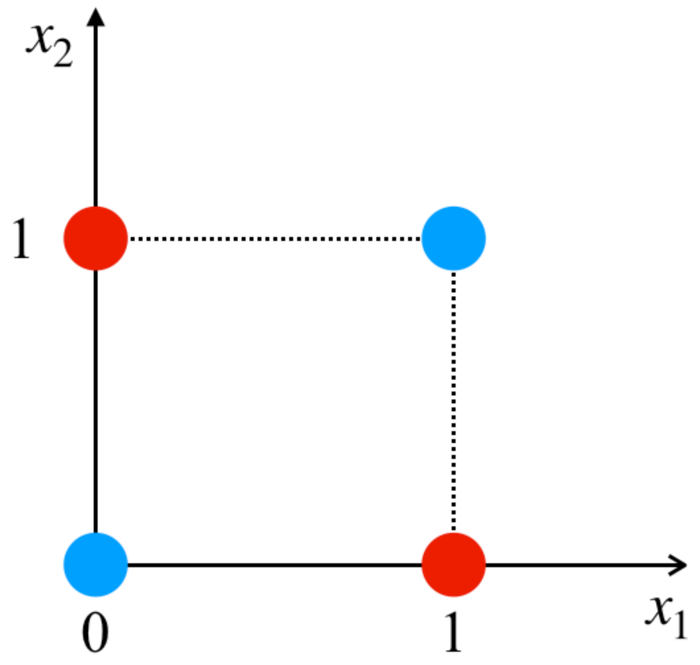
XOR 문제(XOR Problem)



x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

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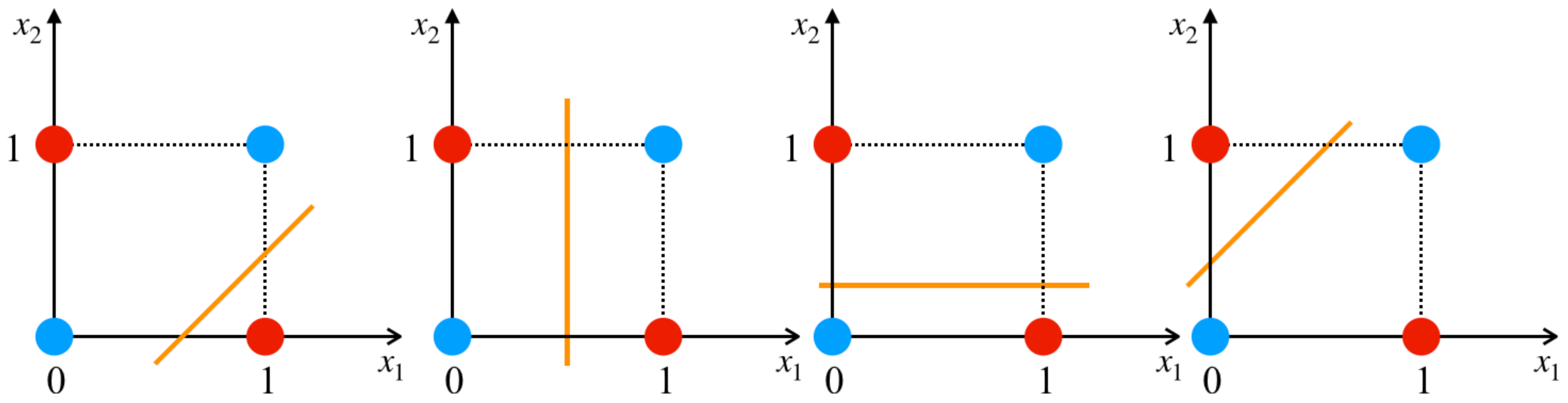
XOR 문제(XOR Problem)



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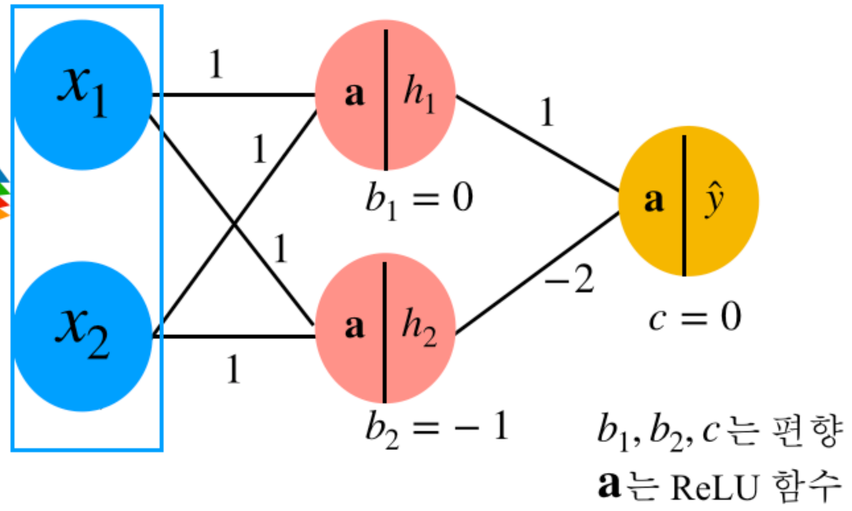
0	0	0
0	1	1
1	0	1
1	1	0



$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = ReLU\left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right) = ReLU\begin{pmatrix} x_1 + x_2 \\ x_1 + x_2 - 1 \end{pmatrix} = \begin{pmatrix} max(0, x_1 + x_2) \\ max(0, x_1 + x_2 - 1) \end{pmatrix}$$

$$\hat{y} = ReLU\left(\begin{pmatrix} 1 & -2 \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + 0\right) = ReLU(h_1 - 2h_2) = max(0, h_1 - 2h_2)$$

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알아두기 2.3.1 — 전치행렬(transposed matrix). 행과 열의 위치를 바꾼 행렬

행렬 A의 성분 a_{ij}를 a_{ji}로 위치를 바꾼 행렬을 A^T라고 표현하며 A의 전치행렬이라고 한다.

예를 들어 A가 2×3 행렬 $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ 이면 전치행렬 $A^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{pmatrix}$ 이다.

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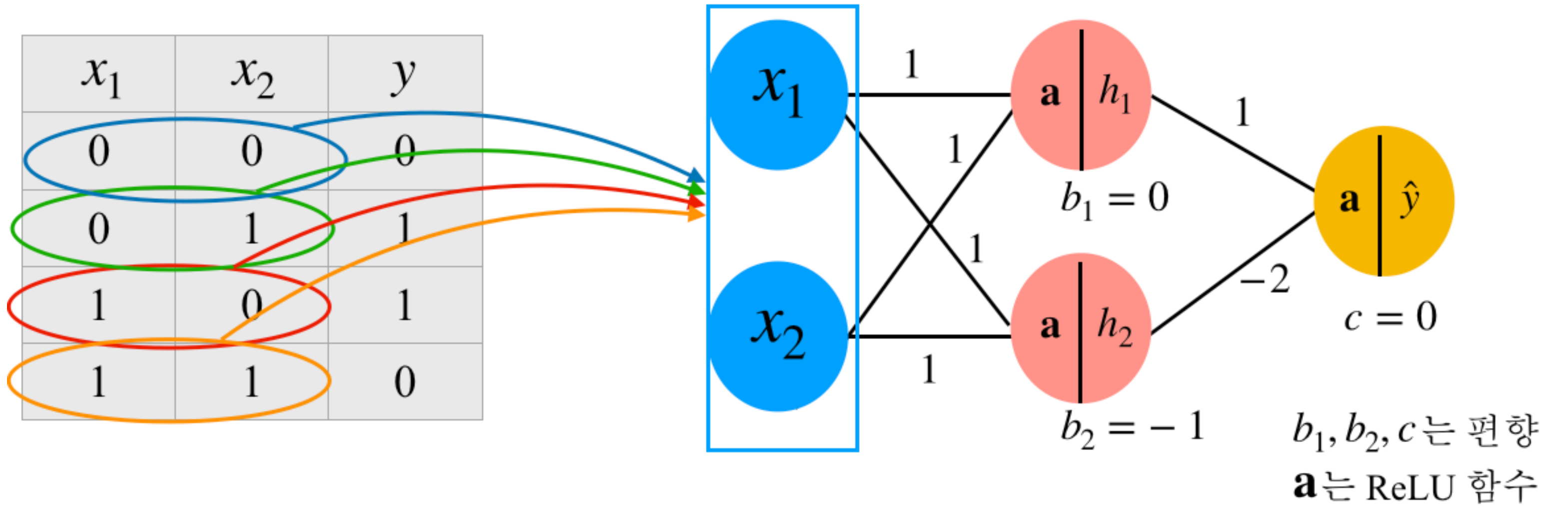

입력 값의 모임 $X = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$, 각 가중치 $H = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 & -2 \end{pmatrix}$로 정의하면 은닉층의 노드값 $B = ReLU\left(XH^T + b\right)$, 결과값 $\hat{\mathbf{y}} = ReLU\left(\mathbf{B}\mathbf{v}^T + c\right)$으로 표현할 수 있다.

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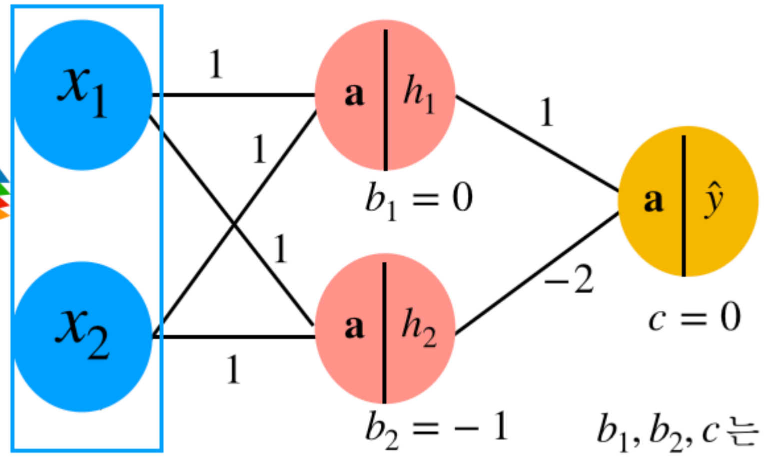
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



b_1, b_2, c는 편향
a는 ReLU 함수

$$B = ReLU\left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -1 \end{pmatrix}\right) = ReLU\left(\begin{pmatrix} 0+0 & 0-1 \\ 1+0 & 1-1 \\ 1+0 & 1-1 \\ 2+0 & 2-1 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$\hat{\mathbf{y}} = ReLU\left(\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + 0\right) = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

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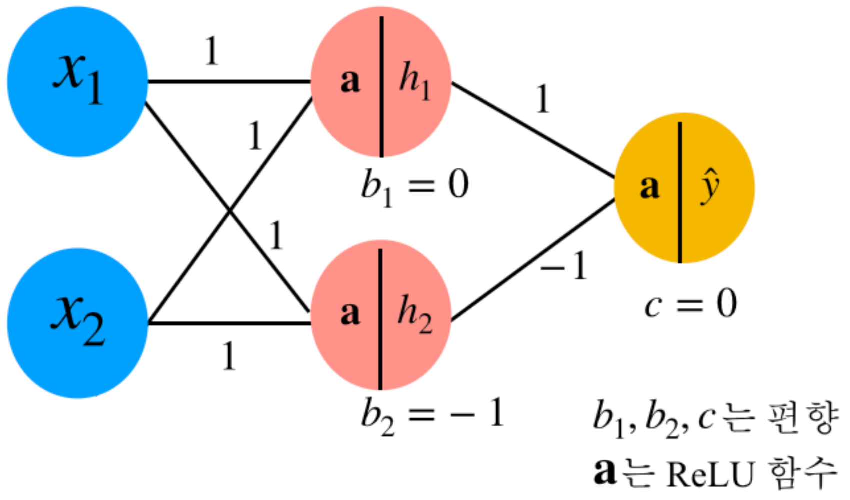
추가 예제

x_1	x_2	y
-1	0	1
0	1	1
1	2	3



$$B = ReLU\left(\begin{pmatrix} -1 & 0 \\ 0 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -1 \end{pmatrix}\right) = ReLU\left(\begin{pmatrix} -1 & -2 \\ 1 & 0 \\ 3 & 2 \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 3 & 2 \end{pmatrix}$$

$$\hat{\mathbf{y}} = ReLU\left(\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + 0\right) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

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