ASTR 210 HW 1

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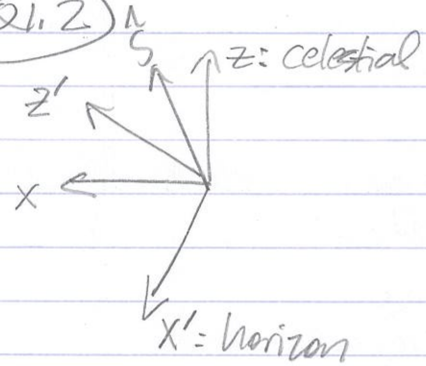
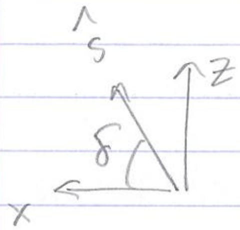
Q1.2

3 pts a)

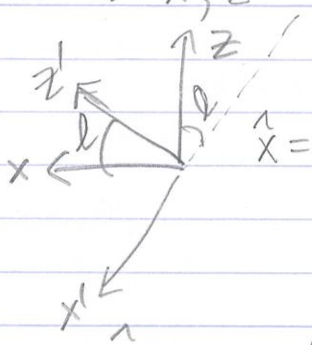


$\hat{s}$ vs $\hat{x}, \hat{z}$



$\hat{s} = \cos\delta\,\hat{x} + \sin\delta\,\hat{z}$

3 pts b) $\hat{x}$ vs $\hat{x}', \hat{z}'$



$\hat{x} = \sin l\,\hat{x}' + \cos l\,\hat{z}'$

3 pts c) $\hat{z} = -\cos l\,\hat{x}' + \sin l\,\hat{z}'$

6 pts d) plug c and d into a:

$$\hat{s} = \cos\delta(\sin l\,\hat{x}' + \cos l\,\hat{z}') + \sin\delta(-\cos l\,\hat{x}' + \sin l\,\hat{z}')$$

$$= (\cos\delta\sin l - \sin\delta\cos l)\hat{x}' + (\cos\delta\cos l + \sin\delta\sin l)\hat{z}'$$

$$= \sin(l-\delta)\hat{x}' + \cos(l-\delta)\hat{z}'$$

6 pts e) transit altitude in $(0, \pi/2)$ = distance to closest N or S horizon point

angle between $\hat{s}$ and $\hat{x}'$ is $\delta + 90° - l$

or its complement $180° - (\delta - l + 90°)$

$= 90° - \delta + l$

e) cont'd

i) $\delta > l$, then $\delta - l > 0$ and altitude is $90° + l - \delta$

ii) $\delta < l$, $\delta - l < 0$ so altitude is $90° + \delta - l$

4 pts f) NCP ($\delta = 90°$) has a transit altitude of

$$\theta = 90° + l - 90° = l$$

3 pts g) Southernmost Dec observable from C-U.

Translation: transit altitude > 0. Use case ii above.

$$\theta_{min} = 90° + \delta - l > 0$$

$$l = 40° \, 07'$$

$$\delta_{lim} = 0 + l - 90° = -49° \, 53'$$

ω Cen is at Dec $= -47° \, 28'$ so is technically visible but in practice will be very difficult.

2 pts h) l ... by inspection

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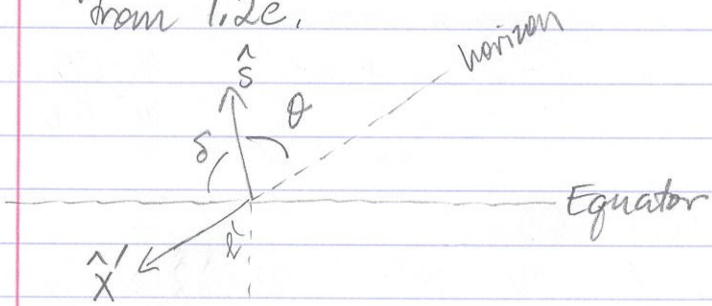
Q1.3

Key result needed here is the meridional altitude θ from 1.2e.



we had, for $\delta > l$: $\theta = 90° + l - \delta$

$\delta < l$: $\theta = 90° + \delta - l$

3 pts a) LMC has $\delta = -69° 45'$. Want $\theta > 0$; find the largest l that still satisfies $\theta > 0$. Will use the $\delta < l$ case.

$$\theta = 90° + \delta - l > 0$$
$$90° + \delta > l$$
$$l < 90° - 69° 45' = +20° 15' \text{ or } +20.25°$$ highest latitude from which LMC can (barely) be glimpsed.

2 pts b) So how far away is that? $1° \approx 111$ km (longitude is more complicated).

$(40° 07' - 20° 15') = 19° 52'$ or if you prefer,
$40.117° - 20.25° = 19.87°$
$\times \frac{111 \text{ km}}{1°} = 2200$ Km South of here.

6 pts c) Galactic Center $\delta = -29° 0'$
meridional altitude θ observed from KPNO ($l = +31.97°$) is $\theta = 90° + \delta - l = 90° - 29° - 31.97° = 29.03°$

$29° 02'$ also OK

GC altitude θ measured from summit of Mauna Kea (ignoring the fact that you're on top of a mountain) is

$$\theta = 90° + \delta - \ell = 90° - 29° - \underbrace{19°49.4'}_{\text{aka } 19.82°} = 41.18°$$

$41° 10.6'$ also ok

from ALMA in northern Chile:

$$\theta = 90° + \delta - \ell = 90° - 29° - (-23.02°) = 84.02°$$

$84° 1.2'$ also ok

5 pts

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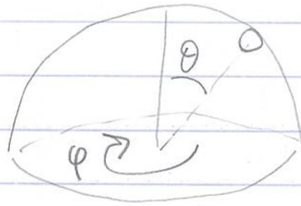
Q1.4

$d\Omega = \sin\theta \, d\theta \, d\varphi$



Solid angle $= \int d\Omega = \int_0^{2\pi} d\varphi \int_0^{\theta_r} \sin\theta \, d\theta = 2\pi(-\cos\theta)\Big|_0^{\theta_r}$

$$\Omega = 2\pi(1 - \cos\theta_r)$$

When θ_r is small, $\cos\theta_r \approx 1 - \frac{\theta_r^2}{2}$ so

$$\Omega \approx 2\pi\left(1 - \left(1 - \frac{\theta_r^2}{2}\right)\right) = 2\pi \cdot \frac{\theta_r^2}{2} = \pi\theta_r^2,$$

and when $\theta_r = \pi/2$, $\Omega = 2\pi$ as requested.