An Explicit Artin Map

Nir Elber

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Abstract

We track through the Artin map, given the local fundamental class $u \in H^2\left(\mathrm{Gal}(L/K), L^\times\right)$, via the cup product.

1 Via Tate's Theorem

It is possible to simply track through the proof of Tate's theorem by hand in order to recover the result. The point is that we end up needing to compute

$$\mathrm{N}_\Gamma(T_\sigma) = \sum_{g\in\Gamma} gT_\sigma.$$

However, the action is given by

$$gT_\sigma = T_{g\sigma} - T_g + u(g,\sigma).$$

Thus, we get

$$\prod_{g\in G} u(g,\sigma)$$

at the end of the computation.