ASTR 210 HW 8

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Q8.1

Binary stars · max sep is 6.0", $P_{orb} = 80$ yr.
Parallax 0.4". Circular orbits.

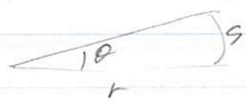
7 pts

a) Distance from parallax: 1 pc $\Leftrightarrow$ 1" parallax

$\frac{1}{0.4}$ pc $\Leftrightarrow$ 0.4"

dist = $1/0.4$ pc = 2.5 pc

At that distance, how big is 6.0"? That's the semimajor axis



$\theta = s/r$

$s = r\theta$, convert 6.0" to radians

$$a = 2.5\text{pc} \times 6.0'' \times \frac{\pi \text{ rad}}{180 \times 3600''} = 7.3 \times 10^{-5}\text{pc}$$

$$\times 3.09 \times 10^{16} \text{ m/pc}$$

$$= 2.2 \times 10^{12} \text{ m}$$

$$\times \text{AU} / 1.496 \times 10^{11} \text{ m}$$

$$= 15. \text{ AU} \quad [\text{This is } a_A + a_B]$$

Now Kepler: $M_A + M_B = \frac{4\pi^2}{G}\frac{a^3}{P^2}$

or if a in AU and P in years it's easier

$$\left(\frac{M_A + M_B}{M_\odot}\right) = \left(\frac{a}{\text{AU}}\right)^3 \left(\frac{P}{\text{yr}}\right)^{-2}$$

Thus $M_A + M_B = (15)^3 / 80^2 = 0.53 M_\odot$

4 pts

b) In order to measure the masses independently we'd need some extra information on the components' distances to the COM or their velocities. We could use the distance of one object to the COM or a measure of one object's maximum velocity with respect to the COM (and an assumption or measurement of inclination).

Q8.2

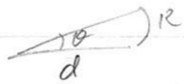
Name	Sp Type	m_B	m_V	(10^{-3}″) Diam	(″) π''	$M_\odot$ mass
A	M2 Ia	2.27	0.42	43.43	0.00451	19.6
B	M2 V	8.96	7.52	1.43	0.393	0.46

2 pts (1 each) a) distances from parallax

$A: \ d = (1/0.00451)\,pc = 222\,pc$

$B: \ d = (1/0.393)\ pc = 2.54\,pc$

4 pts (2 each) b) radii from diameters and distance



here $R = \theta \cdot d$ but you want θ in radians and you want $\left(\frac{\text{angular diameter}}{2}\right)$

$$R_A = \frac{43.43}{2} \times 10^{-3\,''} \times \frac{\pi \text{ rad}}{180 \times 3600''} \times 222\,pc \times \frac{3.09 \times 10^{16}\,m}{pc}$$

$$= 7.21 \times 10^{+11}\ m \times \frac{R_\odot}{6.96 \times 10^{8}\,m}$$

$$= 1035\ R_\odot$$

Similar Calc for R_B gives

$$R_B = 2.72 \times 10^{8}\ m = 0.39\ R_\odot$$

5 pts c) Surface gravities $g = \frac{GM_☆}{R_☆^2}$

$$g_A = \frac{6.67 \times 10^{-11}\ kg^{-1} m^3 s^{-2} \times 19.6 \times 1.99 \times 10^{30}\ kg}{(7.21 \times 10^{11}\ m)^2} = 0.0050\ m/s^2$$

g_B = same calc with $0.46\,M_\odot$ and $2.72 \times 10^{8}\,m$
$= 824\ m/s^2$

4 pts d) Giant stars are supposed to have smaller surface gravities than main sequence stars, because of the influence of that $R_☆^{-2}$ term. These examples follow that prediction; star A is a supergiant and has much lower g than B (main sequence).

4 pts e) use 13.35 and 13.36 in text:

B-V color $= m_B - m_V$

$$T \approx \frac{9000\,K}{(B-V)+0.93}$$

For star A this becomes: $T_A = \dfrac{9000\,K}{(2.27-0.42)+0.93} = 3237\,K$

Similar T_B with $B-V = 8.96-7.52 \rightarrow T_B = 3797\,K$

5 pts f) Blackbody luminosities are $L_\star = 4\pi R_\star^2 \sigma_{SB} T_\star^4$

eg for A this is $L_A = 4\pi (7.21\times10^{11}\,m)^2\, 5.67\times10^{-8} \frac{W}{m^2\cdot K^4} (3237\,K)^4$

$$= 4.06\times10^{31}\,W \times \frac{L_\odot}{3.83\times10^{26}\,W}$$

$$= 1.06\times10^5\,L_\odot$$

Similarly for B : $L_B = 1.09\times10^{25}\,W = 0.029\,L_\odot$

5 pts g) With temps ~ 3500K these stars should be somewhere between Spectral Classes K and M. Star A, $10^5 L_\odot$, is indeed a supergiant and star B's luminosity is consistent with the Main sequence. These data all hold together.

Star	Dist (pc)	R(m)	R($R_\odot$)	g(m/s^2)	T(K)	L($L_\odot$)
A	222.	7.21e11	1035.	0.0050	3237.	1.06e5
B	2.54	2.72e8	0.39	824.	3797.	0.029

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Q8.3

Estimate Main sequence lifetimes assuming stars are pure H and all of the H is available for fusion. Constant luminosity.

5 pts a) $M = 100 M_\odot$, $L = 10^6 L_\odot$.

$m_p = 1.67 \times 10^{-27}$ kg ; $M_\odot = 1.99 \times 10^{30}$ kg ; $L_\odot = 3.84 \times 10^{26}$ W
$\Delta E = 4.1 \times 10^{-12}$ J per reaction (= 4 H nuclei)

$$\tau_{100 M_\odot} = \left[\frac{100 \times 1.99 \times 10^{30}\ \text{kg}}{(4 \times 1.67 \times 10^{-27}\ \text{kg})} \times \frac{4.1 \times 10^{-12}\ \text{J}}{10^6 \times 3.84 \times 10^{26}\ \text{W}} \right] = 3.2 \times 10^{14}\ \text{s} = \boxed{1.0 \times 10^{7}\ \text{yr, aka. } 10\ \text{Myr}}$$

5 pts b) Similarly for $0.5 M_\odot$ and $0.1 L_\odot$

$$\tau_{0.5 M_\odot} = 1.6 \times 10^{19}\ \text{s} = 5.1 \times 10^{11}\ \text{yr, aka. } \boxed{510\ \text{Gyr}}$$

Aside: compare to what you get out of eq 15.55

which is $\tau \sim 10\ \text{Gyr} \left(\frac{M}{M_\odot}\right)^{-3}$

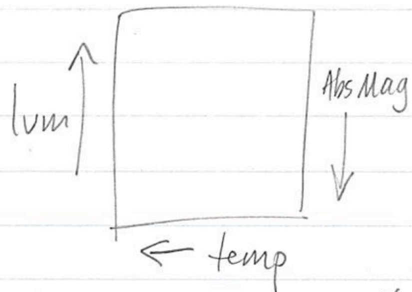
and that predicts $10\ \text{Gyr} (100)^{-3} = \frac{10 \times 10^9\ \text{yr}}{10^6} = 10^4\ \text{yr}$

and $10\ \text{Gyr} (0.5)^{-3} = 80\ \text{Gyr} = 8 \times 10^{10}\ \text{yr}$

This approximate relation is not very accurate at the extremes of mass and luminosity — so the values quoted above are better.

Q8.4

Uncertainties in the HR diagram



3 pts a) Errors in parallax primarily affect the vertical coordinate, not the horizontal one.
The horizontal one is based on color or the strengths of spectral lines, so it is independent of distance.
The vertical coordinate comes from absolute luminosity, and if we're getting L from $L = 4\pi D^2 F$ then the luminosity does depend on distance.
(If we're getting L from a spectroscopic parallax based on line widths and g, that won't depend on distance — but from context we assume that isn't the case here.)

4 pts b) $D = 1/\pi''$

Suppose $\pi''_{meas.} = 0.9\, \pi''_{true}$.

then $D_{meas} = 1/\pi''_{meas} = 1/0.9\pi''_{true} = \left(\frac{1}{0.9}\right) D_{true}$

and $L = 4\pi D^2 F \rightarrow L_{meas} = 4\pi \left(\frac{1}{0.9}\right)^2 D_{true}^2 F$

$= \left(\frac{1}{0.9}\right)^2 L_{true} = 1.23\, L_{true} = L_{meas}$

So you will place the star on the HR diagram with a vertical position 23% higher than it should be.

cont'd →

magnitude offset is a more difficult calculation and most of the credit for (b) should be assigned to the magnitude part.

defn of magnitudes goes like this: $m_1 - m_2 = \frac{5}{2}\log_{10}(F_2/F_1)$

or for luminosities and abs. mags it's similar - see also eq 13.21

$$M_1 - M_2 = \frac{5}{2}\log_{10}(L_2/L_1)$$

$$M_{true} - M_{meas} = \Delta M = \frac{5}{2}\log_{10}\left(L_{meas}/L_{true}\right)$$

$$\Delta M = \frac{5}{2}\log_{10}\left(\frac{(1/0.9)^2 \cancel{L_{true}}}{\cancel{L_{true}}}\right)$$

$$= 0.23 \text{ mag}$$

Since Abs Mag increases downwards on the diagram, this offset will also place the "wrong" position above the "true" position.

Aside: the width of the Main Sequence is substantially larger than 0.23 mag or 23%, especially if you are looking at a complex population of stars with different abundances and compositions.