Names: ______________________________

NetIDs: ______________________________

Discussion section: D ________ **Group number:** ________

MATH 257 - WORKSHEET 1

Today you and your group will review some topics from previous math classes: sets, systems of equations, plotting in three dimensions, and operations on functions.

What is set builder notation?
Set builder notation expresses a set, a collection of mathematical objects (in this course, usually numbers or vectors). One kind of set builder notation, called set comprehension, consists of two or three parts:

$$\{\textbf{template} : \textbf{constraints}\}$$
$$(\textbf{domains} \text{ on either side})$$

- **Template**: What the set's elements look like.
- **Domains**: Try assigning all the values from these sets to the variables...
- **Constraints**: ...but only keep the elements that satisfy these constraints.

Example 1:

$$\{(x, y) \in \mathbb{R}^2 : y = x\}$$

- The part before the ":" says to try all vectors in $\mathbb{R}^2$ (**domain**), calling the first component x and the second component y (**template**).
- The part after the ":" says to include only the elements where those two components are equal (**constraint**).
- The result is the 45-degree diagonal line from the third quadrant, through the origin, to the first quadrant.

Example 2:

$$\{(-2, -2) + a(1, 1) : a \in \mathbb{R}\}$$

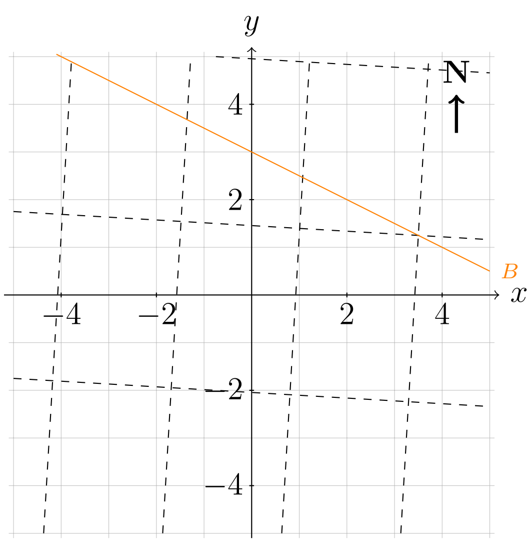
- The part before the ":" says to start with the vector $(-2, -2)$, then add the vector $(1, 1)$ to it scaled by the variable a (**template**).
- The part after the ":" says to plug every real number (**domain**) into a and include the resulting vector in the set (there is no **constraint**).
- The resulting set is the same as in Example 1!

Your team is trying to recover city map data after a data center fire. Planners' notes that survived the fire describe proposed subway routes and electrical lines in set builder notation, but in different formats.

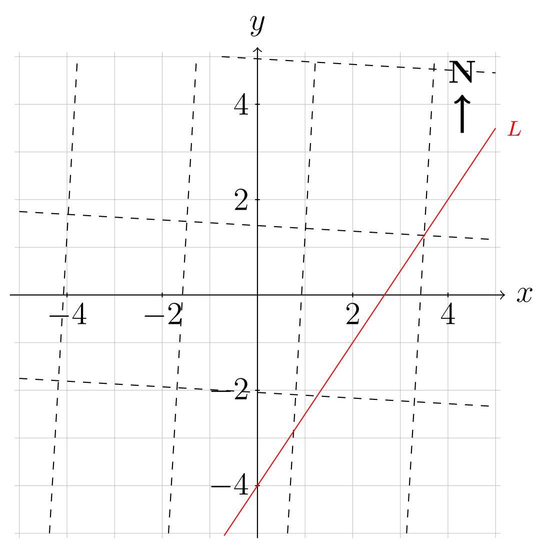
Subway routes		Electrical lines	
Subway route A	$\{(x, y) \in \mathbb{R}^2 : 2y + 8 = 3x\}$	Electrical line J	$\{(2, 2) - a(2, -1) : a \in \mathbb{R}\}$
Subway route B	$\{(x, y) \in \mathbb{R}^2 : y = -(1/2)x + 3\}$	Electrical line K	$\{(2.5, -1) + a(-2, 4) : a \in \mathbb{R}\}$
Subway route C	$\{(x, y) \in \mathbb{R}^2 : 7(y + 2) = -(x - 1)\}$	Electrical line L	$\{(2, -1) - a(-4, -6) : a \in \mathbb{R}\}$

(1) Your team is told that **two of the three subway routes and two of the three electrical lines** were chosen for construction, but you're not sure which two out of each set of three. What you do know is that the subways operate on electric power, so **the electrical lines need to be built on top of the subway routes**. As a group, **plot and label** the remaining subway routes and electrical lines on the corresponding street maps below. Then, use the plots to discuss and decide which **TWO PAIRS** of a subway route with an electrical line should be built.

Subway routes



Electrical lines

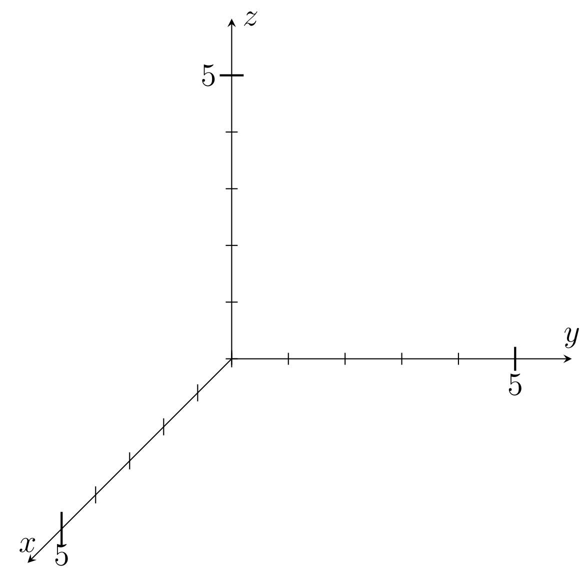


Note: Dashed lines already plotted above represent the street grid.

(2) A subway station needs to be built where the two subway routes intersect.

(a) As a group, set up and solve a system of equations to determine the coordinates of the intersection point.

(b) The subway will be 3 units under street level. Using the x–y plane as the subway level, plot the coordinates of the planned street-level station on the plot below.



(c) An elevator shaft needs to be built to connect the street-level station to the subways. Work together to come up with **two different ways** of writing the set, in set builder notation, representing the vertical line containing the elevator shaft.

(3) **Reflection**. Take a few minutes to pause the problem solving and discuss what you have learned from it.

(a) Another kind of set builder notation lists the elements separated by commas, like this: $\{(1,1),(2,2),(3,-1)\}$. What is useful about set comprehensions compared to listing the elements? What is challenging?

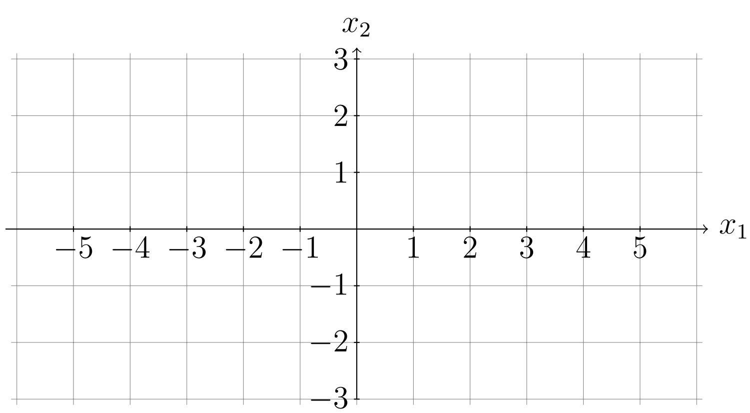
(b) What are some strategies you can use to determine if two sets written using set comprehensions are the same? Can you do it without physically plotting them? Summarize your discussion below in 3–4 bullet points.

Nice work! Your team has helped the city recover its subway project plans.

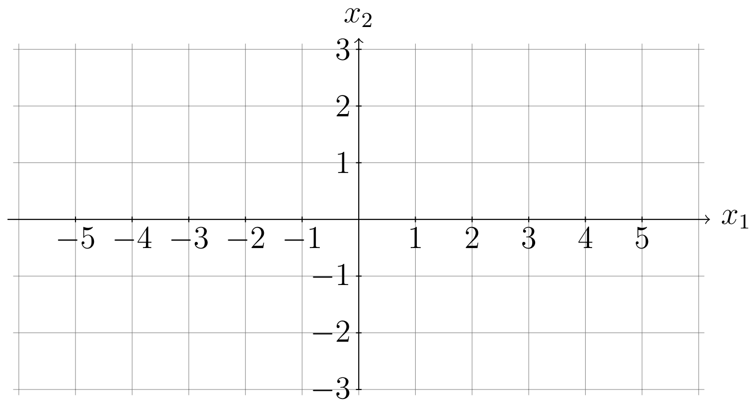
The previous parts use an applied context so that you have real-world analogies for the concepts you're learning. Now let's take the context away and try some abstract problems. Your team will now review working with functions.

In the problems below, let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) := x+1$, and let $g : \mathbb{R} \to \mathbb{R}$ be the function given by $g(x) := \sin(x) - 1$.

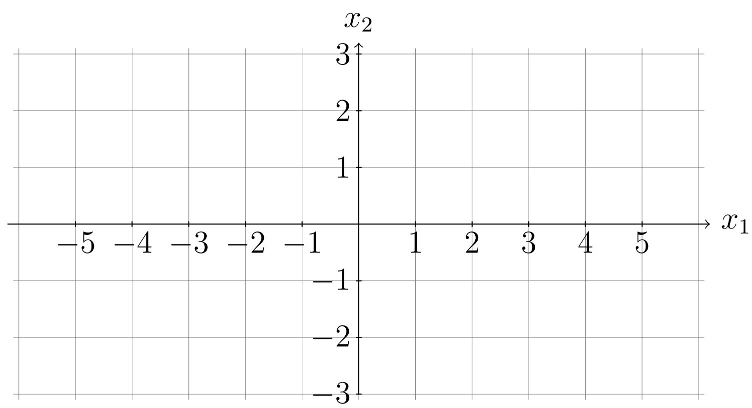
(4) Draw the graphs of f and g.



(5) Let $2g$ be the function given by $(2g)(x) := 2g(x)$. Draw the graph of g and $2g$.



(6) Let $f + g$ be the function given by $(f + g)(x) := f(x) + g(x)$. Draw the graph of $f + g$.



(7) Let $f \circ g$ be the function given by $(f \circ g)(x) := f(g(x))$, and let $g \circ f$ be the function given $(g \circ f)(x) := g(f(x))$. Is $f \circ g = g \circ f$? Explain.