HW 12 cont. . .   11/29/23

3) 3ϵ, 2ϵ, ϵ, 0 $\epsilon = 10^{-20}$ J

$T_{env} = 370$ K

1. Most probable energy of oscillator?

$P(i) = \frac{f_i}{\sum_j f_j}$ $f = e^{-E_i/kT}$ $\Rightarrow P(i) = \frac{\sum_j e^{E_i/kT}}{e^{E_i/kT}}$ Max when $i = 0$: $\boxed{0\epsilon}$

2. $\frac{P(E=2\epsilon)}{P(E=0)} = \frac{e^{-E_2/kT}}{e^{E_0/kT}} = \frac{1}{e^{2\epsilon/kT}} \approx \boxed{0.0108}$

4) $\updownarrow \Delta E$ $E_0 = 0$eV $E_1 = 0.1$ eV (Doubly degenerate, have same energy levels)

1. $T = 300$ K $P(E = 0.1\text{eV}) = ?$

$= \frac{2e^{-0.1\text{eV}/kT}}{2e^{-0.1\text{eV}/kT} + e^{-0/kT}} \approx \boxed{0.0401}$

2. $P(E=0)$ as $T \to \infty$? $P(0) = \frac{1}{2e^{-0.1/kT}+1} = \frac{1}{\frac{2}{e^{0.1/kT}}+1} \Rightarrow \frac{1}{\frac{2}{e^0}+1} = \frac{1}{2+1} = \boxed{\frac{1}{3}}$

3. $P(E=0) = P(E=0.1) = 0.5$

$\frac{1}{\frac{2}{e^{0.1/kT}}+1} = \frac{1}{2}$ $\frac{2}{e^{0.1/kT}} = 1$ $2 = e^{0.1/kT}$ $\ln 2 = \frac{0.1}{kT}$ $T = \frac{0.1}{k\ln 2} \approx \boxed{1674 \text{ K}}$

4. Entropy of N such dots at very low T?

$S = k_B \ln \Omega = k_B \ln \binom{N}{N/2} = k_B \ln\left(\frac{N!}{\frac{N}{2}!\frac{N}{2}!}\right) = k_B \ln\left(\frac{N/2!}{N/2!}\right) = \boxed{0}$

5) 2s, 2p; 1s $E_{1s} = 0$ $E_2 = 10.2$eV

1. Frac of H is in 2p when $T = 5900$ K?

$P(10.2\text{eV}) = \frac{4e^{-10.2/kT}}{4e^{-10.2/kT} + e^{0/kT}} \approx 7.74 \times 10^{-9}$

$\Rightarrow \cdot \frac{3}{4} \approx 5.81 \times 10^{-9}$

2. $T = 4300$ K?

$P(10.2) \approx 4.4338 \times 10^{-12} \Rightarrow 3.325 \times 10^{-12}$

HW 13: Semiconductors   11/29/23

1) $T_0 = 270$ K $R_0 = 0.01\Omega$

$T_1 = 300$ K $R_1 = 2.591 \times 10^{-4}\Omega$ Energy gap $\Delta = ?$

$\rho = \rho_0 e^{\Delta/2kT} \Rightarrow \rho_0 = \rho e^{-\Delta/2kT}$ $R_1 e^{-\Delta/kT_1} = R_0 e^{-\Delta/kT_0}$

$\frac{R_1}{R_0} = e^{\frac{\Delta}{kT_1} - \frac{\Delta}{kT_0}}$ $\Delta T_0 - \Delta T_1 = 2kT_1T_0 \ln\left(\frac{R_1}{R_0}\right)$ $\Delta = \frac{2kT_1T_0\ln\left(\frac{R_1}{R_0}\right)}{T_0 - T_1} \approx \boxed{1.7\text{eV}}$

2) $\Delta = 1.1$ eV

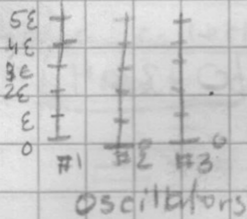
$R(T=300\text{K}) = 2\Omega$ $\rho_0 = 2e^{-1.1/2k(300)}$

$R(T=77\text{K}) = ?$ $\rho = \rho_0 e^{\Delta/2kT} \Rightarrow \sim 1.14 \times 10^{27}\ \Omega$

* Use the more accurate k and eV-J vals *

HW 12: Boltzmann II

1)



1. Number of microstates for $U = 3\varepsilon$
$= E_1 + E_2 + E_3$

$x + y + z = 3$ $\quad \binom{3+2}{2}$

$* \# * \# *$ $\quad = \binom{5}{2} = \boxed{10 \text{ states}}$

Alternatively:

$E_1 = 3\varepsilon$	$E_2 + E_3 = 0$	1 state	
$E_1 = 2\varepsilon$	$E_2 + E_3 = 1\varepsilon$	$2 \cdot 1 = 2$ states	10 states
$E_1 = \varepsilon$	$E_2 + E_3 = 2\varepsilon$	$3 \cdot 1 = 3$ states	
$E_1 = 0$	$E_2 + E_3 = 3\varepsilon$	$4 \cdot 1 = 4$ states	

2. Suppose $U = 5\varepsilon$

How many microstates?

$x + y + z = 5$ $\quad \binom{7}{2} = \boxed{21}$

3. Probability that $U_1 = 0$?

$y + z = 5$ $\quad \binom{6}{1}$

$$\frac{\Omega(E_1 = 0, U = 5\varepsilon)}{\Omega(U = 5\varepsilon)} = \frac{6}{21} \approx \boxed{0.2857}$$

4. $P(E_1 = 4\varepsilon) = ?$

$E_2 + E_3 = 1$ $\quad$ 2 microstates $\quad (E_2 = 0,\ E_3 = 0)$

$$\frac{2}{21} \approx \boxed{0.0952}$$

2) $f = 2.1 \times 10^{12}$ Hz

$\varepsilon = hf$ $\quad$ 2ε ——

$h = 6.626 \times 10^{-34}$ J/s $\quad$ ε ——, 0 ——

1. $\frac{P_1}{P_0} = \frac{1}{2}$ $\quad P_1 = P(E = \varepsilon)$ $\quad P_0 = P(E = 0)$ $\quad T = ?$

$$P = \frac{f_i}{\sum_j f_j} \Rightarrow \frac{f_1 / \cancel{\sum f_j}}{f_0 / \cancel{\sum f_j}} = \frac{f_1}{f_0} = \frac{e^{-\varepsilon/kT}}{e^0} = e^{-\varepsilon/kT} = \frac{1}{2} \quad -\frac{\varepsilon}{kT} = \ln\left(\tfrac{1}{2}\right)$$

$$T = -\frac{\varepsilon}{k \ln(\frac{1}{2})} \approx \boxed{145.4\ \text{K}}$$

2. $T = 10\%$ T of part (1) $\quad P_1/P_0 = ?$

$$\frac{P_1}{P_0} = e^{-\varepsilon/k(14.54)} \approx \boxed{0.000977}$$

3. Part 2 temp, $T_2 = 14.54$ K

$$\frac{P_2}{P_1} = \frac{e^{-2\varepsilon/kT}}{e^{-\varepsilon/kT}} = e^{-\varepsilon/kT} \approx \boxed{0.000977}$$

4. $\frac{\langle E \rangle}{kT} = ?$ $\quad$ At low T $(kT \ll \varepsilon)$: $\langle E \rangle \approx \varepsilon e^{-\varepsilon/kT} \ll kT$

$$\frac{\langle E \rangle}{kT} \approx \frac{\varepsilon e^{-\varepsilon/kT}}{kT} \approx \boxed{0.00677}$$

At high T: $\langle E \rangle \approx kT$

Since:
$$\langle E \rangle = \sum E_i P_i = \frac{\varepsilon}{e^{\varepsilon/kT} - 1}$$

... HW 12 TBCont.