Homework 1

Due Tuesday, April 8

The homework consist of two parts:

(i) Do the following recommended exercises from the book:

Section	**Exercises**
1.3	3, 5, 11, 21, 27
1.4	5, 7, 9, 11, 13, 15, 35
1.5	1,3
1.6	3, 7, 19
1.7	1, 9, 17, 29

Note that your solutions to the book's exercises should NOT be SUBMITTED.

(ii) Follow the instructions below and solve the problems on this paper!

Instructions

- Work together in your study group and solve the problems together.
- Write down the solutions individually and submit them.
- It is fine to submit scanned handwritten solutions, but you are responsible for these being readable. Submit your solutions as ONE pdf-file.
- The solutions will be presented at the seminar on April 9 (April 10 for seminar group D2). In the beginning of the seminar you will be asked to inform the teacher which problem you have solved (it is only these problems you will be asked to present solutions for). Students are responsible for correcting their own solutions. No written solutions will be published.
- The score of the homework will be $\frac{\#\text{ solved problems}}{\#\text{ problems}}$.
- If a student has marked a problem as solved and cannot present a prepared solution on the seminar then the score will be 0.

Problems

1. Show that $A \to B \wedge C$ is a logical consequence of $A \to B$ and $B \to C$
   (a) by using a truth table.
   (b) by using logical inferences (a proof sequence by using conditional proof).
2. Show by using logical inference that C is a logical consequence of $C \vee \neg B$, $\neg B \to A \vee D$, $D \to C$ and $A \to C$.
3. Transform $A \vee B \to (C \leftrightarrow B)$ into

(a) Disjunctive normal form.

(b) Conjunctive normal form.

4. Let $M(x)$ be the predicate *"x is studying mathematics."* and let the domain be *Students at Linnaeus University*. Express the following statements in English and determine their truth values.
   (i) $\forall x M(x)$
   (ii) $\exists x M(x)$
   (iii) $\neg \forall x M(x)$
   (iv) $\neg \exists x M(x)$
   (v) $\forall x \neg M(x)$
   (vi) $\exists x \neg M(x)$

   What will happen to the truth values if we change the domain to *Computer science students*? We can assume that all computer science students study mathematics.

5. Let us define the predicates
   $I(x)$: x is an instructor,
   $S(y)$: y is a student,
   $T(x, y)$: x is teaching y.
   Let the domain be all humans. Use quantifiers and this predicates to describe

   (i) All humans are instructors.
   (ii) There are instructors and students.
   (iii) There are instructors that teach students.
   (iv) All instructors are teaching students.
   (v) There are instructors that are teaching all students.

6. Show that $\exists x(P(x) \wedge R(x))$ is a logical consequence of $\forall x(Q(x) \rightarrow P(x))$ and $\exists x(Q(x) \wedge R(x))$.

7. Let n and m be integers. Show that if $m + n \geq 4$ then $m \geq 2$ or $n \geq 2$. Is the converse true? Prove or disprove.

8. (a) Let n be an integer. Show $3n + 1$ is even if n is odd.
   (b) Let n be an integer. Show that the n^2 is even if and only if n is even.