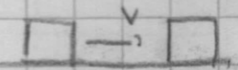
Lecture 14: More elastic collisions + Avg force

Big ideas:
1. Rate of approach = rate of separation
2. An impulse causes a change in momentum

In CM frame, the speed of an object before an elastic collision is the same as the speed of the object after.
So the magnitude of the difference of velocities is the same.
* The rate of approach = rate of separation in any reference frame

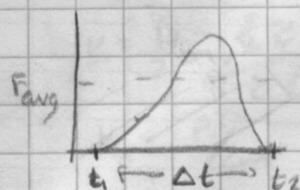
Ex:



V/2 because speed of approach is the same as speed of separation.

Forces during Collisions - Impulse

$\vec{F}_{Tot} = m\vec{a}$ $\quad \vec{F}_{Tot} = \frac{dP}{dt}$ $\quad \int_{t_1}^{t_2} F_{Tot}\, dt = \vec{P}(t_2) - \vec{P}(t_1)$



$\Delta P = \vec{F}_{avg} \Delta t$ $\qquad$ Impulse: $\int_{t_1}^{t_2} \vec{F}_{tot}\, dt = F_{avg} \Delta t$

Elastic Collisions HW

2) 

$m_1 = 3.5$ kg $\quad m_2 = 1.7$ kg $\quad \mu = ?$
$v_1 = 6.3$ m/s $\quad v_2 = 0$

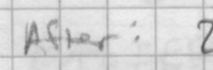
$\Sigma p = p_1 + p_2 = m_1 v_1 + m_2 v_2 = (3.5)(6.3) + 0 = 22.05$ kg m/s

After: $22.05 = 5.2\text{kg} \cdot v \qquad v = 4.24038$

$KE = \frac{5.2}{2} \cdot v^2 = 46.75024$ J

Work $= F \cdot d = f_k \cdot d = \mu_k N \cdot d = \mu_k ((m_1 + m_2) \cdot g) d$

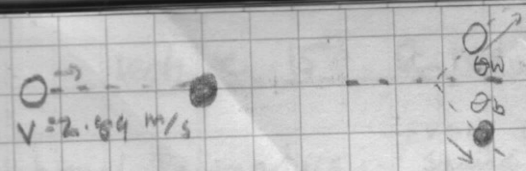
$46.75024 = \mu_k ((5.2) \cdot 9.8)(1.85) \qquad \mu_k = 0.495887$

3) 

$m_1 = 103$ kg $\quad m_2 = 91$ kg
$v_1 = 4$ m/s $\quad v_2 = -3.4$ m/s

1. $\frac{(103)(4) + (91)(-3.4)}{194} \approx 0.53$ m/s
2. v_{c1} in CM frame? $4 - 0.53 = 3.47$ m/s
3. $\vec{v}_{f_1}$? After elastic, flipped -3.47 m/s
4. v_{f_1} in orig frame? $-3.47 + 0.53 = -2.94$ m/s
5. v_{f_2} in orig? $-3.4 - 0.53 = -3.93 \rightarrow 3.93 + 0.53 = 4.46$

4)



$m_w = 1.32$ kg $\quad \theta_w = 68°$

$m_b = 1.32$ kg $\quad \theta_b = 22°$

1. $v_{wf} = ?$

$p_0 = 2.89 \cdot 1.32 = 3.8148$ kg m/s

$\Sigma p_f = p_{wf} + p_{bf} = p_{wy_f} + p_{wx_f} - p_{by_f} - p_{bx_f} = m_w v_w \cos\theta_w + m_b v_b \cos\theta_b$

$\frac{1}{2} m_w v_w^2 = \frac{1}{2} m_w v_{wf}^2 + \frac{1}{2} m_b v_{bf}^2$ $\quad 0 = m_w v_w \sin\theta_w - m_b v_b \sin\theta_b$

$3.8148 = m_w v_w \cos\theta_w + m_b v_b \cos\theta_b$

$m_w v_w^2 = m_w v_{wf}^2 + m_b v_{bf}^2$

$v_w^2 = v_{wf}^2 + v_{bf}^2$ $\quad 2.89 - v_{wf}\cos 68° = v_{bf}\cos 22°$

$8.3521 = v_{wf}^2 + \left(\frac{2.89 - v_w \cos 68°}{\cos 22°}\right)^2$

$8.3521 = v_{wf}^2 + (3.1169 - 0.404 v_w)^2$

$8.3521 = v_{wf}^2 + 9.715 - 2.5186 w_f + 0.16323 w_f^2$

$7.180048 = v_{wf}^2 - 2.165226 w_f - 8.352099$

$0 = v_{wf}^2 - 2.1652261 w_f + 1.17205$

$v_{wf} = 1.08266 \approx 1.08$ m/s

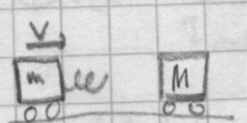
2. $0 = 1.08 \cdot 1.32 \sin 68° - v_{bf} \cdot 1.32 \sin 22°$

$v_{bf} = \frac{1.08 \sin 68°}{\sin 22°} \approx \boxed{2.673 \text{ m/s}}$

3. $p_f = p_0 = 2.89 \cdot 1.32 = \boxed{3.8146 \text{ kg m/s}}$

4. $E_f = \Sigma KE = \frac{1}{2}(1.32)(1.08)^2 + \frac{1}{2}(1.32)(2.673)^2$
$= 0.769824 + 4.715.. = 5.485477$ J $\approx \boxed{5.49 \text{ J}}$

5)



$m_1 = 3.2$ kg $\quad v_{1,i} = 2.1 \frac{m}{s}$ $\quad M_2 = 4.3$ kg

v_{1f} in CM frame = ?

$v_{cm} = \frac{3.2 \cdot 2.1 + 4.3 \cdot 0}{7.5} = 0.896$ m/s

$v_{1,i}$ in CM: $2.1 - 0.896 = 1.204$ m/s

$v_{2,i}$ in CM: $0 - 0.896 = -0.896$

In CM frame, just flip sign. So:

$v_{1,f} = -v_{1,i} = \boxed{-1.204 \text{ m/s}}$