ASTR 210 HW2

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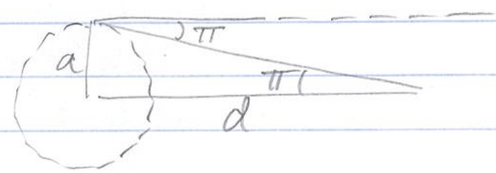
Q2.1

a) $\tan \pi'' = \dfrac{a}{d}$



radians

Here a and d can be in any units as long as they are the same;

$d = \dfrac{a}{\tan \pi''}$, Small angles: $\tan \pi'' \approx \pi''$ [rad]

$d \approx \dfrac{a}{\pi'' \text{[rad]}}$

$d \text{[pc]} \approx \dfrac{a \text{[pc]}}{\pi'' \text{[rad]}}$

$\pi'' \text{[arcsec]} = \pi'' \text{[rad]} \times \dfrac{\pi \text{ rad}}{180 \times 3600 \text{ arcsec}}$

For observers on Earth, $a = 1$ AU

$a \text{[pc]} = 1 \text{ AU} \times \dfrac{1.496 \times 10^{8} \text{ km}}{\text{AU}} \times \dfrac{\text{pc}}{3.086 \times 10^{13} \text{ km}}$

$= 4.847 \times 10^{-6}$ i.e. $a = 4.8 \times 10^{-6}$ pc

$d \text{[pc]} = \dfrac{4.847 \times 10^{-6}}{\pi'' \text{[arcsec]}} \times \dfrac{180 \times 3600}{\pi} = \dfrac{1.0}{\pi'' \text{[arcsec]}}$

b) if you could detect parallax of 3 arcmin (180")
you could see the parallax of a star at
$d = \dfrac{1.0}{180}$ pc $= 0.0056$ pc $= 4.3 \times 10^{-3}$ of

But the closest star is 1.3 pc away – we could
Not see parallax!

c) Now we can measure parallaxes of 0.1×10^{-3} arcsec

$\Rightarrow$ distances $\dfrac{1}{0.1 \times 10^{-3}}$ pc $= 10^4$ pc

$\Rightarrow$ we can go slightly beyond the Center of our Galaxy.

An Earth-centered cosmology has no annual parallax, and the fact that nobody (until recently) could detect any parallax seems like confirmation of the Ptolemaic model. But it's actually more like absence of evidence than evidence of absence.

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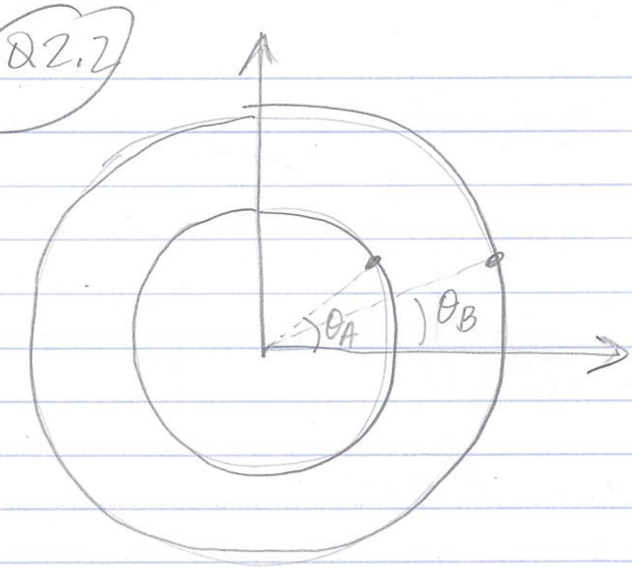
Q2.2



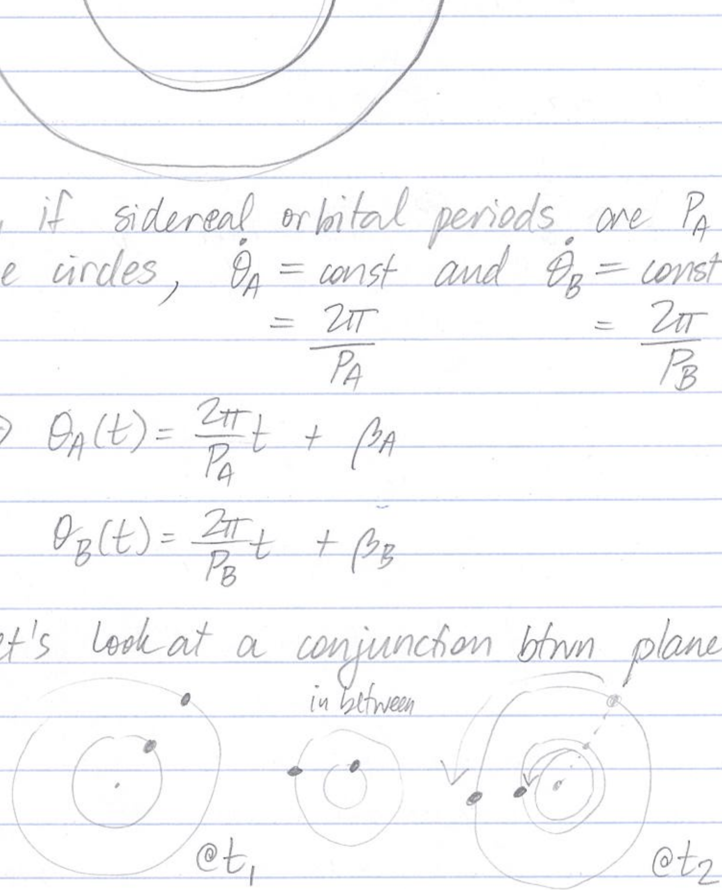
a) ok, if sidereal orbital periods are P_A and P_B and orbits are circles, $\dot{\theta}_A = \text{const}$ and $\dot{\theta}_B = \text{const}$

$$= \frac{2\pi}{P_A} \qquad\qquad = \frac{2\pi}{P_B}$$

$$\rightarrow \theta_A(t) = \frac{2\pi}{P_A} t + \beta_A$$

$$\theta_B(t) = \frac{2\pi}{P_B} t + \beta_B$$

b) let's look at a conjunction btwn planets: $\theta_A(t_1) - \theta_B(t_1) = 0$



A went around exactly one time more than B

So the next conjunction is at t_2 where $\theta_A(t_2) - \theta_B(t_2) = 2\pi$

synodic period is $P_{syn} = t_2 - t_1$. Find P_{syn}.

ok: $\theta_A(t_1) - \theta_B(t_1) = 0 = \frac{\cancel{2\pi}}{P_A} t_1 + \beta_A - \frac{\cancel{2\pi}}{P_B} t_1 - \beta_B$

$\theta_A(t_2) - \theta_B(t_2) = \cancel{2\pi} = \frac{\cancel{2\pi}}{P_A} t_2 + \beta_A - \frac{\cancel{2\pi}}{P_B} t_2 - \beta_B$

2 equations
2 unk ($t_2 - t_1$, $\beta_A - \beta_B$)

subtract! linear system

rewrite $$0 = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)t_1 + \beta_A - \beta_B$$

$$1 = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)t_2 + \beta_A - \beta_B$$

$$1 = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)(t_2 - t_1) = \left(\frac{1}{P_A} - \frac{1}{P_B}\right)P_{syn}\,.$$

c) Earth is farther out: $B \rightarrow \oplus$

$$\frac{1}{P_{syn}} = \frac{1}{P_p} - \frac{1}{P_\oplus} \quad \text{so} \quad \frac{1}{P_p} = \frac{1}{P_{syn}} + \frac{1}{P_\oplus}$$

d) Earth is closer in: $A \rightarrow \oplus$

$$\frac{1}{P_{syn}} = \frac{1}{P_\oplus} - \frac{1}{P_p} \quad \text{so} \quad \frac{1}{P_p} = \frac{1}{P_\oplus} - \frac{1}{P_{syn}}$$

e) Find P_{syn} in terms of P_p and $P_\oplus$ for case d and let $P_p \rightarrow \infty$

$$P_{syn} = \left(\frac{1}{P_\oplus} - \frac{1}{P_p}\right)^{-1} = \left(\frac{P_p - P_\oplus}{P_\oplus P_p}\right)^{-1} = \frac{P_\oplus P_p}{P_p - P_\oplus}$$

lim of that as $P_p \rightarrow \infty$ is $P_\oplus$

which makes sense because outer planet gets v far away and is basically stationary so $\oplus$ motion is the only relevant bit

f) Similarly for case c and let $P_p \rightarrow P_\oplus$ from below

$$P_{syn} = \left(\frac{1}{P_p} - \frac{1}{P_\oplus}\right)^{-1} = \left(\frac{P_\oplus - P_p}{P_p P_\oplus}\right)^{-1} = \frac{P_p P_\oplus}{P_\oplus - P_p}$$

as P_p approaches $P_\oplus$ this blows up to ∞

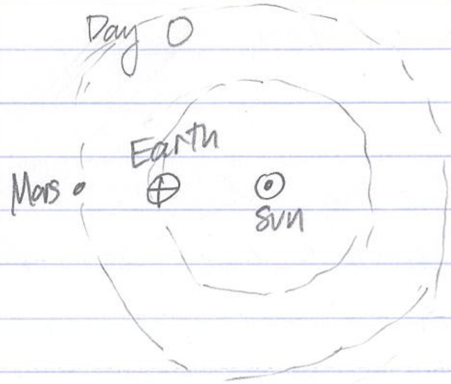
which I suppose makes sense for the same reason that it's safest to match speeds on hwy

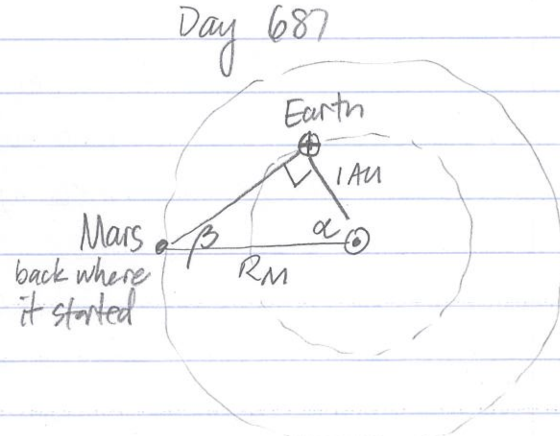
g) Mars is $P = 1.881 P_\oplus$ so $P_{syn,mars} = \frac{1.881 P_\oplus^{2}}{(1.881-1)P_\oplus} = \frac{1.881}{0.881} P_\oplus$, larger than for Neptune

Neptune $P = 164.79 P_\oplus$ $\quad P_{syn,Nep} = \frac{164.79}{163.79} P_\oplus \approx P_\oplus$ ok

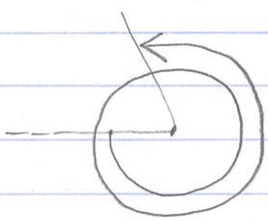
Q2.3

a)





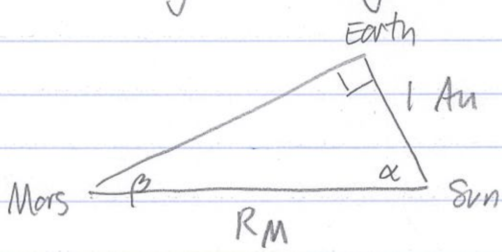
b) In 687 days Mars completed one full orbit wrt fixed stars. Earth completed not quite two orbits



Earth's angular velocity $\omega_E = \frac{2\pi}{365.25d}$

Thus in those 687 days it moved through an angle given by $\theta_E = \left(\frac{2\pi}{365.25d}\right) \times 687d$

redrawing the triangle of interest:



Referencing the above picture I see that the angle α must be $4\pi - \theta_E = 4\pi - 2\pi\left(\frac{687}{365.25}\right) \approx 0.748\text{ rad} = \alpha$

So $\beta = \pi/2 - \alpha = 0.822$ rad (picture isn't to scale!)

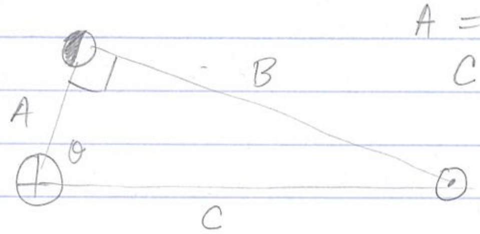
c) From the right triangle: $\cos\alpha = 1/R_M$

$R_M = 1/\cos\alpha = 1.36$ AU

as advertised, less than 1.5 AU.

Mars was near perihelion on day 0 and 687.

Q2.4
~~HW2~~ extra credit



$A = C \cos\theta$ see Fig 2.1
$C = A/\cos\theta$
let's treat A as const and just work out unc in C due to unc in θ.

uncertainty $\sigma_C = \left|\frac{dC}{d\theta}\right| \sigma_\theta$ At least that's the one component of σ_C

but $C = A/\cos\theta = A(\cos\theta)^{-1}$
$dC/d\theta = A(-1)(\cos\theta)^{-2}(-\sin\theta) = \frac{A\sin\theta}{\cos^2\theta}$

eval:

θ	$\sin\theta/\cos^2\theta \propto \sigma_C$
87°	364
88°	820
89°	3282 wow, up by ×10.