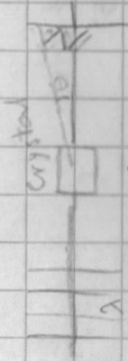


HW 6: The wave function

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- 1)  $d = 0.5 \text{ nm}$
 $\theta = 0.27^\circ$

1. $a \sin(\theta) = m\lambda$ $\psi(x) = 5.5 \text{ nm} \cdot \sin(0.27) = \lambda$
 $(\lambda \approx 0.0259 \text{ nm})$

2. $E_{\text{electron}} = ?$ $E \approx mc^2$

$\lambda = \frac{h}{mv} = \frac{h^2}{2mE} = \frac{h^2}{2 \cdot 9.1 \times 10^{-31} \text{ kg} \cdot E}$

$0.0259 \text{ nm} = \frac{h}{mv} \approx 15262.8 \text{ m/s}$ $K \approx 1.95 \times 10^{-19} \text{ J}$

3. $E = hf = \frac{1240 \text{ eV nm}}{\lambda}$ $E_{\text{photon}} = \frac{1240 \text{ eV nm}}{0.0259 \text{ nm}} = 47843 \text{ eV}$

2) $\psi(x) = A^* \sin(2\pi x/\lambda)$ $\omega = \frac{2\pi}{T} \leq x \leq \frac{T}{2}$ $\psi = 0$ elsewhere

$\lambda = 2.5 \text{ nm}$

1. $p(x) = |\psi(x)|^2 = A^2 \sin^2(\frac{2\pi x}{2.5 \text{ nm}})$

$\int_{-1.25}^{1.25} A^2 \sin^2(\frac{2\pi x}{2.5}) dx$ $\int_{-1.25}^{1.25} \sin^2(\frac{2\pi x}{2.5}) dx = 2.5$

$A^2 = \frac{1}{2.5} \Rightarrow A \approx 0.894 \text{ nm}^{-1/2}$

2. $\psi(x) = A e^{ikx}$ $k = 2\pi/\lambda$ $-\frac{\pi}{L} \leq x \leq \frac{\pi}{L}$ $\psi = 0$ elsewhere

$\int_{-\pi/L}^{\pi/L} (A e^{ikx})(A e^{-ikx}) dx = \int_{-\pi/L}^{\pi/L} A^2 dx = \frac{2\pi}{L} A^2 = 1$ $A^2 = \frac{L}{2\pi}$

$A^2 = \frac{2 \text{ nm}}{2\pi} \Rightarrow A \approx 0.564$

3) $\psi(x) = Nx$ $0 \leq x \leq L/2$ $p = (Nx)^2$ $0 \leq x \leq L/2$

$\psi(x) = N(L-x)$ $\frac{L}{2} \leq x \leq L$ $p = (N(L-x))^2$ $\frac{L}{2} \leq x \leq L$

$\psi(x) = 0$ else

$L = 3.5 \text{ nm}$

$1 = \int_0^{L/2} N^2 x^2 dx + \int_{L/2}^L N^2 (L-x)^2 dx$ $N \approx 0.529 \text{ nm}^{-1}$

2. probability $|\psi(x)|^2 = 1$ $N^2 x^2 \Rightarrow |\psi(\frac{3.5}{2})|^2 \approx 0.381 \text{ (nm)}^{-1}$

3. $\int_0^{L/4} N^2 x^2 dx = \int_0^{0.875} (0.529)^2 x^2 dx \approx 0.0625$