$d = 0.17m$

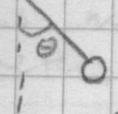
$T = ?$

$W^2 = \frac{mgR_{cm}}{I}$ $\qquad I_{rod} = \frac{1}{12}mL^2 + mR^2$

$= \frac{1}{12}m(1)^2 + m(0.33)^2 = \frac{1}{12}m + 0.1089m$

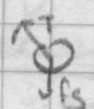
Angular f $\omega = \sqrt{\frac{2.5g(0.33)}{0.1089(2.5)}} = \sqrt{\frac{0.33}{0.1089}} \approx 4.104 \text{ rad/s}$

$T = \frac{1}{f} = \frac{2\pi}{\omega} \approx \boxed{1.53s}$

4)  $m = 1.4 \text{ kg}$ $\quad L = 2.78m$ $\quad \theta = 8.2°$ $\quad t = 0 =$ released

a) $T = ?$ $\qquad T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\frac{2.78}{9.81}} \approx \boxed{3.34s}$

b) $\perp F$ to string on bob @ $t=0$



$F_g = mg = 1.4kg \cdot 9.81 \text{ m/s}^2 = 13.734N$

$F_g \cdot \sin\theta \approx \boxed{1.959N}$

c) max speed?

$mgh = \frac{1}{2}mv^2 \rightarrow 0.0284 \cdot 9.81 \cdot 1.4 = \frac{1}{2}(1.4)v^2$

$h = \sqrt{2.78^2 - (2.78\sin(8.2))^2} \approx 0.0284m$ $\qquad v \approx 0.747 \text{ m/s}$

OR $v_{max} = r\omega A = L\omega\theta_{max} = 2.78 \cdot 8.2° \cdot \pi/180 \approx 0.747$

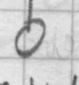
d) angular displacement @ $t = 3.81s$?

$\theta = \theta_{max}\cos(\omega t)$ $\qquad \omega = \sqrt{g/L} = 1.8785 \text{ rad/s}$

$\theta = 8.2° \cos(1.8785 \text{ rad/s} \cdot (3.81))$

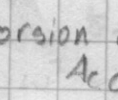
$\theta = 8.2° \cdot 0.6418 \approx \boxed{5.263°}$

e) tan acceleration through equil position $(\theta = 0)$

 $F = ma$ $\qquad a = \frac{F}{m}$ $\qquad 0 = \Sigma F_{ext}$ $\qquad a = 0 \text{ m/s}$

f) radial acceleration?

$a = V^2/R \rightarrow (0.747)^2/2.78 = 0.2009 \text{ m/s}^2$

5)  $m = 5.5kg$ $\quad R = 0.79m$

$F = 40.7N$ on edge rotates it 1/4 revolutions from eq

a) Torsion constant k?

According to notes $\tau = I\alpha = -k\theta$

$rF\sin\theta = -k(\pi/2) \rightarrow (0.79m)(40.7N) = -\frac{\pi}{2}k$

$k \approx 20.5 \text{ Nm/rad}$

b) τ_{min} for 1 full rev? $\qquad \tau = (20.5)(2\pi) = 128.612 \text{ Nm}$

c) $\omega = \sqrt{\frac{k}{I}} = \sqrt{\frac{20.5 \text{ N/m}}{1.716 \text{ kgm}^2}} = \boxed{3.45 \text{ rad/s}}$

Lecture 22: Simple Harmonic Motion

Torsion Pendulum:

$\tau = I\alpha \rightarrow I\frac{d^2\theta}{dt^2} \rightarrow \frac{d^2\theta}{dt^2} = -\frac{k}{I}\theta = -\omega^2\theta$

* κ kappa not k

similar to spring: $-k\theta$

$\omega = \sqrt{k/I}$

don't confuse with angular velocity

$\theta(t) = \theta_{max}\cos(\omega t + \phi)$ ← from IC

wire

θ

I

Pendulum: $\tau = I\alpha$

$-Mg X_{cm}$ → $I\frac{d^2\theta}{dt^2}$

$-Mg R_{cm}\theta$

$\vec{\tau} = r \times F$

$= R_{cm}(mg\sin\theta)$

R_{cm}

mg

$\frac{d^2\theta}{dt^2} = -\frac{MgR_{cm}\theta}{I}$ Approximate for θ small

$\frac{d^2\theta}{dt^2} = -\omega^2\theta$ $\boxed{\omega = \sqrt{\frac{MgR_{cm}}{I}}} \rightarrow \sim\sqrt{\frac{MgR_{cm}}{M f(R)}}$ Independent of mass

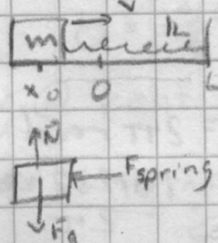
For simple pendulums: $\omega = \sqrt{\frac{MgL}{ML^2}} = \sqrt{\frac{g}{L}}$ so $T = 2\pi\sqrt{L/g}$

We can see ω of a stick as $\omega = \sqrt{\frac{g}{\frac{2}{3}L}}$ if we substitute I.

We also see that for a hoop, $\omega = \sqrt{\frac{g}{D}}$ where D is the diameter.

Homework 24

2)



x_0 0 L

N

F_{spring}

F_g

$m = 5\text{ kg}$

@ $t = 0$: $x = x_0 = -0.33\text{ m}$ $V = 2\text{ m/s}$

$k = 61.2\text{ N/m}$ When will $x = 0$?

$x(t) = A\cos(\omega t + \phi)$ The problem doesn't start at max or min displacement.

To find A: Use x_{max} and IC.

To find ϕ: use A

frequency $\omega = \sqrt{k/m} = \sqrt{\frac{61.2\text{ N/m}}{5\text{ kg}}} = 3.49857\ \frac{1}{s}$

$\sum E = K_{block} + E_{spring} = \frac{1}{2}mv^2 + kx^2/2 = \frac{kA^2}{2}$

$mv^2 + kx^2 = kA^2$

$20 + 61.2(0.33)^2 = 61.2A^2$ → $A = 0.66\text{ m}$

$x(0) = -0.33 = 0.66\cos(3.4986(0) + \phi)$

$-0.33 = 0.66\cos(\phi) \rightarrow -1/2 = \cos\phi$ $\phi = \pm 2\pi/3 \approx 2.094$

only $-2\pi/3$ is correct

$x(t) = 0.66\cos(3.4986t - \frac{2\pi}{3})$

$0 = 0.66\cos(3.4986t - \frac{2\pi}{3})$

$3.4986t = \frac{7\pi}{6}$ or $\frac{\pi}{6}$

$t \approx 1.0475$ or $\boxed{0.14966}$

$3.4986t - \frac{2\pi}{3} = \pm\frac{\pi}{2}$

Period