University of Illinois **Fall 2023**

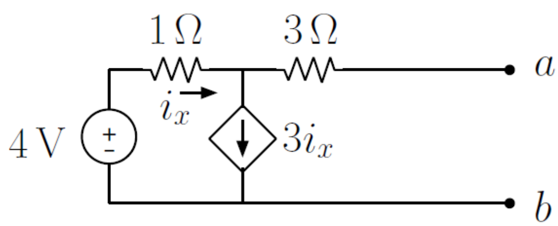
ECE 210/211 - Homework 3: Problems and Solutions

Due: Tuesday, September 12 by 11.59pm

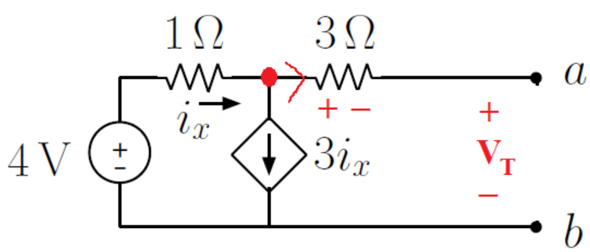
Problems:

1. Sign acknowledging you will abide by this course's and the University's Academic Integrity policies or face sanctions for not doing so. These policies include, among others, plagiarism: representing the words or ideas of others as your own; cheating: using unauthorized materials and/or information. The possible sanctions include, among others, reduced letter grades and an F in the course. If your solution upload does not include your signature, your homework will NOT be graded, resulting in a zero.
   sign: ______________________.

2. Consider the following circuit. Determine the Thevenin and Norton equivalent circuits between nodes a and b, **and** also determine the maximum power of the network.



Solution: Thevenin's voltage is the open circuit voltage, which in this case is the voltage V_{ab}.



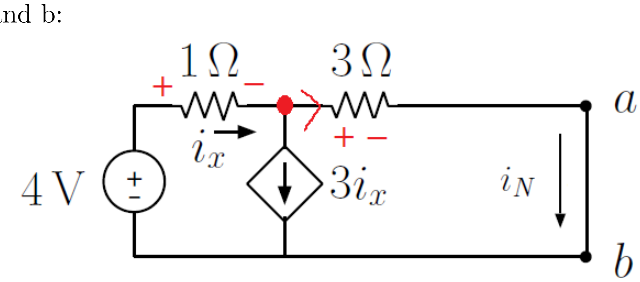
Applying KCL at the upper central node and realizing that there is no current through the 3Ω resistor, we obtain:

$$i_x = 3i_x + 0 \quad \Rightarrow \quad i_x = 0\text{A}.$$

That means no current is flowing through the resistors, consequently no voltage drop in them. Hence,

$$v_T = V_{ab} = 4\,\mathrm{V}.$$

Norton's current is the short-circuit current. To get the Norton current we short-circuit terminals a and b:



This time, the 3Ω resistor does have current, and it is equal to i_N, hence the KCL at the upper central node yields:

$$i_x \;=\; 3i_x \;+\; i_N \quad\Rightarrow\quad i_N = -2i_x.$$

To obtain i_x we can do a KVL around the outer loop:

$$4 \;=\; (1)i_x \;+\; (3)i_N \quad\Rightarrow\quad i_x \;=\; 4 - 3i_N.$$

Substituting this i_x in the KCL equation we obtain

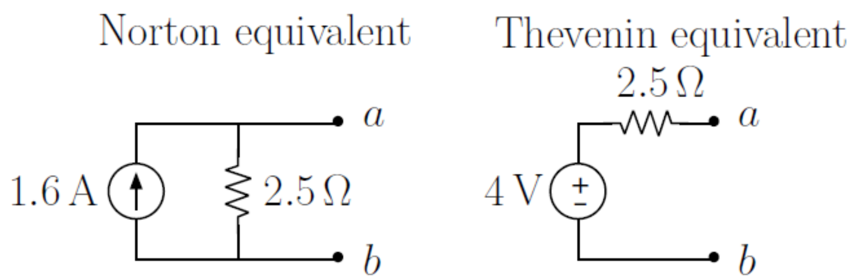
$$i_N \;=\; -2(4 - 3i_N) \;=\; -8 \;+\; 6i_N \quad\Rightarrow\quad i_N \;=\; \frac{8}{5}\mathrm{A} \;=\; 1.6\,\mathrm{A}.$$

Notice that the value of the dependent variable, $i_x = 4/5$A, is different in the short-circuit calculation than in the open-circuit calculation. This will usually be the case because they are different circuits.

To obtain the equivalent resistor we can use the relation

$$R_T \;=\; \frac{v_T}{i_N} \;=\; \frac{4\,\mathrm{V}}{\frac{8}{5}\,\mathrm{A}} \;=\; \frac{5}{2}\,\Omega \;=\; 2.5\,\Omega.$$

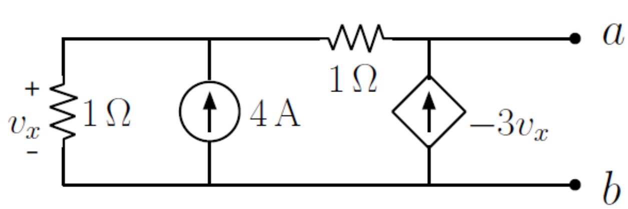
The equivalent circuits are therefore:



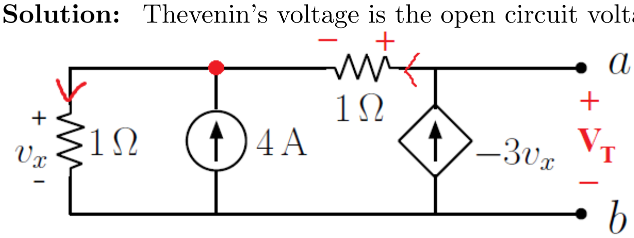
Finally, the maximum power is the same as the available power:

$$P_a \;=\; \frac{V_T^2}{4R_T} \;=\; \frac{(4)^2}{4\left(\frac{5}{2}\right)} \;=\; \frac{8}{5}\mathrm{W} \;=\; 1.6\mathrm{W}.$$

3. Determine the Thevenin equivalent of the following network between nodes a and b, **and** also determine the available power of the network.



Solution: Thevenin's voltage is the open circuit voltage, which in this case is the voltage V_{ab}.



Notice that the current through the 1Ω resistor is the same as the one from the dependent current source because terminal a is open. Applying KCL at the top left node and Ohm's law in the 1Ω resistor yields

$$4 \; + \; (-3v_x) \; = \; \frac{v_x}{1} \quad \Rightarrow \quad v_x \; = \; 1\,\text{V}.$$

The KVL equation around the outer loop gives

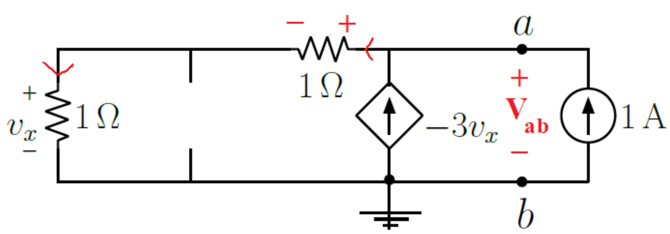
$$v_x \; + \; (1)\,(-3v_x) \; = \; v_T \quad \Rightarrow \quad v_{\text{T}} \; = \; -2v_x \; = \; -2\,\text{V},$$

To obtain R_T we set all independent sources to zero, which means opening the terminals of the 4A source. We cannot simplify the circuit then to a single resistor due to the dependent source, hence we must use the test signal method:

(a) Inject 1A into terminal a.

(b) Determine V_{ab}.

(c) Obtain $R_T = V_{ab}/(1\text{A})$.



Writing the KCL equation for node a and Ohm's law for the resistor on the far left

$$1 \; + \; (-3v_x) \; = \; \frac{v_x}{1} \quad \Rightarrow \quad v_x \; = \; \frac{1}{4}\,\text{V}.$$

Notice that the value of the dependent variable, v_x, is different in the short-circuit calculation than in the open-circuit calculation. This will usually be the case because they are different circuits.

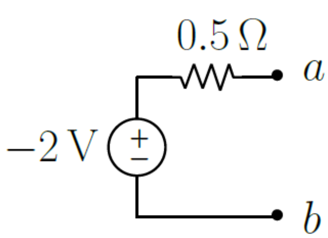
Applying a KVL around the outer loop and using the fact that both resistors have the same current, $\frac{v_x}{1}$, yields

$$v_x \; + \; 1\left(\frac{v_x}{1}\right) \; = \; v_{ab} \quad \Rightarrow \quad v_{ab} \; = \; 2v_x \; = \; \frac{1}{2}\,\text{V}.$$

Therefore, the Thevenin resistance is

$$R_T \; = \; \frac{v_{ab}}{1\text{A}} \; = \; \frac{1}{2}\,\Omega \; = \; 0.5\,\Omega.$$

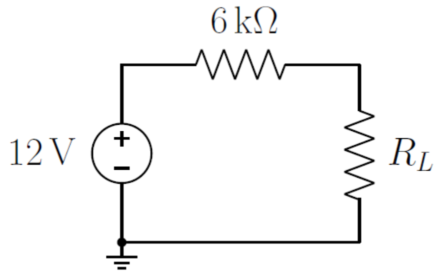
Hence the Thevenin equivalent circuit becomes



Finally, the available power is:

$$P_a \; = \; \frac{v_T^2}{4R_T} \; = \; \frac{(-2)^2}{4\left(\frac{1}{2}\right)} \; = \; 2\text{W}.$$

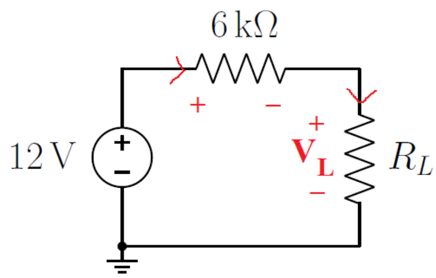
4. Consider the circuit below.



Calculate the absorbed power in R_L for:

(a) $R_L = 3\,\text{k}\Omega$.
Solution: We can use the absorbed power formula for a resistor, in terms of its voltage:

$$P_R = \frac{V^2}{R}.$$



To obtain the voltage, V_L, across resistor R_L we can use voltage division:

$$V_L = 12\left(\frac{R_L}{R_L + 6000}\right).$$

Hence,

$$P_L = \frac{V_L^2}{R_L} = \frac{1}{R_L}\left(\frac{12R_L}{R_L + 6000}\right)^2 = \frac{144\,R_L}{(R_L + 6000)^2}.$$

In particular, when $R_L = 3\,\text{k}\Omega$,

$$P_L = \frac{144\,(3000)}{(3000 + 6000)^2} = \frac{2}{375}\text{W} \approx 5.3\,\text{mW}.$$

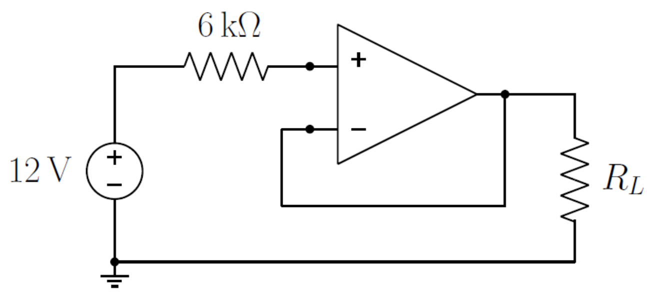
(b) $R_L = 6\,\text{k}\Omega$.
Solution: Substituting $R_L = 6000\Omega$ into the absorbed power equation from part (a):

$$P_L = \frac{144\,(6000)}{(6000 + 6000)^2} = \frac{3}{500}\text{W} = 6\text{mW}.$$

(c) $R_L = 18\,\text{k}\Omega$.
Solution: Substituting $R_L = 18000\Omega$ into the absorbed power equation from part (a):

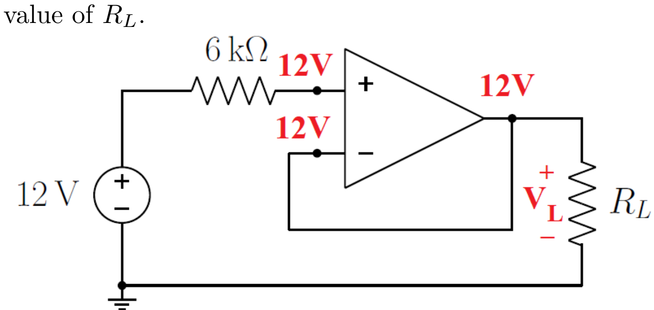
$$P_L = \frac{144\,(18000)}{(18000 + 6000)^2} = \frac{9}{2000}\text{W} = 4.5\text{mW}.$$

Next, consider the circuit below, which implements a buffer between the source and the load. Assume the circuit behaves linearly and make use of the ideal op-amp approximations.



Calculate the absorbed power in R_L for:

(d) $R_L = 3\,\mathrm{k}\Omega$.
Solution: Given the ideal op-amp assumptions ($i_+ = i_- = 0\,\mathrm{A}$, $v_+ = v_-$), we see that there is no voltage nor current at the $6k\Omega$ resistor, and hence $12\mathrm{V} = v_+ = v_- = V_L$, regardless of the value of R_L.



Hence, using the absorbed power formula for a resistor, in terms of its voltage:

$$P_L = \frac{V_L^2}{R_L} = \frac{144}{R_L}.$$

In particular, when $R_L = 3\,\mathrm{k}\Omega$,

$$P_L = \frac{144}{3000} = \frac{6}{125} = 48\,\mathrm{mW}.$$

(e) $R_L = 6\,\mathrm{k}\Omega$.
Solution: Substituting $R_L = 6000\Omega$ into the absorbed power equation from part (d):

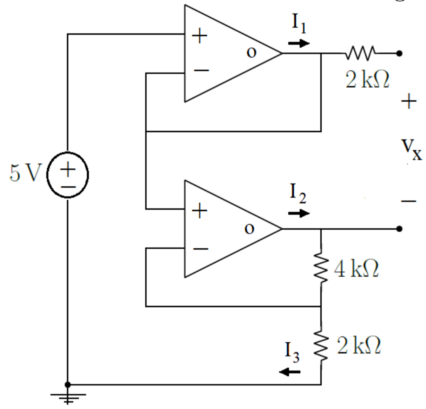
$$P_L = \frac{144}{6000} = \frac{3}{125} = 24\,\mathrm{mW}.$$

(f) $R_L = 18\,\mathrm{k}\Omega$.
Solution: Substituting $R_L = 18000\Omega$ into the absorbed power equation from part (d):

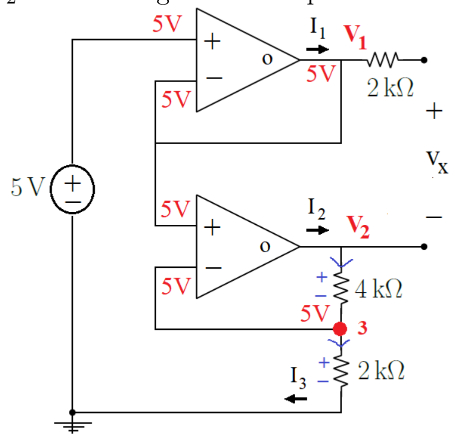
$$P_L = \frac{144}{18000} = \frac{1}{125} = 8\,\mathrm{mW}.$$

(g) Compare your answers from parts (a)-(c) to parts (d)-(f) and comment on why the power absorbed by the load is different for the two circuit designs (e.g., consider why one circuit delivers more power to the load and where that power comes from).
Solution: For the circuit with the op-amp, the load is buffered from the circuit, which holds the voltage across the load to be always $12\,\mathrm{V}$, regardless of the load resistance. This differs from the first circuit, where the load voltage depends on the load resistance according to the voltage divider rule. The maximum power for the first circuit will occur when the load is "matched" (i.e. $R_L = 6\,\mathrm{k}\Omega$), but the load voltage will only be $6\,\mathrm{V}$ for this resistance, which explains why even at the maximum power delivered to the first circuit, the op-amp circuit will deliver more power to the load. It is important to notice that the extra power delivered to the load is coming from the bias voltages of the op-amp, $\pm V_b$ (not shown in the Figure).

5. Consider the op-amp circuit shown below operating in the linear operation under ideal op-amp approximations. Determine the voltage V_x, and the currents I_1, I_2 and I_3.



Solution: We can obtain $V_x = V_1 - V_2$, where V_1 is the voltage at the output of the top op-amp and V_2 is the voltage at the output of the bottom op-amp, as indicated in the figure below.



Considering ideal op-amps, $v^+ = v^- = 5\,\text{V}$ in both op-amps, as well as at node 3, between the two resistors of the bottom op-amp. Hence,

$$V_1 \;=\; V_3 \;=\; 5\text{V}.$$

Also, from ideal op-amp assumptions, $i_+ = i_- = 0\,\text{A}$, at both op-amps. Hence, applying a KCL at node 1:

$$I_{4k\Omega} \;=\; I_{2k\Omega} \;+\; 0.$$

Using Ohm's law at the 2Ω bottom resistor:

$$5 \;-\; 0 \;=\; 2000 I_{2k\Omega} \quad\Rightarrow\quad I_{2k\Omega} \;=\; \frac{5}{2000} \;=\; \frac{1}{400}\text{A} \;=\; I_{4k\Omega}.$$

Therefore, V_2, in the bottom op-amp is

$$V_2 \;=\; 2000 I_{2k\Omega} \;+\; 4000 I_{4k\Omega} \;=\; 2000\left(\frac{1}{400}\right) \;+\; 4000\left(\frac{1}{400}\right) \;=\; 15\text{V}.$$

Finally we obtain

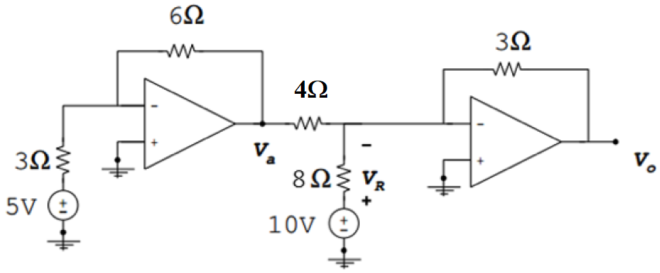
$$v_x \;=\; V_1 - V_2 \;=\; 5 - 15 \;=\; -10\,\text{V}.$$

Now, for the currents,

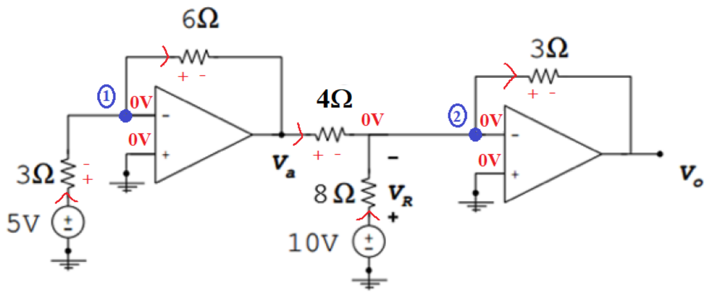
$$I_3 \;=\; I_{2k\Omega} \;=\; \frac{1}{400}\text{A} \;=\; 2.5\text{mA}.$$

$$I_2 \;=\; I_{4k\Omega} \;=\; \frac{1}{400}\text{A} \;=\; 2.5\text{mA}.$$
$$I_1 \;=\; i_- \;=\; 0\text{A}.$$

6. In the op-amp circuit shown below, determine the voltages V_a, V_R and V_o. Assume the circuit behaves linearly and make use of the ideal op-amp approximation.



Solution: Considering ideal op-amps, $v_+ = v_- = 0\,\text{V}$ and $i_+ = i_- = 0\,\text{A}$ in both op-amps.



Hence,

$$V_R \;=\; 10 \;-\; 0 \;=\; 10\,\text{V},$$

To get V_a, apply a KCL at node 1, the inverting terminal of the first op-amp, with Ohm's law at the resistors:

$$\frac{5-0}{3} \;=\; 0 \;+\; \frac{0-V_a}{6} \qquad \Rightarrow \qquad V_a \;=\; -10\text{V}.$$

Finally, to get V_o apply a KCL at node 2, the inverting terminal of the second op-amp, with Ohm's law at the resistors:

$$\frac{V_a-0}{4} \;+\; \frac{10-0}{8} \;=\; \frac{0-V_0}{3} \qquad \Rightarrow \qquad V_o \;=\; -\frac{3}{4}V_a \;-\; \frac{30}{8} \;=\; \frac{15}{4}\text{V} \;=\; 3.75\text{V}.$$