e) $a_c = ?$ @ $R/3$ $a_c = \frac{v^2}{r}$, $v = \omega R$

$$a_c = \frac{\left(2.73 \cdot \frac{1.56}{3}\right)^2}{1.56/3} = \boxed{3.863 \ m/s^2}$$

HW 22: Precession

1) $R = 1.85$ m $\quad \omega_0 = 1.55$ rad/s $\quad m = 52$ kg

$I = 187 \ kg \cdot m^2 \quad v = 5 m/s$

a) L_0 of merry-go-round:

$L_0 = I\omega = 187 \cdot 1.55 = \boxed{289.85} \ kg \ m^2/s$

b) L_p 2m before jumping on?

$L = r \times p \rightarrow m \cdot v \cdot 1.85 = 52 \cdot 5 \cdot 1.85 = \boxed{481} \ kg \ m^2/s$

c) L_p just before jumping on? (481)

d) $\omega_f = ?$

$L_f = \omega_f \times I_{TOT} = L_p + L_0 = 289.85 + 481 = 770.85 \ kg \ m^2/s$

$I_{TOT} = 187 + (52)(1.85)^2 = 364.97$

$\omega_f = 770.85 / 364.97 \approx \boxed{2.112}$ rad/s

e) What force do they need to hold on? $\Sigma F = F_c + F_{hold} = 0 \quad F_c = a_c m = F_{hold}$

$a_c = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r \quad a_c = (2.112)^2(1.85) = 8.25272 \ m/s^2$

$F = a_c \cdot 52 \approx \boxed{429 \ N}$

f) halfway they let go. What is $|v|$?

$L = I\omega_0 = 375.89 \ kg \ m^2 = p \cdot r = mvr \quad v = \frac{375.89}{1.85 \cdot 52} \approx \boxed{3.907 \ m/s}$

g) $\omega_{mgr_f} = ?$

$L_{TOT} = 770.85 \ kg \ m^2$

$L_{mgr} = L_{TOT} - L_p = 770.85 - 375.89 = 394.961 \ kg \ m^2/s$

$\omega_{mgr_f} = \frac{L_{mgr}}{I} = \frac{394.961}{187} \approx 2.112$ rad/s

2) Gyroscope

rope $m = 5.4$ kg $\quad R = 0.38$ m $\quad r$ (from rope) $= 1.4$ m

$f = 19.9$ rev/s

1 rev $= 2\pi r \quad \omega = 125$ rad/s

a) $L = ?$

$I = \frac{1}{2}MR^2 \quad L = \frac{1}{2}(5.4)(0.38)^2 \cdot 125 = \boxed{48.7488 \ kg \ m^2/s}$

b) $\tau_g = ?$

$\tau = r \times F = rF\sin\theta \Rightarrow 1.4 \cdot (5.4)(9.8) = 74.088$ Nm

c) Period of precession?

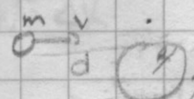
$T = \frac{2\pi}{\Omega} \quad \Omega = \frac{\tau_{ext}}{L} \Rightarrow \frac{74.088}{48.7488} \approx 1.526$

$T = \frac{2\pi}{1.52} \approx 4.13$ s

HW 21 : Angular Momentum.

1) $I = 215 kgm^2$ $R = 3.1 m$ $\omega_0 = 1.75$ rad/s

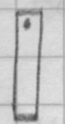
a) $|\vec{L}| = I\omega = 215 \cdot 1.75 = 376.25$ $kg\,m^2/s$

c)  $m = 52 kg$ $v = 3.1 m/s$

$L_{obj} = ?$ when $d = 6.35 m$

$L = r \times p = pD$

$L = (52 \cdot 3.1) \cdot R \approx 500 \frac{kg\,m^2}{s}$

2)  $m = 0.75 kg$ $M = 2 kg$ $\frac{KE_{lost}}{KE_{init}} = ?$

$v = 2.5 m/s$ $L = 0.9m$

$k = \frac{L^2}{2I}$ $K_i = \frac{1}{2}mv^2 = \frac{1}{2}(0.75)(2.5)^2 = 2.34375 J$

$K_f = \frac{(mvD)^2}{2I} = \frac{(1.6875)^2}{2(\frac{1}{3}MR^2 + mr^2)} = \frac{(1.6875)^2}{2(0.54 + 0.6075)} = 1.2408 J$

for stick around end

$K_{loss} = K_I - K_f = 1.1029 J$

$K_{lost}/K_i = 0.470588$

3)  $m_1 = 9.1 kg$ $R = 0.18$ $\omega = 35$ rad/s

$m_2 = 3.6 kg$ $L = 0.36m$

a) $L_0 = I\omega_0 = \frac{1}{2}MR^2 \cdot \omega = \frac{1}{2}(9.1)(0.18)^2 \cdot 35 = \boxed{5.1597}$ $kg\,m^2/s$

b) $K_0 = ?$

$K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I} \Rightarrow \frac{1}{2}(0.14742) \cdot 35^2 = 90.29475 J$

c) $\omega_f = ?$

$L_f = L_0 \rightarrow 5.1597 \frac{kg\,m^2}{s} = (I_1 + I_2)\omega = (0.14742 + \frac{1}{12}(3.6)(0.36)^2)\omega$

$\omega = 27.69565$ rad/s

d) $L_f \approx 5.16$

e) $K_f = \frac{1}{2}(0.1863)(27.696)^2 \approx \boxed{71.45 J}$

f) $t = 6.9 s$ to accelerate rod to ω_f. What is τ_{net}?

$\tau = I_{rod}\alpha$ $\alpha = \frac{\omega}{t} = \frac{27.69}{6.4} = 4.01386$ rad/s² $\tau = \frac{1}{12}(36)(0.36)^2 \cdot 4.01 = \boxed{0.156}$

4)  $m_p = 71 kg$ $R = 1.56 m$ $m_a = 192 kg$

$\omega_0 = 1.7$ rad/s

a) I when person on rim?

$I_d + I_p = \frac{1}{2}MR^2 + mR^2 = \frac{1}{2}(192)(1.56)^2 + 71(1.56)^2 \approx 406\ kg\,m^2$

b) when @ 2/3 towards center ? $\rightarrow R_p = \frac{1.56}{3}$

$\boxed{I \approx 252.824}$

c) $\omega_f = ?$ $L_0 = 406 \cdot 1.7 = 690.9$ $kg\,m^2/s = L_f$

$\omega_f (252.824) = 690.9$ $\omega_f = 2.7327$ rad/s

d) $\Delta K = ?$

$K_0 = \frac{1}{2}I\omega^2 = \frac{1}{2}(406)(1.7)^2 = 587.26 J$ $\Delta K = 356.755$

$K_f = \frac{1}{2}(252.8)(2.73)^2 = 944.019 J$