University of Illinois **Fall 2023**

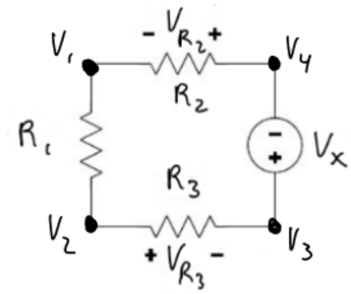
ECE 210/211 - Homework 1: Problems and Solutions

Due: Tuesday, August 29 by 11.59pm

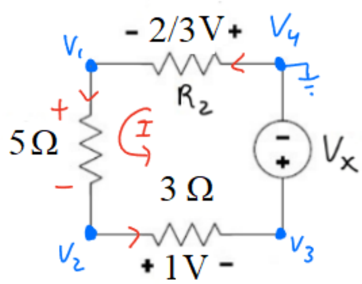
Problems:

1. Sign acknowledging you will abide by this course's and the University's Academic Integrity policies or face sanctions for not doing so. These policies include, among others, plagiarism: representing the words or ideas of others as your own; cheating: using unauthorized materials and/or information. The possible sanctions include, among others, reduced letter grades and an F in the course. If your solution upload does not include your signature, your homework will NOT be graded, resulting in a zero.
sign: ______________________________.

2. Consider the circuit below, where $R_1 = 5\Omega$, $R_3 = 3\Omega$, $V_{R_2} = \frac{2}{3}$V and $V_{R_3} = 1$V. Let the top-right node be the reference node. Determine the resistance R_2 and the voltage V_x in the following circuit, as well all the node voltages V_1, V_2, V_3 and V_4.



Solution:



Applying Ohm's law to 3Ω resistor,

$$I = \frac{V_{3\Omega}}{3} = \frac{1}{3}\text{A},$$

where we have used SRS for the current direction.
Current is the same everywhere in the single loop, so apply Ohm's law to R_2:

$$R_2 = \frac{V_{R_2}}{I} = \frac{2/3}{1/3} = 2\Omega.$$

Using KVL, we have

$$V_x = -\frac{2}{3} - 5I - 1 = -\frac{2}{3} - 5\left(\frac{1}{3}\right) - 1 = -\frac{10}{3}V$$

Then taking the top-right node as the reference node we can determine the node voltages:

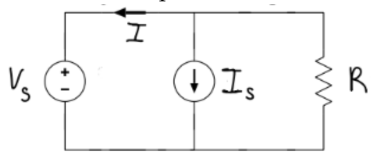
$$V_1 = -\frac{2}{3}\text{V},$$

$$V_2 = V_1 - 5I = -\frac{2}{3} - 5\left(\frac{1}{3}\right) = -\frac{7}{3}\text{V}$$

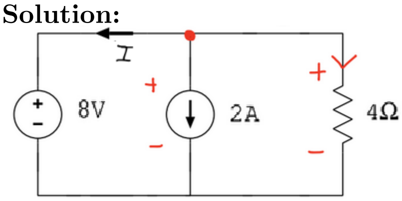
$$V_3 = V_x = -\frac{10}{3}\text{V}.$$

$$V_4 = 0\text{V}.$$

3. Consider the circuit below, where $V_s = 8$V, $I_s = 2$A and $R = 4\Omega$. Determine the current I and calculate the absorbed power at each circuit element. Clearly indicate if the power is absorbed or injected.



Solution:



Applying KCL at the upper node yields

$$\begin{aligned} I + 2 + I_{4\Omega} &= 0 \\ I + 2 + \frac{8}{4} &= 0 \\ I &= -4\,\text{A}, \end{aligned}$$

where we have used SRS for Ohm's law at the 4Ω resistor and the fact that all three elements are in parallel, so the 4Ω resistor has 8V.

To determine the absorbed power of any element, it is important to follow SRS in order to use the formula $P - VI$.

Therefore, the absorbed power at the 8V voltage source is

$$P_{8\text{V}} = V_{8\text{V}} I_{8\text{V}} = (8)(-4) = -32\,\text{W},$$

which is to say that the 8V voltage source is delivering 32W of power.

The absorbed power at the 2A current source is

$$P_{2A} = V_{2A} I_{2A} = (8)(2) = 16\,\text{W},$$

which is to say that the 2A current source is absorbing 16W of power.

The absorbed power at the $4\,\Omega$ resistor is

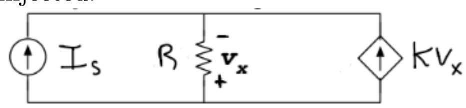
$$P_{4\Omega} = \frac{V^2}{R} = \frac{V_{4\Omega}^2}{4} = \frac{(8)^2}{4} = 16\,\text{W},$$

which is to say that the $4\,\Omega$ resistor is absorbing 16W of power.

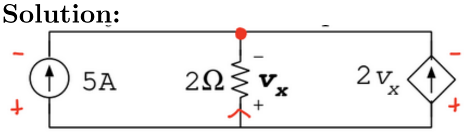
You may verify the result by considering the conservation of power: the 32 W delivered by the independent voltage source matches the sum of the absorbed powers of the resistor and the current source.

4. Consider the circuit below, where $I_s = 5$A, $R = 2\Omega$ and $K = 2$A/V. Determine the voltage V_x and calculate the absorbed power at each circuit element. Clearly indicate if the power is absorbed or

injected.



Solution:



The KCL equation at the top node of v_x can be written as

$$\begin{aligned} 5 \;+\; I_{2\Omega} \;+\; 2\,V_x &= 0 \\ 5 \;+\; \frac{V_x}{2} \;+\; 2\,V_x &= 0 \\ V_x &= -2\,\mathrm{V}, \end{aligned}$$

where we have used SRS for Ohm's law at the 2Ω resistor. Notice that all elements are in parallel, so they have the same voltage.

Absorbed/injected powers:

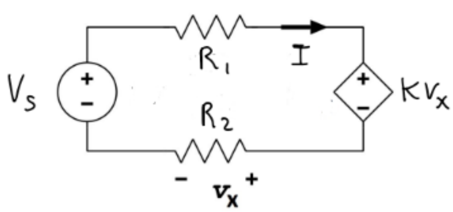
$$P_{2\Omega} \;=\; \frac{V^2}{R} \;=\; \frac{V_{2\Omega}^2}{2} \;=\; \frac{V_x^2}{2} \;=\; 2\,\mathrm{W}\ \text{(absorbs energy)}$$

$$P_{5\mathrm{A}} \;=\; V_{5\mathrm{A}}(5) \;=\; (-2)(5) \;=\; -10\,\mathrm{W}\ \text{(injects energy)}$$

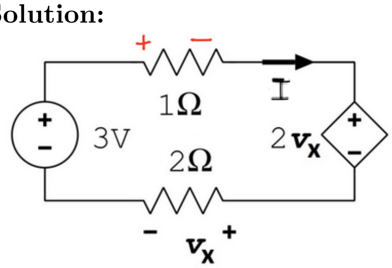
$$P_{2v_x} \;=\; V_{2V_x}(2V_x) \;=\; (-2)(-4) \;=\; 8\,\mathrm{W}\ \text{(absorbs energy)}$$

The independent current source is injecting the energy. Notice that the 10 W delivered by the independent source matches the sum of the absorbed powers of the resistor and the dependent source.

5. Consider the circuit below, where $V_s = 3\mathrm{V}$, $R_1 = 1\Omega$, $R_2 = 2\Omega$ and $K = 2$. Determine the current I and the voltage V_x.



Solution:



Applying KVL we obtain

$$3 \;-\; V_x \;-\; 2V_x \;-\; V_{1\Omega} = 0,$$

but from Ohm's law in the 2 Ω resistor using SRS we know that

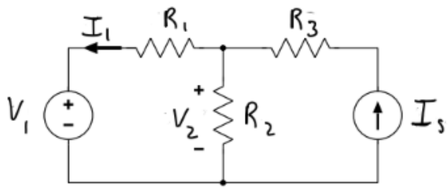
$$V_x \;=\; 2I.$$

Inserting this V_x in the KVL equation yields

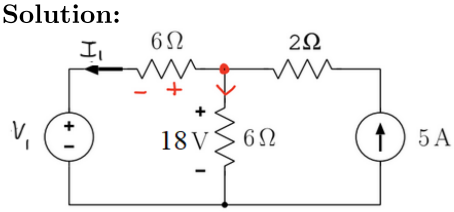
$$
\begin{aligned}
3 \; - \; 2I \; - \; 2\,(2I) \; - \; (1)(I) \;\; &= \;\; 0 \\
I \;\; &= \;\; \frac{3}{7}\,\text{A}.
\end{aligned}
$$

Therefore, $V_x = 2I = \frac{6}{7}\,$V.

6. Consider the circuit below, where $R_1 = 6\Omega,\ R_2 = 6\Omega,\ R_3 = 2\Omega,\ V_2 = 18$V and $I_s = 5$A. Determine the current I_1 and the voltage V_1.



Solution:



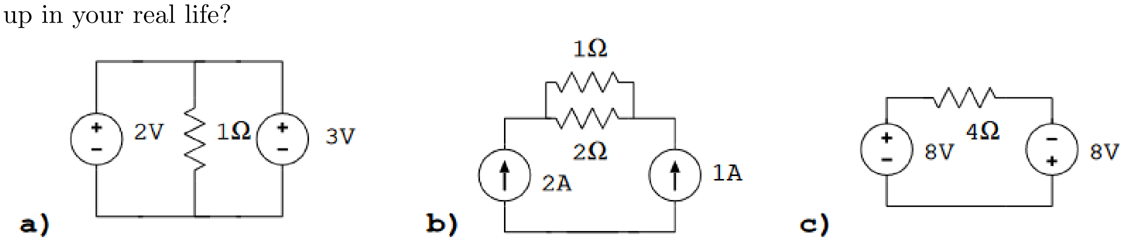
Applying KCL at the upper central node, yields

$$
\begin{aligned}
I_1 \; + \; \frac{18}{6} \;\; &= \;\; 5 \\
I_1 \;\; &= \;\; 2\,\text{A}.
\end{aligned}
$$

Applying KVL in the left loop, we obtain

$$
\begin{aligned}
V_1 \; + \; 6I_1 \; - \; 18 \;\; &= \;\; 0 \\
V_1 \;\; &= \;\; 18 \; - \; 6(2) \\
V_1 \;\; &= \;\; 6\,\text{V}
\end{aligned}
$$

7. Some of the following circuits violate KVL/KCL and/or basic definitions of two-terminal elements given in Section 1.3. For each one of the circuits, determine if it is correct or ill-specified. If it is ill-specified, explain the problem and indicate what will happen if the incorrect circuit has been built up in your real life?



Solution:

(a) This circuit violates KVL, top node cannot be both 2V and 3V. Most likely, the 2V source will burn out in this configuration.

(b) This circuit violates KCL. Consider the bottom node that connects both sources, based on KCL, the overall current which flows into the node must be 0. Here it cannot be 3A. Most likely, the 1A source will burn out.

(c) Circuit is correct!

8. Let $A = \sqrt{3} - j2\sqrt{3}$ and let $B = -3 - j\sqrt{3}$.

(a) Express A in exponential form.
Solution:

$$|A| = \sqrt{(\sqrt{3})^2 + (-2\sqrt{3})^2} = \sqrt{15},$$

$$\angle A = arctan\left(\frac{-2\sqrt{3}}{\sqrt{3}}\right) = -arctan(2) \text{ rad}$$

Therefore

$$A = \sqrt{15}e^{-jarctan(2)}$$

(b) Express B in exponential form.
Solution:

$$|B| = \sqrt{(-3)^2 + (-\sqrt{3})^2} = 2\sqrt{3},$$

$$\angle B = -\pi + arctan\left(\frac{-\sqrt{3}}{3}\right) = -\pi + arctan\left(\frac{1}{\sqrt{3}}\right) = -\pi + \frac{\pi}{6} = -\frac{5\pi}{6} \text{ rad}$$

Recall π must be subtracted because B is in the third quadrant. therefore

$$B = 2\sqrt{3}e^{-j\frac{5\pi}{6}}$$

(c) Determine the magnitudes of $A + B$ and $A - B$.
Solution:

$$|A+B| = \left|\sqrt{3} - j2\sqrt{3} - 3 - j\sqrt{3}\right| = \sqrt{\left(\sqrt{3}-3\right)^2 + (-3\sqrt{3})^2} = \sqrt{3 - 6\sqrt{3} + 9 + 27} = \sqrt{39 - 6\sqrt{3}}$$

$$|A-B| = \left|\sqrt{3} - j2\sqrt{3} + 3 + j\sqrt{3}\right| = \sqrt{\left(\sqrt{3}+3\right)^2 + (-\sqrt{3})^2} = \sqrt{3 + 6\sqrt{3} + 9 + 3} = \sqrt{15 + 6\sqrt{3}}$$

(d) Express AB and A/B in rectangular form.
Solution:

$$\begin{aligned} AB &= (\sqrt{15}e^{-jarctan(2)})(2\sqrt{3}e^{-j\frac{5\pi}{6}}) = 6\sqrt{5}e^{j(-\frac{5\pi}{6} - arctan(2))} \\ &= 6\sqrt{5}cos\left(-\frac{5\pi}{6} - arctan(2)\right) + j6\sqrt{5}sin\left(-\frac{5\pi}{6} - arctan(2)\right) \end{aligned}$$

This expression is good enough for full points, but you can further simplify that using the following trigonometric identities:

$$\begin{aligned} \cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \\ \sin(a+b) &= \sin(a)\cos(b) - \cos(a)\sin(b) \\ \sin(-a) &= -\sin(a) \\ \cos(-a) &= \cos(a) \\ \sin(a) &= \sin(\pi - a) \\ \cos(a) &= -\cos(a - \pi) \\ \cos\left(arctan\left(\frac{b}{a}\right)\right) &= \frac{a}{\sqrt{a^2+b^2}} \end{aligned}$$

$$\sin\left(arctan\left(\frac{b}{a}\right)\right) = \frac{b}{\sqrt{a^2+b^2}}$$

$$\begin{aligned}
AB &= 6\sqrt{5}cos\left(-\frac{5\pi}{6} - arctan(2)\right) + j6\sqrt{5}sin\left(-\frac{5\pi}{6} - arctan(2)\right) \\
&= 6\sqrt{5}\left[\cos\left(-\frac{5\pi}{6}\right)\cos(-arctan(2)) - \sin\left(-\frac{5\pi}{6}\right)\sin(-arctan(2))\right] \\
&\quad +j6\sqrt{5}\left[\sin\left(-\frac{5\pi}{6}\right)\cos(-arctan(2)) + \cos\left(-\frac{5\pi}{6}\right)\sin(-arctan(2))\right] \\
&= 6\sqrt{5}\left[\cos\left(\frac{5\pi}{6}\right)\cos(arctan(2)) - \sin\left(\frac{5\pi}{6}\right)\sin(arctan(2))\right] \\
&\quad +j6\sqrt{5}\left[-\sin\left(\frac{5\pi}{6}\right)\cos(arctan(2)) - \cos\left(\frac{5\pi}{6}\right)\sin(arctan(2))\right] \\
&= 6\sqrt{5}\left[-\cos\left(\frac{\pi}{6}\right)\cos\left(arctan\left(\frac{2}{1}\right)\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(arctan\left(\frac{2}{1}\right)\right)\right] \\
&\quad +j6\sqrt{5}\left[-\sin\left(\frac{\pi}{6}\right)\cos\left(arctan\left(\frac{2}{1}\right)\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(arctan\left(\frac{2}{1}\right)\right)\right] \\
&= 6\sqrt{5}\left[-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{5}}\right) - \left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{5}}\right)\right] + j6\sqrt{5}\left[-\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{5}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{5}}\right)\right] \\
&= \left(-3\sqrt{3} - 6\right) \ + \ j\left(6\sqrt{3} - 3\right)
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{A}{B} &= \frac{\sqrt{15}e^{-jarctan(2)}}{2\sqrt{3}e^{-j\frac{5\pi}{6}}} = \frac{\sqrt{5}}{2}e^{j\left(\frac{5\pi}{6} - arctan(2)\right)} = \frac{\sqrt{5}}{2}cos\left(\frac{5}{6}\pi - arctan(2)\right) + j\frac{\sqrt{5}}{2}sin\left(\frac{5}{6}\pi - arctan(2)\right) \\
&= \frac{\sqrt{5}}{2}\left[\cos\left(\frac{5\pi}{6}\right)\cos(-arctan(2)) - \sin\left(\frac{5\pi}{6}\right)\sin(-arctan(2))\right] \\
&\quad +j\frac{\sqrt{5}}{2}\left[\sin\left(\frac{5\pi}{6}\right)\cos(-arctan(2)) + \cos\left(\frac{5\pi}{6}\right)\sin(-arctan(2))\right] \\
&= \frac{\sqrt{5}}{2}\left[-\cos\left(\frac{\pi}{6}\right)\cos\left(arctan\left(\frac{2}{1}\right)\right) + \sin\left(\frac{\pi}{6}\right)\sin\left(arctan\left(\frac{2}{1}\right)\right)\right] \\
&\quad +j\frac{\sqrt{5}}{2}\left[\sin\left(\frac{\pi}{6}\right)\cos\left(arctan\left(\frac{2}{1}\right)\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(arctan\left(\frac{2}{1}\right)\right)\right] \\
&= \frac{\sqrt{5}}{2}\left[-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{5}}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{5}}\right)\right] + j\frac{\sqrt{5}}{2}\left[\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{5}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{5}}\right)\right] \\
&= \left(-\frac{\sqrt{3}}{4} + \frac{1}{2}\right) \ + \ j\left(\frac{1}{4} + \frac{\sqrt{3}}{2}\right)
\end{aligned}$$

The expression at the end of the first line is good enough for full points.

9. This last problem will introduce the basic python programming concepts that will be useful to you in this course, and others.
   - You will use an IPython notebook via PrairieLearn with unlimited attempts.
   - Make sure that you complete the PrairieLearn question before this homework's due date to get credit for it.
   - You do not need to submit an answer for this question on Gradescope, we will download this problem's grade from PrairieLearn.

- You can access the question by clicking on this link:
  https://us.prairielearn.com/pl/course$_i nstance/139840/assessment/2357834$.

- If you do not have access to the question, please email Prof. Alvarez.

Conflict exam requests:

- Our exam schedule is posted since before the first day of the semester.
- Some of you either have classes, labs, other exams or work that overlap with our exams. We will offer conflict exams for those of you who are in these situations but you need to get Prof. Alvarez's approval in order to be able to take the conflict exams.
- As indicated in the student code, conflict exams are to be granted if the student informs the instructor of the conflict within one week after being informed of the examination schedule. Priority will be given to the examination announced in class the earliest in the semester.
- If you have conflicts with one or more of our exams and want to request a conflict exam, you must complete this form by September 6 at 11.59pm:
  https://forms.illinois.edu/sec/2082281966.
- If you do not complete the form by the deadline, we are not required to grant a conflict exam so you might not get one.
- If you do not have access to the form, please email Prof. Alvarez.

