ASTR 210 HW 11

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Q11.1 Redshifts and Hubble law

$H\beta$ $\lambda_{rest} = 486.135\,nm$ and $\lambda_{obs} = 497.97\,nm$

3 pts a) $z \equiv \dfrac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = 0.024$

5 pts b) $m - M = 5\log\left(d/10\,pc\right) \Rightarrow$ rewrite using z instead of d

nonrelativistic limit of that Hubble relation is

$cz = H_0 \cdot d$ or $d = cz/H_0$

$$m - M = 5\log\left(\frac{cz/H_0}{10\,pc}\right)$$

we now want to plug in for c, H_0

$$\frac{c}{H_0 \cdot 10pc} = \frac{2.998\times10^8\,m/s}{70\times10^3\,m/(s\cdot Mpc)\times 10pc} = \frac{2.998\times10^8\,m/s}{70\times10^3\,m/s} \cdot \frac{10^6\,pc}{10\,pc}$$

$$= 4.28\times10^7$$

$$\Rightarrow m - M = 5\log\left(4.28\times10^7 \cdot z\right) = 5\log z + 5\log\left(4.28\times10^7\right)$$

$$= 5\log z + 43.16$$

or $m - M = 5\log z + 43.2$

3 pts c) galaxy $m_V = 12.7$ and $z = 0.024$

$$M_V = m - 5\log(0.024) - 43.2 = -22.4$$

2 pts d) $M_V = -22.4$

Text reports that the most luminous Es have $M_V \sim -23$ whereas bright spirals like M31 have $M_V \sim -21$

So this galaxy we are studying has a very high luminosity and is more likely to be a giant elliptical.

10 pts

Q11.2 Eddington luminosity and massive stars

The Eddington luminosity – Theoretically – is the highest luminosity that an accreting object can have without its own radiation pressure overcoming its own self-gravity.

$$L_{Edd} = \frac{4\pi G m_p c}{\sigma_e} \cdot M_{bh}$$ or in this case it'll be M_* instead of M_{bh}

Massive stars are observed to obey a mass-luminosity relation $\frac{L}{L_\odot} = 1.02\left(\frac{M}{M_\odot}\right)^{3.92}$ eq 13.78

Now let's require $L < L_{Edd}$ and see what that says about mass.

$$L = 1.02 L_\odot \left(\frac{M}{1 M_\odot}\right)^{3.92} \text{ must be } < \frac{4\pi G m_p c}{\sigma_e}\left(\frac{M}{1 M_\odot}\right) \cdot 1 M_\odot$$

$$1.02 L_\odot \left(\frac{M}{1 M_\odot}\right)^{2.92} < \frac{4\pi G m_p c}{\sigma_e} \cdot 1 M_\odot$$

$$\left(\frac{M}{1 M_\odot}\right)^{2.92} < \frac{4\pi G m_p c}{\sigma_e} \cdot \frac{1 M_\odot}{1.02 L_\odot}$$

$$\left(\frac{M}{1 M_\odot}\right) < \left[\frac{4\pi\ 6.67\times10^{-11}\ kg^{-1} m^3 s^{-2} \cdot 1.67\times10^{-27}\ kg \cdot 2.998\times10^{8}\ m/s \cdot 1.99\times10^{30}\ kg}{6.65\times10^{-29}\ m^2 \cdot 1.02 \cdot 3.84\times10^{26}\ W}\right]^{1/2.92}$$

$$\left(\frac{M}{1 M_\odot}\right) < 35.0 \quad \text{aka} \quad M < 35\ M_\odot$$

Our text shows the most massive stars in Fig 13.11 to be not quite $20 M_\odot$, which suggests the Eddington limit could be playing a role for massive stars. However a quick Wikipedia search lists a number of stars (mostly variable stars) with masses upto $\sim 200 M_\odot$, so evidently the simple Eddington analysis here is not the whole story.

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Q11.3 Stellar close encounters

5 pts

a) Stars can disrupt each others' orbits and planetary systems if they approach within about the gravitational scattering radius $R_{eff} \sim 6GM_*/V^2$

Aside: Notice that this relation can be rearranged as

$$V^2 \sim \frac{6GM_*}{R_{eff}} \quad \text{or} \quad M_* V^2 \sim \frac{6GM_* M_*}{R_{eff}}$$ so it is clearly

based on roughly equating kinetic energy and grav. potential energy.

Estimate $R_{eff} \sim \frac{6GM_*}{V^2} \sim \frac{6 \cdot 6.67\times10^{-11}\ kg^{-1}m^3s^{-2} \cdot 1.99\times10^{30}\ kg}{(30\times10^3\ m/s)^2}$

$R_{eff} \sim 8.8\times10^{11}\ m$
$\sim 5.9\ AU$

That's much larger than $R_\odot$, of course, but still only about the radius of Jupiter's orbit.

6 pts

b New estimate of collision time using this value instead of $R_\odot$.

Original estimate came from eq 22.16: $t_* \sim \frac{1}{nV\pi(2R_\odot)^2}$

Another aside: this looks like the mean free path material in Chapter 5 and that's not a coincidence

Anyway here we can plug $n \sim 0.1$ star/pc^3 and $V \sim 30$ km/s

$$t_* \sim \left[0.1\ pc^{-3} \times 30\ km/s \times \pi \times 4 R_{eff}^2\right]^{-1}$$
$$= \left[0.1 pc^{-3} \left(\frac{pc}{3.09\times10^{16} m}\right)^3 30\times10^3\ m/s \cdot 4\pi \cdot (8.8\times10^{11} m)^2\right]^{-1}$$

$t_* \sim 9.9\times10^{20}\,s \sim 3.2\times10^{13}\,yr = 3.2\times10^{4}\,Gyr$

Also ok to scale from eq 22.17 by just plugging in the new value $R_{eff} \approx 1272\,R_\odot$

Though note this route is less accurate because the scaling constant 5×10^{10} Gyr is only quoted to 1 digit.

3 pts c) by analogy with what's in the text: if the time between these sorts of soft collisions is only 3.2×10^{4} Gyr, and stars in the solar neighborhood are on average about 5 Gyr old, the probability that each one of them will have suffered a "collision" is

$$\sim \frac{5\,Gyr}{3.2\times10^{4}\,Gyr} \sim 1 \text{ in } 6400.$$

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Q11.4

Newtonian universe whose density matches the critical density

5 pts a) eq 23.32 (Friedmann eqn) was $H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{2k}{r_0^2 a^2}$

but if $\rho = \rho_c$ then $k = 0$
and we also note that matter density drops with a^3,
ie $\rho(t) = \rho_0 a^{-3}$

So that's how we end up with the relation

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho_{c,0}}{3a^3}.$$

Students do not have to do this preceding material; start here.

$$\text{mult by } a^2 \rightarrow \quad \dot{a}^2 = \frac{8\pi G\rho_{c,0}}{3a}$$

$$\text{or} \quad \dot{a} = \left(\frac{8\pi G\rho_{c,0}}{3}\right)^{1/2} a^{-1/2} = da/dt$$

$$\left(\frac{8\pi G\rho_{c,0}}{3}\right)^{1/2} \int_0^t dt = \int_0^a a^{1/2}\, da$$

$$\left(\frac{8\pi G\rho_{c,0}}{3}\right)^{1/2} t = \frac{2}{3} a^{3/2}$$

$$a = \left[\frac{3}{2}\left(\frac{8\pi G\rho_{c,0}}{3}\right)^{1/2}\right]^{2/3} t^{2/3} = \left(\frac{3}{2}\right)^{2/3}\left(\frac{8\pi G\rho_{c,0}}{3}\right)^{1/3} t^{2/3}$$

In this universe we also have

$$H^2 = \frac{8\pi G\rho_{c,0}}{3a^3} \quad \text{or} \quad H_0^2 = \frac{8\pi G\rho_{c,0}}{3} \quad \text{since } a = 1 \text{ now}$$

So you can also rewrite the above as

$$a(t) = \left(\frac{3}{2}\right)^{2/3} H_0^{2/3} t^{2/3}.$$

3 pts b) t_0 is now, when $a=1$; so $1 = \left(\frac{3}{2}\right)^{2/3} H_0^{2/3} t_0^{2/3}$

or $1 = \frac{3}{2} H_0 t_0$ or $t_0 = \left(\frac{2}{3}\right) H_0^{-1}$

5 pts

c) We just found, above, that the age (t_0) of this Newtonian universe where $\rho_0 = \rho_{c,0}$ and $K=0$

is $t_0 = \frac{2}{3H_0}$. How old is that?

$$t_0 = \frac{2}{3H_0} = \frac{2 \cdot \text{Mpc}}{3 \cdot 70 \text{ km/s}} = \frac{2 \cdot 10^6 \cdot 3.09 \times 10^{16} \text{ m}}{3 \cdot 70 \times 10^3 \text{ m/s}} = 2.9 \times 10^{17} \text{ s}$$

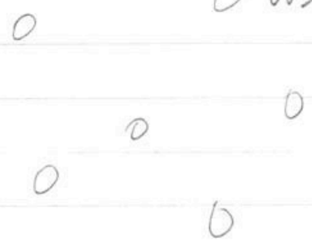
$$= 9.3 \times 10^9 \text{ yr}$$

That value is too short to be consistent with the known ages of some stars, which are 13 Gyr.

Q11.5 Olbers's paradox

2 pts

a) o WD cinders, $M \sim 0.7 M_\odot$ and $R \sim 0.01 R_\odot$



Critical density now is $\rho_{c,0} = \dfrac{3 H_0^2}{8\pi G} = 9.2 \times 10^{-27}$ kg/m³ or $1.4 \times 10^{11}\, M_\odot/\text{Mpc}^3$. } eq 23.34

if each of them is $0.7 M_\odot$ and $\rho = \underbrace{n}_{\#/\text{vol}} \cdot \underbrace{m}_{\text{mass of each}}$

This means $n_{WD} = \rho/m = \dfrac{1.4 \times 10^{11}\, M_\odot/\text{Mpc}^3}{0.7 M_\odot} = 2 \times 10^{11}\ \text{Mpc}^{-3}$

3 pts

b) Average distance to which you can see before your l.o.s. intersects a WD cinder is given by Olbers's radius

$$r_{Olb} \approx \frac{1}{n_* \pi R_*^2} \qquad (\text{eq } 23.7)$$

and that would be $r_{Olb} \sim (n_{WD} \pi R_{WD}^2)^{-1}$

$$r_{Olb} = \left(\frac{2 \times 10^{11}\ \text{stars}}{\text{Mpc}^3} \times \left(\frac{\text{Mpc}}{3.09 \times 10^{16} \times 10^{6}\ \text{m}} \right)^3 \pi \times \left(10^{-2} \times 6.96 \times 10^{8}\ \text{m}\right)^2 \right)^{-1}$$

$$r_{Olb} = 9.7 \times 10^{41}\ \text{m} = 3.1 \times 10^{19}\ \text{Mpc}$$

2 pts

c) We can see galaxies out to 4000 Mpc. But if the universe were filled with WD cinders at the critical density, we would still expect to be able to see farther than 4000 Mpc.

No, the fact that we see galaxies to 4000 Mpc does not produce any useful constraints on the number density of WD cinders. They're just too small and too far apart to be interesting.

3 pts

d) What about if they were basketballs, $M = 0.6$ kg and $R = 0.12$ m?

$$n_{bb} = \rho/m = \frac{9.2\times10^{-27}\ \text{kg/m}^3}{0.6\ \text{kg}} = 1.5\times10^{-26}\ \text{bb/m}^3$$

Average distance to which you could see would be

$$r_{bb} \sim \left(1.5\times10^{-26}\ \frac{\text{bb}}{\text{m}^3} \times \pi \times (0.12\,\text{m})^2\right)^{-1}$$
$$= 1.5\times10^{27}\ \text{m}$$
$$= 49\,000\ \text{Mpc}$$

Still not small enough to be interesting, but much closer to being interesting than for the case of the WD cinders. Of course that is because WD cinders are much more dense than basketballs.
In the case of the basketballs there might occasionally be a basketball in front of a galaxy.