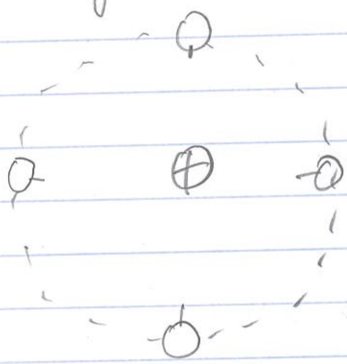
Hw 4: Ch 4

Q4.1

5 pts a) Tidally locked moon –



Moon spins once for every orbit around $\oplus$
so $\omega_{spin} = \omega_{orb}$

sidereal orbital period is 27.32d
and, for uniform circular motion,
$v = r\omega$. Here $V_{eq,M} = R_M \cdot \omega_{spin}$

$$V_{eq,M} = R_M \cdot \omega_{spin} = 1.737\times10^6\,m \cdot \frac{2\pi}{27.32d} \times \frac{1d}{24\times3600s} = 4.6\ m/s$$

$$V_{eq,\oplus} = \frac{2\pi \cdot R_\oplus}{0.997\times24\times3600s} = \frac{2\pi\cdot6.371\times10^6 m}{0.997\times24\times3600s} = 465\ m/s$$

↑ Not too fussy about the difference btwn sidereal day and solar day in this context

Moon's equatorial speed is much slower than Earth's

5 pts b) orbital speeds. For this we want to use orbital radii, not physical radii

$$V_{orb,M} = \frac{2\pi \cdot a_M}{27.32d} = \frac{2\pi \cdot 384.4\times10^6 m}{27.32\times24\times3600s} = 1020\ m/s$$

$$V_{orb,\oplus} = \frac{2\pi \cdot a_\oplus}{365.256d} = \frac{2\pi \cdot 1.496\times10^{11} m}{365.256\times24\times3600s} = 2.98\times10^4\ m/s$$

As expected, all the earth speeds are larger than corresponding Moon ones and orbital speeds are many times larger than rotational (spin) speeds. As an aside, the orbital speed of our Sun around the center of the Galaxy is about 2×10^5 m/s

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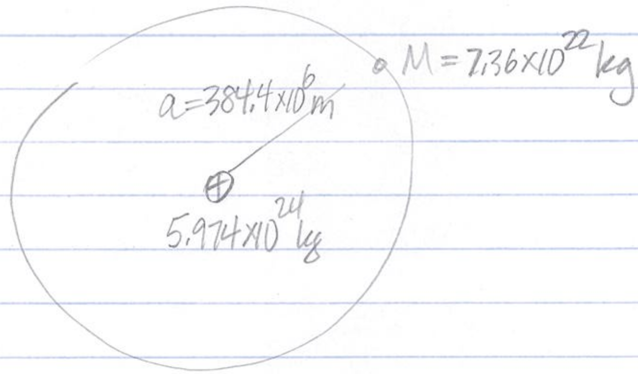
4.2



4 pts a) Assuming we're supposed to use one of Kepler's laws here

$P^2 = \frac{4\pi^2 a^3}{GM_\oplus}$ and orbital period relates to ang. vel

ω as $\omega = 2\pi/P$. So $\left(\frac{2\pi}{\omega}\right)^2 = \frac{4\pi^2 a^3}{GM_\oplus}$.

$\omega^2 = GM_\oplus/a^3 \rightarrow \omega_{orb} = (GM_\oplus/a^3)^{1/2}$

4 pts b) orbital Ang. Mom $\vec{L} = m\vec{r}\times\vec{V}$. In nice circular orbits this becomes

$L = mr^2\omega$ or in this case $L_{orb} = M_M\, a^2 \frac{\sqrt{GM_\oplus}}{a^{3/2}}$

$L_{orb} = M_M\sqrt{GM_\oplus a}$

5 pts c) Earth as a sphere with $M_\oplus$ and $R_\oplus$ and ω_{sid}

spin ang. mom is $L_{rot} = I\omega_{sid} = \frac{2}{5}M_\oplus R_\oplus^2\,\omega_{sid}$

7 pts d) $L_{tot} = L_{rot} + L_{orb} = \text{const}$ so $dL_{tot}/dt = 0$.

$\frac{dL_{rot}}{dt} = -\frac{dL_{orb}}{dt}$.

$$\left.\begin{aligned}\frac{dL_{rot}}{dt} &= \frac{2}{5}M_\oplus R_\oplus^2\frac{d\omega_{sid}}{dt}\\ \frac{dL_{orb}}{dt} &= M_M\sqrt{GM_\oplus}\,\frac{1}{2}a^{-1/2}\frac{da}{dt}\end{aligned}\right\}\quad \frac{2}{5}M_\oplus R_\oplus^2\frac{d\omega_{sid}}{dt} = -\frac{M_M}{2}\frac{\sqrt{GM_\oplus}}{a^{1/2}}\,\dot{a}$$

continued →

d) from prev. side $\frac{2}{5} M_\oplus R_\oplus^2 \frac{d\omega_{sid}}{dt} = \frac{-M_M}{2} \frac{\sqrt{GM_\oplus}}{a^{1/2}} \dot{a}$

want $\frac{\dot{a}}{a}$ = something

$$\frac{\dot{a}}{a^{1/2}} = \frac{-4}{5} \frac{M_\oplus}{M_m} R_\oplus^2 \frac{d\omega_{sid}}{dt} \cdot \frac{1}{\sqrt{GM_\oplus}}$$

From (a): $\omega_{orb} = \sqrt{GM_\oplus} \cdot a^{-3/2}$ so $\sqrt{GM_\oplus} = \omega_{orb} \cdot a^{3/2}$

$$\boxed{\frac{\dot{a}}{a^{1/2}} = \frac{-4}{5} \frac{M_\oplus}{M_M} \left(\frac{d\omega_{sid}}{dt}\right) \frac{R_\oplus^2}{\omega_{orb} a^{3/2}}} \times \frac{1}{a^{1/2}}$$

$$\frac{\dot{a}}{a} = \frac{-4}{5} \frac{M_\oplus}{M_M} \left(\frac{d\omega_{sid}}{dt}\right) \frac{R_\oplus^2}{a^2} \cdot \frac{1}{\omega_{orb}}$$

6 pts e) $dP_{rot}/dt = 5.2 \times 10^{-13}$ s/s $\Rightarrow$ use for $d\omega_{sid}/dt$

here we are using P_{rot} to refer to $\oplus$ sidereal day

so $\omega_{sid} = 2\pi / P_{rot}$

$$\frac{d\omega_{sid}}{dt} = 2\pi (-1) P_{rot}^{-2} \frac{dP_{rot}}{dt} \cdot \quad P_{rot} = 0.997\,d \ \left(\begin{matrix}\text{Tab}\\ \text{A3}\end{matrix}\right)$$

$\omega_{orb} = \frac{2\pi}{P_{orb}}$ for which we'll use sidereal month 27.32 d

$$\dot{a} = +\frac{4}{5}\left(\frac{5.974 \times 10^{24}\,\cancel{kg}}{7.36 \times 10^{22}\,\cancel{kg}}\right) \frac{\cancel{2\pi}}{(0.997\,d)^2} \left(5.2 \times 10^{-13} \frac{\cancel{s}}{\cancel{s}}\right) \left(\frac{(6.378 \times 10^6\,m)^2}{384.4 \times 10^6\,m}\right) \cdot \frac{27.32\,d}{\cancel{2\pi}}$$

= units will be $\frac{m^2}{m} \cdot \frac{d}{d^2} = m/\text{day}$ ok

$$\dot{a} = 9.82 \times 10^{-5} \frac{m}{d} \times \frac{100\,cm}{m} \times \frac{365.256\,d}{yr} = 3.6\ cm/yr!$$

4.3

4 pts a) Tab A.3 Jupiter radius $10.97\,R_\oplus = 10.97 \times 6.378\times10^6\,m$

mass $317.8\,M_\oplus = 317.8 \times 5.974\times10^{24}\,kg$

$$\text{density} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3\times317.8\times5.974\times10^{24}\,kg}{4\pi\,(10.97\times6.378\times10^6\,m)^3} = 1.32\times10^3\,\frac{kg}{m^3}$$

4 pts b) ice density 1000 kg/m³ fairly similar to Jupiter value

Roche Limit for Jupiter's moons is $2.44\left(\frac{\rho_{big}}{\rho_{small}}\right)^{1/3} R_{big}$

here ρ_{big} and R_{big} refer to Jupiter
ρ_{small} to the small body (ice)

$\Rightarrow$ Roche Limit is $2.44\left(\frac{1320}{1000}\right)^{1/3} R_J = 2.68\,R_J$

ok to use 2.5 here

$\times\ 10.97\times6.378\times10^3\,km = 1.9\times10^5\,km$

3 pts c) Innermost moons have orbital radii 128,000 km
vs Roche Limit at 190,000 km (approx)
These moons are inside the Roche Limit and they are vulnerable to tidal disruption

3 pts d) Kepler's laws work: $P^2 = \frac{4\pi^2}{GM_J}a^3$

$$\Rightarrow P = 2\pi\left[\frac{(1.28\times10^8\,m)^3}{6.67\times10^{-11}\,m^3\,kg^{-1}\,s^{-2}\cdot317.8\times5.974\times10^{24}\,kg}\right]^{1/2}$$

$P = 2.56\times10^4\,s$

$\times \frac{hr}{3600} = 7.1\,hr$

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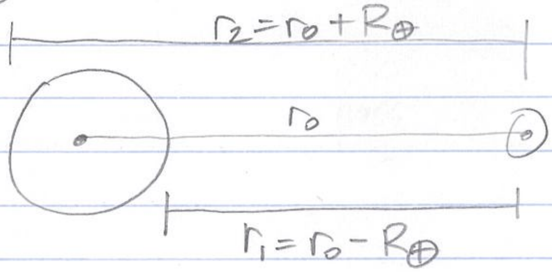
5 pts

Q4.4 extra credit



a) m @ center of $\oplus$

F_C by moon is $\dfrac{GM_M m}{r_0^2}$

b) m on near side of $\oplus$

F_R by moon is $\dfrac{GM_M m}{r_1^2} = \dfrac{GM_M m}{(r_0 - R_\oplus)^2}$

c) $\Delta F = F_R - F_C = GM_M m\left(\dfrac{1}{r_0^2} - \dfrac{1}{(r_0 - R_\oplus)^2}\right)$

$$= \frac{GM_M m}{r_0^2}\left(1 - \frac{1}{(1 - R_\oplus/r_0)^2}\right)$$

Call $R_\oplus/r_0 \equiv x$ and $x \ll 1$.

use $f(x) \approx f(0) + x f'(0)$

$f(x) = (1-x)^{-2} \Rightarrow f(0) = 1$

$f'(x) = -2(1-x)^{-3} \Rightarrow f'(0) = -2$

$$\Delta F \approx \frac{GM_M m}{r_0^2}\left(1 - [1 + x(-2)]\right) = \frac{GM_M m}{r_0^2} \cdot 2\frac{R_\oplus}{r_0}$$

$$\Delta F \approx \frac{2GM_M m R_\oplus}{r_0^3}$$