Homework 4: Entropy of Physical systems

1) $T = 296\,K$, $\Delta Q = 10^{-6}\,J$, $T_0 \approx T_f$

1. $\Delta S = ?$ $\quad \Delta S = \int_{U_1}^{U_2} \frac{1}{T} dU = \frac{1}{T}(10^{-6}) \approx \boxed{3.38 \times 10^{-9}}$

2. Entropy increase if block was 1000 times larger? 1 times. Not size dependent

2) $m = 1.4\,kg$ gold, $T_A = 300\,K$, C_V, $m_C = 2.2\,kg$ copper, $T_C = 400\,K$, $c_v = 386\,\frac{J}{kg \cdot K}$

$C_V = 126\,\frac{J}{kg\,K}$

1. $T_f = ?$ $\quad -\Delta Q_A = \Delta Q_C \quad -c_{v_1}(m_A)(T_f - T_A) = c_{v_2}(m_C)(T_f - T_C)$

$-176.4(T_f - T_A) = 849.2(T_f - T_C)$

$849.2\,T_C + 176.4\,T_A = 1025.6\,T_f \quad \boxed{T_f \approx 382.8\,K}$

2. $\Delta S = \int_{U_1}^{U_2} \frac{1}{T} dU \quad C = \frac{dU}{dT} \quad \Delta S_C = ?$

$dU = C\,dT \rightarrow \Delta S = \int_{T_1}^{T_2} \frac{C}{T} dT = 849.2\,[\ln T]_{T_1}^{T_2} = 849.2\,[\ln(382.8) - \ln(400$

$\boxed{\Delta S_C \approx -37.33\,\frac{J}{K}}$

3. ΔS_{TOT} $\quad \Delta S_A = \int_{T_1}^{T_2} \frac{C}{T} dT \approx 42.994 \quad \Delta S_{TOT} = \Delta S_A + \Delta S_C \approx \boxed{5.67\,J/K}$

3)

m_A	m_B
T_A	T_B

$m_A = 3\,kg \quad m_B = 6\,kg$

$T_A = 0.5\,K \quad T_B = 0.2\,K$

No heat to environment.

$C_V(T) = 0.001T\,\frac{J}{kg \cdot K^2}$

$\Delta S_{TOT} = ?$

$C = \frac{dQ}{dT}$

$C\,dT = dQ$

$\int C\,dT = \Delta Q$

$Q_B = \int_{T_B}^{T_{eq}} \frac{bT}{1000} dT = \frac{3}{1000} T^2 \Big|_{T_B}^{T_F}$

$Q_A = \int_{T_A}^{T_F} \frac{3T}{1000} dT = \frac{3}{1000} T^2 \Big|_{T_A}^{T_F}$

$-\Delta Q_A = \Delta Q_B$

$Q_A + Q_B = 0 \rightarrow \frac{3}{2000}(T_f^2 - T_A^2) + \frac{3}{1000}(T_f^2 - T_B^2) = 0$

$\rightarrow m_A T_f^2 - m_A T_a^2 + m_B T_f^2 - m_B T_B^2 = 0$

$T_f^2 = \frac{m_A T_a^2 + m_B T_B^2}{m_A + m_B} \rightarrow T_f \approx 0.33166$

$\Delta S = \Delta S_A + \Delta S_B$

$= \int_{0.5}^{0.33} \frac{3}{1000} \frac{T}{T} dT + \int_{0.2}^{0.33} \frac{6}{1000} dT$

$\approx \boxed{0.000285\,J/K}$