Homework 6 cont.

3) $S = nR\ln(V-nb) + \text{const}$ $\qquad \frac{dS}{dV} = \frac{nR}{V-nb}$

1. $\frac{dS}{dV} = \frac{P}{T} = \frac{nR}{V-nb}$ $\qquad p(V-nb) = nRT$

$p(V-nb) = NkT$

$\boxed{T \approx 393.7\,K}$

2. $pV - pnb = nRT$

$pV = nRT + pnb$

$V = \frac{nRT + pnb}{p}$

$V = \frac{1.5\,mol \cdot R \cdot 350K + 101300\,Pa \cdot 1.5\,mol \cdot 8\times10^{-4}}{101300\,Pa}$

$\boxed{V \approx 0.0443\,m^3}$

4) $p(V-Nb) = NkT$

$b = 1.1\times10^{-28}\,m^3$

$n = 3$ moles

$T = 300\,K$

$V_i = 0.001\,m^3$

1. $P_i = ?$

$P = \frac{NkT}{V-Nb} \approx \boxed{9334290\ Pa}$

2. Isothermal expansion. $V_f = 0.002\,m^3$.

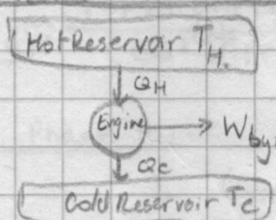
$\boxed{P \approx 4152241\ Pa}$

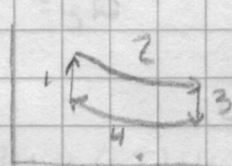
3. $W_{by} = ?$

$W_{by} = \int p\,dV = NkT\left[-\frac{1}{(V-nb)^2}\right]_{0.001}^{0.002} \approx \boxed{6058.6\ J}$

Lecture 7: Heat Engines

10/30/23





A heat engine uses the spontaneous flow of heat to do some work.

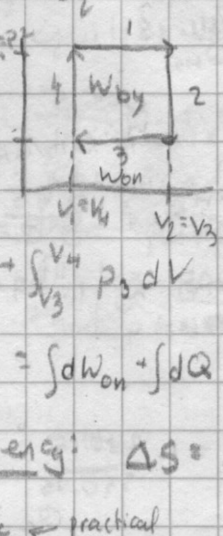
$\Delta S_{hot} > 0 \qquad \Delta S_{cold} > 0$

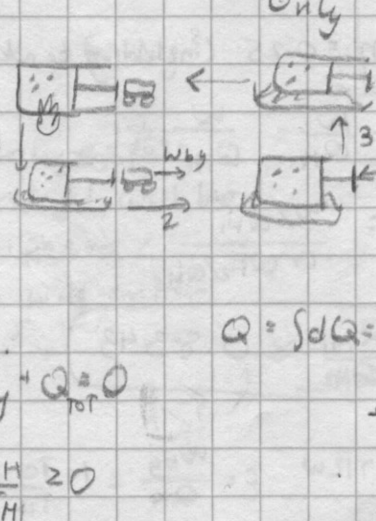
$\Delta S_{hot} + \Delta S_{cold} \geq 0$

Only some heat is turned into

Important Eq: $dS = \frac{dQ}{T}$

A very simple engine





$\frac{Q_C}{T_C} - \frac{Q_H}{T_H} \geq 0$

$W_{by} = \int_{V_1}^{V_2} p_1\,dV + \int_{V_3}^{V_4} p_3\,dV$

$Q = \int dQ = \int_{T_4}^{T_1} C_V\,dT + \int_{T_1}^{T_2} C_p\,dT \;\Big]\, Q_H$

$+ \int_{T_2}^{T_3} C_V\,dT + \int_{T_3}^{T_4} C_p\,dT \;\Big]\, -Q_C$

$\Delta U = \int dU = \int dW_{on} + \int dQ = -W_{by} + Q_{tot} = 0$

Carnot Efficiency: $\Delta S = \frac{Q_C}{T_C} - \frac{Q_H}{T_H} \geq 0$

$\varepsilon = \frac{W_{by}}{Q_H} = 1 - \frac{Q_C}{Q_H}$ ← practical $\qquad \frac{Q_H}{T_H} \geq \frac{T_C}{T_H}$

$\varepsilon \leq 1 - \frac{T_C}{T_H}$ ← limit $\qquad Q_H = W + Q_C$

$\Delta S = \int \frac{1}{T} dU + \int \frac{p}{T} dV$

Monatomic

Ideal gas law:

$pV = NkT$

$p = \frac{NkT}{V}$

Equipartition:

$U = \frac{N_{DOF}}{2} NkT + \text{const}$

$= \frac{3}{2} NkT + \text{const}$

$\Delta S = \int_{T_i}^{T_f} \frac{1}{T} \frac{3}{2} Nk\, dT + \int_{V_i}^{V_f} \frac{NkT}{VT} dV$

$\frac{dU}{dT} = \frac{3}{2} Nk \Rightarrow dU = \frac{3}{2} Nk\, dT$

$= \frac{3}{2} k \ln(T_f/T_i) + Nk \ln\left(\frac{V_f}{V_i}\right)$

Homework 6: Quasistatic Processes

10/27/23

1) Adiabatic

$\boxed{V_i \rightarrow V_f}$

$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) + \alpha nR \ln(T_f/T_i)$

$\alpha = \frac{N_{DOF}}{2}$

Argon $\quad p_i = 100\text{ kPa} \quad T_i = 300\text{ K} \quad V_i = 0.01\text{ m}^3 \quad V_f = 0.029\text{ m}^3$

1. ΔS due to ΔV (ignoring ΔU)

$R = k N_A \leftarrow$ avogadro

$p = \frac{NkT}{V} \Rightarrow N = \frac{PV}{kT} = \frac{100\text{ kPa} \cdot 0.01\text{ m}^3}{k \cdot 300\text{ K}} \approx 2.45 \times 10^{24}$

$nk \cdot N_A = nR \approx 3.\overline{3}$

$\Delta S_v = 3.\overline{3} \cdot \ln\left(\frac{0.029}{0.01}\right) \approx 3.55$

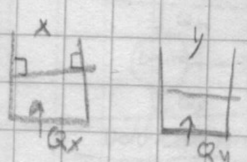
2. $PV^{\gamma} = \text{const} \quad \gamma = \frac{\alpha+1}{\alpha} \leftarrow$ final cond. $\quad T_f = ?$

$p = \frac{c}{V^{\gamma}}$

$P_i V_i^{5/3} = P_f V_f^{5/3} \quad P_f = \frac{P_i V_i^{5/3}}{V_f^{5/3}} = 16956.5\text{ Pa}$

$PV = NkT \quad T_f = \frac{PV}{Nk} \approx 147.5\text{ K}$

2)



$T_i = 30°C \quad V_i = 0.5\text{ L}$

$P_i = 1\text{ atm}$

$Q_x \rightarrow$ raise T_x by $10°C$

$Q_y \rightarrow$ raise T_y by $10°C$

$\Delta U = Q - pdV$

$Q_x = \Delta U + \cancel{pdV}$

$Q_y = \Delta U + pdV$

$Q_x - Q_y = -pdV = -W_{by}$

$W_{by} = \int_{V_i}^{V_f} p\, dV$

$= p(V_f - V_i)$ since y is at const pressure

$PV = NkT$: $\quad N = \frac{1\text{ atm} \cdot 0.0005\text{ m}^3}{303.15°K \cdot k} \approx 1.2107 \times 10^{22}$

$V_f = \frac{NkT}{P_f} = \frac{N \cdot k \cdot 313.15}{1\text{ atm}} \approx 0.000516\text{ m}^3 \approx 0.516\text{ L}$

$W = \frac{-P V_{30} \Delta T}{T_{30}} \approx -1.67\text{ J}$