HW 11: Boltzmann Factors

1) $E_C = 5E_A$. $\quad P(E_B) = ?$ $\quad E_A = kT$

$E_B = 3E_A$

E_A

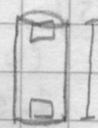
$$P_B = \frac{2e^{-E_B/kT}}{e^{-E_A/kT} + 2e^{-E_B/kT} + 3e^{-E_C/kT}} = \frac{2e^{-3}}{e^{-1} + 2e^{-3} + 3e^{-5}} \approx \boxed{0.204}$$

2) $N = 10^{24}$

$m = 3\times10^{-26}\,kg$

$T = 300\ K$

V is constant



$h = 6\times10^3\,m$ $\quad \frac{P_{TOP}}{P_{bot}} = ?$

Well, we know $\frac{P_{TOP}}{P_{bot}} = \frac{P(top)}{P(bot)}$ pressure vs probability

$$\frac{P(h_2, m)}{P(h_1, m)} = e^{-\frac{mg(h_2 - h_1)}{kT}} \leftarrow \text{potential energy}$$

$$= e^{-m\cdot g\,(6\times10^3 - 0)/k(300)} \approx \boxed{0.653}$$

3) $E_{down} = \mu B$

$E_{up} = -\mu B$ $\quad \mu = 9.3\times10^{-24}\ J/Tesla$

1. Assume 55% point up in therm eq bit

$N_{down} = Ce^{(-\mu B/kT)}$

$N_{up} = Ce^{\mu B/kT}$

$$\frac{N_{up}}{N_{down}} = \frac{0.55}{0.45} \approx \boxed{1.22}$$

2. $T = 24°C$

$E_{down} - E_{up} = ?$

$$\frac{N_{up}}{N_{down}} = \frac{Ce^{E_{down}/kT}}{Ce^{E_{up}/kT}} = 1.222$$

$$E_{down} - E_{up} = kT\ln(1.22)$$

$\boxed{E_{down} - E_{up} \approx 8.23\times10^{-22}\ J}$

3. What magnetic field would give that difference?

$\mu B + \mu B = 8.23\times10^{-22}\ J$ $\quad \boxed{B \approx 44.26\ Tesla}$

4)

$$C(T) = \frac{dU}{dT} = \frac{N\Delta^2}{kT^2}\cdot\frac{e^{\Delta/kT}}{(1+e^{\Delta/kT})^2}$$

$N = 6.022\times10^{23}$ $\quad \epsilon = 2.07\times10^{-21}\ J$ other state has $E = 0\ J$

1. $U(200K) = ?$

$$U = N(\cancel{-\mu B p_L} + \mu B p_H) = N\mu B p_H = N\epsilon\cdot\frac{e^{-\epsilon/kT}}{e^{-\epsilon/kT} + e^0}$$

$$= N\epsilon\cdot\frac{e^{-\epsilon/kT}}{e^{-\epsilon/kT} + 1} \approx \boxed{400\ J}$$

2. $C(200K) = ?$

Remember, Δ is the change in energy.

$\Delta = \epsilon - 0 = \epsilon$

Plugging into our above formula $C(200) \approx \boxed{1.01\ J/K}$

3, 4: Plug and chug.
Code a function if you're lazy.