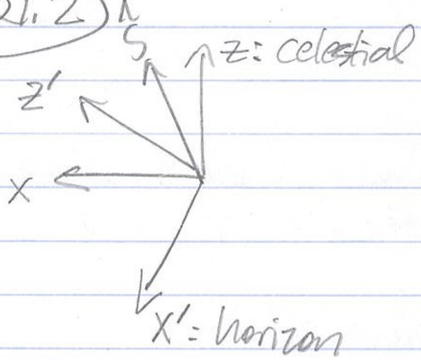
ASTR 210 HW 1

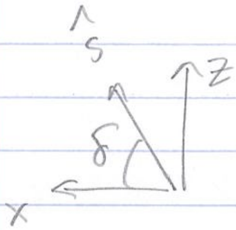
©LYoung 2023

Q1.2

a)

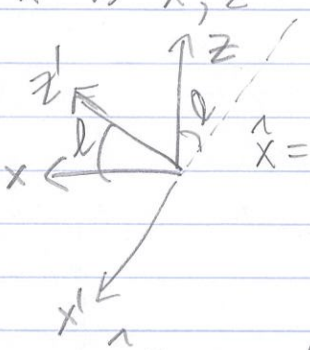


$\hat{s}$ vs $\hat{x}, \hat{z}$



$\hat{s} = \cos\delta\,\hat{x} + \sin\delta\,\hat{z}$

b) $\hat{x}$ vs $\hat{x}', \hat{z}'$



$\hat{x} = \sin l\,\hat{x}' + \cos l\,\hat{z}'$

c) $\hat{z} = -\cos l\,\hat{x}' + \sin l\,\hat{z}'$

d) plug c and d into a:

$$\hat{s} = \cos\delta(\sin l\,\hat{x}' + \cos l\,\hat{z}') + \sin\delta(-\cos l\,\hat{x}' + \sin l\,\hat{z}')$$

$$= (\cos\delta\sin l - \sin\delta\cos l)\hat{x}' + (\cos\delta\cos l + \sin\delta\sin l)\hat{z}'$$
$$= \sin(l-\delta)\hat{x}' + \cos(l-\delta)\hat{z}'$$

e) transit altitude in $(0, \pi/2)$ = distance to closest N or S horizon point

angle between $\hat{s}$ and $\hat{x}'$ is $\delta + 90° - l$

or its complement $180° - (\delta - l + 90°)$
$= 90° - \delta + l$

e) cont'd

i) $\delta > l$, then $\delta - l > 0$ and altitude is $90° + l - \delta$

ii) $\delta < l$, $\delta - l < 0$ so altitude is $90° + \delta - l$

f) NCP ($\delta = 90°$) has a transit altitude of

$$\theta = 90° + l - 90° = l$$

g) Southernmost Dec observable from C-U.

Translation: transit altitude > 0. Use case ii above.

$$\theta_{min} = 90° + \delta - l > 0$$

$$l = 40° \, 07'$$

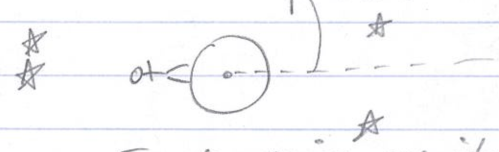
$$\delta_{lim} = 0 + l - 90° = -49° \, 53'$$

ω Cen is at Dec $= -47° \, 28'$ so is technically visible but in practice will be very difficult.

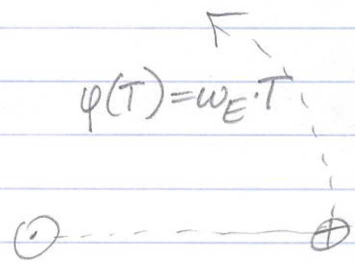
h) l ... by inspection

Q1.3

a) $\theta(T) = \omega_{sid} \cdot T$

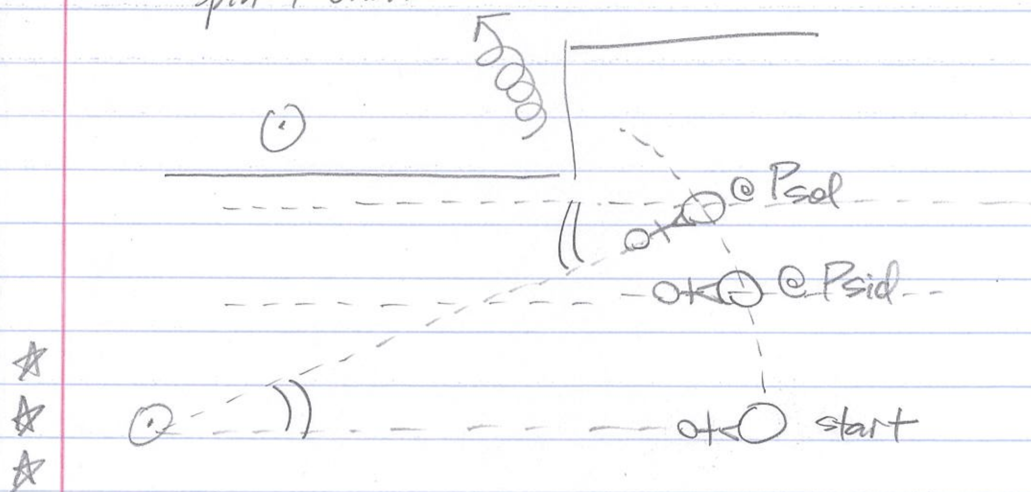


Earth spins on its axis at a rate $\omega_{sid} = 2\pi / P_{sid}$



Also orbits sun at a rate $\omega_E = 2\pi / P_E$

Combined motion of a point on the surface is spin + orbit



$\theta(0) = 0$

$\theta(P_{sid}) = 2\pi$ by definition

$\theta(P_{sol}) = \theta(P_{sid}) + \measuredangle$

$\measuredangle = \varphi(P_{sol}) = \omega_E P_{sol}$

$\Rightarrow \theta(P_{sol}) = 2\pi + \omega_E P_{sol}$

b) $\Delta\theta = \theta(P_{sol}) = 2\pi + \omega_E P_{sol}$ and also $= \omega_{sid} P_{sol}$ (see top of page)

$\therefore \omega_{sid} = (2\pi + \omega_E P_{sol}) / P_{sol}$

b) continued

$$P_{sid} = \frac{2\pi}{\omega_{sid}} = \frac{2\pi P_{sol}}{(2\pi + \omega_E P_{sol})} = \frac{P_{sol}}{\left(1 + \frac{P_{sol}}{P_E}\right)}$$ ✓

c) Taylor series: $x \equiv P_{sol}/P_E \ll 1$

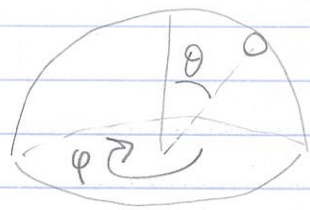
$$f(x) \approx f(0) + x f'(0)$$

$$(1+x)^{-1} \approx 1 + x(-1)(1+x)^{-2}\Big|_{x=0} \approx 1 - x$$

so $P_{sid} \approx P_{sol}\left(1 - \frac{P_{sol}}{P_E}\right)$

Q1.4

$d\Omega = \sin\theta \, d\theta \, d\varphi$



solid angle $= \int d\Omega = \int_0^{2\pi} d\varphi \int_0^{\theta_r} \sin\theta \, d\theta = 2\pi(-\cos\theta)\Big|_0^{\theta_r}$

$$\Omega = 2\pi(1-\cos\theta_r)$$

When θ_r is small, $\cos\theta_r \approx 1 - \frac{\theta_r^2}{2}$ so

$$\Omega \approx 2\pi\left(1-\left(1-\frac{\theta_r^2}{2}\right)\right) = 2\pi \cdot \frac{\theta_r^2}{2} = \pi\theta_r^2,$$

and when $\theta_r = \pi/2$, $\Omega = 2\pi$ as requested.