University of Illinois **Fall 2023**

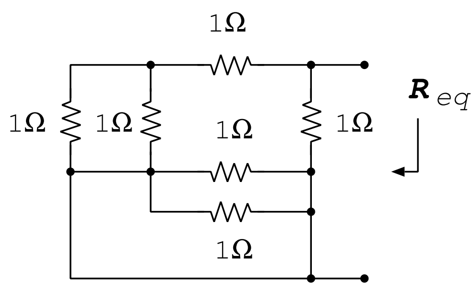
ECE 210/211 - Homework 2: Problems and Solutions

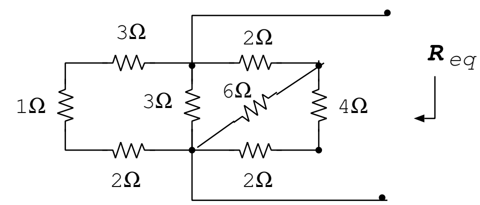
Due: Wednesday, September 6 by 11.59pm

Problems:

1. Sign acknowledging you will abide by this course's and the University's Academic Integrity policies or face sanctions for not doing so. These policies include, among others, plagiarism: representing the words or ideas of others as your own; cheating: using unauthorized materials and/or information. The possible sanctions include, among others, reduced letter grades and an F in the course. If your solution upload does not include your signature, your homework will NOT be graded, resulting in a zero.
sign: ______________________.

2. For each one of the following two circuits, obtain R_{eq}.

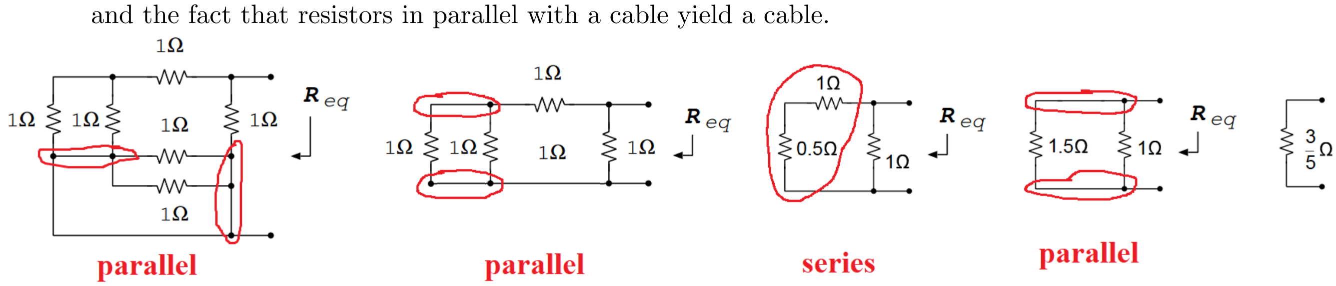
a) 

b) 

Solution:
a) Simplify the circuit as shown below, where we have used the parallel and series formulas

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \quad \text{and} \quad R_s = R_1 + R_2,$$

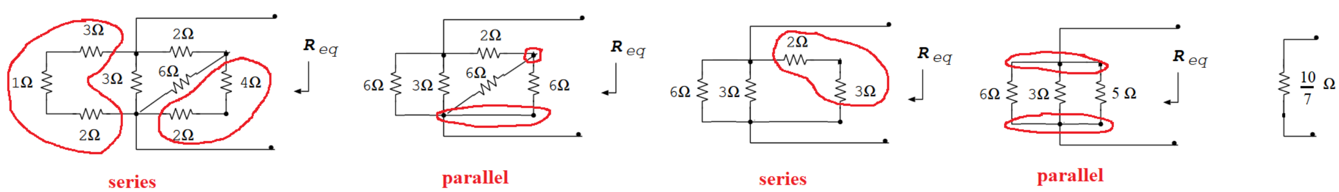
and the fact that resistors in parallel with a cable yield a cable.



Thus, $R_{eq} = \frac{3}{5}\Omega$.

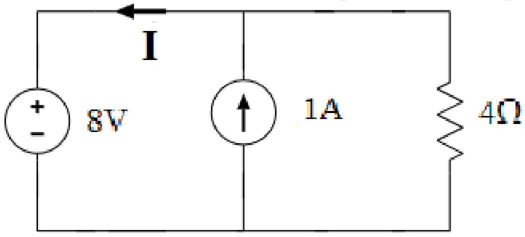
b) Simplify the circuit as shown below, where we have used the parallel and series formulas

$$R_s = R_1 + R_2 + \cdots + R_n \quad \text{and} \quad R_p^{-1} = \frac{1}{R_1} + \frac{1}{R_2},$$

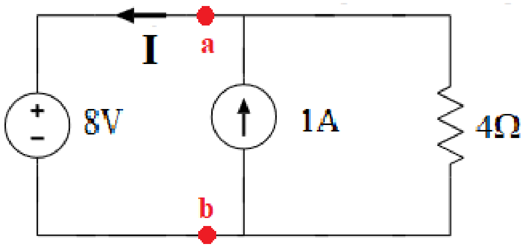


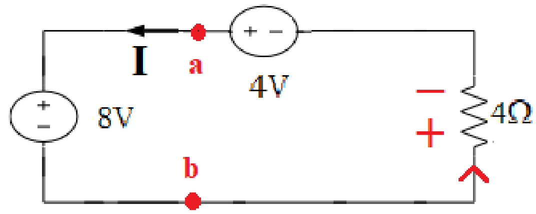
Thus, $R_{eq} = \frac{10}{7}\Omega$.

3. Determine the current I in the following circuit using source transformation.



Solution: Transforming the current source with parallel resistor at terminals a and b into a voltage source with resistor in series, we have the new circuit shown as following:





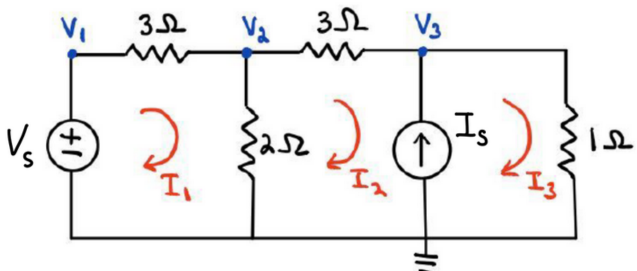
where the voltage source value is obtained using Ohm's law:

$$V_s = RI_s = 4(1) = 4\text{V}.$$

Apply KVL to the circuit with single loop counter-clockwise from the top right, we have:

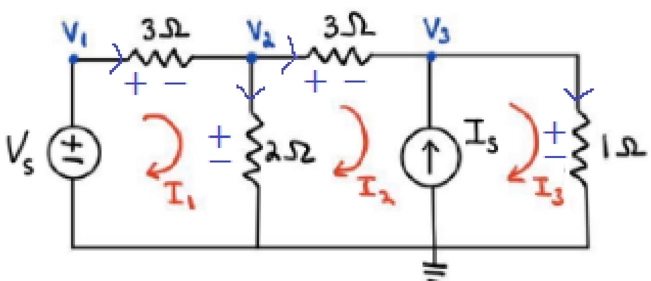
$$\begin{aligned} 8 + 4I - 4 &= 0 \\ I &= \frac{4-8}{4} = -1\,\text{A}. \end{aligned}$$

4. Consider the circuit below,



(a) Use the loop-current method to obtain a set of three linearly independent equations, in terms of the loop currents I_1, I_2 and I_3, and the sources V_s and I_s, but no other variables, that can be used to determine the loop currents. Simplify your equations and write them with integer-valued coefficients.

Solution: One possible assignment of polarities and current directions on the resistors is:



From the current source we get

$$I_3 \; - \; I_2 \; = \; I_s.$$

The KVL for loop I_1 is

$$3I_1 \; + \; 2\,(I_1 - I_2) \; = \; V_s$$

The KVL for super loop I_2-I_3,

$$2\,(I_2 - I_1) \; + \; 3I_2 \; + \; 1(I_3) \; = \; 0$$

Simplifying the equations, we obtain

$$\begin{aligned} -I_2 \; + \; I_3 \; &= \; I_s, \\ 5I_1 \; - \; 2I_2 \; &= \; V_s, \\ -2I_1 \; + \; 5I_2 \; + \; I_3 \; &= \; 0 \end{aligned}$$

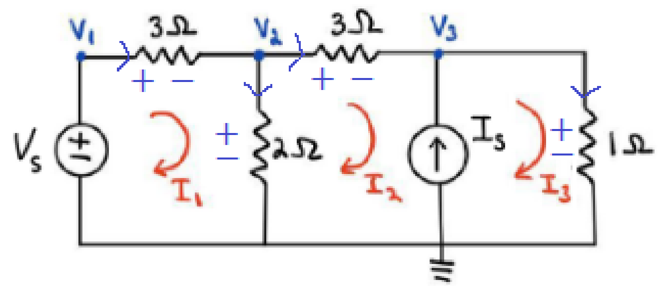
We could have also done a KVL on super loop I_1-I_2-I_3 :

$$3I_1 + 3I_2 + 1(I_3) \; = \; V_s,$$

instead of the any of the other two KVL equations, but not instead of the equation from the source because it is a linear combination of the other two.

(b) Use the node-voltage method to obtain a set of three linearly independent equations, in terms of the node voltages V_1, V_2 and V_3, and the sources V_s and I_s, but no other variables, that can be used to determine the node voltages. Simplify your equations and write them with integer-valued coefficients.

Solution: One possible assignment of polarities and current directions on the resistors is:



Using the voltage source:

$$V_1 \; = \; V_s$$

The KCL equation at node V_2

$$\frac{V_1 - V_2}{3} \; = \; \frac{V_2}{2} + \frac{V_2 - V_3}{3}$$

The KCL equation at node V_3

$$\frac{V_2 - V_3}{3} + I_s \; = \; \frac{V_3}{1}$$

Simplifying the equations, we obtain

$$\begin{aligned} V_1 \; &= \; V_s, \\ 2V_1 \; - \; 7V_2 \; + \; 2V_3 \; &= \; 0, \\ V_2 \; - \; 4V_3 \; &= \; -3I_s. \end{aligned}$$

(c) Is it known that $V_3 = k_1 V_s + k_2 I_s$. Use superposition to determine the values of k_1 and k_2.

Solution: Using superposition method, we first obtain k_1 by setting $I_s = 0$, which is equivalently an open circuit. We can apply the node-voltage method, as in part (b), which is equivalent to substituting $I_s = 0$ into the equations in (b), to obtain

$$\begin{aligned}
\frac{V_1 - V_2}{3} &= \frac{V_2}{2} + \frac{V_2 - V_3}{3}, \\
\frac{V_2 - V_3}{3} &= \frac{V_3}{1}. \\
\Rightarrow V_3 &= \frac{1}{13} V_s, \\
\Rightarrow k_1 &= \frac{1}{13}.
\end{aligned}$$

Next, we obtain k_2 by setting $V_s = 0$, which is equivalently a short circuit. Substituting $V_s = 0$ into the equations in (b), we have

$$\begin{aligned}
V_1 &= 0, \\
\frac{V_1 - V_2}{3} &= \frac{V_2}{2} + \frac{V_2 - V_3}{3}, \\
\frac{V_2 - V_3}{3} + I_s &= \frac{V_3}{1}. \\
\Rightarrow V_3 &= \frac{21}{26} I_s, \\
\Rightarrow k_2 &= \frac{21}{26} \text{ V/A}.
\end{aligned}$$

(d) Assuming that $V_s = 10$V and $I_s = 1$A, use your equations from part (a) to obtain the loop currents I_1, I_2 and I_3. It is fine to use calculators, software, or python to solve this problem but keep in mind that you might need to do it in an exam.

Solution: Given $V_s = 10$V and $I_s = 1$A and solving the equations above for I_1, I_2, and I_3, we have

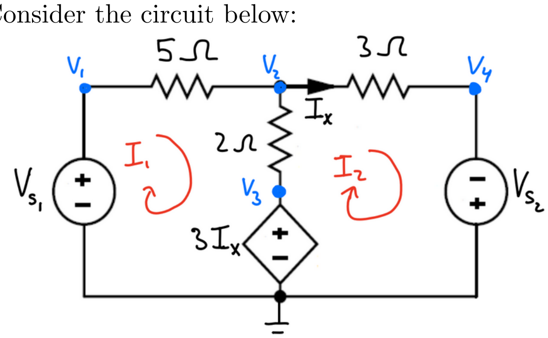
$$\begin{aligned}
I_1 &= \frac{29}{13} \text{ A}, \\
I_2 &= \frac{15}{26} \text{ A}, \\
I_3 &= \frac{41}{26} \text{ A}.
\end{aligned}$$

(e) Assuming that $V_s = 10$V and $I_s = 1$A, use your equations from part (b) to obtain node voltages V_1, V_2, and V_3. It is fine to use calculators, software, or python to solve this problem but keep in mind that you might need to do it in an exam.

Solution: Given $V_s = 10$V and $I_s = 1$A and solving the equations above for V_1, V_2, and V_3, we have

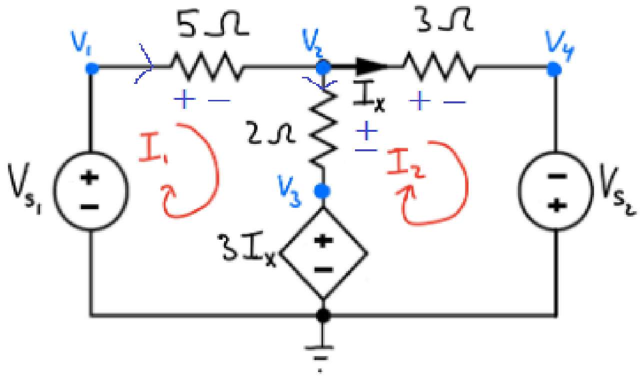
$$\begin{aligned}
V_1 &= 10 \text{ V}, \\
V_2 &= \frac{43}{13} \text{ V}, \\
V_3 &= \frac{41}{26} \text{ V}.
\end{aligned}$$

5. Consider the circuit below:



(a) Use the loop-current method to obtain a set of two linearly independent equations, in terms of the loop currents I_1 and I_2, and the sources V_{s_1} and V_{s_2}, but no other variables, that can be used to determine the loop currents. Simplify your equations and write them with integer-valued coefficients.

Solution: One possible assignment of polarities and current directions on the resistors is:



The KVL equations can be written for loops I_1 and I_2, such that we have

$$\begin{aligned} 5I_1 \ + \ 2\,(I_1 - I_2) \ + \ 3I_x &= V_{s_1}, \\ 2\,(I_2 - I_1) \ + \ 3I_2 &= V_{s_2} \ + \ 3I_x. \end{aligned}$$

Due to the dependent variable, we need one more equation to remove it:

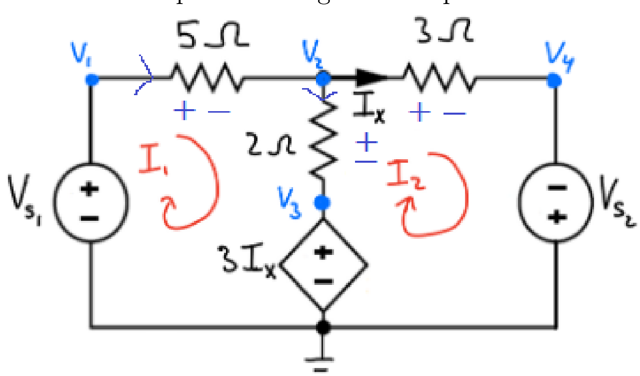
$$I_x = I_2.$$

Substituting I_x into the other two equation and simplifying the equations, we obtain

$$\begin{aligned} 7I_1 \ + \ I_2 &= V_{s_1}, \\ -2I_1 \ + \ 2I_2 &= V_{s_2}. \end{aligned}$$

(b) Use the node-voltage method to obtain a set of four linearly independent equations, in terms of the node voltages V_1, V_2, V_3 and V_4, and the sources V_{s_1} and V_{s_2}, but no other variables, that can be used to determine the node voltages. Simplify your equations and write them with integer-valued coefficients.

Solution: One possible assignment of polarities and current directions on the resistors is:



Using the voltages sources, we get the three equations

$$\begin{aligned} V_1 &= V_{s_1}, \\ V_4 &= -V_{s_2}, \\ V_3 &= 3I_x. \end{aligned}$$

The KCL equation for node V_2, is

$$\frac{V_1 - V_2}{5} = \frac{V_2 - V_3}{2} + \frac{V_2 - V_4}{3}.$$

We need to get rid of the dependent variable:

$$I_x = \frac{V_2 - V_4}{3}$$

Substituting I_x and simplifying the equations, we obtain

$$\begin{aligned} V_1 &= V_{s_1}, \\ V_4 &= -V_{s_2}, \\ -V_2 + V_3 + V_4 &= 0, \\ 6V_1 - 31V_2 + 15V_3 + 10V_4 &= 0. \end{aligned}$$

(c) It is known that $V_3 = k_1 V_{s_1} + k_2 V_{s_2}$. Use superposition to determine the values of k_1 and k_2.
Solution: Using superposition method, we first obtain k_2 by setting $V_{s_1} = 0$. We can apply the node-voltage method, as in part (b), which is equivalent to substituting $V_{s_1} = 0$ into the equations in (b), to obtain

$$\begin{aligned} V_1 &= 0, \\ V_4 &= -V_{s_2}, \\ -V_2 + V_3 + V_4 &= 0, \\ 6V_1 - 31V_2 + 15V_3 + 10V_4 &= 0, \\ \Rightarrow V_3 &= \frac{21}{16} V_{s_2}, \\ \Rightarrow k_2 &= \frac{21}{16}. \end{aligned}$$

Next, we set $V_{s_2} = 0$ t obtain k_1. Substituting it into the equations in (b), we have

$$\begin{aligned} V_1 &= V_{s_1}, \\ V_4 &= 0, \\ -V_2 + V_3 + V_4 &= 0, \\ 6V_1 - 31V_2 + 15V_3 + 10V_4 &= 0, \\ \Rightarrow V_3 &= \frac{3}{8} V_{s_1}, \\ \Rightarrow k_1 &= \frac{3}{8}. \end{aligned}$$

(d) Assuming that $V_{s_1} = 10$V and $V_{s_2} = 4$V, use your equations from part (a) to obtain the loop currents I_1 and I_2.
Solution: Given $V_{s_1} = 10$V and $V_{s_2} = 4$V and solving the loop equations for I_1 and I_2:

$$\begin{aligned} I_1 &= 1 \text{ A}, \\ I_2 &= 3 \text{ A}. \end{aligned}$$

(e) Assuming that $V_{s_1} = 10$V and $V_{s_2} = 4$V, use your equations from part (b) to obtain node voltages $V_1,\ V_2,\ V_3,$ and V_4.
Solution: Given $V_{s_1} = 10$V and $V_{s_2} = 4$V and solving the node equations above for $V_1, V_2, V_3,$ and V_4:

$$\begin{aligned} V_1 &= 10 \text{ V}, \\ V_2 &= 5 \text{ V}, \\ V_3 &= 9 \text{ V}, \\ V_4 &= -4 \text{ V}. \end{aligned}$$

6. This problem will introduce the python programming concepts to solve linear systems of equations.

   - You will use an IPython notebook via PrairieLearn with unlimited attempts.
   - Make sure that you complete the PrairieLearn question before this homework's due date to get credit for it.
   - You do not need to submit an answer for this question on Gradescope, we will download this problem's grade from PrairieLearn and upload it to Canvas.
   - You can access the question by clicking on this link:
     https://us.prairielearn.com/pl/course$_i nstance/139840/assessment/$2357835.
   - If you do not have access to the question, please email Prof. Alvarez.

7. Fill out the following survey before this homework's deadline. We will use these data to improve the course over time.

   - It is completely your choice if you opt into allowing the use of your data for research. For this reason, the survey will first ask you to decide whether you want to be included in the anonymous research study, but you should complete the survey after the consent form, even if you say "no".
   - Your grade for this HW problem will not depend on the content of your survey responses, only on the completion of the survey.
   - The link to the survey is here:
     https://illinois.qualtrics.com/jfe/form/SV$_a 9818IddYYaNUhM$.
   - You do not need to submit an answer for this question on Gradescope, we will be notified of your submission and will upload the points to Canvas. Be sure to enter your netID correctly into the survey so your submission is recorded.
   - If you do not have access to the survey, please email Prof. Alvarez.

Conflict exam requests:

- Our exam schedule has been posted since before the first day of the semester.
- Some of you either have classes, labs, other exams or work that overlap with our exams. We will offer conflict exams for those of you who are in these situations but you need to get Prof. Alvarez's approval in order to be able to take the conflict exams.
- As indicated in the student code, conflict exams are to be granted if the student informs the instructor of the conflict within one week after being informed of the examination schedule. Priority will be given to the examination announced in class the earliest in the semester.
- If you have conflicts with one or more of our exams and want to request a conflict exam, you must complete this form by September 6 at 11.59pm:
  https://forms.illinois.edu/sec/2082281966.
- If you do not complete the form by the deadline, we are not required to grant a conflict exam so you might not get one.
- If you do not have access to the form, please email Prof. Alvarez.

Quiz 1 is coming up:

- The first quiz begins next Thursday, September 7. You can already register for the quiz through PrairieTest.
- We strongly recommend you sign up for the first day, in case you get sick. If you sign up for the last couple of days and you get sick, you might not be granted an extension.