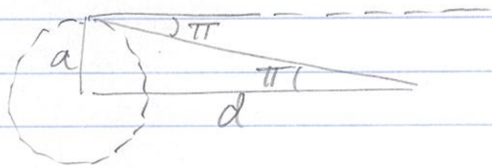
ASTR 210 HW2

Q2.1

a) $\tan \pi'' = \dfrac{a}{d}$



radians

Here a and d can be in any units aslong as they are the same;

$d = \dfrac{a}{\tan \pi''}$, Small angles: $\tan \pi'' \approx \pi''$ [rad]

$d \approx \dfrac{a}{\pi'' \,[rad]}$

$d\,[pc] \approx \dfrac{a\,[pc]}{\pi''\,[rad]}$

$\pi''\,[arcsec] = \pi''\,[rad] \times \dfrac{\pi \text{ rad}}{180 \times 3600 \text{ arcsec}}$

For observers on Earth, $a = 1$ AU

$a\,[pc] = 1 \text{ AU} \times \dfrac{1.496\times10^{8} \text{ km}}{\text{AU}} \times \dfrac{\text{pc}}{3.086\times10^{13}\text{ km}}$

$= 4.847\times10^{-6}$ i.e. $a = 4.8\times10^{-6}$ pc

$d\,[pc] = \dfrac{4.847\times10^{-6}}{\pi''\,[arcsec]} \times \dfrac{180\times3600}{\pi} = \dfrac{1.0}{\pi''\,[arcsec]}$

b) if you could detect parallax of 3 arcmin (180")
you could see the parallax of a star at

$d = \dfrac{1.0}{180}$ pc $= 0.0056$ pc $= 4.3\times10^{-3}$ of

But the closest star is 1.3 pc away – we could Not see parallax!

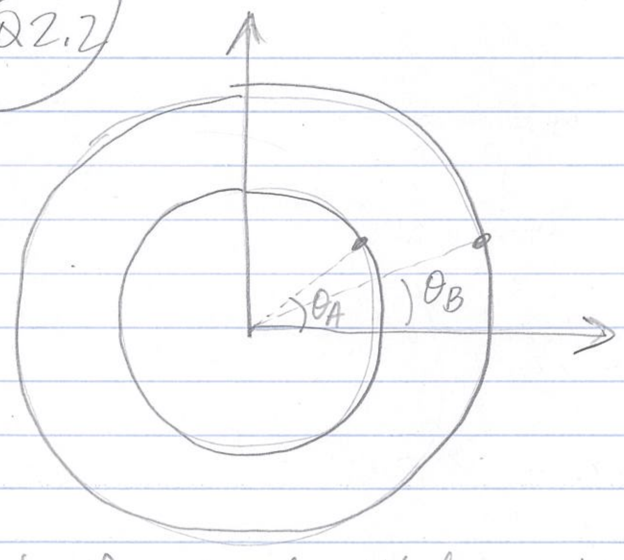
c) Now we can measure parallaxes of 0.1×10^{-3} arcsec

$\Rightarrow$ distances $\frac{1}{0.1 \times 10^{-3}}$ pc $= 10^4$ pc

$\Rightarrow$ we can go slightly beyond the center of our Galaxy.

d) An Earth-centered cosmology has no annual parallax, and the fact that nobody (until recently) could detect any parallax seems like confirmation of the Ptolemaic model. But it's actually more like absence of evidence than evidence of absence.

Q2.2



a) ok, if sidereal orbital periods are P_A and P_B and orbits are circles, $\dot{\theta}_A = \text{const}$ and $\dot{\theta}_B = \text{const}$

$$\dot{\theta}_A = \frac{2\pi}{P_A} \qquad \dot{\theta}_B = \frac{2\pi}{P_B}$$

$$\rightarrow \theta_A(t) = \frac{2\pi}{P_A} t + \beta_A$$

$$\theta_B(t) = \frac{2\pi}{P_B} t + \beta_B$$

b) let's look at a conjunction btwn planets: $\theta_A(t_1) - \theta_B(t_1) = 0$

A went around exactly one time more than B

So the next conjunction is at t_2 where $\theta_A(t_2) - \theta_B(t_2) = 2\pi$

synodic period is $P_{syn} = t_2 - t_1$. Find P_{syn}.

ok: $\theta_A(t_1) - \theta_B(t_1) = 0 = \frac{\overset{1}{\cancel{2\pi}}}{P_A} t_1 + \beta_A - \frac{\overset{1}{\cancel{2\pi}}}{P_B} t_1 - \beta_B$

$\theta_A(t_2) - \theta_B(t_2) = \overset{1}{\cancel{2\pi}} = \frac{\overset{1}{\cancel{2\pi}}}{P_A} t_2 + \beta_A - \frac{\overset{1}{\cancel{2\pi}}}{P_B} t_2 - \beta_B$

2 eq
2 unk ($t_2 - t_1$, $\beta_A - \beta_B$)

subtract! linear system

rewrite $$0 = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)t_1 + \beta_A - \beta_B$$

$$1 = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)t_2 + \beta_A - \beta_B$$

$$1 = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)(t_2 - t_1) = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)P_{syn}.$$

c) Earth is farther out: $B \to \oplus$

$$\frac{1}{P_{syn}} = \frac{1}{P_p} - \frac{1}{P_\oplus} \quad \text{so} \quad \frac{1}{P_p} = \frac{1}{P_{syn}} + \frac{1}{P_\oplus}$$

d) Earth is closer in: $A \to \oplus$

$$\frac{1}{P_{syn}} = \frac{1}{P_\oplus} - \frac{1}{P_p} \quad \text{so} \quad \frac{1}{P_p} = \frac{1}{P_\oplus} - \frac{1}{P_{syn}}$$

e) Find P_{syn} in terms of P_p and $P_\oplus$ for case d and let $P_p \to \infty$

$$P_{syn} = \left(\frac{1}{P_\oplus} - \frac{1}{P_p}\right)^{-1} = \left(\frac{P_p - P_\oplus}{P_\oplus P_p}\right)^{-1} = \frac{P_\oplus P_p}{P_p - P_\oplus}$$

lim of that as $P_p \to \infty$ is $P_\oplus$

which makes sense because outer planet gets v far away and is basically stationary so $\oplus$ motion is the only relevant bit

f) Similarly for case c and let $P_p \to P_\oplus$ from below

$$P_{syn} = \left(\frac{1}{P_p} - \frac{1}{P_\oplus}\right)^{-1} = \left(\frac{P_\oplus - P_p}{P_p P_\oplus}\right)^{-1} = \frac{P_p P_\oplus}{P_\oplus - P_p}$$

as P_p approaches $P_\oplus$ this blows up to ∞

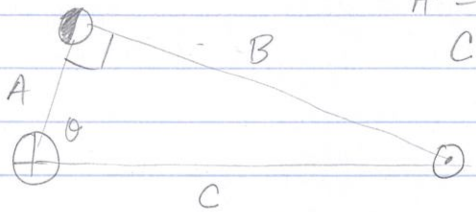
which I suppose makes sense for the same reason that it's safest to match speeds on thwy

g) Mars is $P = 1.881 P_\oplus$ so $P_{syn, mars} = \frac{1.881 P_\oplus^{\cancel{2}}}{(1.881-1)\cancel{P_\oplus}} = \frac{1.881}{0.881} P_\oplus$

Neptune $P = 164.79 P_\oplus$ $\quad P_{syn, Nep} = \frac{164.79}{163.79} P_\oplus$ ok

Q2.3

HW2 extra credit



$A = C \cos\theta$ see Fig 2.1

$C = A/\cos\theta$

let's treat A as const and just work out unc in C due to unc in θ.

uncertainty $\sigma_C = \left|\frac{dC}{d\theta}\right| \sigma_\theta$ At least that's the one component of σ_C

but $C = A/\cos\theta = A(\cos\theta)^{-1}$

$dC/d\theta = A(-1)(\cos\theta)^{-2}(-\sin\theta) = \frac{A\sin\theta}{\cos^2\theta}$

eval:

θ	$\sin\theta/\cos^2\theta$
87°	364
88°	820
89°	3282 wow, up by ×10.