HW 7: Momentum + Position 9/11/23

1) $\psi(x) = \frac{1}{\sqrt{2a}}$ for $|x| \le a$ $a = 2\,nm$ Estimate Δx w/ $(\Delta x)^2 = \int x^2 p(x)\,dx$

1. $p = \frac{1}{2a}$ $(\Delta x)^2 = \int_{-a}^{a} \frac{x^2}{2a}\,dx = \frac{x^3}{6a}\Big|_{-a}^{a} = \frac{a^3}{6a} - \frac{-a^3}{6a} = \frac{a^3}{3a} = \frac{a^2}{3}$

$\Delta x = \frac{a}{\sqrt{3}} \approx \boxed{1.15\,nm}$

2. Minimum spread?

Heisenberg uncertainty: $\Delta x\,\Delta p \ge \frac{\hbar}{2} = \frac{h}{4\pi}$

$\Delta p \ge \frac{0.655\times10^{-15}\,eV\cdot s}{1.15\,nm} \approx \boxed{2.58\times10^{-16}\,\frac{eV\cdot s}{nm}}$

2) $\psi_1 \to p_1 = 10\,eV/c$ w/ $p = 0.2$

$\psi_2 \to p_2 = 16\,eV/c$ $p = 0.3$

$\psi_3 \to p_3 = 34\,eV/c$ $p = 0.5$

1. $|a| = ?$ $a^2 = 0.2$ $a = \sqrt{0.2} \approx \boxed{0.447}$ $\frac{|a|^2}{|a|^2+|b|^2+|c|^2} = \frac{0.2}{1}$

2. $|b| \approx 0.547$

3. $|c| \approx 0.707$

4. $p_{avg} = 0.2p_1 + 0.3p_2 + 0.5p_3 = \boxed{23.8\,eV/c}$

5. $p = 16\,eV/c$ $KE = ?$ $p = \hbar k = \frac{h}{\lambda}$ $E = \hbar\omega = \frac{p^2}{2m}$

$E = \frac{p^2}{2m} = \frac{m^2v^2}{2m} = \frac{1}{2}mv^2 \to \frac{256\,(eV/c)^2}{2(0.511\,MeV/c^2)} = \frac{256}{1022000}\,eV$

$= \boxed{2.504\times10^{-4}\,eV}$

3) $\psi(x) = A\cos(k_1x) + B\cos(k_2x)$ $A = 2$ $B = 1+3i$

1. $P(+\hbar k_1)$

$e^{ikx} = \cos(kx) + i\sin(kx)$ $e^{-ikx} = \cos(kx) - i\sin(kx)$

$\psi = \frac{A}{2}(e^{ik_1x} + e^{-ik_1x}) + \frac{B}{2}(e^{ik_2x} + e^{-ik_2x})$

$2\left(\frac{e^{ik_1x} + e^{-ik_1x}}{2}\right), (1+3i)\left(\frac{e^{ik_2x} + e^{-ik_2x}}{2}\right)$

$\frac{1}{1+1+\frac{10}{4}+\frac{10}{4}} = \frac{1}{\frac{28}{4}} = \boxed{1/7}$

2. $p_{avg} = ?$ $\frac{1}{7}\hbar k_1 + \frac{1}{7}(-\hbar k_1) + \frac{5}{14}\hbar k_2 + \frac{5}{14}(-\hbar k_2) = \boxed{0\,eV/c}$

3. $P(p_{avg}) = ?$ Probability of the measured momentum $= p_{avg}$?

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