Constraint Satisfaction Problems (CSP) pt 2

- Reading: Russell and Norvig, ch. 6
  - The material here is an edited version of Russell’s slides
- Min-conflicts

Iterative algorithms for CSP

- Hill climbing, simulated annealing
  - Work for states with all variables assigned values
- For CSP, allow states where not all constraints are satisfied
  - Operators (“actions”) change variable values
- Choosing which variable to change
  - Randomly choose a variable involved in a constraint that is not satisfied
- Choosing a new value
  - Pick a new value to minimize violated constraints
  - Leads to heuristic function h(n) = # violated constraints (for a given set of values)

Example: N Queens

- [Pretty much the same as for Local Search notes, but treating as a constraint satisfaction problem]
- State: assignment of positions to queens
- Operators: move one queen to different row in same column
- Goal test: no queens attacking each other

Performance of min-conflicts

- Nearly constant (O(1)) time starting from arbitrary state for any N
- True in general for CSP problems, except at or near *critical ratio*
  - *R = number of constraints / number of variables*
- Ex: Williams, Explorations in Quantum Computing, p. 300
  - 3-SAT problem: determining if a conjunction of disjunctions of 3 Boolean variables can be satisfied (used by Karp to show other problems are NP-Hard)
    - Ex: (x or y or z) and (!x or !y or !z) satisfied by x = True, y = False
  - Critical ratio of ~4.25 clauses / variables
    - Much harder to solve problem close to this ratio
    - Phase transition from very likely satisfiable to very likely unsatisfiable

History of min-conflicts

- https://en.wikipedia.org/wiki/Min-conflicts_algorithm
  - Johnston and Adorf developed neural network algorithm to help schedule use of Hubble telescope
    - 1024-Queens was the “proof of concept” problem, but able to solve 1M-Queens in 50 steps
  - Minton and Philips analyzed algorithm into two phases: greedy phase for initial attempt at solution, then min-conflicts phase
  - Min-conflicts helped reduce scheduling time for Hubble from 3 weeks to 10 minutes