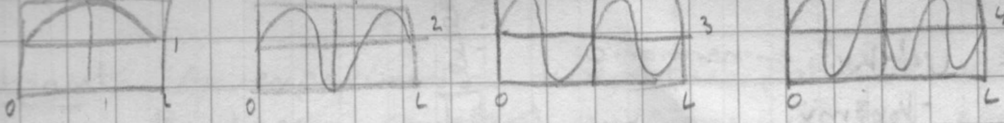
HW 8: Time Independent Schrodinger Equation

9/14/23

1)



$U(x) = 0$
$L = 0.05\,nm$
$0 \le x \le L$
$N\sin(2\pi x/\lambda)$

1. $\lambda_1 = 2L$: $\boxed{\lambda_1 = 0.1\,nm}$

2. $E_1 = ?$

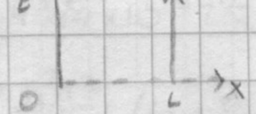
$$E\Psi(x) = -\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2} + U(x)\Psi(x)$$

$$E = -\frac{\hbar^2}{2m}\cdot\frac{-(\frac{2\pi}{\lambda})^2\sin(\frac{2\pi x}{\lambda})\cdot N}{N\sin(\frac{2\pi x}{\lambda})} = \frac{4\hbar^2\pi^2}{2m\lambda^2} = \frac{2\hbar^2\pi^2}{m\lambda^2} \approx \boxed{150\,eV}$$

3. $\lambda_2 = 0.033\,nm$
4. $E_2 \approx 1352\,eV$
5. $\lambda_3 \approx 0.025\,nm$
6. $E_3 \approx 2405\,eV$
7. $\lambda_4 \approx 0.0166\,nm$
8. $E_4 \approx 5455\,eV$

9.) $N = \sqrt{\frac{2}{L}} = \sqrt{\frac{2}{0.05}}\,nm^{-1/2} = \sqrt{40} \approx \boxed{6.32\,nm^{-1/2}}$

2) $L = 0.12\,nm$



1. The electron is in the lowest energy eigenstate. What is the energy of the electron

$$E_n = \frac{\hbar^2 n^2\pi^2}{2mL^2} \overset{n=1}{\Rightarrow} E_1 \approx \boxed{26.0976\,eV}$$

2. What is λ for this eigenstate?

$$E = hf = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} \qquad 26.0976\,eV = \frac{h^2}{\lambda^2\cdot 2m} \qquad \boxed{\lambda \approx 0.24\,nm}$$

3. Probability that the electron is located $0.048\,nm \le x \le 0.072\,nm$?

$$\Psi_1(x) = \sqrt{\frac{2}{0.12}}\sin\left(\frac{\pi x}{0.12}\right) \qquad p = |\Psi_1(x)|^2 = \frac{2}{0.12}\sin^2\left(\frac{\pi x}{0.12}\right)$$

$$\int_{0.048}^{0.072} p\,dx \approx \boxed{0.38}$$

$* \sin^2(x) = \frac{1-\cos 2x}{2}$

4. $\Psi_2(x) = \sqrt{\frac{2}{0.12}}\sin\left(\frac{2\pi x}{0.12}\right)$

$$p = |\Psi_2(x)|^2 = \frac{2}{0.12}\sin^2\left(\frac{2\pi x}{0.12}\right) \qquad \int_{0.048}^{0.072} p\,dx \approx \boxed{0.049}$$

3) $L = 5\times10^{-6}\,nm$

proton: $mc^2 = 9.3827\times10^8\,eV$

1. $E_1 = \frac{\hbar^2\pi^2}{2mL^2} \approx \boxed{8186590\,eV}$

2. Ejects photon. $E_{photon} = 4.0959\times10^7\,eV$. $n = ?$

$$E_{photon} = E_n - E_{n-1} = \frac{\hbar^2\pi^2}{2mL^2}(n^2 - (n-1)^2) \qquad 5.00318 = n^2 - (n^2 - 2n + 1) = 2n - 1$$

$\boxed{n = 3}$

3. Ground state for trapped electron?

$$E_1 = \frac{\pi^2\hbar^2}{2mL^2} \approx \boxed{1.5E10}$$