Electric Drive Fundamentals

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MCI

B.Sc - Drive Systems

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Introduction

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Introduction

- The goal of this lecture is to give you the fundamentals of electric machinery using theory and engineering practice.
- This lecture is a total of 2 SWS with a total of thirty (30) UE.
  - With 28 UE is devoted to lectures.
- There is a written exam at the end of the module worth two (2) UE.
- There is one (1) assignment for this course:

  1st will be a pre-defined work which is individual based.

Introduction

- The individual assignments focus on understanding electric machinery.
- The assignment is uploaded to SAKAI for you to work on along with what is required of you for submission.
  - The assignments contain electric drive problems to solve.
- The strict deadline is the end day of the **last lecture**.
- Any submission after this date will not be accepted.
- If all submissions are sent early we can do a statistical analysis and go through questions before the final exam.

Introduction

- Lecture materials and all possible supplements will be present in its Github Repo.
  - You can easily access the link to the web-page from here.

> Github is chosen for easy access to material management and CI/CD capabilities and allowing hosting websites.

- In the lecture some exercises are solved using Python and other examples and can be accessed from the Repo website.

Introduction

- At the end of the lectures there will be a final examination which you will tested.
- You will be asked three (3) questions related to electric drives.
- The exam will be ninety (90) minutes.
- You are NOT allowed a personal formula sheet or any kind of supporting material.
- You are allowed a calculator.

Assessment Type	Overall Points	Breakdown	%
Individual Assignment	40		
		Report	20
		Solution(s)	80
			20
Final Exam	60		
		Question 1	40
		Question 2	30
		Question 2	30

Table 1: Assessment Grade breakdown for the lecture.

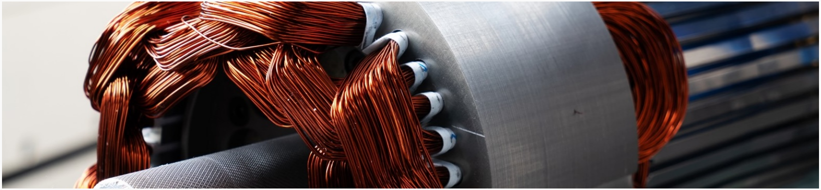
Introduction

Covered Topic	Appointment
Return to Fundamentals	1
Magnetic Circuits & Materials	1
Transformers	2
Electromechanical Energy Conversion	2-3
Rotating Magnetic Fields	3
DC Drives	4
Poly-phase Induction Drives	4-5
Single-phase Induction Drives	5-6
Linear Induction Drives	6
Poly-phase Synchronous Drives	6-7
Solid-state Commutation Drives	7

Table 2: Distribution of materials across the semester.

Return to Fundamentals

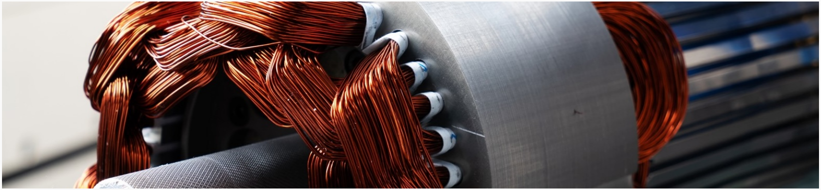
- Complex number notation,
- Multi-phase systems,
- Phasors and Wave-forms



Introduction

Transformers

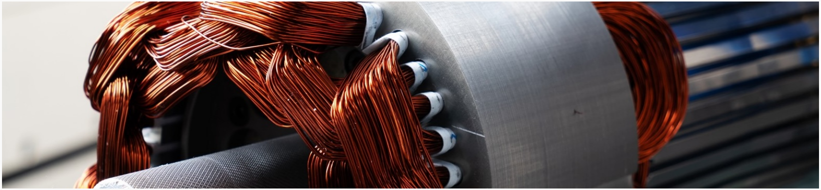
- Construction & Physical Properties
- Modelling
- Parameter Tests
- Connection Types



Introduction

Rotating Magnetic Fields

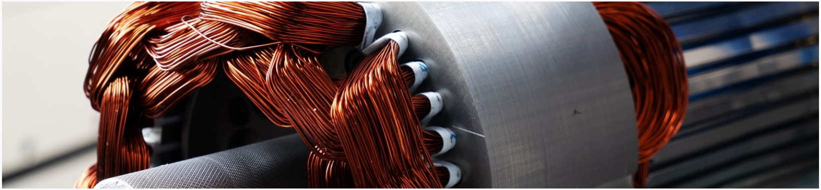
- MMF in Winding
- Torque Generation
- Generated Voltage



Introduction

DC Drives

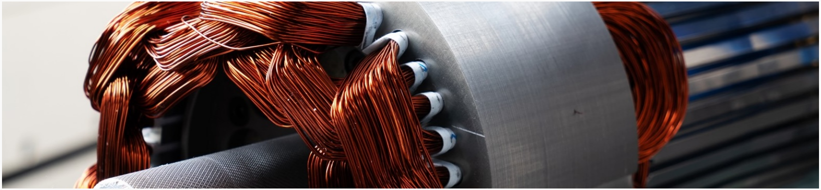
- Construction
- Physical laws governing its operation
- Types of Connections used in industry and commercial applications.
- Applications in Industrial/Commercial venues.



Introduction

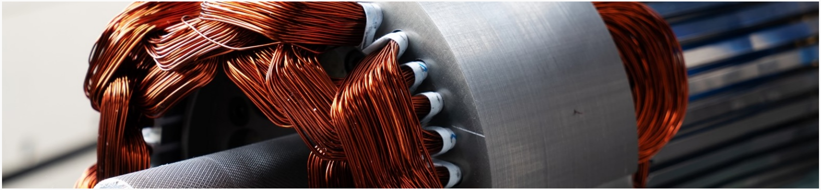
Poly-phase Induction Drives

- Construction
- Physical laws governing its operation principle.
- Modelling an induction drive mathematically
- Methods of starting an induction drive.



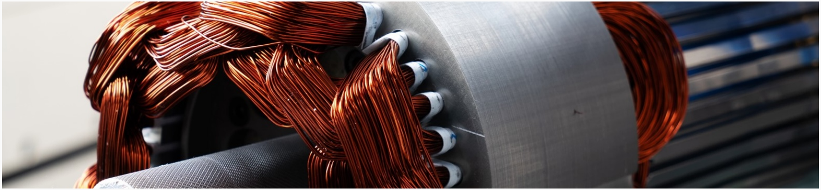
Single-Phase Induction Drives

- Creating a Rotating Magnetic Field in a single phase
- Types of single-phase types
- Salient-pole



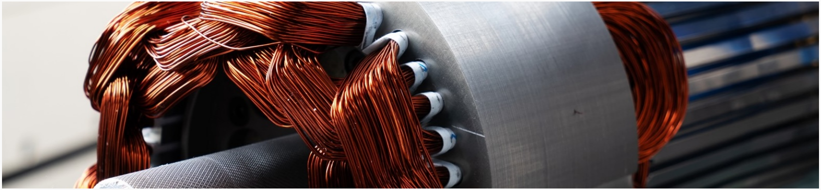
Poly-phase Synchronous Drives

- Construction and Rotor types,
- Operation principles,
- Regulatory behaviours,
- V-curves,



Solid-State Commutation Drives

- Solid State Commutation
- BLDC & PMSM Drives
- Switched Reluctance Motor (SRM)
- Stepper Drives



Books

- Mohan Ned. "*Advanced electric drives: analysis control and modeling using MATLAB/Simulink*" John Wiley & Sons 2014.
- Krause Paul C. et. al. "*Analysis of electric machinery and drive systems*" Vol. 2 IEEE Press 2002.
- Pyrhonen Juha et. al "*Design of rotating electrical machines*" John Wiley & Sons 2013.
- Stephen J. Chapman. "*Electric Machinery Fundamentals (5th Edition)*" (2012).
- Fitzgerald A. E. et. al. "*Electric Machinery*" McGraw Hill (2003).

Books

- Hughes A. et. al. "*Electric Motors and Drives: Fundamentals Types and Applications*" Newnes 2019.
- Melkebeek A. "*Electrical Machines and Drives: Fundamentals and Advanced Modelling*" Springer 2018.
- Wildi T. "*Electrical machines, drives, and power systems*" Pearson Education 2006.
- Veltman A. et. al. "*Fundamentals of Electrical Drives*" Springer 2007.

White Papers

- Maddox Transformer "*Guide to transformer cores: types, construction, & purpose*"
- Control Engineering *Springtime for Switched-Reluctance Motors?* .

Introduction

Lecture Notes

- Power Transformers "*ESE 470 Energy Distribution Systems*" Oregon State University,
- Principles of Electromechanical Energy Conversion "*Actuators & Sensors in Mechatronics Electromechanical Motion Fundamentals*" NYU,

Return to Fundamentals

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Learning Outcomes

(LO1) An Overview of Poly-phase circuits,

(LO2) Definitions on Active-Reactive power,

(LO3) Polar Coordinate System.

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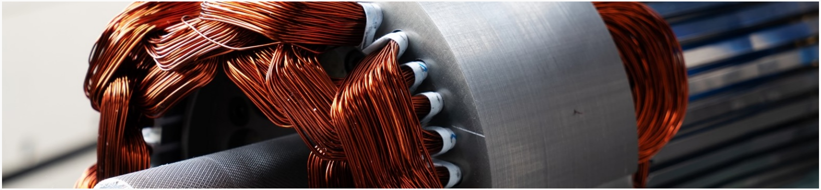


Return to Fundamentals

- A rotating magnetic field is a magnetic field with moving polarities.
  - Which its opposite poles rotate about a **central point** or **axis**.
- To create a rotating magnetic field (RMF) you need at least a 2-phase system.
- In the industry, RMFs are mostly produced using a 3-phase supply [1].
  - There are also economic reasons, as it is cheaper to design a 3-phase system with minimal cost to wiring [2].
- Before starting with electric drives, it is a good time to look at some fundamental concepts in power engineering.

[1] 3-phase supply produces smoother operation of motors compared to 2-phase. This is due to the power transfer in 3 phase supply being less pulsating than in 2 phase supply.

[2] However, there have been test and feasibility on 6-phase power in 1970s with minimal success.

Return to Fundamentals

- Generation, transmission, and heavy-power utilisation of AC electric energy almost invariably involve a poly-phase circuit.
- In such a system, each voltage source consists of a group of voltages having related magnitudes and phase angles.
- Thus, an n-phase system employs voltage sources which typically consist of n voltages substantially equal in magnitude and successively displaced by a phase angle of $360°/n$.
- Therefore a 3-phase system would have 3 different voltage sources separated 120 degrees apart.

Return to Fundamentals

- The three individual voltages of a three-phase source may each be connected to its own independent circuit.
- We would then have three separate single-phase systems.
- Alternatively, symmetrical electric connections can be made between the three voltages and the associated circuitry to form a three-phase system.
- It is the latter alternative that we are concerned.
- Note that the word phase now has two distinct meanings.
  - It may refer to a portion of a pol-yphase system or circuit,
  - It may be used in reference to the angular displacement between voltage or current phasors.

Return to Fundamentals

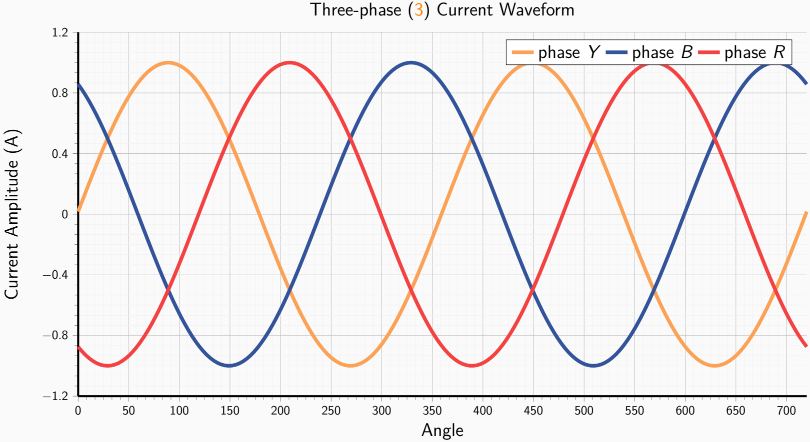
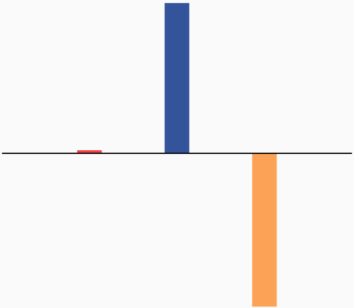


Figure 1: A three phase waveform of currents **R**, **B**, and **Y**.



$i_R = 0.01746$

$i_B = 0.85716$

$i_Y = \text{-}0.87462$

$$i_R + i_B + i_Y = 0.0$$

Figure 2: A three phase waveform of currents **R**, **B**, and **Y**.

Return to Fundamentals

- There are **no neutral connection** available.
- Phase voltage appears **across the windings**.
- In a delta connection, Line voltage is equal to Phase voltage:

$$v_{\text{line}} = v_{\text{ph}}$$

- i_{YB}, i_{YR}, i_{BR} are also called **Line current** (i.e., i_{line}).
- Line current is $\sqrt{3}$ times that of the phase current.

$$i_{\text{line}} = \sqrt{3} i_{\text{subs}}$$

- Power related definitions are:

$$P = \sqrt{3} v_{\text{line}} i_{\text{line}} \cos\varphi, \quad \text{and} \quad P = 3 v_{\text{ph}} i_{\text{ph}} \cos\varphi.$$

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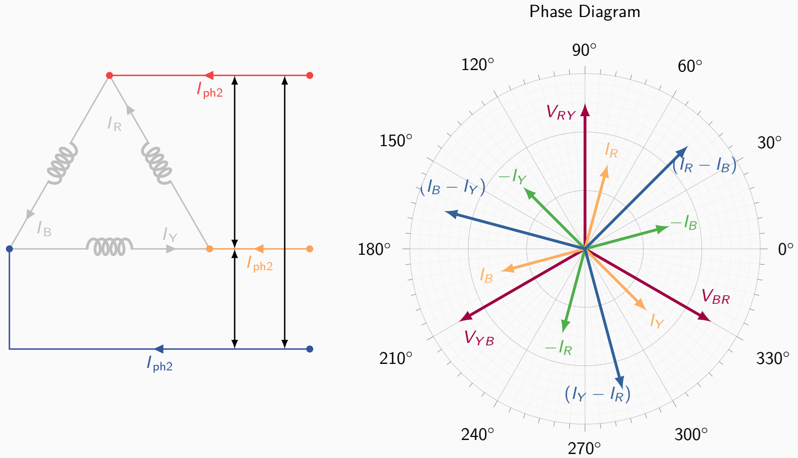


Figure 3: The connection and the phasor diagram of a 3-phase delta connection.

Example

Three impedance with a value $Z_{\Delta} = 12.00 + \mathbf{j}\,9.00 = 15.00\angle 36.9\ \Omega$ are connected in Δ.

For balanced line-to-line voltages of 208 V, find the line current, the power factor, and the total power.

Return to Fundamentals

- There is a neutral connection available.
- Phase voltage appear across windings.

$$v_{\text{line}} = \sqrt{3} v_{\text{ph}},$$

- i_{YB}, i_{YR}, i_{BR} are also known as line current (i_{line}).
- In a star connection, line current is equal to phase current.

$$i_{\text{line}} = i_{\text{ph}}.$$

- Power related equations for 3-phase:

$$\begin{aligned}
P &= \sqrt{3} v_{\text{line}} i_{\text{line}} \cos\varphi, \\
P &= \sqrt{3}\sqrt{3} \left(v_{\text{ph}}\right) \left(i_{\text{ph}}\right) \cos\varphi, \\
P &= 3 v_{\text{ph}} i_{\text{ph}} \cos\varphi.
\end{aligned}$$

Return to Fundamentals

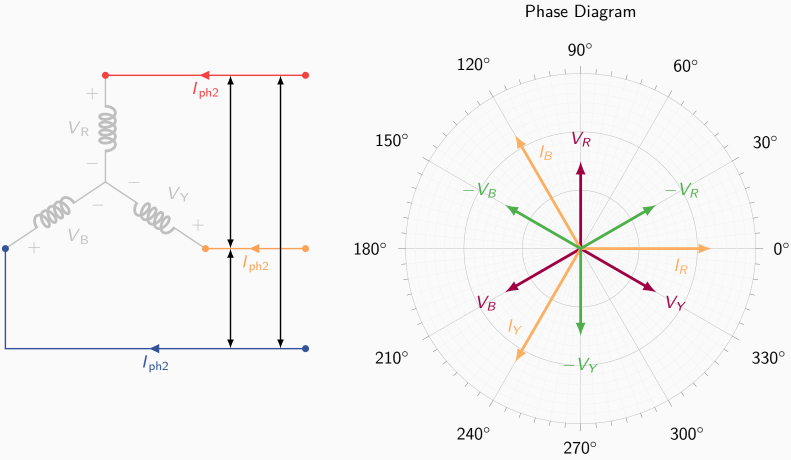


Figure 4: The connection and the phasor diagram of a 3-phase wye connection.

Example

A Y-connected 120 V source feeds a Δ-connected load through a distribution line having an impedance of $0.3 + \mathbf{j}0.9\ \Omega$. The Y-source impedance is $0.2 + \mathbf{j}0.5\Omega$

The load impedance is $118.5 + \mathbf{j}85.8\ \Omega/o$.

Use the a-phase internal voltage of the generator as the reference.

(a) Construct a single-phase equivalent circuit of the three-phase system,

(b) Calculate the line currents,

(c) Calculate the phase voltages at the load terminals,

(d) Calculate the phase currents of the load,

(e) Calculate the line voltages at the source terminals.

Return to Fundamentals

$$Z = x + \mathbf{j}\,y = A\angle\theta,$$

where:

- Z is the complex vector,
- A is vector magnitude,
- x is real/active part,
- $\mathbf{j}$ is defined as $\sqrt{-1}$.
- y is imag/reactive part,
- θ is the complex angle.

$$\theta = {}^{y}/_{x}.$$

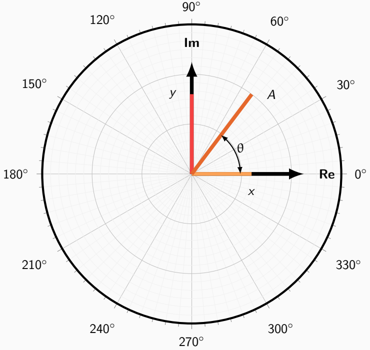


Figure 5: A polar coordinate system.

Example

Express:

$$\frac{3-\mathbf{j}}{7-3\mathbf{j}},$$

in the form of $a+\mathbf{j}\,b$.

Solution

Answer is:

$$-\frac{12}{29} - \mathbf{j}\left(\frac{1}{29}\right) \quad \blacksquare$$

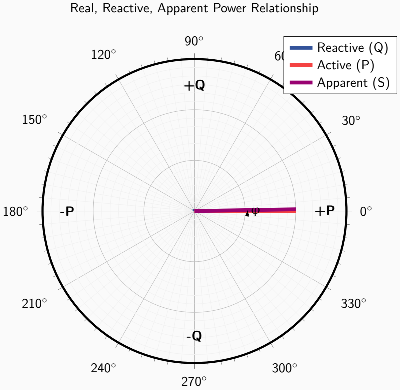
When it comes to power in AC, there are three (3) definitions:

- If energy is used/generated by an active element it is called **Active** or **Real** (P) power and measured in W.
- If energy is used/generated by an reactive element it is called **Reactive** power (Q)and measured in $\mathrm{V} \cdot \mathrm{A}r$.
- The combination of these two values is called **Apparent** power (S) and measured in $\mathrm{V} \cdot \mathrm{A}$.

$$S = \sqrt{P^2 + Q^2}$$

- Finally the angle difference between the voltage and current waveform is defined as the **phase** and tells how reactive/active a circuit is.

$$\varphi = \varphi_V - \varphi_I.$$



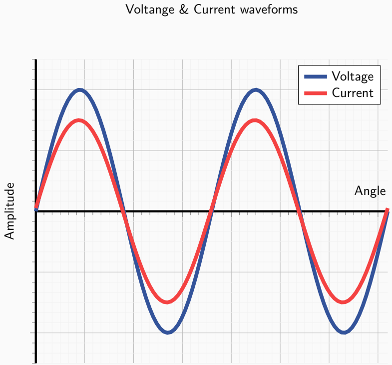


Figure 6: An animation showing the relations between Active, Reactive and Apparent power along with phase angle.

Example

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8.

The load is fed from a line having an impedance of $0.005 + \mathbf{j}\,0.025\Omega$.

The line voltage at the terminals of the load is 600 V.

(a) Construct a single-phase equivalent circuit of the system.

(b) Calculate the magnitude of the line current.

(c) Calculate the magnitude of the line voltage at the sending end of the line.

(d) Calculate the power factor at the sending end of the line.

Magnetic Circuits and Materials

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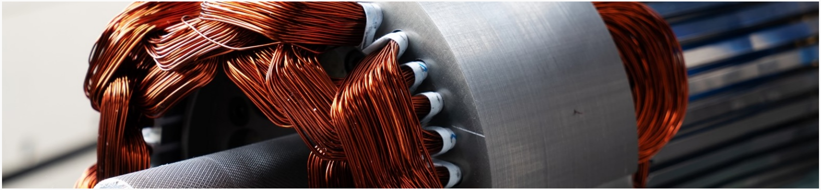
Learning Outcomes

(LO1) An Overview of Maxwell's Equations,

(LO2) Introduction to Magnetic Circuits,

(LO3) Brief look on Magnetic Materials,

(LO4) Magnetic Material Losses.



Magnetic Circuits and Materials

- Almost all electric drives use ferromagnetic material for shaping and directing **B**-fields.
  - These fields act as the medium for transferring and converting energy.

> Permanent-magnets are also widely used in drive design.

- Without these materials, practical implementations of most familiar EEC devices would not be possible.
- Analysing and describing systems containing them is essential for designing effective drives.
- We start by looking at Maxwell's equations.

Magnetic Circuits and Materials

- Maxwell's equations are a set of coupled partial differential equations which form the foundation of electric and magnetic circuits.
- In their PDE form, they are written as [8]:

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right).
\end{aligned}
$$

where **E** is the electric field, **B** is the magnetic field, ρ is the electric charge density and **J** the current density, ε_0 is the vacuum permittivity and μ_0 is the vacuum permeability.

Magnetic Circuits and Materials

- Solution to Maxwell's equation can be hard if not attainable and usually simplifying assumptions are made to reach practical solutions.
- First, the displacement-current term ($\varepsilon_0 \, \partial \mathbf{E}/\partial t$) can be neglected.
    - This term accounts for **B**-fields produced in space by time-varying **E**-fields (i.e., electromagnetic radiation) [20].
- Neglecting $\varepsilon_0 \, \partial \mathbf{E}/\partial t$ results in the magneto-static form which relates **B**-fields to the currents (**J**) which produce them.
- In integral form [5]:

$$\oint_{\mathcal{C}} \mathbf{H} \, \mathrm{d}\mathbf{l} = \int_{\mathcal{S}} \mathbf{J} \cdot \, \mathrm{d}\mathbf{a}, \quad \text{and} \quad \oint_{\mathcal{S}} \mathbf{B} \cdot \, \mathrm{d}\mathbf{a} = 0.$$

$$\oint_{\mathcal{C}} \mathbf{H}\,\mathrm{d}\mathbf{l} = \int_{\mathcal{S}} \mathbf{J} \cdot \,\mathrm{d}\mathbf{a}, \quad \text{and} \quad \oint_{\mathcal{S}} \mathbf{B} \cdot \,\mathrm{d}\mathbf{a} = 0.$$

The line integral ($\oint$) of the tangential component of the magnetic field intensity ($\mathbf{H}$) around a closed contour ($\mathcal{C}$) is equal to the total current ($\mathbf{J}$) passing through any surface ($\mathcal{S}$) linking that contour.

- The second one states the magnetic flux density ($\mathbf{B}$) is conserved, i.e., no net flux enters or leaves a closed surface (i.e, $\nabla \cdot \mathbf{B} = 0$).

These simplifications have allowed us to remove the effect of the $\mathbf{E}$ field on our calculations.

Magnetic Circuits and Materials

- A second simplifying assumption involves the concept of magnetic circuits.
- The general solution for the **H** and the **B** in a structure of complex geometry is extremely difficult if not practically impossible.
- However, a 3D field problem can often be reduced to what is essentially a circuit equivalent of magnetic elements,
  - with acceptable engineering accuracy [23].

Magnetic Circuits and Materials

- A magnetic circuit is a structure composed of highly permeable ($\mu_r \gg 0$) material(s).
- The presence of high-permeability causes magnetic flux to be confined to the paths defined by the structure.

> This is similar to how currents are confined to the conductors of an electric circuit.

Magnetic Circuits and Materials

- The core is assumed to be composed of magnetic material with permeability much greater than of surrounding air (i.e., $\mu_{\mathrm{r}} \gg \mu_0$).
- The core is of uniform cross section and is excited by a winding of N turns carrying a current of i amperes.

- This winding produces a **B**-field in the core.

Magnetic Circuits and Materials

- Due to high permeability of the material (i.e., $\mu_{\mathrm{r}} \gg \mu_{0}$), the magnetic flux is confined almost entirely to the core,
- The **B**-field lines follow the path defined by the core,
- The flux density is essentially uniform over a cross section as the cross-sectional area is uniform.
- The **B**-field can be visualised in terms of flux lines which form closed loops interlinked with the winding.

Magnetic Circuits and Materials

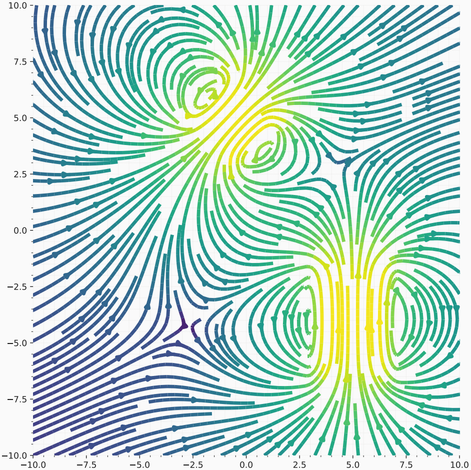


Figure 7: An example demonstration of flux paths passing highly permeable material.

Magnetic Circuits and Materials

- The source of the **B**-field in the core is the ampere-turn Ni.
- In magnetic circuit terminology Ni is the magnetomotive force (mmf) acting on the magnetic circuit.
- In systems with more than one winding, Ni must be replaced by the algebraic sum of the ampere-turns of all the windings:

$$\sum_{n=0}^{\infty} N_i i_i.$$

- The magnetic flux (ϕ) crossing a surface $\mathcal{S}$ is the surface integral of the normal component of **B**:

$$\phi = \int_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$$

where the unit of flux (ϕ) is Wb.

$$\phi = \int_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$$

- This equation states that all flux entering the surface enclosing a volume must leave the volume as they form closed loops.
- This is enough to justify the uniformity of the magnetic flux density ($\mathbf{B}$) across the cross section of a magnetic circuit (A_{c}):

$$\phi_{\mathrm{c}} = B_{\mathrm{c}} A_{\mathrm{c}},$$

  where ϕ_{c} is the flux, B_{c} is the magnetic flux density magnitude and A_{c} is the cross-sectional core area.

Magnetic Circuits and Materials

- Using the original magneto-static simplifications (i.e., no **E**-fields), we can come up to:

$$\mathcal{F} = Ni = \oint \mathbf{H}\, \mathrm{d}\mathbf{l}.$$

- As the path of the flux line is close to the mean length[1] of the core (l_c), this is simplified to:

$$\mathcal{F} = Ni = H_\mathrm{c} l_\mathrm{c},$$

where H_c is the average magnitude of **H**.

[1] The centre path going through the magnetic circuit.

Magnetic Circuits and Materials

- The relationship between the **H** and **B** is a property of the material.
- It is common to assume a linear relationship shown as:

$$\mathbf{B} = \mu \mathbf{H},$$

  where μ is the magnetic permeability.
- In SI units, **H** is measured in $\mathrm{A} \cdot \mathrm{m}^{-1}$ and **B** is in $\mathrm{Wb} \cdot \mathrm{m}^{-2}$ or T.

Magnetic Circuits and Materials

- Transformers are wound on closed cores.
- However, EEC devices which incorporate a moving element must have air gaps in their magnetic circuits.
- When the air-gap (g) is much smaller than the dimensions of the adjacent core faces, the magnetic flux will follow the path defined by the core and the air gap and the techniques of magnetic-circuit analysis can be used.

> If g becomes excessively large, the flux will be leak out of the sides of the air gap and the techniques of magnetic-circuit analysis will no longer be strictly applicable.

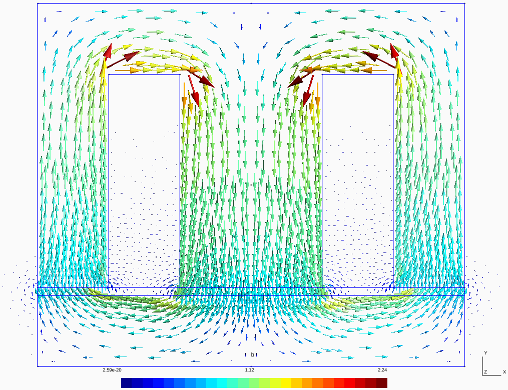


Figure 8: A Magnetic Circuits with two mmf sources.

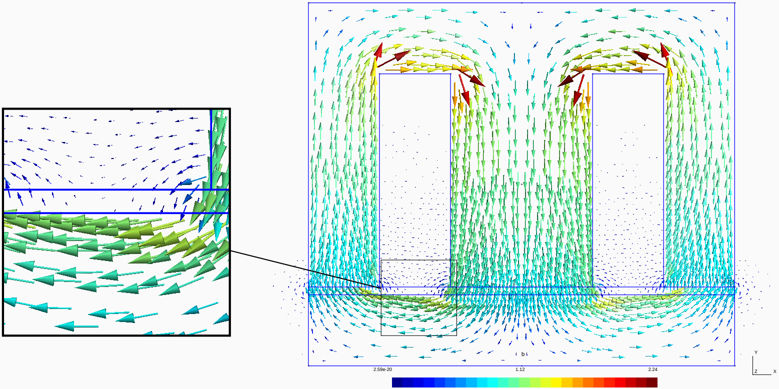


Figure 9: A closeup of the fringe paths caused by the air gap.

Magnetic Circuits and Materials

- Provided the air-gap length g is sufficiently small, the configuration can be analysed as a magnetic circuit with two (2) series components:
  1. Magnetic core (μ), cross-sectional area A_c, and mean length l_c,
  2. Air gap (μ_0), cross-sectional area A_g, and length l_g.
- In the core the flux density can be assumed uniform and ($\mu \gg \mu_0$):

$$B_c = \frac{\phi}{A_c}, \quad \text{and in the air gap,} \quad B_g = \frac{\phi}{A_g},$$

Magnetic Circuits and Materials

- Applying this to the magnetic circuit gives:

$$\mathcal{F} = H_c l_c + H_g g,$$

using the **B-H** relationship:

$$\mathcal{F} = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g.$$

- Here, $\mathcal{F}$ is the mmf applied to the magnetic circuit where $\mathcal{F}_c = H_c l_c$ is the magnetic field in the core and $\mathcal{F}_g = H_g l_g$ is the magnetic field in the air-gap.
- Using ϕ we can re-write this equation as:

$$\mathcal{F} = \phi \left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g} \right).$$

Magnetic Circuits and Materials

- The term which multiplies mmf to flux is called Reluctance.

$$\mathcal{F} = \phi \left(\overbrace{\frac{l_c}{\mu A_c}}^{\mathcal{R}_c} + \overbrace{\frac{g}{\mu_0 A_g}}^{\mathcal{R}_g} \right) \quad \text{or} \quad \mathcal{F} = \phi \left(\mathcal{R}_c + \mathcal{R}_g \right)$$

> Reluctance can be seen as analogous to resistance to electrical circuits.

Example

The magnetic circuit shown has dimensions:

$$A_c = A_g = 9 \ \text{cm}^2,$$
$$g = 0.05 \ \text{cm},$$
$$l_c = 30 \ \text{cm},$$
$$N = 500 \,\text{Turns}.$$

Assume the value $\mu_0 = 70000 \ \text{H} \cdot \text{m}^{-1}$ for core material.

(a) Find the reluctance values $\mathcal{R}_c$ and $\mathcal{R}_g$.

(b) For the value of $B_c = 1$ T, find the flux ϕ and the current i.

Solution

(a) 3,789.4

442,097.06

(b) 0.0009

(c)

Example

The following magnetic structure has infinite permeability ($\mu \to \infty$). Find the air-gap flux ϕ and flux density B_{g}.

Parameters: $I = 10$ A, $N = 1000$ Turns, $g = 1$ cm, and $A_{\mathrm{g}} = 2000$ cm^2.

Solution

(a) 0.13

- When a **B**-field varies with time, an **E**-field is produced in space as determined by Faraday's law [8]:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a}.$$

The line integral of the electric field intensity E around a closed contour C is equal to the time rate of change of the magnetic flux linking (i.e. passing through) that contour.

In electric drives, the effects of **E** field can be neglected.

- This simplification allows us to cancel the LHS of the equation to just the induced voltage (i.e., electromotive force)

$$\underbrace{\oint_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{s}}_{\mathcal{E}} = -\frac{d}{dt}\int_{S} \mathbf{B} \cdot d\mathbf{a}.$$

- Whereas on the RHS, the flux is dominated by the core flux (ϕ).

$$\mathcal{E} = -\frac{d}{dt}\underbrace{\int_{S} \mathbf{B} \cdot d\mathbf{a}}_{\phi}.$$

- As winding links the core flux N times, our equation is simplified to:

$$\mathcal{E} = N\frac{d\phi}{dt} = \frac{d\lambda}{dt} \quad \text{where} \quad \lambda = N\phi \quad \blacksquare$$

where λ is flux linkage, measured in Weber-turns,

- For a magnetic circuit composed of magnetic material of constant magnetic permeability, the relationship between ϕ and i will be linear and we can define the inductance L as:

$$L = \frac{\lambda}{i} \quad \text{which can be written as} \quad L = \frac{N^2}{\mathcal{R}_{\text{tot}}}$$

where L is the inductance measured in Henry (H).

Inductance requires a linear relationship between flux and mmf.

Example

The magnetic circuit consists of an N-turn winding on a magnetic-core of infinite permeability with two parallel air gaps of lengths g_1 and g_2 and areas A_1 and A_2 respectively. Find:

(a) The winding inductance,

(b) The flux density B_1 in gap 1 when the winding is carrying current i.

Note: Neglect fringing effects at the air gap.

- The mmf ($\mathcal{F}$) is given by the total ampere-turns.
- The reference directions for the currents (i_1, i_2) have been chosen to produce flux in the same direction.
- The total mmf is therefore:

$$\mathcal{F} = N_1 i_1 + N_2 i_2.$$

- With the reluctance of the core neglected and assuming that $A_c = A_g$, the core flux ϕ is:

$$\phi = (N_1 i_1 + N_2 i_2) \frac{\mu_0 A_c}{g},$$

where ϕ is the resultant core flux produced by the total mmf of the two windings.

- This resultant ϕ determines the operating point of the core material.
- If ϕ is broken up into terms attributable to the individual currents, the resultant flux linkages of coil 1 can be expressed as:

$$\lambda_1 = N_1\phi = \overbrace{N_1^2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{11}} i_1 + \overbrace{N_1 N_2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{12}} i_2,$$

and this is simplified to:

$$\lambda_1 = L_{11} i_1 + L_{12} i_2,$$

where L_{11} is the self inductance and L_{12} is the mutual inductance of the coil.

- Similarly, the flux linkage on coil 2 is:

$$\lambda_2 = N_2\phi = \overbrace{N_1 N_2 \left(\frac{\mu_0 A_c}{g}\right)}^{L_{21}} i_1 + \overbrace{N_1^2 \left(\frac{\mu_0 A_c}{g}\right)}^{L_{22}} i_2$$

and this is simplified to:

$$\lambda_1 = L_{21} i_1 + L_{22} i_2$$

where $L_{21} = L_{12}$ is the mutual inductance and L_{22} is the self inductance.

> It is important to note the ϕ is calculated based on the superposition of i_1, i_2 which implies a linear relationship between flux and mmf.

Magnetic Circuits and Materials

- It cannot be rigorously applied in situations where the nonlinear characteristics of magnetic materials dominate .
- However, in many situations of practical interest, the reluctance of the system is dominated by that of an air gap (which is of course linear) and the nonlinear effects of the magnetic material can be ignored.
- In other cases it may be perfectly acceptable to assume an average value of magnetic permeability for the core material.
- Using this it is possible to calculate a corresponding average inductance which can be used for calculations of reasonable engineering accuracy.

- EEC[1] devices require two (2) important properties:
  - Obtain large magnetic flux densities ($\uparrow \mathbf{B}$)
  - Requiring relatively low levels of magnetizing force ($\downarrow \mathbf{H}$).

> Since magnetic forces and energy density increase with increasing flux density, this effect plays a large role in the performance of energy-conversion devices.

[1] Electro-mechanical Energy Conversion

Magnetic Circuits and Materials

- Magnetic materials can be used to constrain and direct **B**-fields in well-defined paths.
- In a transformer, they are used to maximize the coupling between the windings as well as to lower the excitation current required for transformer operation.
- In electric drives, they are used to shape the fields to obtain desired torque-production and electrical terminal characteristics.

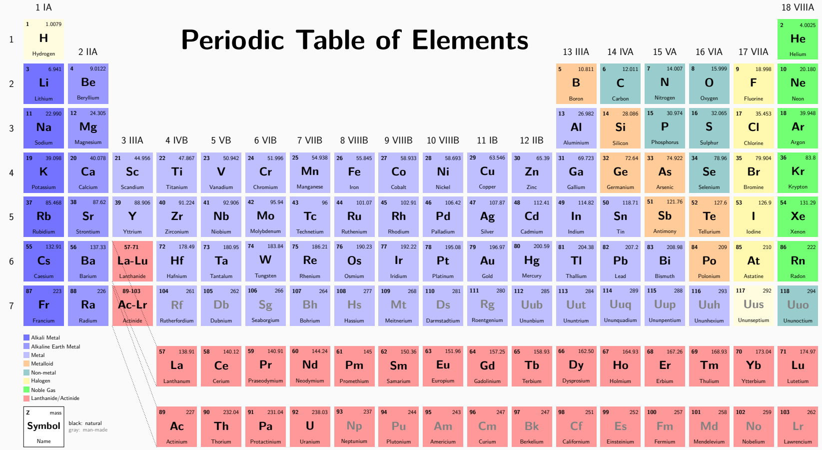


Figure 10: The periodic table of elements.

- Typically composed of iron (**Fe**) and alloys of iron with:
  - Cobalt, Tungsten, Nickel, Aluminium, and other metals
- These materials are characterised by a wide range of properties,
  - Phenomena responsible for their properties are the same.
- Ferro-magnetic materials are found to be composed of a large number of domains.
  - Regions where the magnetic moments of all the atoms are parallel, giving rise to a net magnetic moment for that domain.
- In an un-magnetised sample of material, the domain magnetic moments are randomly oriented, and the net resulting magnetic flux in the material is zero (0).

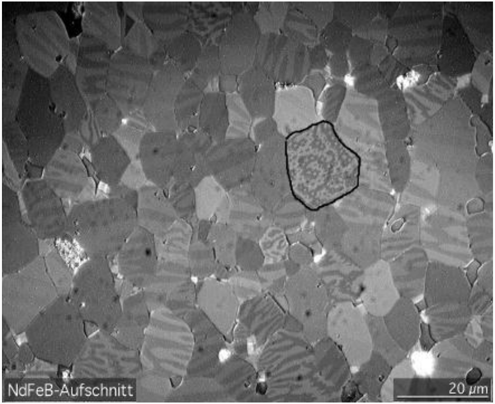


Figure 11: Microcrystalline grains within a piece of Nd2Fe14B (the alloy used in neodymium magnets) with magnetic domains made visible with a Kerr microscope. The domains are the light and dark stripes visible within each grain. The outlined grain has its magnetocrystalline axis almost vertical, so the domains are seen end-on [7].

Magnetic Circuits and Materials

- When **B**-field is applied to the material, the magnetic domain moments tend to align with the applied **B**-field.
- As a result, the magnetic moments add to the **B**-field, producing a higher flux density than would exist due to the magnetizing force alone.
- Therefore the effective permeability μ, equal to the ratio of the total magnetic flux density to the applied magnetic-field intensity, is large compared with the permeability of free space (μ_0).
- As the magnetising force (**H**) is increased, this continues until all the magnetic moments are aligned with the applied field.
  - at this point they can no longer contribute to increasing the magnetic flux density

Material becomes fully saturated.

- Without an external **B**-field, magnetic domains naturally align along certain directions associated with the crystal structure of the domain.
  - This is known as axes of easy magnetisation [13].
- If the applied **B** is reduced, the domain magnetic moments relax to the direction of easy magnetism nearest to the applied field.
- As a result, **B** is reduced to zero (0), although they will tend to relax towards their initial orientation, the magnetic dipole moments will no longer be totally random in their orientation;
- They will retain a net magnetisation along the applied field.
- It is this effect which is responsible for the phenomenon known as magnetic hysteresis [29].

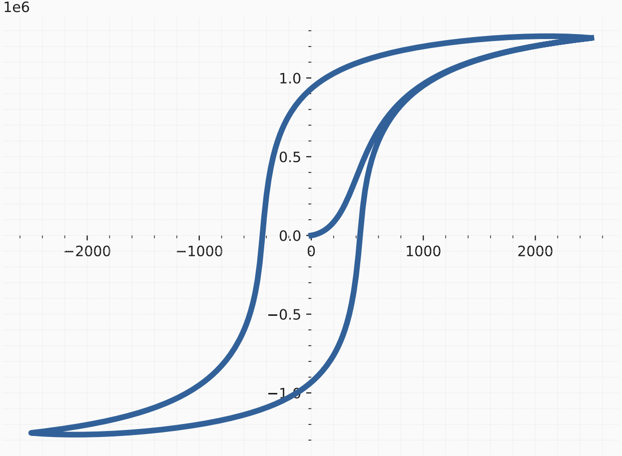


Figure 12: A Hysteresis Curve (B-H) Generated using the Jiles-Artherton model.

1^{st} Quadrant

- The domains are very small, below the single domain size where there is resistance to demagnetization [3].
- The domains start to increase in size.

For a small interval, the magnetization will be reversible.

- As the field increases, magnetization will no longer reverse to zero but move on a minor hysteresis loop.
- Eventually the curve starts to bend over to the right.
  - It will still increase as more magnetic domains reach their full size and their magnetisation become parallel to the external field [19].

1st Quadrant - Saturation

- Eventually the **B**-field will become high enough where no more change in the magnetisation occurs.
  - This is called technical saturation.
- It is possible to reach 99+% saturation.
- As the field is backed off from "saturation", the magnetisation declines very slightly to the B_r point.
  - This is the remanence, or residual induction [16].

> All of the magnetic energy is now in the magnet and its field.

2nd Quadrant - Demagnetisation

- In this quadrant the applied field opposes the materials **B**-field.
- As the external **B**-field increases in magnitude, some domains will reverse.
- At the knee of the demagnetisation curve, this increase has become rapid and the magnetisation will fall to the H_{ci} point.
- At H_{ci}, the number of domains aligned with the original magnetization is the same as the number aligned with the opposing magnetic field.

> The net magnetisation is zero (0).

3rd Quadrant - Re-magnetisation

- The total magnetization of the part will be reversed.
- If we go far enough, magnetization will reach the saturation level in the negative direction.

4th Quadrant - Demagnetisation

- After fully reversing the magnet and removing the field in the third quadrant, magnetization will recoil to a point that is the negative of the B_r observed when in the first and second quadrant.
- If we apply additional field in the positive direction, we duplicate the second quadrant curve.

- In AC power systems, voltage and flux wave-forms closely approximate sinusoidal functions over time.
- Time to describe the excitation characteristics and losses associated with steady-state ac operation of magnetic materials.
- Assume a closed-core magnetic circuit (i.e., with no air gap),
- And a sinusoidal variation of the core flux (ϕ) with the following:

$$\phi(t) = \phi_{\text{max}} \sin(wt) = A_{\text{c}} B_{\text{max}} \sin(wt)$$

where:

ϕ_{max} amplitude of core flux in Wb,

B_{max} Amplitude of flux density B_{c} in T,

ω Angular frequency in rad/s,

f Frequency in Hz.

- From, the voltage induced in the N-turn winding is:

$$e(t) = \omega N \phi_{\text{max}} \cos(wt) = E_{\text{max}} \cos(wt)$$

where:

$$E_{\text{max}} = \omega N \phi_{\text{max}} = 2\pi f N A_{\text{c}} B_{\text{max}}$$

- In steady-state AC, it is more important to use rms rather than instant values.
- Generally, the rms value of a periodic function of time $f(t)$, of T is:

$$F_{\text{rms}} = \sqrt{\left(\frac{1}{T}\int_0^T f^2(t)\,\mathrm{d}t\right)}$$

- The rms value of a sine wave can be shown to be $1/\sqrt{2}$ times its peak value. Therefore the induced voltage rms is:

$$E_{\mathrm{rms}} = \frac{2\pi}{\sqrt{2}} f N A_{\mathrm{c}} B_{\mathrm{max}} = \sqrt{2}\pi N A_{\mathrm{c}} B_{\mathrm{max}} = 4.44 N A_{\mathrm{c}} B_{\mathrm{max}} \quad \blacksquare$$

- To produce magnetic flux in the core requires current in the exciting winding known as the exciting current, i_{ϕ},

> The nonlinear magnetic properties of the core require that the waveform of the exciting current differs from the sinusoidal waveform of the flux.

- When the hysteresis curve saturates the excitation current spikes.

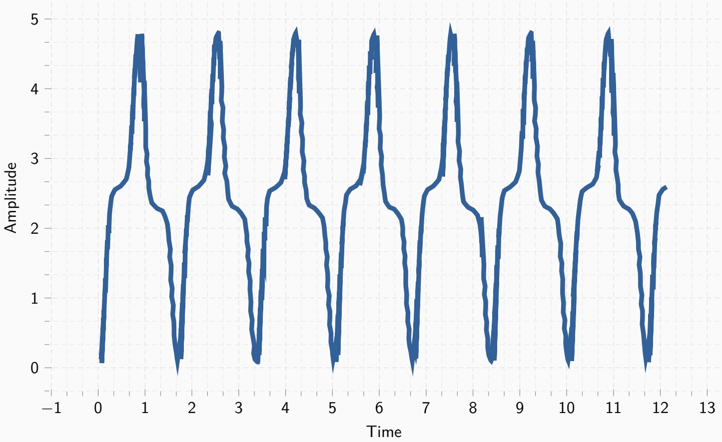


Figure 13: An Example Visualisation of the excitation current experienced by a magnetic circuit.

- The exciting current supplies the mmf required to produce the core flux and the power input associated with the energy in the magnetic field in the core.
- Part of this energy is dissipated as losses and results in heating of the core.
- The rest appears as reactive power associated with energy storage in the magnetic field.

> This reactive power is not dissipated in the core as it is cyclically supplied and absorbed by the excitation source.

- There are two (2) mechanisms associated with time varying **B**-fields.

Ohmic Heating

- Associated with induced currents in the core material.
- From Faraday's law, we see that **B**-fields give rise to **E**-fields.
- In magnetic materials these **E**-fields result in induced currents,
  - These are known as eddy currents, circulating in the core material, opposing the material's internal **B**-field [6].

- To counteract the corresponding demagnetizing effect by the eddy currents, the current in the exciting winding must increase.
- Therefore the resultant "dynamic" B-H loop under ac operation is somewhat "fatter" than the hysteresis loop for slowly varying conditions, and this effect increases as the excitation frequency is increased.
- It is for this reason that the characteristics of electrical steels vary with frequency and hence manufacturers typically supply characteristics over the expected operating frequency range of a particular electrical steel.

- To reduce the effects of eddy currents, magnetic structures are usually built of thin sheets of laminations of the magnetic material.
- These laminations, which are aligned in the direction of the field lines, are insulated from each other by an oxide layer on their surfaces or by a thin coat of insulating enamel or varnish.
- This greatly reduces the magnitude of the eddy currents since the layers of insulation interrupt the current paths; the thinner the laminations, the lower the losses.
- In general, eddy-current loss tends to increase as the square of the excitation frequency and also as the square of the peak flux density.

- Due to the hysteretic nature of magnetic material.
- In a magnetic circuit or the transformer, a time-varying excitation (i_ϕ) will cause the magnetic material to undergo a cyclic variation described by a hysteresis loop.
- The energy input W to the magnetic core as the material undergoes a single cycle is shown to be:

$$W = \oint i_\phi \, d\lambda = \oint \left(\frac{H_c l_c}{N} \right) (A_c N \, dB_c) = A_c l_c \oint H_c \, dB_c$$

- Notice $A_c l_c$ is the core volume and the integral is the area of the ac hysteresis loop, we see that each time the magnetic material undergoes a cycle, there is a net energy input into the material.

- This is the required energy to move around the magnetic dipoles in the material and is dissipated as heat in the material.
- Therefore for a given flux level, the corresponding hysteresis losses are proportional to the area of the hysteresis loop and to the total volume of material.
- As there is an energy loss per cycle, hysteresis power loss is proportional to the frequency of the applied excitation. (Data Sheet)

- In general, both losses depend on the metallurgy of the material as well as the flux density and frequency.
- Information on core loss is typically presented in graphical form.
- It is plotted in terms of watts per unit weight as a function of flux density;
  - Often a family of curves for different frequencies are given.
  - Generally it is either 50 or 60 Hz.

- Nearly all transformers and certain sections of electric machines use sheet-steel material that has highly favorable directions of magnetization along which the core loss is low and the permeability is high.
- This material is termed grain-oriented steel [25].
- The reason lies in the atomic structure of a crystal of the silicon-iron alloy,
  - which is a body-centred cube.

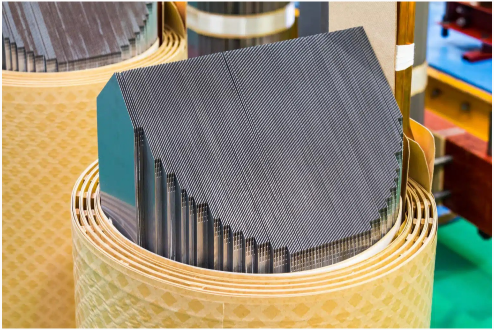


Figure 14: An example of Grain Oriented Steel used in industry [15].

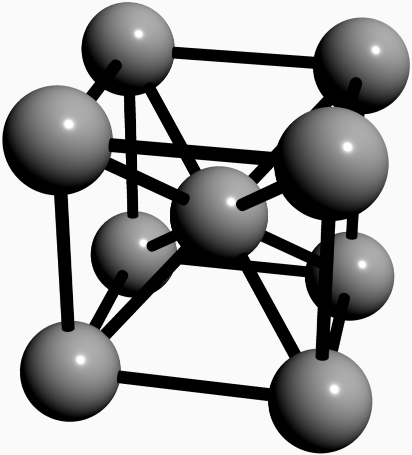


Figure 15: The atomic structure of grain oriented steel.

- Each crystalline cube has an atom at each corner as well as one in the center of the cube.
- In the cube, the easiest axis of magnetization is the cube edge.
- The diagonal across the cube face is more difficult
- By suitable manufacturing techniques most of the crystalline cube edges are aligned in the rolling direction to make it the favorable direction of magnetisation.

- The behaviour in this direction is superior in core loss and permeability to non-oriented steels in which the crystals are randomly oriented to produce a material with characteristics which are uniform in all directions.
- As a result, oriented steels can be operated at higher flux densities than the nonoriented grades.
- Non-oriented electrical steels are used in applications where the flux does not follow a path which can be oriented with the high-Temperature rolling direction or where low cost is of importance.
- In these steels the losses are somewhat higher and the permeability is very much lower than in grain-oriented steels.

Transformers

Table of Contents

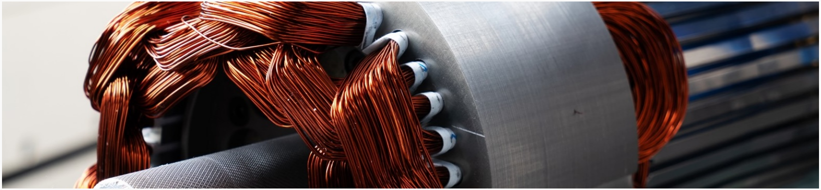
Learning Outcomes

(LO1) Modelling an Ideal Transformer,

(LO2) Construction methods,

(LO3) Physical properties,

(LO4) Types of transformers.



Transformers

- Although transformers are not an energy conversion device, it is an indispensable component in many energy conversion systems.
- In AC power systems, it makes it possible:
  1. Electric generation at the most economical generator voltage,
  2. Power transfer at the most economical transmission voltage,
  3. Power utilisation at the most suitable voltage.
- The transformer is also widely used in:
  - Impedance matching of a source and its load for maximum power transfer,
  - Isolating one circuit from another,
  - Isolating DC while maintaining ac continuity between two circuits[3]

[3] Galvanic isolation.

Figure 16: An example of a arc furnace reactor. They are used to improve operating regime of furnace transformers in iron and steel plants [26].

Figure 17: Delta Transformers specializes in the design and manufacture of dry-type power transformers up to 15,000 kVA, 34.5 kV class [27].

Transformers

- Transformer is one of the simpler devices comprising two or more electric circuits coupled by a common magnetic circuit.
- Its analysis involves many of the principles essential to the study of electric machinery.
- Thus, our study of the transformer will serve as a bridge between the introduction to magnetic-circuit analysis of Chapter 1 and the more detailed study of electric machinery to follow.

Transformers

- Essentially, a transformer consists of two (2) or more windings coupled by mutual magnetic flux.
- If primary is connected to an AC-voltage source, an alternating flux will be produced whose amplitude will depend on:
  - The primary voltage (v_{in}),
  - The primary voltage frequency (f_1),
  - Number of turns (N_1),
- The mutual flux will link the secondary, and will induce a voltage in it whose value will depend on the number of secondary turns as well as the magnitude of the mutual flux and the frequency.

> By changing the number of primary and secondary turns, almost any voltage ratio can be obtained.

Transformers

- The essence of transformer action requires only the existence of time-varying mutual flux linking two windings.
- Such action can occur for two windings coupled through air.
- This would be highly inefficient, therefore an iron-core or other ferromagnetic materials are used.

> This forces the flux path to be confined to the material.

- Such a transformer is commonly called an iron-core transformer.

> Most transformers are of this type.

- We will focus almost wholly with iron-core transformers.

Transformers

An ideal transformer is a lossless device with:

- An input winding (N_1)
- An output winding (N_2)

It has the following properties:

- No iron and copper losses.
- No leakage fluxes.
- A core of infinite permeability and infinite electrical resistivity.
- Flux is confined to the core and winding resistance are negligible.

Transformers

- The ideal transformer may be defined from Faraday's law:

> Change in magnetic field will create emf ($\mathcal{E}$)

$$v_1 = \mathcal{E}_1 = -\frac{d\lambda}{dt} = -\frac{d(N_1\phi)}{dt} = -N_1\frac{d\phi}{dt}.$$

- The core flux also links the secondary and produces an induced emf $\mathcal{E}_2$, and an equal secondary terminal voltage v_2, given by:

$$v_2 = \mathcal{E}_2 = -\frac{d\lambda}{dt} = -\frac{d(N_2\phi)}{dt} = -N_2\frac{d\phi}{dt}.$$

Transformers

- As flux is present in both v_1, v_2 we can derive the following:

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \quad \blacksquare$$

> An ideal transformer transforms voltages in the direct ratio of the turns in its windings.

Transformers

- Let us add a load to the transformer.
- This creates current i_2 and mmf $N_2 i_2$ on the secondary.
- As it is ideal, it has infinite permeability ($\mu \to \infty$) which means the flux is unchanged between primary and secondary.

$$N_1 i_1 - N_2 i_2 = 0$$

- Rearranging it we derive:

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} \quad \blacksquare$$

> This means in an ideal transformer $P_{\text{in}} = P_{\text{out}}$.

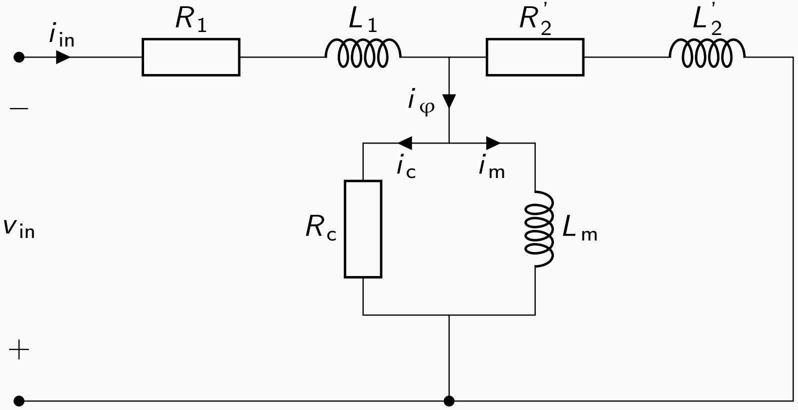


Figure 18: The equivalent circuit for a transformer.

- The open-circuit test (no-load test), is one of the methods used to determine the no-load impedance in the excitation branch of a transformer.
- The test is done on the primary side of the transformer.
- The wattmeter, ammeter and the voltage are connected to their primary winding.
- The nominal rated voltage is supplied to their primary winding.

> As the no-load current is very small, the copper losses experienced are negligible.

- The secondary winding of the transformer is kept open, and the voltmeter is connected to their terminal.

> This voltmeter measures the secondary induced voltage.

- As the secondary of the transformer is open, thus no-load current flows through the primary winding.
- The value of no-load current is very small as compared to the full rated current.
- The copper loss occurs only on the primary winding of the transformer because the secondary winding is open.
- Wattmeter reading only represents the core and iron losses.
- The core loss of the transformer is the same for all types of loads.

Calculation

- Let:

$$\begin{aligned} W_0 &\ \text{wattmeter reading,} \\ V_1 &\ \text{voltmeter reading,} \\ I_0 &\ \text{ammeter reading.} \end{aligned}$$

- We can state, the iron loss of the transformer (P_{C}) as:

$$W_0 = P_{\text{C}} = V_1 I_0 \cos \varphi_0$$

The no-load power factor ($\cos \varphi_0$) is then:

$$\cos \varphi_0 = \frac{W_0}{V_1 I_0}$$

Transformers

- The real part of the current $\hat{I}_0$ is:

$$\hat{I}_0 = I_0 \cos\varphi_0, \qquad \text{and} \qquad \hat{I}_0 = I_0 \sin\varphi_0,$$

- Using these derivations, the core resistance and magnetising inductances are:

$$R_\mathrm{C} = \frac{V_1}{I_\mathrm{m}} \qquad \text{and} \qquad X_\mathrm{m} = \frac{V_1}{I_\mathrm{C}} \quad \blacksquare$$

- Determines the copper loss occur on the full load.
- The copper loss is used for finding the efficiency of the transformer.
- The equivalent resistance, impedance, and leakage reactance are known by the short circuit test.

Performed on the secondary or the high voltage winding.

- Wattmeter, voltmeter and ammeter are connected to the high voltage winding.
- Using a variac, the applied voltage is slowly increased until the ammeter gives a reading equal to the rated current of the HV side.
- The full load current is measured by the ammeter connected across their secondary winding.

- The low voltage source is applied across the secondary winding, which is approximately 5 to 10% of the normal rated voltage.
- The flux is set up in the core of the transformer.

> The magnitude of the flux is small as compared to the normal flux.

- The iron loss of the transformer depends on the flux.
- This is negligible as the generated flux is very low.
- Wattmeter only determines the copper loss occurred, in their windings.

Example

A single phase, 100 kVA, 480 / 120 V transformer is subjected to short-circuit and open-circuit test to determine model parameters:

Open Circuit	Short Circuit
$I_{1,\,\mathrm{OC}} = 0.05$ A	$P_{\mathrm{OC}} = 0.1$ W
$V_{1,\,\mathrm{SC}} = 80$ V	$P_{\mathrm{SC}} = 10$ kW

Table 3: Results of the Tests

Using this information, please determine the model parameters: R_{s}, X_{s}, R_{c}, X_{m}.

Solution

A code solution is as follows:

```
import numpy as np

I_1OC, V_1OC = 0.05, 480
P_OC = 0.1

Z_1OC =  V_1OC / I_1OC
R_C = V_1OC**2 / P_OC
X_m = round(np.sqrt(1 / Z_1OC ** 2 - 1 / R_C ** 2) ** (-1),2)

print(f"{Z_1OC = } Ohm, {R_C = } Ohm, {X_m = } Ohm")
```

```
Z_1OC = 9600.0 Ohm, R_C = 2304000.0 Ohm, X_m = 9600.08 Ohm
```

Transformers

- During its operation, a transformer experiences unavoidable losses.
- These can be categorised as:
  - Iron (Core) Loss,
  - Copper Loss,
  - Stray Loss,
  - Dielectric Loss.
- These losses appear in the form of heat and causes two (2) problems:
  - Increases transformer temperature,
  - Reduces overall efficiency.

- Heat is generated in the transformer while running and is produced by the excitation of the windings and core.
- If the temperature of the transformer continues to increase rapidly, it results in the degradation of the various parts, specifically the insulation materials, and could lead to the failure of the equipment.
- Depending on the cooling methods used, transformers can be divided into two (2) types:
  1. dry,
  2. oil.

Transformers

Dry-Type Cooling

- Dry-type transformers, are normally cooled by air.
- The following two (2) methods adopted in dry-type transformers.

Air Natural Cooled by surrounding air via convection.

Air Force Forced air circulation using fans and blowers.

Oil-Type Cooling

- Uses either:
    1. Oil-air cooling,
    2. Oil-water cooling.
- Looking at them in more detail:

Oil Natural Air Natural

- The core and coils are cooled by surrounding in oil.
- Heat transfer of oil by natural air convection.

(Non-Mineral) Oil Natural Air Natural

- The core and coils are cooled by surrounding in synthetic oil.
- Heat transfer of oil by natural air convection.

Oil-Type Cooling

- Uses either:
    1. Oil-air cooling,
    2. Oil-water cooling.
- Looking at them in more detail:

Oil Natural Air Forced

- Cooled by surrounding in oil.
- Forced air circulation using pumps, fans and blowers.

Oild Forced Air Forced

- Forced oil and air circulation using fans and blowers.

Oil-Type Cooling

- Uses either:
  1. Oil-air cooling,
  2. Oil-water cooling.
- Looking at them in more detail:

Oil Natural Water Forced

- Cooled by surrounding in oil.
- Forced water circulation using heat exchanges.

Oil Forced Water Forced

- Forced oil and water circulation using oil-to-water heat exchanges.

Transformers

Figure 19: An example of oil cooled transformer which uses radiators to exchange heat [22].

- A transformer core is composed of limbs and yokes joined together to form a single structure around which the coils are placed.

> The manner in which the respective yoke and limbs join together will depend on the type and design of the core.

Limb Vertical sections which the coils are formed around. The limbs can also be located on the exterior of the outermost coils in the case of some core designs. The limbs on a transformer core can also be referred to as legs.

Yoke The yoke is the horizontal section of the core which joins the limbs together. The yoke and limbs form a pathway for magnetic flux to flow freely.

Transformers

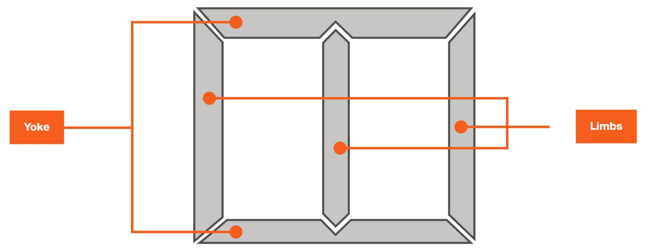


Figure 20: Diagram of the transformer geometry and its parts [12].

Transformers

- The earliest transformer cores utilized solid iron.
- Methods developed over the years to refine raw iron ore into more permeable materials such as silicon steel, which is used today for transformer core designs due to its higher permeability.
- Also, the use of many densely packed laminated sheets reduces issues of circulating currents and overheating caused by solid iron core designs.
- Further increases in core design are made through cold rolling, annealing, and using grain oriented steel.

Transformers

Cold Rolling

- Silicon steel is a softer metal.
- They also have small hysteresis area, low core loss and high μ.
- They are used in making sheets less than 2mm thick.
- Cold rolling silicon steel will increase its strength–making it more durable when assembling the core and coils together.
- Permeability can be increased even further by orienting the grain of the steel in the same direction.

Annealing

- Involves heating the core steel up to a high temperature to remove impurities.
- This process will increase the softness and ductility of the metal by decreasing the internal stress of the material.

Shell-Type

- The core surrounds the windings.
- Creates a closed pathway surrounding the windings for magnetic flux.
- This design also typically yields less energy loss than a core type design.
- A shell type design is the classification for most distribution class padmounts and substations with a wrapped 5-legged core.

Core-Type

- A core type design is where the windings surround the core steel.
- In this design, there is no return path (or closed loop) for the magnetic flux around the coils.
- This design typically yields more energy losses, and it requires more copper or aluminum winding material than a shell-type configuration.

Transformers

- For low-frequency, high-power applications, laminated cores are an economical way of reducing eddy current loss.
- Note that the laminations are in parallel with the magnetic flux, while the eddy currents are perpendicular.
- This arrangement restricts the eddy currents to the width of the laminations

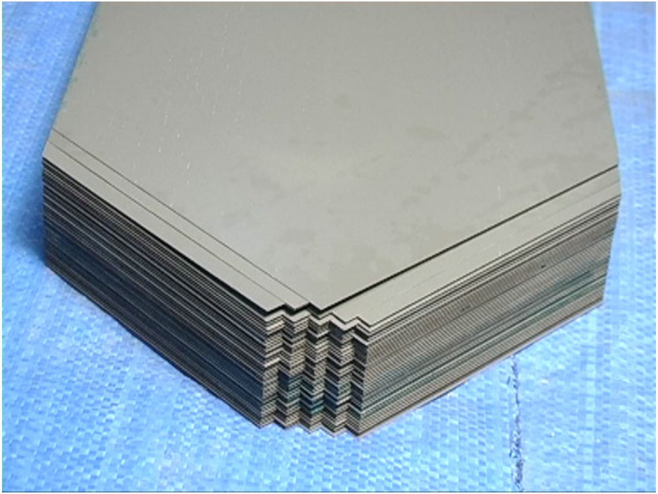


Figure 21: Lamination is done to restrict the eddy current flow.

Auto Transfomer

- It is a transformer with only one (1) winding.
- portions of the same winding act as both the primary winding and secondary winding sides of the transformer.
- Has higher efficiency than two winding transformer.
  - Less ohmic loss and core loss due to reduction of transformer material.

> They are often smaller, lighter, and cheaper than typical dual-winding transformers, but the disadvantage of not providing electrical isolation between primary and secondary circuits.

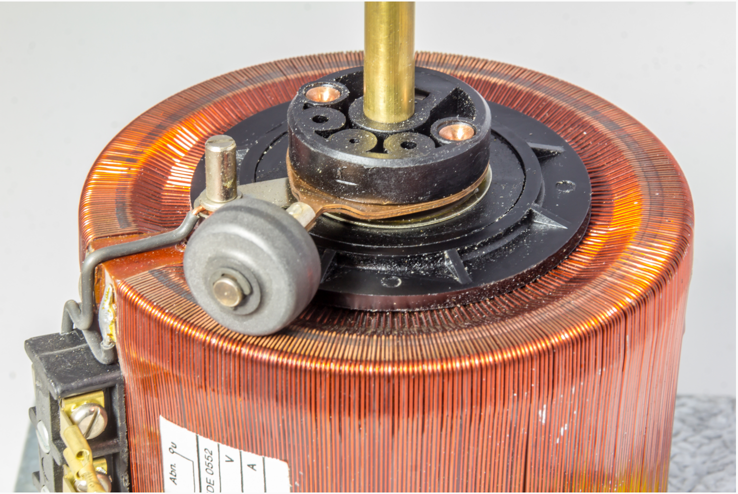


Figure 22: Variable autotransformer 0-220 V, 4 A, 880 VA.

Isolation (Galvanic) Transformer

- Used in transferring power from an AC source to an equipment while isolating the powered device from the power source.
  - Usually for safety reasons or to reduce transients and harmonics.

> They provide galvanic isolation
> no conductive path between source and load.

- This isolation is used to protect against electric shock, to suppress electrical noise in sensitive devices, or to transfer power between two circuits which must not be connected.

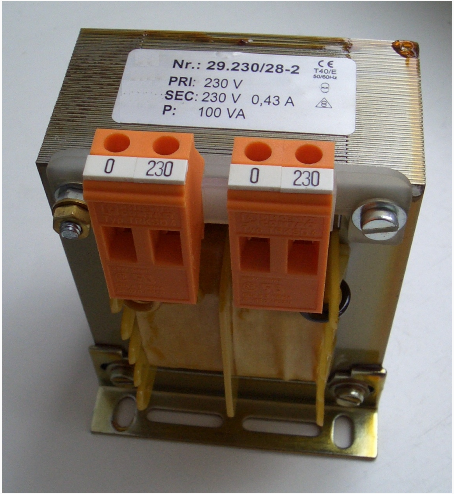


Figure 23: A 230 V isolation transformer.

- Three single-phase transformers can be connected to form a three-phase transformer bank in any of the four (4) ways:
  - Y-Δ Connection,
  - Δ-Y Connection,
  - Δ-Δ Connection,
  - Y-Y Connection.
- The windings at the left are the primaries,
- The ones at the right are the secondaries,

Rated V and I at the primary and secondary of the three-phase transformer bank depends upon the connection used but that the rated kVA of the three-phase bank is three times that of the individual single-phase transformers, regardless of the connection.

Star-Delta Connection

- Y-Δ connection is commonly used in stepping down from a high voltage to a medium or low voltage due to grounding on the high-voltage side.
- This is due to the neutral connection being available on the high-voltage side for safety reasons.

Delta-Star Connection

- Δ-Y connection is commonly used for stepping up to a high voltage.

> Having a neutral connection on the high-voltage side is always a benefit.

Delta-Delta Connection

- The $\Delta - \Delta$ connection has the advantage where one transformer can be removed for repair or maintenance while the remaining two continue to function as a three-phase bank with the rating reduced to 58 percent of that of the original bank
- This is known as the open-delta, or V, connection.

> An open delta transformer is a three phase transformer that only has two primary and secondary windings, with one side of the delta phase diagram "open".

Star-Star Connection

- Seldom used due to significant disadvantages.

Disadvantages

- If the neutral connection is not provided and an unbalanced load is connected, the phase voltages tend to become severely unbalanced.
- Magnetising current varies non-sinusoidal having a third harmonic.
  - In balance, the 3rd harmonic primary winding magnetising currents are equal in magnitude and in phase with each other.
  - These will be directly additive at the neutral point.
- These components will distort the magnetic flux which induces a voltage having a third harmonic component in both windings.
- This third harmonic component of the induced voltage may be as large as the fundamental voltage.

Transformers

- Instead of three single-phase transformers, a three-phase bank may consist of one three-phase transformer having all six windings on a common multi-legged core and contained in a single tank. Advantages of three-phase transformers over connections of three single-phase transformers are that they cost less, weigh less, require less floor space, and have somewhat higher efficiency.

Electromechanical Energy Conversion

Table of Contents

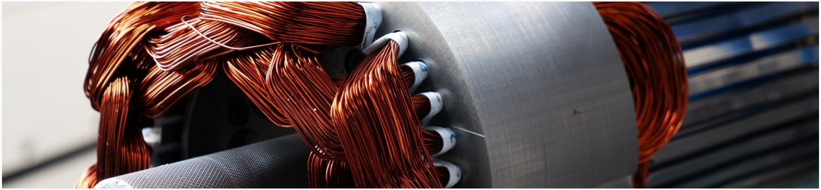
Learning Outcomes

(LO1) Definition of Electromechanical devices,

(LO2) Understanding the conversion process,

(LO3) Describing the energy co-energy relationship,

(LO4) Looking at multiply-excited system..



Electromechanical Energy Conversion

- The electromechanical energy conversion (EEC) process, takes place through the medium of the electric or magnetic field of the conversion device.
- We can classify these items into three (3) categories:

1. Devices for measurement and control are frequently referred to as transducers.
   - Generally operate under linear input-output conditions and with relatively small signals.
   - i.e., microphones, pickups, sensors, and loudspeakers
2. Devices encompasses force-producing motions: solenoids, relays, and electromagnets.
3. Continuous energy-conversion equipment: motors and generators.

- The purpose of studying EEC is threefold:

1. To understand how energy conversion takes place,
2. To provide techniques for designing and optimising the devices for specific requirements,
3. To develop models of EEC devices that can be used in analysing their performance as components in engineering systems.

- The Lorentz force law states:

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

where $\mathbf{F}$ is the force on a particle, q is the charge. $\mathbf{B}$ is the magnetic field, $\mathbf{E}$ is the electric field and $\mathbf{v}$ is the speed in which the particle is moving.

- In a pure $\mathbf{E}$ field, the force is simply:

$$\mathbf{F} = q\mathbf{E}$$

- In a $\mathbf{B}$ field the force is slightly more complex.

$$\mathbf{F} = q\left(\mathbf{v} \times \mathbf{B}\right)$$

Example

A nonmagnetic rotor containing a single-turn coil is placed in a uniform magnetic field of magnitude B_0. The coil sides are at radius R and the wire carries current I.

Find the θ-directed torque as a function of rotor position α when $I = 10$ A, $B_0 = 0.02$ T and $R = 0.05$ m.

Assume the rotor is of length $l = 0.3$m.

Example

The force per unit length on a wire carrying current I can be found by multiplying Eq. 3.6 by the cross-sectional area of the wire. When we recognize that the product of the cross-sectional area and the current density is simply the current I, the force per unit length acting on the wire is given by

$$\mathbf{F} = \mathbf{I} \times \mathbf{B}$$

Thus, for wire ℓ carrying current I into the paper, the θ-directed force is given by:

$$F_{1_\theta} = -IB_0\ell \sin \alpha$$

Solution

And for wire 2 (which carries current in the opposite direction and is located 180° away from wire 1)

$$F_{2_\theta} = -IB_0 \ell \sin\alpha$$

where l is the length of the rotor. The torque T acting on the rotor is given by the sum of the force-moment-arm products for each wire

$$T = -2IB_0 R\ell \sin\alpha = 2(10)(0.02)(0.05)(0.3)\sin\alpha = -0.006\sin\alpha$$

Electromechanical Energy Conversion

- The principle of conservation of energy states that energy is neither created nor destroyed.
  - It is only a change in form.
- For isolated systems with clear boundaries, this allows to keep track of energy:

> the net flow of energy into system across its boundary is equal to the sum of the time rate of change of energy stored in the system.

- For the case of electro-mechanical systems we can write:

$$\begin{pmatrix}\text{Energy input}\\ \text{from electric}\\ \text{sources}\end{pmatrix} = \begin{pmatrix}\text{Mechanical}\\ \text{energy}\\ \text{output}\end{pmatrix} + \begin{pmatrix}\text{increase in energy}\\ \text{stored in}\\ \text{magnetic field}\end{pmatrix} + \begin{pmatrix}\text{energy}\\ \text{converted}\\ \text{into heat}\end{pmatrix}$$

Electromechanical Energy Conversion

- For the lossless magnetic energy storage system:

$$\mathrm{d}W_{\text{elec}} = \mathrm{d}W_{\text{mech}} + \mathrm{d}W_{\text{fld}} \tag{1}$$

- In time $\mathrm{d}t$, W_{elec} is given as :

$$\mathrm{d}W_{\text{elec}} = ei\,\mathrm{d}t$$

  where e is the voltage induced in the electric terminals by the changing magnetic stored energy.

- It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals.

- Let's rewrite this equation:

- Consider the electromagnetic relay shown schematically

Electromechanical Energy Conversion

- The excitation coil resistance is shown as R,
- The mechanical terminal variables are shown as a force f_{fld} produced by the B-field directed from the relay to the external mechanical system and a displacement x.
- mechanical losses can be included as external elements connected to the mechanical terminal. Similarly, the moving armature is shown as being massless; its mass represents me- chanical energy storage and can be included as an external mass connected to the mechanical terminal. As a result, the magnetic core and armature constitute a lossless magnetic-energy-storage system,

Electromechanical Energy Conversion

- As you know, and inductance L is a function of the geometry magnetic structure and material.
- EEC devices contain air-gaps in their circuits for moving parts.

> The reluctance of the air gap is significantly higher than that of the magnetic material which forces most of the energy storage to occur in the air gap.

- This allows us to disregard the non-linear relations in the system for practical devices.
- Our second assumption is flux and MMF are directly proportional to the entire magnetic circuit.

- The flux linkages λ and current i are considered linearly dependent by a geometrically defined **inductance**:

$$\lambda = L(x)\, i \tag{2}$$

- As magnetic force (f_{fld}) has been defined as acting from the relay upon the external mechanical system and $\mathrm{d}W_{\mathrm{mech}}$ is defined as the mechanical energy output of the relay, we can write

$$\mathrm{d}W_{\mathrm{mech}} = f_{\mathrm{fld}}\, \mathrm{d}x \tag{3}$$

- Using Eq. (3), and substituting $\mathrm{d}W_{\text{elec}} = i\,\mathrm{d}\lambda$, we can write:

$$\mathrm{d}W_{\text{fld}} = \mathrm{d}W_{\text{elec}} - \mathrm{d}W_{\text{mech}},$$
$$\mathrm{d}W_{\text{fld}} = i\,\mathrm{d}\lambda - f_{\text{fld}}\,\mathrm{d}x.$$

- As the magnetic energy storage system is considered lossless, it is a **conservative system**.
- The value of W_{fld} uniquely specified by the values of λ and x.
- λ and x are referred to as state variables since their values uniquely determine the state of the system.

- We see that W_{fld} is uniquely defined by the value of λ and x.

> This means the value of W_{fld} is the same regardless of how λ and x are brought to their final values.

- Consider the following paths:

Electromechanical Energy Conversion

- These two separate paths are shown to be integrated to find W_{fld} at the point (λ_0, x_0).
- The curvy path is the general case and it is difficult to integrate unless both i and f_{fld} are known as a function of λ and x.
- As integration is path independent Path 3 gives the same result and is much easier to integrate:

$$W_{\text{fld}}(\lambda_0, x_0) = \int_{\text{path 2a}} \mathrm{d}W_{\text{fld}} + \int_{\text{path 2b}} \mathrm{d}W_{\text{fld}}$$

Example

The relay drawn is made infinitely-permeable magnetic material with a movable plunger, also of infinitely-permeable material.

The height of the plunger is much greater than the air-gap length (h » g).

Calculate the magnetic stored energy W_{fld} as a function of plunger position ($0 < x < d$) for $N = 1000$ turns, $g = 2.0$ mm, $d = 0.15$ m, $l = 0.1$ m, and $i = 10$ A.

Solution

We can solve for W_{fld} when λ is known.

For this situation, i is held constant, and thus it would be useful to have an expression for W_{fld} as a function of i and x.

This can be obtained quite simply by:

$$W_{\text{fld}} = \frac{1}{2} L(x)\, i^2 \qquad \text{where} \qquad L(x) = \frac{\mu_0 N^2 A_{\text{gap}}}{2g}$$

where Agapis the gap cross-sectional area. From Fig. 3.6b, Agapcan be seen to be

$$A_{\text{gap}} = l(d - x) = ld\left(1 - \frac{x}{d}\right)$$

Solution

Therefore:

$$L(x) = \frac{\mu_0 N^2 ld\,(1 - x/d)}{2g}$$

and:

$$W_{\text{fld}} = \frac{1}{2}\frac{N^2\mu_0 \ell d\,(1 - x/d)}{2g} i^2$$

$$= \frac{1}{2}\frac{\left(1000^2\right)}{2(0.002)} \times 10^2\,(1 - x/d)$$

$$236\left(1 - \frac{x}{d}\right) \quad \blacksquare$$

Practice

The relay of from the previous question is modified, making the air gaps surrounding the plunger are no longer uniform. The top air gap length is increased to $g_{\text{top}} = 3.5$ mm and that of the bottom gap is increased to $g_{\text{bot}} = 2.5$ mm.

The number of turns is increased to $N = 1500$.

Calculate the stored energy as a function of plunger position ($0 < x < d$) for a current of $i = 5$ A.

$$W_{\text{fld}} = 88.5\,(1 - x/d)$$

- For a lossless magnetic-energy-storage system, the magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x.
- Rewriting Eq. (165) in the following form gives us:

$$\mathrm{d}W_{\text{fld}}(\lambda, x) = i\,\mathrm{d}\lambda - f_{\text{fld}}\,\mathrm{d}x. \tag{4}$$

- For any state function of two (2) independent variables, W_{fld} can be written as:

$$\mathrm{d}W_{\text{fld}}(\lambda, x) = \left.\frac{dW_{\text{fld}}}{d\lambda}\right|_{x} \mathrm{d}\lambda + \left.\frac{dW_{\text{fld}}}{dx}\right|_{\lambda} \mathrm{d}x, \tag{5}$$

- As λ and x are independent variables, Eq. (4) and Eq. (5) **must** be equal for all values of $d\lambda$ and dx.
- Taking the partial derivative while holding x constant:

$$i = \left.\frac{dW_{\text{fld}}(\lambda, x)}{d\lambda}\right|_x \tag{6}$$

- Taking the partial derivative while holding λ constant:

$$f_{\text{fld}} = \left.\frac{dW_{\text{fld}}(\lambda, x)}{dx}\right|_\lambda$$

- This is the results we needed.
- Once we know W_{fld} as a function of λ and x, we can solve Eq. (6) as a function for $i(\lambda, x)$

Example

The magnetic circuit consists of a single-coil stator and an oval motor. As the air-gap is nonuniform, the coil inductance varies with rotor angular position, measured between the magnetic axis of the stator coil and the major axis of the rotor as:

$$L(\theta) = L_0 + L_2 \cos 2\theta$$

where $L_0 = 10.6$ mH and $L_2 = 2.7$ mH.

> Note the 2nd harmonic variation of inductance with rotor angle θ. This is consistent with inductance being unchanged if the rotor is rotated through an angle of 180°.

Find the torque as a function of θ for a coil current of 2 A.

Solution

$$T_{\text{fld}}(\theta) = \frac{i^2}{2}\frac{dL(\theta)}{d\theta} = \frac{i^2}{2}\left(-2L(2)\sin(2\theta)\right)$$

Notes on Solution

```python
import sympy as sy

# Define symbols for symbolic calculation
i, theta, L0, L2 = sy.symbols('i theta L0 L2')

L = L0 + L2 * sy.cos(2*theta)
T = i ** 2 / 2 * sy.diff(L, theta)

T.subs([(L2, 2.7e-3), (i, 2)])

print(T) # Print the output

del i, theta, L0, L2  # Remove the used variables from memory
```

```
#+RESULTS: ELECTROMECHANICAL-OVAL-ROTOR
```

Example

The coil inductance coil on a magnetic circuit is found to vary with rotor position as:

$$L(\theta) = L_0 + L_2 \cos(2\theta) + L_4 \sin(4\theta)$$

where L_0= 25.4 mH, L_2 = 8.3 mH and L_4 = 1.8 mH. Using these values:

1. Find the torque as a function of θ (i.e., $T(\theta)$) for a winding current of 3.5 A. 2. Find a rotor position θ_{max} that produces the largest negative torque.

Solution

```
import sympy as sy

i, theta, L0, L2, L4 = sy.symbols('i theta L0 L2 L4')  # Define
↪  symbols for symbolic calculation

L = L0 + L2 * sy.cos(2*theta) + L4 * sy.sin(4*theta)
T = i ** 2 / 2 * sy.diff(L, theta)

T.subs([(L2, 8.3e-3), (i, 3.5), (L4, 1.8e-3)])

print(T)

del i, theta, L0, L2, L4  # Remove the used variables from memory
```

```
#+RESULTS: COIL-INDUCTANCE
```

Example

Consider a plunger whose inductance varies as with position as:

$$L(x) = L_0(1 - (x/d)^2)$$

Find the force on the plunger as a function of x when the coil is driven by a controller which produces a current as a function of x of the form

$$i(x) = I_0 \left(\frac{x}{d}\right)^2 A$$

Solution

```python
import sympy as sy

i, L0, I0, x, d, mu0, N, l, g = sy.symbols('i L0 I0 x d mu0 N l g')

i = I0 * (x / d)

L = L0*( 1 - (x/d)**2)

f = i ** 2 / 2 * sy.diff(L, x)

W_fld = i ** 2 / 2 * L

print(f)
```

```
#+RESULTS: PLUNGER
```

Example

In a drive with a rotow with Hlight(non-uniform air gap), the inductances in henrys are given as:

$$L_{11} = (3 + \cos 2\theta) \times 10^{-3}, L_{12} = 0.3 \cos\theta, L_{22} = 3\theta + 10 cos2\theta.$$

Using this information, please find and plot the torque $T_{fld}(\theta)$ for current $I_1 = 0.8$ A and $I_2 = 0.01$ A.

Solution

```
import sympy as sy
import numpy as np

theta = sy.symbols('theta')

L = np.array([[(3 + sy.cos(2 * theta))*1e-3, 0.3 *
↪  sy.cos(theta)],[0, (30 + 10 * sy.cos(2 * theta))]])

i = [0.8, 0.01]

T_fld = i[0]**2 / 2 * sy.diff(L[0,0], theta) + i[1]**2 / 2 *
↪  sy.diff(L[1,1], theta) + i[0] * i[1] * sy.diff(L[0,1], theta)

sy.plot(T_fld, (theta, -10, 10), adaptive=True, depth =50 )
```

- To obtain force, directly as a function of current, we can use co-energy.

> Co-energy does not exist in reality. It is a mathematical convenience [2].

- The selection of energy or coenergy as the state function is purely a matter of convenience.
  - They both give the same result, but one or the other may be simpler analytically, depending on the desired result and the characteristics of the system being analysed.

- The coenergy (W'_{fld}) is defined as a function of λ and x:

$$W'_{\text{fld}}(\lambda, x) = i\lambda - W_{\text{fld}}(\lambda, x)$$

- The desired derivation is carried out by using the differential of $i\lambda$:

$$\mathrm{d}(i\lambda) = i\,\mathrm{d}\lambda + \lambda\,\mathrm{d}i$$

Electromechanical Energy Conversion

- The analysis of a system with multiple excitation is similar.
- Assume a system having:
  - a mechanical terminal with the values T_{fld} and
  - and two (2) electrical terminals (i.e., λ_1, λ_2, i_1, i_2)

> This could be a system where rotor and stator are **both** excited.

- Using the fluxes, the differential energy function $\mathrm{d}W_{\text{fld}}(\lambda_1, \lambda_2, \theta)$ can be written as:

$$\mathrm{d}W_{\text{fld}}(\lambda_1, \lambda_2, \theta) = i_1\,\mathrm{d}\lambda_1 + i_2\lambda_2 - T_{\text{fld}}\,\mathrm{d}\theta$$

- Using our previous analogies:

$$i_1 = \left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1}\right|_{\lambda_2, \theta}$$

$$i_2 = \left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2}\right|_{\lambda_1, \theta}$$

$$T_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \theta}\right|_{\lambda_1, \lambda_2} \qquad \blacksquare$$

The partial derivative with respect to each independent variable must be taken holding the other variables **constant**.

Electromechanical Energy Conversion

- Using co-energy definition, we can re-write our equation as:

$$W'_{\text{fld}}(i_1, i_2, \theta) = \lambda_1 i_1 + \lambda_2 i_2 - W_{\text{fld}}.$$

- Taking the derivative of this statement gives us:

$$\mathrm{d}W'_{\text{fld}}(i_1, i_2, \theta) = \lambda_1 \,\mathrm{d}i_1 + \lambda_2 \,\mathrm{d}i_2 - T_{\text{fld}} \,\mathrm{d}\theta\,.$$

- Through significant simplification we arrive at:

$$T_{\text{fld}} = \frac{i_1^2}{2} \frac{d^{L_{11}()}}{d^{L_{11}()}}$$

Example

In the following system, the inductance in H are given as:

- $L_{11} = (3 + \cos 2\theta) \times 10^{-3}$,
- $L_{12} = 0.3 \cos \theta$,
- $L_{22} = 30 + 10 \cos 2\theta$.

Find the torque equation for currents $i_1 = 0.8$ A and $i_2 = 0.01$ A.

Solution

The torque is simply determined by:

$$
\begin{aligned}
T_{\text{fld}} &= \frac{i_1^2}{2}\frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta} \\
&= \frac{i_1^2}{2}\left(-2\times 10^{-3}\right)\sin 2\theta + \frac{i_2^2}{2}\left(-20\sin 2\theta\right) - i_1 i_2 (0.3)\sin\theta\,.
\end{aligned}
$$

For $i_1 = 0.8$ A and $i_2 = 0.01$ A, the torque is:

$$
T_{\text{fld}} = -1.64\times 10^{-3}\sin 2\theta - 2.4\times 10^{-3}\sin\theta \quad \blacksquare
$$

Notes on the Solution

The torque expression consists of terms worth discussing.

- The term proportional to $i_1 i_2 \sin\theta$, is due to the mutual interaction between the rotor and stator currents.
- This acts in a direction to align the rotor and stator so as to maximize their mutual inductance.

> It can be thought of as being due to the tendency of two magnetic fields (in this case those of the rotor and stator) to align.

Notes on the Solution

The torque expression consists of terms worth discussing.

- Two terms each proportional to $\sin 2\theta$ and to the square of one of the coil currents $(i_1^2,\ i_2^2)$.
- Due to the action of the individual winding currents alone and correspond to the torques one sees in singly-excited systems.
- The torque is due to self inductances being a function of rotor position and the corresponding torque acts in a direction to maximize each inductance so as to maximize the co-energy.
- The $\sin 2\theta$ variation is due to the corresponding variation in the self inductances caused by the variation of the air-gap reluctance.

Rotating Magnetic Fields

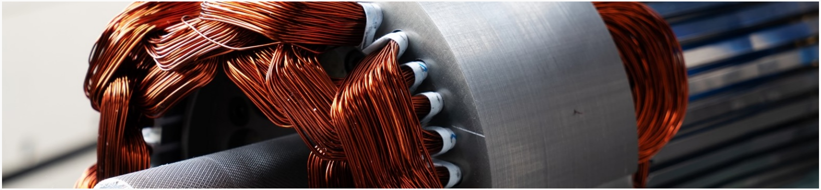
Table of Contents

Learning Outcomes

(LO1) Types of Rotor Constructions,

(LO2) Flux paths through the rotor,

(LO3) MMF Waves.

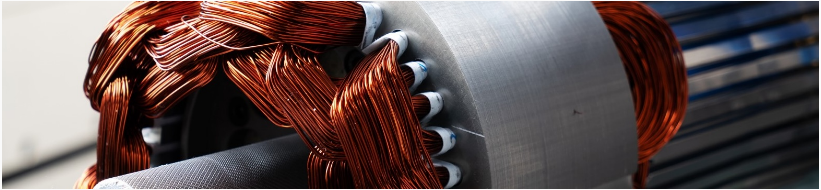


Rotating Magnetic Fields

- The topic is to introduce and discuss some of the principles underlying the performance of electric machinery.

> Following principles are common to both Alternating Current (AC) and Direct Current (DC) machines.

- Various techniques and approximations involved in reducing the machine to simple mathematical models will be discussed.



Rotating Magnetic Fields

- To determine the voltages induced by time-varying magnetic fields we can use Faraday's law:

$$\mathcal{E} = \frac{d\lambda}{dt},$$

- Electromagnetic Energy Conversion (EEC) occurs when changes in λ results from mechanical motion.
- $\mathcal{E}$ are generated in windings or groups of coils by rotating these windings mechanically through a **B**-field, either:
  1. mechanically rotating a magnetic field past the winding,
  2. designing the magnetic circuit so that the reluctance varies with rotation of the rotor (i.e., $\mathcal{R}(\theta)$),

> A coil's λ is changed cyclically generating time-varying voltage.

Rotating Magnetic Fields

- Coils connected together is referred to as an **armature winding**.
- The term is used to refer to a winding or a set of windings on a rotating machine which carry AC.
- In AC machines, the armature winding is typically on the stationary (i.e., stator).
  - Which case these windings may also be referred to as stator windings.

Rotating Magnetic Fields

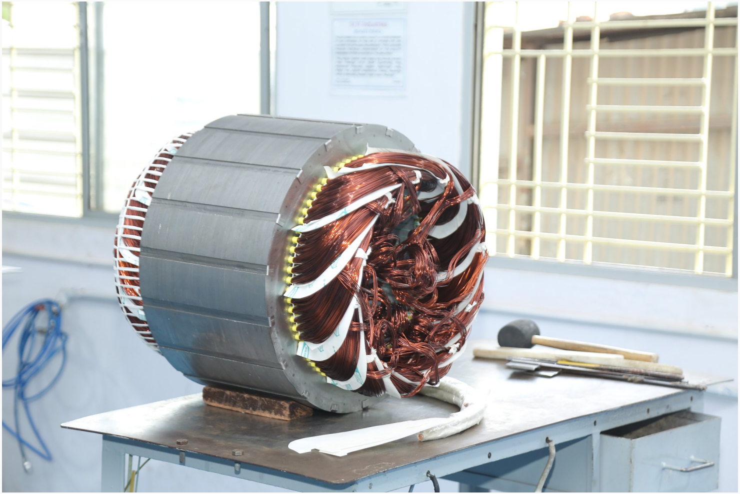


Figure 24: The armature of a AC is the stator as the stator has AC flowing during operation [10].

Rotating Magnetic Fields

- In DC machines, armature winding is found on the rotating member (i.e., rotor).
- The armature winding of a DC machine consists of numerous coils connected together to form a closed loop.
- A rotating mechanical contact is used to supply current to the armature winding as the rotor rotates.
  - This is known as a **commutator**.

Rotating Magnetic Fields

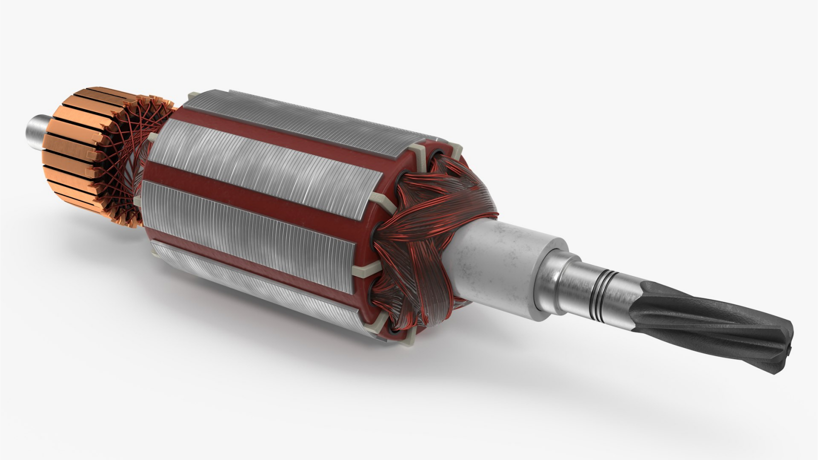


Figure 25: The armature of a DC is the rotor as the rotor has AC flowing during operation [28].

Rotating Magnetic Fields

- Synchronous Machine (SynM) and DC machines usually include a second winding(s) which carry DC and which are used to produce main operating flux in the machine.
- Such a winding is typically referred to as **field winding**.
  - On DC machines, it is found on the stator,
  - On SynM machines, it is found on the rotor.

> Permanent Magnet (PM)s also produce DC magnetic flux and are used in the place of field windings in some machines.

Rotating Magnetic Fields

- Some machines, such as variable reluctance machines and stepper motors, have no windings on the rotor.
- Operation of these depends on the non-uniform air-gap reluctance associated with variations in rotor position in conjunction with time-varying currents applied to their stator windings.
- For these, both the stator and rotor structures are subjected to time-varying magnetic flux.
  - Both may require lamination to reduce eddy-current losses.

Rotating Magnetic Fields

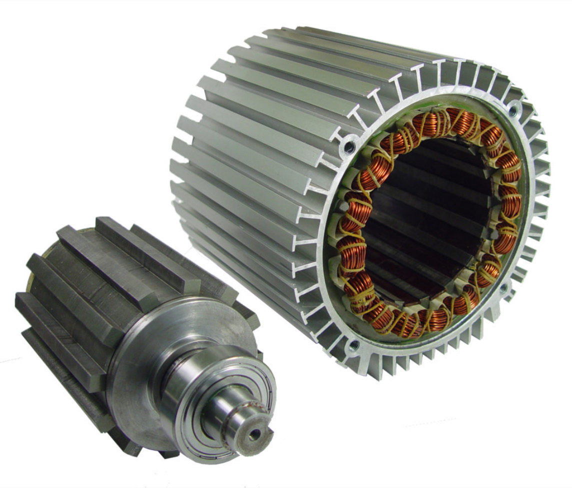


Figure 26: An example of a switched reluctance drive with its rotor (left) and stator (right).

Rotating Magnetic Fields

- Start with the voltage induced in the armature of the simplified salient-pole ac synchronous generator.

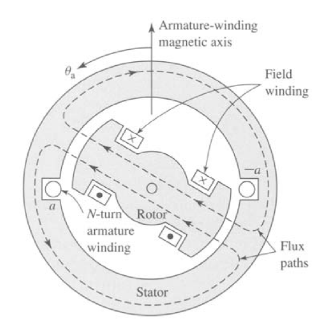


Figure 27: A simple diagram of a 2-pole salient generator.

Rotating Magnetic Fields

Figure 28: A unique characteristics of salient-pole machine is its protrusions from the core.

Rotating Magnetic Fields

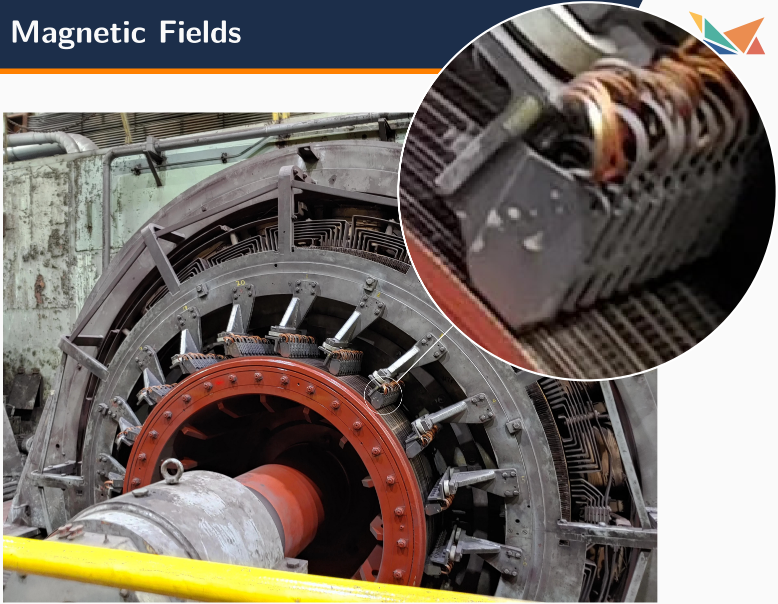
- The field-winding of this machine produces one pair of magnetic pole.
  - Referred to as a two-pole machine.
- With rare exceptions, the armature winding of a SynM is on the stator, and the field winding is on the rotor.
- The field winding is excited by DC conducted to it by means of stationary carbon brushes which contact rotatating slip rings or collector rings.
- Practical factors usually dictate this orientation of the two windings:

> It is advantageous to have the single, low-power field winding on the rotor while having the high-power, typically multiple-phase, armature winding on the stator.

Rotating Magnetic Fields

Figure 29: A 10,000 HP SynM.

Rotating Magnetic Fields



Rotating Magnetic Fields

- The armature winding, consisting of only a single coil of N turns placed in slots on the inner periphery of the stator.
- The conductors forming these coil sides are parallel to the shaft of the machine and are connected in series by end connections.
- The rotor is turned at a constant speed by a source of mechanical power connected to its shaft.
- The armature winding is assumed to be open-circuited and hence the flux in this machine is produced by the field winding alone.
- An idealised analysis of this machine would assume a sinusoidal distribution of magnetic flux in the air gap.

> The resultant radial distribution of B is a function of the spatial angle θ_a (respect to the magnetic axis of the armature winding) around the rotor periphery.

Rotating Magnetic Fields

- In practice, the air-gap flux-density of practical salient-pole machines can be made to approximate a sinusoidal distribution by properly shaping the pole faces.
- As the rotor rotates, the λ of the armature winding change with time.
- Under the assumption of a sinusoidal flux distribution and constant rotor speed, the resulting coil voltage will be sinusoidal in time.
- The coil voltage passes through a complete cycle for each revolution of the two-pole machine.
- Its frequency (f) in cycles per second (Hz) is the same as the speed of the rotor in revolutions per second: the electric frequency of the generated voltage is synchronized with the mechanical speed, and this is the reason for the designation "synchronous" machine. video
- Thus a two-pole synchronous machine must revolve at 3600 revolutions per minute to produce a 60-Hz voltage.

Rotating Magnetic Fields

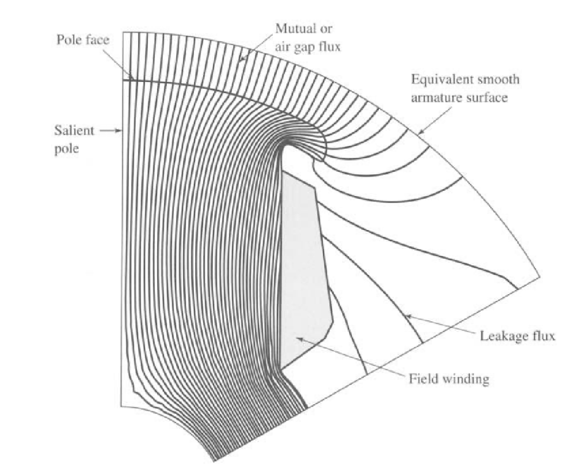


Figure 30: Finite-element solution for the flux distribution around a salient pole.

Rotating Magnetic Fields

- Many SynM have more than two poles.
- As a specific example, a schematic form a four-pole single-phase generator.

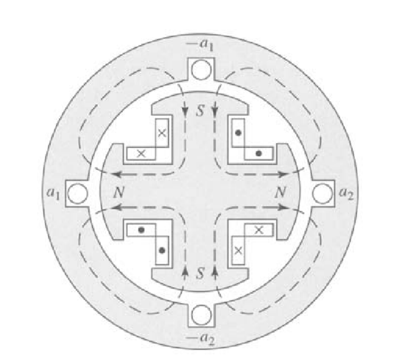


Figure 31

Rotating Magnetic Fields

- The field coils are connected so that the poles are of alternate polarity.
- There are two complete wavelengths, or cycles, in the flux distribution around the periphery.
- The armature winding now consists of two coils connected in series by their end connections.
- The span of each coil is one wavelength of flux.
- The generated voltage now goes through two complete cycles per revolution of the rotor.
- The frequency in hertz will thus be twice the speed in revolutions per second.

Rotating Magnetic Fields

- When a machine has more than two poles, it is convenient to concentrate on a single pair of poles and to recognize that the electric, magnetic, and mechanical conditions associated with every other pole pair are repetitions of those for the pair under consideration.
- For this reason it is convenient to express angles in electrical degrees or electrical radians rather than in physical units.
- One pair of poles in a multipole machine or one cycle of flux distribution equals 360 electrical degrees.
- Since there are poles/2 complete wavelengths, or cycles, in one complete revolution, it follows, for example, that

$$\theta_{\mathrm{e}} = \left(\frac{p}{2}\right)\theta_{\mathrm{a}}$$

Rotating Magnetic Fields

- Armature currents rotates at synchronous speed.
- To produce a steady electromechanical torque, the **B**-fields of the stator and rotor must be constant in amplitude and stationary with respect to each other.
- In a SynM, the steady-state speed is determined by the number of poles and the frequency of the armature current.

> a SynM operated from a constant-frequency AC source will operate at a constant steady-state speed.

Rotating Magnetic Fields

- Similar to SynM, stator winding of an Induction Machine (IM) is excited with AC.
- Unlike a SynM, rotor field windings are excited with AC.
- In IM, AC is applied directly to the stator windings.
- Rotor currents are produced by induction.
- IM may be regarded as a transformer where electric power is transformed between rotor and stator with a change of frequency and flow of mechanical power.

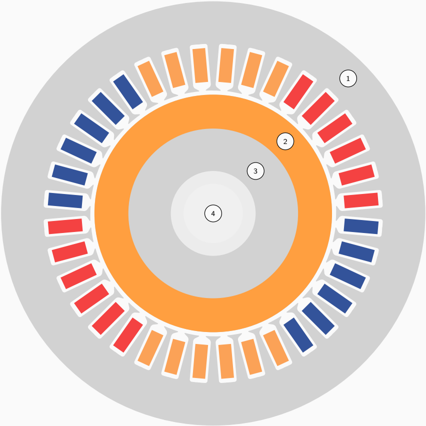
> Most common of all motors, is seldom used as a generator.

- Performance characteristics as a generator are unsatisfactory, although it has been found to be well suited for wind-power applications.

Rotating Magnetic Fields

- In IMs, stator windings are essentially same as those of a SynM.
- The rotor windings are electrically short-circuited and frequently have no external connections.
- currents are induced by transformer action from the stator winding.
- Here the rotor "windings" are actually solid aluminum bars which are cast into the slots in the rotor and which are shorted together by cast aluminum rings at each end of the rotor.
- This type of rotor construction results in IMs which are relatively inexpensive and highly reliable, factors contributing to their immense popularity and widespread application

Rotating Magnetic Fields



① **Stator Frame**
- Usually from silicon steel, or electrical steel.
- An iron alloy with silicon as the main additive element, instead of carbon.
- μ_r is 4,000 times that of vacuum, or 2-3 times of stainless steel.

② **Short circuited-rotor coils**
- End rings are attached to create a short-circuit for induced current to flow through.
- Made from aluminium or copper.

③ **Rotor**
- This is the magnetic part of the rotor.
- Made using electrical steel laminations.

④ **Rotor Shaft**
- Usually made from mild steel
- This is an iron alloy with carbon as its main alloying element.

Figure 32: A Schematic view of a Stator with a squirrel cage rotor.

Rotating Magnetic Fields

- Similar to SynM, the armature flux in the IM leads the rotor and produces torque.
- just as in a SynM, the rotor and stator fluxes rotate in synchronism with each other and that torque is related to the relative displacement between them.
- However, unlike a SynM, the rotor of an IM does not itself rotate synchronously.
- it is the "slipping" of the rotor with respect to the synchronous armature flux that gives rise to the induced rotor currents and hence the torque.
- Induction motors operate at speeds less than the synchronous mechanical speed.

- The armature winding of a DC generator is on the rotor with current conducted from it by means of **carbon brushes**.

Figure 33: Brushes are needed for conventional DC Machines to transfer electricity to the rotor.

Rotating Magnetic Fields

- The field winding is on the stator and is excited by direct current.
- The rotor is normally turned at constant speed by a source of mechanical power connected to the shaft.
- The air-gap flux distribution usually approximates a flat-topped wave, rather than the sine wave found in AC machines.
- Rotation of the coil generates a coil voltage which is a time function having the same waveform as the spatial flux-density distribution.

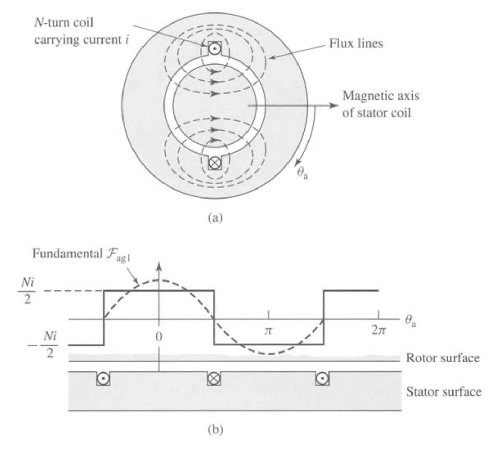
Rotating Magnetic Fields

- While the purpose is the generation of a direct voltage, the voltage induced in an individual armature coil is an alternating voltage, which must therefore be rectified.
- The output voltage of an ac machine can be rectified using external semiconductor rectifiers.
- This is in contrast to the conventional dc machine in which rectification is produced mechanically by means of a commutator, which is a cylinder formed of copper segments insulated from each other by mica or some other highly insulating material and mounted on, but insulated from, the rotor shaft.
- Stationary carbon brushes held against the commutator surface connect the winding to the external armature terminals.

> The need for commutation is the reason why the armature windings of dc machines are placed on the rotor.

Rotating Magnetic Fields

- Most armatures have distributed windings, i.e., windings which are spread over a number of slots around the air-gap periphery.
- The individual coils are interconnected so that the result is a magnetic field having the same number of poles as the field winding.
- The study of the magnetic fields of distributed windings can be approached by examining the magnetic field produced by a winding consisting of a single N-turn coil which spans 180 electrical degrees called a full-pitch coil



N-turn coil
carrying current i
Flux lines
Magnetic axis
of stator coil
θa
(a)
Fundamental $\mathcal{F}_{ag1}$
$\frac{Ni}{2}$
$-\frac{Ni}{2}$
0
π
2π
θa
Rotor surface
Stator surface
(b)

Figure 34

- Fourier analysis shows the air-gap Magneto-Motive Force (MMF) produced by a single coil such as the full-pitch coil of Fig. 4.19 consists of a fundamental space-harmonic component as well as a series of higher-order harmonic components.
- In AC machine design, serious efforts are made to distribute the coils making up the windings so as to minimise higher-order harmonic components and to produce an air-gap MMF wave which consists predominantly of the space-fundamental sinusoidal component.
- It is thus appropriate here to assume that this has been done and to focus our attention on the fundamental component.

Rotating Magnetic Fields

- The rectangular air-gap MMF wave of the concentrated two-pole, full-pitch coil can be resolved into a Fourier series comprising a fundamental component and a series of odd harmonics.
- The fundamental component ($\mathcal{F}_{\mathrm{ag1}}$) is:

$$\mathcal{F}_{\mathrm{ag1}} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta_{\mathrm{a}}$$

where θ_{a} is measured from the **B**-axis of the stator coil.

- It is a sinusoidal space wave of amplitude:

$$\left(\mathcal{F}_{\mathrm{ag1}}\right)_{\mathrm{peak}} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)$$

- with its peak aligned with the magnetic axis of the coil.

Rotating Magnetic Fields

- Electric machinery often contains systems of multiple windings, requiting careful bookkeeping to account for the flux contributions of the various windings. Although the details of such analysis are beyond the scope of this book, it is useful to discuss these effects in a qualitative fashion and to describe how they affect the basic machine inductances.

Air-Gap Space-Harmonic Flux

- although sin- gle distributed coils create air-gap flux with a significant amount of space-harmonic content, it is possible to distribute these windings so that the space-fundamental component is emphasized while the harmonic effects are greatly reduced.

> we can neglect harmonic effects and consider only space-fundamental fluxes in calculat- ing the self and mutual-inductance expressions

- often small, the space-harmonic components of air-gap flux do exist.
- Dc machines they are useful torque-producing fluxes and therefore can be counted as mutual flux between the rotor and stator windings.
- ac machines, however, they may generate time-harmonic voltages or asynchronously rotating flux waves.

Slot Leakage Flux

- there are flux components which cross the slot.
- t also forms a component of the leakage inductance of the winding producing it.

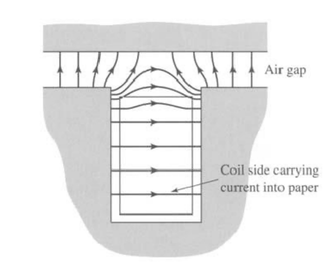


Figure 35: Flux created by a single coil side in a slot.

End-Turn Flux

- The magnetic field distribution created by end turns is extremely complex.
- In general these fluxes do not contribute to useful rotor-to-stator mutual flux, and thus they, too, contribute to leakage inductance.
- This self-inductance expression of Eq. B.26 must, in general, be modified by an additional term Ll, which represents the winding leakage inductance. This leakage inductance corresponds directly to the leakage inductance of a transformer winding as discussed in Chapter 1. Although the leakage inductance is usually difficult to calculate analytically and must be determined by approximate or empirical techniques, it plays an important role in machine performance.

- A set of current are applied to the 3-phase stator coils.

$$i_{\mathrm{RR'}} = I_{max} \sin(wt)\,,$$
$$i_{\mathrm{YY'}} = I_{max} \sin(w\,t - 120^\circ)\,,$$
$$i_{\mathrm{BB'}} = I_{max} \sin(w\,t - 240^\circ)\,.$$

- The magnetic flux densities generated from these currents are;

$$B_{\mathrm{RR'}} = B_{max} \sin(wt)\,,$$
$$B_{\mathrm{YY'}} = B_{max} \sin(w\,t - 120^\circ)\,,$$
$$B_{\mathrm{BB'}} = B_{max} \sin(w\,t - 240^\circ)\,.$$

Rotating Magnetic Fields

- Let's add up all these generated magnetic flux densities.

$$\mathbf{B}_{\text{net}}(t) = \mathbf{B}_{\mathbf{RR'}} + \mathbf{B}_{\mathbf{YY'}} + \mathbf{B}_{\mathbf{BB'}},$$
$$= B_{\text{max}} \sin(wt) + B_{\text{max}} \sin(w\,t - 120^\circ) + B_{\text{max}} \sin(w\,t - 240^\circ)\,.$$

- Each of magnetic field can now be broken to its $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ components.

$$\mathbf{B}_{\text{net}}(t) = B_{\text{max}} \sin(wt)\,\hat{\mathbf{x}} - \left[\frac{1}{2} B_{\text{max}} \sin(w\,t - 120^\circ)\right] \hat{\mathbf{x}}$$
$$+ \left[\frac{\sqrt{3}}{2} B_{\text{max}} \sin(w\,t - 240^\circ)\right] \hat{\mathbf{y}} + \left[\frac{\sqrt{3}}{2} B_{\text{max}} \sin(w\,t - 240^\circ)\right] \hat{\mathbf{y}}$$
$$+ \left[\frac{\sqrt{3}}{2} B_{\text{max}} \sin(w\,t - 240^\circ)\right] \hat{\mathbf{y}}.$$

Rotating Magnetic Fields

- Simplification presents us with the following:

$$\mathbf{B}_{\text{net}}(t) = \frac{3}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} - \cos(wt)\,\hat{\mathbf{y}}\right] \quad \blacksquare$$

For an AC drive with n phases, the net magnetic vector would be:

$$\mathbf{B}_{\text{net}}(t) = \frac{n}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} - \cos(wt)\,\hat{\mathbf{y}}\right] \quad \blacksquare$$

Rotating Magnetic Fields

- If the current in any two of the three coils is swapped, the direction of the magnetic field's rotation will be reversed.
- This means that it is possible to reverse the direction of rotation of an AC motor just by switching the supply connections on any of two of the three coils.
- For a 3-phase motor, the rotation of $\mathbf{B}_{\text{net}}(t)$ is:
  - Clockwise:

$$\mathbf{B}_{\text{net}}(t) = \frac{3}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} - \cos(wt)\,\hat{\mathbf{y}}\right].$$

  - Counter-clockwise:

$$\mathbf{B}_{\text{net}}(t) = \frac{3}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} + \cos(wt)\,\hat{\mathbf{y}}\right].$$

Rotating Magnetic Fields

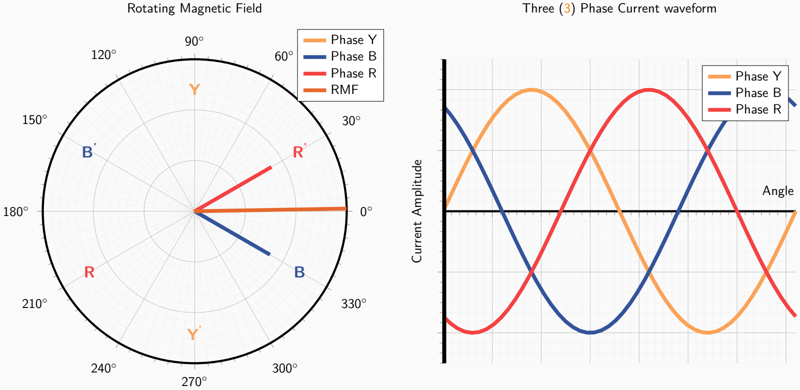


Figure 36: A three phase rotating magnetic field.

Induction Drives

Table of Contents

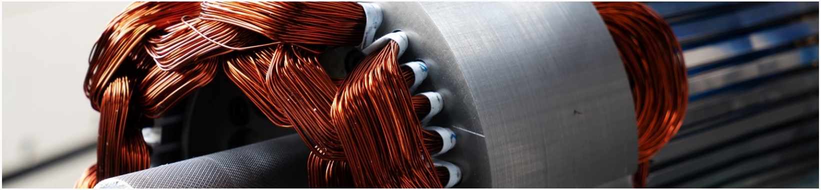
Learning Outcomes

(LO1) IM Operation Principles,

(LO2) Rotor Types,

(LO3) Equivalent Circuit,

(LO4) Starting Methods.



- An induction machine (IM) or asynchronous drive is an AC electric drive in which the torque produced by the rotor is obtained by **electromagnetic induction** from the magnetic field of the stator winding.
- 3-phase squirrel-cage IMs are widely used as industrial drives because they are self-starting, reliable and economical.
- 1-phase IMs are used extensively for smaller loads, such as household appliances like fans[4].

[4] We will cover this topic in detail in Part 3.

Induction Drives

- Three-phase IMs are the most common and frequently encountered machines/drives in industry.

Advantages

- Simple design, rugged, low-price, easy maintenance,
- Wide range of power ratings: from few hundred watts to 10 MW.
- Can run essentially as constant speed from zero to full load.
- Speed is power source frequency dependent.

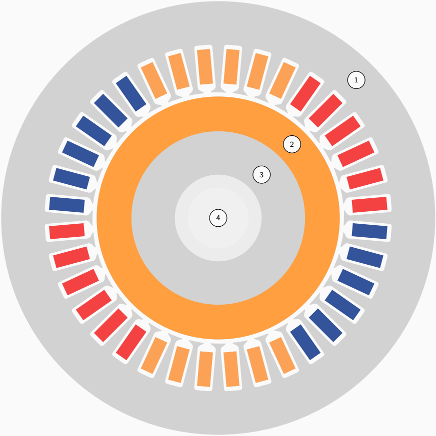
Disadvantages

- Not easy to have variable speed control.
- Requires a variable-frequency drive for optimal speed control.

- Stator is the stationary part of induction motor.
- A stator winding is placed in the stator of induction motor and the three phase supply is given to it.
- The stator consists of three (3) main parts:
  - **Stator frame:** consists of laminations of silicon steel, usually with a thickness of about 0.5 millimetre.
  - **Stator core:** segments of electrical sheet steel, usually 2–3% silicon, 0.35 mm thick, cold rolled and non-oriented[5].
  - **Stator winding:** Copper or sometimes aluminium, but should be thicker to hold a similar load securely due higher resistivity.
    - The copper winding allows for a smaller motor.
  - Windings are typically insulated with mica and glass laminate.

[5] This means the sheet has no magnetic bias to any direction.

Induction Drives



① **Stator Frame**
- Usually from silicon steel, or electrical steel.
- An iron alloy with silicon as the main additive element, instead of carbon.
- μ_r is 4,000 times that of vacuum, or 2-3 times of stainless steel.

② **Short circuited-rotor coils**
- End rings are attached to create a short-circuit for induced current to flow through.
- Made from aluminium or copper.

③ **Rotor**
- This is the magnetic part of the rotor.
- Made using electrical steel laminations.

④ **Rotor Shaft**
- Usually made from mild steel
- This is an iron alloy with carbon as its main alloying element.

Figure 37: A standard stator geometry, produced as laminated sheets to reduce eddy currents.

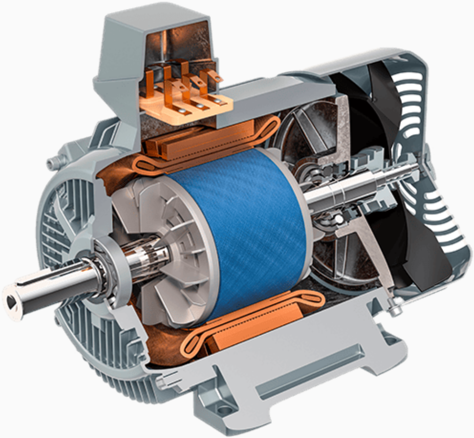


Figure 38: A sliced up view of a squirrel cage induction motor (SCIM).

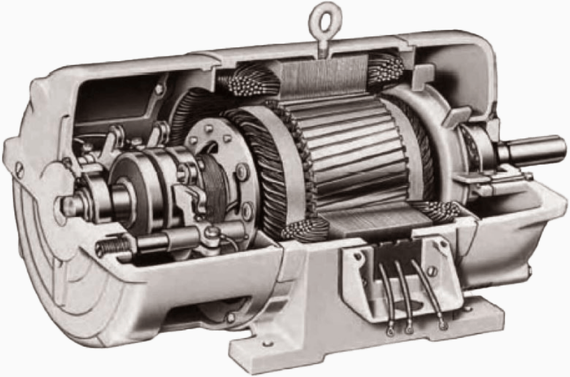


Figure 39: A sliced up view of a wound rotor induction motor (WRIM).

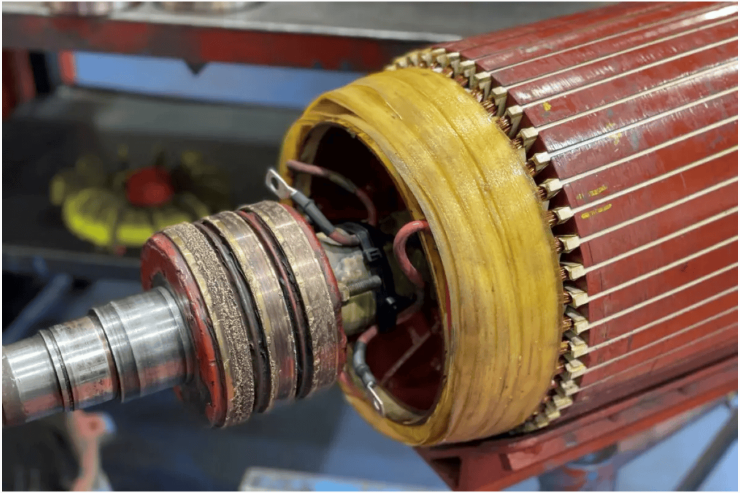


Figure 40: A close-up of a wound rotor induction motor (WRIM).

Induction Drives

Property	SRIM	SCIM
Construction	Complicated	Simple
Rotor Resistance	Can be added	Permanent
Starting Torque	Can be high	Low and can't be changed
Maintenance	Frequent	Minimal
Build Cost	Complicated	Comparably simple
Industrial Use	Rare	Widely used

- The operation principle of a 3-phase IM is based on the ability of a 3-phase winding to create a rotating magnetic field.
- The speed of rotation of this field (n_s) is directly proportional to the AC frequency (f_s) and inversely proportional to the number of poles (p) of the 3-phase winding.

$$n_s = 120\frac{f_s}{p} \quad \mathrm{min}^{-1},$$

where:

- n_s is the stator RMF speed (min^{-1}),
- f_s is the AC frequency (Hz),
- p is the number of poles.

- Place a short-circuited conductor inside the rotating magnetic field.
  - According to **Faraday's law of induction**:

> A changing of the magnetic field will lead to an electromotive force (EMF) in the conductor.

- This EMF will produce a current in the conductor.
  - According to **Ampere's law**:

> In a magnetic field there will be a conductor with a current on which force will act.

- These laws causes the loop to start rotating.
- In practice, instead of a short-circuited loop, IMs use a short-circuited rotor resembling a squirrel-cage in construction[6].
  - A squirrel-cage rotor consists of rods shorted by the end rings.
- 3-phase AC, passing through the stator windings, creates a RMF.
- The stator RMF induces current in the rotor bars, causing the rotor start to rotate.

> This rotation is caused by magnitude change in the magnetic field being **different** in **different pairs of rods**, which is due to rotor bars being in different locations relative to the field.

[6] These rotors are known as squirrel-cage induction motor (SCIM).

- The change in current in the rods will change with time.
- The rotor rods are inclined relative to the axis of rotation.
  - This is called **skewing**.
- This is done in order to reduce the higher harmonics of the EMF and prevent torque fluctuation.

> If the rods perfectly aligned the axis of rotation, then a **pulsating magnetic field** would arise in them.

> These pulses would create energy losses in the drive, and create torque harmonics[7].

[7] Torque harmonics are also generated due to the different phase windings interaction and they can be reduced by applying a stator teeth design

- A distinctive feature of an IM is the rotor speed (n_r) being less than the synchronous speed of stator RMF (n_s).
- The EMF in the rotor winding rods is induced only when the rotation speed is unequal ($n_\mathrm{s} \neq n_\mathrm{r}$).
- Rotor lag from the stator RMF is characterised by (s), called the slip, and defined as:

$$\mathrm{slip} = \frac{n_\mathrm{s} - n_\mathrm{r}}{n_\mathrm{s}}$$

The slip value range is from 0 to 1 (or 0% - 100%).

- Consider where the rotor frequency is **equal** to the stator frequency.
- Here, the relative magnetic field of the rotor will be constant,
  - This stops EMF generation, and in turn, no current will flow.
- If no current flows, this implies no acting force on the rotor.
  - The rotor will start to slow down.
- As rotor slows down, the stator RMF will begin acting on the rotor bars, inducing current and increasing the force.

> Now, there is a relative magnetic field that is non-constant.

> The rotor of an IM will never reach the speed of the stator RMF.

- The rotor will rotate at a speed slightly less than n_s.

- Slip in an IM can vary in the range from 0 to 1 (i.e., 0% to 100%).
  - If s is 0 (i.e., 0%), it is in idling mode, i.e., rotor practically does not experience the load torque.
  - if $s = 1$ (i.e., %100) it is in short circuit mode, i.e., rotor is stationary ($n_r = 0$).

Slip depends on the mechanical load on the shaft.

Higher loads means higher slip.

- The slip of rated load is called the rated (nominal) slip.

For low and medium power IMs, the rated slip varies from 8% to 2%.

Induction Drives

- As rotor turns, **discontinuities** on the rotor and stator surface disrupt the magnetic flux path of the motor.
- The flux path variation shows up in the form of harmonics that affect performance.

> The difference between stator and rotor slots has a significant impact on the harmonics.

- The motor may be noisy, or there may be stray losses lowering the torque during starting or acceleration.

> The stator-rotor slot difference is why a motor winding that is designed for a different speed can have problems.

Induction Drives

- Happens when stator and rotor slots are equal or integer ratio.

> Causes the motor to lock and stops from starting spinning.

- If happens, alignment forces are produced between stator and rotor.
- As a result, the **alignment torque overpowers the accelerating torque**.
- Therefore, a **locking** is created between the stator and rotor teeth.

> This condition is known as cogging or magnetic locking.

- This due to creating a minimum reluctance path when the stator and the rotor teeth face each other.

- We have seen that IMs operate on the principle of induced currents.
- There are two (2) magnetic fields: one from the rotor and the stator.

> The rotor field is **induced** by the stator field.

- Effectively, IM is a rotating transformer.
- The stator is a transformer primary and creates the initial field:
  - induces voltages and currents in the secondary rotor winding.
- The fundamental differences from a stationary transformer are:
  - The secondary part (i.e., rotor) rotates.
- There is an airgap, therefore more MMF[8] is needed for a given **B**.
- The secondary voltage and frequency depend on rotor speed.

[8] Magneto-motive Force

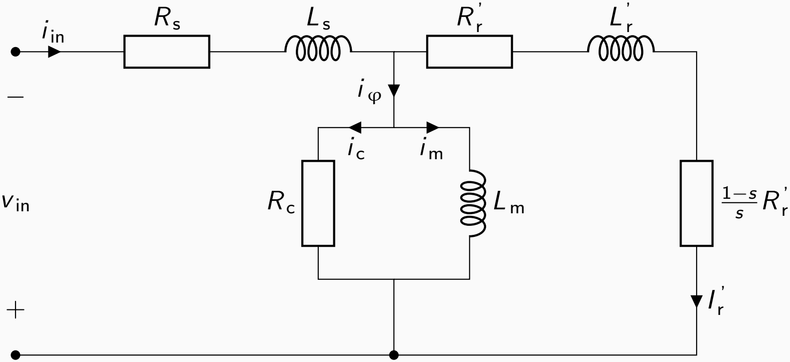


Figure 41: The equivalent circuit of an induction motor in **steady state**.

Symbol	Definition	Unit
$\hat{V}_{\text{in}}$	Stator line-to-neutral (i.e., phase) terminal voltage	V
$\hat{V}_{\text{in}}$	Stator current	A
$\hat{I}_{\varphi}$	Excitation current to create magnetic flux	A
$\hat{I}_{\text{c}}$	Core loss current	A
$\hat{I}_{\text{m}}$	Magnetising current	A
R_{s} & R_{r}	Stator and rotor resistance	Ω
L_{s} & L_{r}	Stator and rotor leakage inductance	H
R_{c}	Core loss resistance	Ω
L_{m}	Magnetising inductance	H

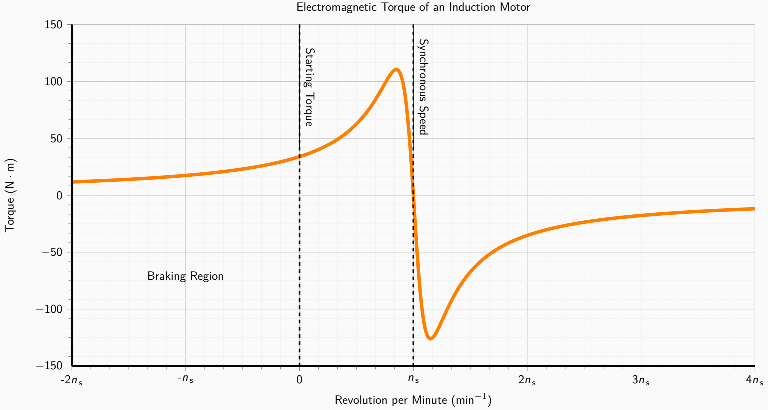


Figure 42: The steady-state torque vs. rpm graph of an induction drive.

A 3-phase IM is inherently **self starting**.

- It can be plugged to the grid without any need of circuits.
- At the time of starting, the starting current is quite high.
- The purpose of a starter is not to just start the motor, but also:
  - to reduce heavy starting currents (i.e., inrush current[9])
  - to provide overload and under voltage protection.
- The 3-phase IM can start by connecting to full voltage of the supply.
- Torque is proportional to the square of the applied voltage.
- Thus, a greater torque is exerted when started on full voltage than on reduced voltage.

[9] The maximal instantaneous current drawn by a device when first turned on.

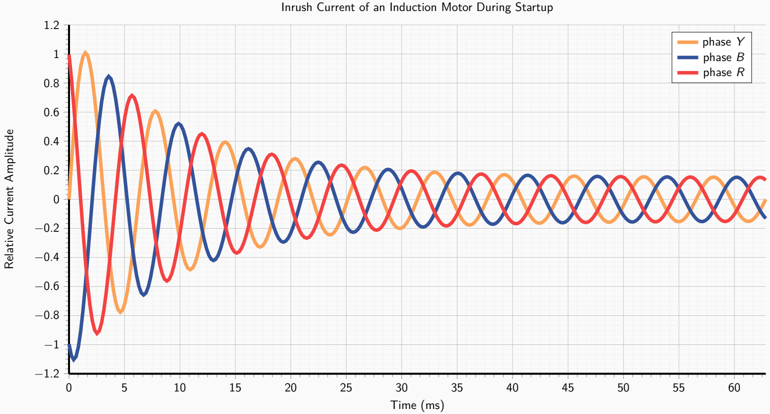


Figure 43: The waveforms encountered when a heavy inductive load is plugged into the grid. This behaviour is known as inrush current, which is a prominent behaviour of IMs and transformers.

- Compared to other methods, DoL is simple and economical.
- Here, the starter is directly connected to the supply voltage.

Small motors up to 5 kW can be started without causing supply voltage fluctuation.

This method does not provide control over the **inrush current**.

- This method also provides the following
  - Short circuit protection (via circuit breaker or fuses)
  - Overload protection (via an overload relay)
  - Under voltage protection[10],

[10] A combination of inrush current and high starting torque creates an excessive voltage drop in the line which supplies power to the motor.

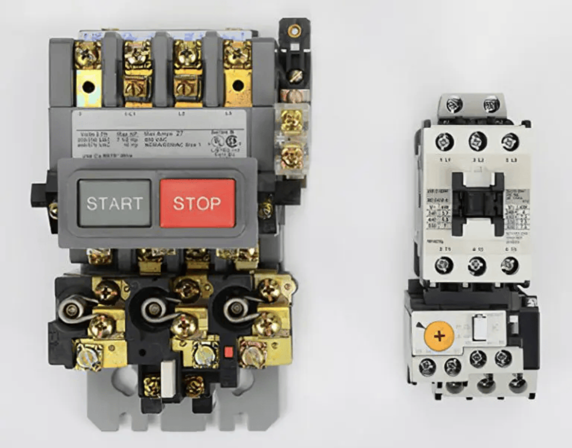


Figure 44: Types of Direct on-Line Starter which can be found in industry.

- The auto transformer is used in both the type of the connections:
  - Star connection (Y),
  - Delta connection (Δ).

Auto transformer is used to limit the starting current of the IM[11].

- The starter is connected to one particular tapping to obtain the most suitable starting voltage.
- A switch is used to connect the auto transformer in the circuit for starting.

[11] i.e., Inrush current

- The primary of the auto transformer is connected to the supply line,
  - The motor is connected to the secondary of the auto transformer.
- When the motor picks up the speed (i.e., 80 % of its rated value), the switch is quickly moved to the **run** position.
- Therefore, the auto transformer is disconnected from the circuit,
  - The motor is directly connected to the line and achieve its full rated voltage.

Figure 45: Types of Direct on-Line Starter which can be found in industry.

- A very common and widely used starter among all the methods.
- Here, the motor runs at delta (Δ) connected stator windings.
- Used to reduce the starting current of the motor,
    - without using any external device or apparatus.

> This is a big advantage of a (Y/Δ) starter, as it typically has around 3^{rd} of the inrush current compared to a DoL starter.

- The starter uses a **triple pole double throw** switch.
- This switch changes stator winding from star to delta.
- During starting condition stator winding is connected in star,
    - During operation it is in delta connection.

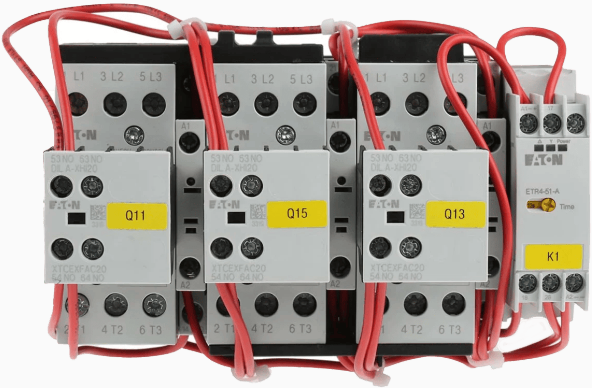


Figure 46: A close-up of Eaton, Star Delta, 22 kW, 110 V AC, 3 Phase, IP55, Star Delta Starter.

- In **Slip-ring Induction Motor** starter, the full supply voltage is connected across the starter.
- Full starting resistance is connected,
  - and supply current to the stator is reduced.
- The rotor begins to rotate, and the rotor resistances are gradually cut out as the speed of the motor increases.
- When the motor is running at its rated full load speed,
  - the starting resistances are cut out completely, and the slip rings are short-circuited.

Figure 47: An ABB Slip-ring Starter used in mining operations.

Linear Induction Motor

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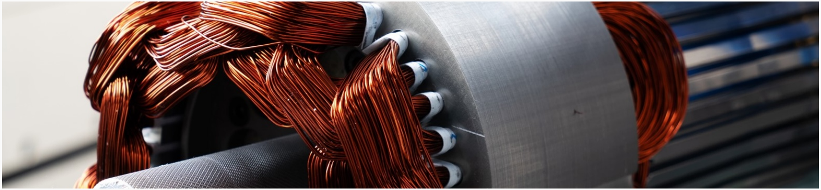
Learning Outcomes

(LO1) Definition of A Linear Machine,

(LO2) Construction,

(LO3) Operation Principles,

(LO4) Magnetic Levitation.



Linear Induction Motor

- A linear induction motor (LIM) is an AC motor with same general principles as other IMs, but produces motion in a straight line.
- LIMs have a **finite** primary or secondary length,
  - which, unfortunately, generate end-effects.
  - whereas traditional IMs are arranged in an endless loop (i.e., circular).
- Despite their name, not all LIMs produce linear motion.
  - Some are employed for generating rotations of large diameters where the use of a continuous primary would be very expensive.

> LIMs are often used where contactless force is required and low maintenance is desirable.

Linear Induction Motor

- LIMs usually run on a **3-phase power supply** and can support high speeds.
- There are end-effects that reduce the motor's force.

> LIMs are less efficient than normal rotary motors for any given required force output.

- LIMs, unlike their rotary counterparts, can give a levitation effect.
- Their practical uses include magnetic levitation, linear propulsion, and linear actuators.

Linear Induction Motor

- LIMs are similar to a 3-phase induction motor.
- If you were to get a rotary IM and slice up from top to its central point, and then bend it into a straight line, you would get a LIM.
  - The bended stator forms the primary of the LIM housing the field system,
  - The bended rotor forms the secondary consisting of flat aluminium conductors with ferromagnetic core.
    - This material choice is for effective flux linkage.

Linear Induction Motor

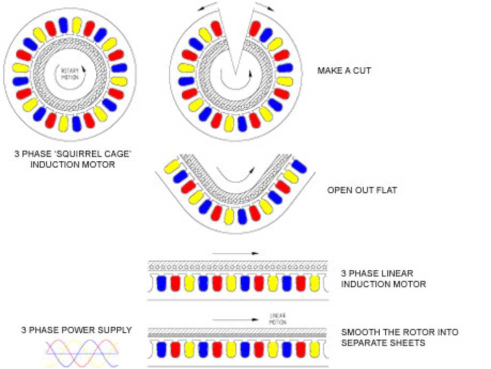


Figure 48: Relation between a rotary and linear induction motor.

Linear Induction Motor

- In this design, the force is produced by a linearly moving magnetic field acting on conductors in the field.
- Any conductor that is placed in this field will have eddy currents induced in it thus creating an opposing magnetic field.
  - This is in accordance with Lenz's law.
- The two opposing fields will repel each other, creating motion as the magnetic field sweeps through the metal.

$$n_s = 120\frac{f_s}{p} \quad \mathrm{min}^{-1},$$

where f_s is supply frequency in Hz, p is the number of poles, and n_s is the magnetic field synchronous speed in min^{-1}.

- There is another variant of LIM for increasing efficiency known as the double sided linear induction motor (DLIM).
- Has primary winding on both sides of secondary (i.e., two stators).
  - This allows for more effective utilization of the induced flux from both sides.

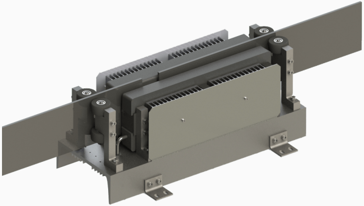


Figure 49: A double sided linear induction motor, rated 5.5KW.

- Thrust is generated as a **function of slip**,
  - similar to how torque is generated on an rotary induction drive.
- The torque generated by LIMs is similar to conventional induction motors; the drive forces show a roughly similar characteristic shape relative to slip, albeit significantly affected by end effects.
- An approach to modelling it could be expressed as:

$$F = m\frac{I^2 R}{Vs} \quad \mathrm{N}$$

Linear Induction Motor

- Goodness factor is a metric developed by **Eric Laithwaite** to determine the **goodness** of an electric motor. Using it he was able to develop efficient magnetic levitation induction motors.

$$G = \frac{\omega}{R\mathcal{R}} = \frac{w\sigma\mu A_e A_m}{l_m l_e}$$

where:

G	Goodness factor (factors above 1 are likely to be efficient),
A_m, A_e	cross sections of the magnetic and electric circuit
l_m, l_e	lengths of the magnetic and electric circuits,
μ	permeability of the core,
w	angular frequency the motor is driven at
σ	conductor conductivity,

Linear Induction Motor

- LIMs are used in situations requiring linear motion, including:
  - overhead travelling cranes,
  - belt-less conveyors, i.e., moving sheet metal.
- They are also used as source of motion in the latest generation of high-speed maglev trains, which promise safe travel at very high speeds,
  - with most research on maglev trains has been carried out in Japan and Germany.

Linear Induction Motor

- Magnetic levitation, maglev, or magnetic suspension is a method which an object is suspended with no support other than magnetic fields.
- Magnetic force is used to counteract the effects of the gravitational and any other accelerations.
- The two primary issues involved in magnetic levitation are

Lift Force To provide an upward force sufficient to counteract gravity

Stability To ensure the system does not spontaneously slide or flip into a configuration where the lift is neutralized.

- Magnetic levitation is used for maglev trains, contactless melting, magnetic bearings and for product display purposes.

Linear Induction Motor

- Magnetic materials are able to attract or repel each other with its strength dependent on the magnetic field and the area of the magnets.
  - For example, a dipole magnet positioned in a magnetic field.
- Essentially all types of magnets can be used to generate lift for magnetic levitation;
  - permanent magnets, electromagnets, ferromagnetism, diamagnetism, superconducting magnets and magnetism due to induced currents in conductors.
- To calculate the amount of lift, a magnetic pressure can be defined.
- For example, the magnetic pressure of a magnetic field on a superconductor can be calculated by:

$$P_{mag} = \frac{B}{2\mu} \qquad \text{Pa.}$$

Linear Induction Motor

Figure 50: A live frog levitates inside a *32* mm diameter vertical bore of a Bitter solenoid in a magnetic field of about 16 teslas.

Figure 51: A magnetically levitated (maglev) train developed by Central Japan Railways Co. operates a test run on May 11, 2010 in Tsuru, Japan.

Figure 52: High-speed maglev train, Shanghai, China. It can reach to a cruising speed was 431 km/h within a track length of 30 km.

Single Phase Induction Drivess

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Learning Outcomes

(LO1) How to Rotate a 1-phase Motor,

(LO2) Types of Starting Methods,

(LO3) Shaded Pole Motor,

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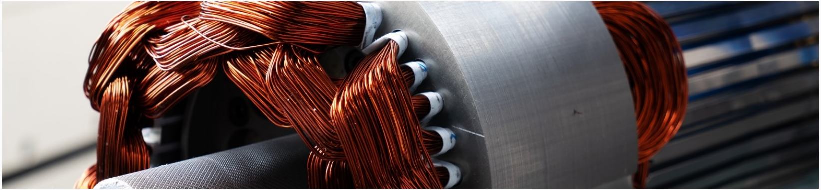


Single Phase Induction Drivess

- A three-phase motor may be run from a single-phase power source.

However, the motor will not start on its own.

- It is possible to spin the motor in either direction, coming up to speed in a few seconds.

This is done by giving the motor a push.

- It will only develop *2/3* of the 3 phase power rating because **one winding is not used**.

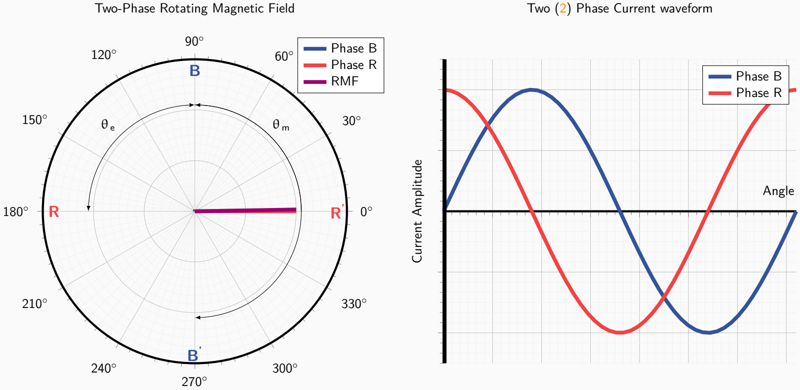


Figure 53: An animation showing two-phase RMF. This is the smallest number of phases needed to create a rotating field. Here, θ_e is the electrical angle, and θ_m is the mechanical angle.

Single Phase Induction Drivess

- A single coil of a single-phase IM cannot produce a RMF.

> It creates a pulsating field reaching maximum intensity at 0° and 180° electrical angles.

- Another view is the single-coil excited by a single-phase current produces two counter-RMFs, coinciding twice at 90° and 270°.
- When the phasors rotate to 0° and 180° they cancel.
- At 45° and -45° they are additive along the $\hat{\mathbf{y}}$ and cancel along $\hat{\mathbf{x}}$.

> The sum of these two phasors is a phasor **stationary in space**, but **alternating polarity in time**.

- Therefore, no starting torque can be developed.

Single Phase Induction Drivess

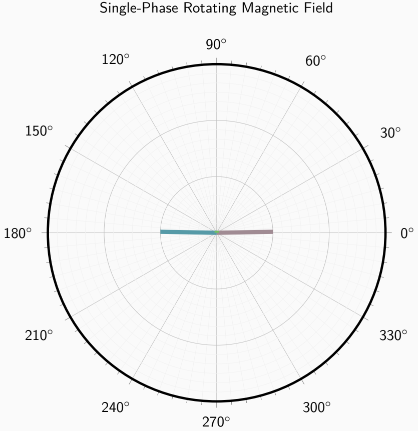


Figure 54: An animation showing the one phase rotating magnetic field. Here the resultant magnetic field pulsates between 90 and 270 degrees. This is not enough to create a rotating motion.

Single Phase Induction Drivess

- If rotor is rotated forward at a bit less than the synchronous speed, It will develop maximum torque at 10% slip.

> Less torque will be developed above or below 10% slip.

- Little torque other than a double frequency ripple is developed from the counter-rotating phasor.
  - The single-phase coil will develop torque once the rotor starts.
- If rotor starts in reverse, it will develop torque as it nears the speed of the backward rotating phasor.
- Construction wise, 1-phase rotor is similar to 3-phase.

- A way to solve the single phase problem is to build a 2-phase motor.
  - Achieved by deriving the 2-phase power from single-phase.
- This requires a motor with:
  - Two windings spaced apart 90° electrical,
  - Fed with two phases of current displaced 90° in time.
- This is called a **permanent-split capacitor** motor.
  - It suffers increased current magnitude and backward time shift as the motor speeds up, with torque pulsations at full speed.

> The solution is keeping the capacitor (impedance) small as possible to minimize losses due to reactive power and low power factor.

Single Phase Induction Drivess

- This connection type works well up to 200 watts.
- The direction of the motor is easily reversed by switching the capacitor in series with the other winding.
- This type of motor can be adapted for use as a servo motor.
- Single-phase IMs may have coils embedded into the stator for larger size motors.

Advantages

- No centrifugal switch is required.
- High efficiency (Relative to other single-phase) .
- As the capacitor is connected permanently in the circuit, the power factor is high.
- It has a higher pull-out torque[12].

Disadvantages

- Paper capacitor is required, which increases the cost and the size of the motor.
- It has low starting torque, less than full load torque.

[12] The maximum torque an induction motor take with out going to instability region.

Single Phase Induction Drivess

- Uses a large capacitor to start a single-phase IM via auxiliary winding.
  - The capacitor is switched out by a centrifugal switch once the motor is up to speed.
- The auxiliary winding may be many more turns of heavier wire than used in a resistance split-phase motor to mitigate excessive temperature rise.
- This results in higher starting torque.
- This motor configuration works well in multi-kilowatt applications.
  - i.e., heavy loads like air conditioning compressors.
- A starting capacitor may be a double-anode non-polar electrolytic capacitor which could be two series-connected polarized electrolytic capacitors.

- An electrical switch found in single-phase and some shaded pole IMs.
- Provides a switching operation when specified speed is generated.
- Works on the concept of centrifugal force.

> As its operation is identical to a centrifugal clutch used in vehicles, the switch is generally known as a clutch.

- Single-phase IMs have centrifugal switches, attached to the shaft.
  - When the motor is off and motionless, the switch is closed.
- When the IM is on, the switch drives electricity to the capacitor and the extra coil winding, increasing its starting torque.
- As the IM speeds up, the switch opens, as the IM no longer needs a boost.

Single Phase Induction Drivess

Figure 55: A centrifugal switch used in single phase or shaded pole induction motor. Due to their similarity to a clutch pedal, they are also known as clutch switch.

Single Phase Induction Drivess

- Similar to capacitor-start, uses a relatively large capacitor to produce high starting torque.
  - Difference is to have a smaller value capacitor in place after starting to improve running performance while not drawing excessive current.
- The additional complexity of the capacitor-run motor is justified for larger size motors.
- Such AC rated electrolytic capacitors have such high losses that they can only be used for intermittent duty (1 second on, 60 seconds off) like motor starting.
- A capacitor for motor running must not be of electrolytic construction, but a lower loss polymer type.

Single Phase Induction Drivess

- Achieved with an auxiliary winding of much fewer turns and smaller wire being placed at 90° mechanical to the main winding.
- With lower inductance and higher resistance, the current will experience less phase shift than the main winding.
- About 30° of phase difference may be obtained. This coil produces a moderate starting torque, which is disconnected by a centrifugal switch at 75% of synchronous speed.
- This simple (no capacitor) arrangement serves well for motors up to 250 watts, driving easily started loads.

> The current density in the auxiliary winding is so high during starting that the consequent rapid temperature rise precludes frequent restarting or slow starting loads.

Single Phase Induction Drivess

Induction motors are inefficient at less than full load.

- This inefficiency correlates with a low power factor.

$$0.7 < \cos\varphi < 0.9$$

- Caused by the magnetizing current required by the stator.
- This fixed current is a larger proportion of total motor current as the motor load is decreased.
- At light load, the full magnetizing current is not required.
- It could be reduced by decreasing the applied voltage, improving the power factor and efficiency.
- The power factor corrector senses power factor, and decreases motor voltage, thus restoring a higher power factor and decreasing losses.

Single Phase Induction Drivess

- Not self-starting without an auxiliary winding driven near $\theta_{\mathsf{e}} = 90^\circ$.

> Once started the auxiliary winding is optional.

- Permanent split capacitor IM has a capacitor in series with it during starting and running.
- Capacitor-start IM only has a capacitor in series during starting.
- Capacitor-run IM typically has a large non-polarized electrolytic capacitor in series for starting, afterwards, a smaller non-electrolytic capacitor during running.
- Resistance split-phase IM develops a phase difference versus the main winding during starting by virtue of the difference in resistance.

Single Phase Induction Drivess

- The shaded pole induction motor is a self-starting single-phase IM where one of the pole is shaded by the copper ring.

> The copper ring is also called the shaded ring.

- This copper ring act as a **secondary winding for the motor**.
- The shaded pole motor rotates only in one particular direction, and the reverse movement of the motor is not possible.
- This motor is characterised by **high losses** and a **considerably low power factor**.

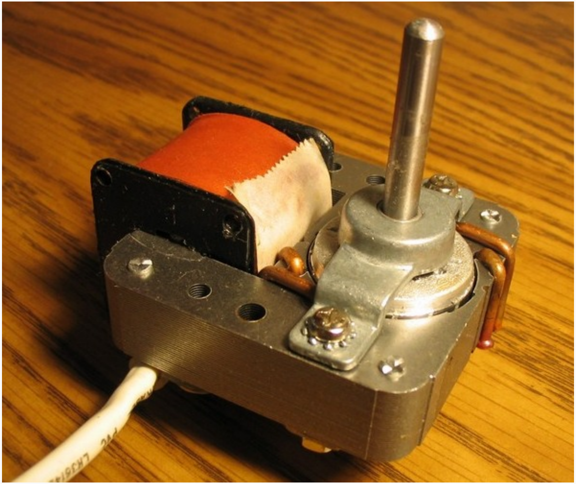


Figure 56: A Shaded pole induction motor.

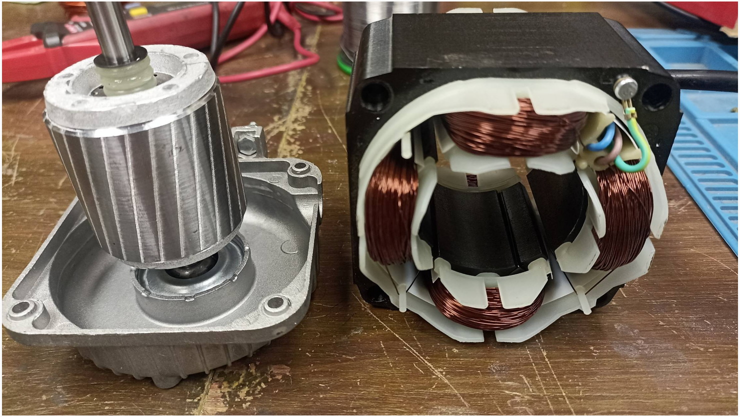


Figure 57: A Shaded pole induction motor.

- The stator has salient poles.
  - This means the poles are projected **towards** the rotor.
- Each pole of the motor is excited by its exciting coil.
- The copper rings shade the loops.
  - The loops are known as the shading coil.
- The stator is laminated.
- The slot is constructed at some distance apart from the edge of the poles.
  - The short-circuited copper coil is placed in this slot.
- The part which is covered with the copper ring is called the shaded part and which are not covered by the rings are called unshaded part.

Single Phase Induction Drivess

- Similar to poly-phase, squirrel cage is used.
- The bars of the rotor is skewed at an angle of 60°.
  - The skew is done for obtaining the better starting torque.
- The shaded pole induction motor does not have any centrifugal switch.

- When the supply is connected, Alternating flux is induced in the stator core.
- The small portion of the linkage flux is generated with the shaded coil.
  - Caused by being **short-circuited**.
- The variation in the flux induces the voltage inside the ring, causing circulating currents .
- The circulating current develops the flux in the ring which opposes the main flux of the motor.
- The flux induces in the shaded and the unshaded portions have a alertphase difference.

Single Phase Induction Drivess

- The main and the shaded ring flux have a space displacement of $\theta_{\mathrm{m}} = 90^{\circ}$.
- As there is **time** and **space** displacement between the two fluxes, RMF is created.
- This develops the starting torque in the motor.
- The field rotates from the unshaded portion to the shaded portion of the motor.

Single Phase Induction Drivess

- Suitable for small devices like relays and fans because of its low cost and easy starting.
  - exhaust fans, hair dryers, table fans, air conditioning, refrigeration, cooling fans, record players, photocopiers ....
- This type of motor is used to drive the devices which require low starting torque.

Synchronous Machines

Table of Contents

Learning Outcomes

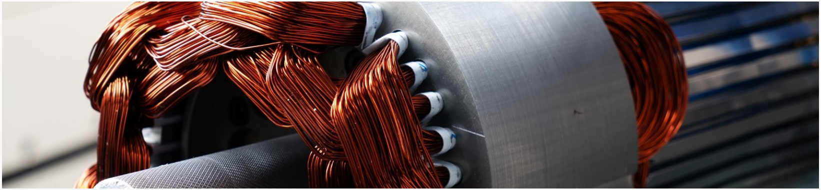
(LO1) An Overview of SynM,

(LO2) Understanding Construction Types,

(LO3) Studying SynM Characteristics,

(LO4) Analysing Salient-Pole Performance,

(LO5) Reading a Nameplate.



Synchronous Machines

- A SynM is a motor/generator where rotational speed of the stator magnetic field (n_s) is **equal** to the rotor magnetic field (n_r).

> The rotation of the shaft is synchronised with the frequency of the supply current.

- SynM are predominantly used as generators in industry.
- To achieve synchronicity, the rotor is constructed considerably different compared to an induction rotor.

The main electromagnetic parts of a SynM are:

- The primary with a poly-phase (in most cases three-phase) winding, mostly on the stator.

> This is almost identical to an induction drive stator.

- The excitation aspect happens to the rotor.
  - This is done either by a DC field winding or PM excitation.
- In industry there are two (2) common synchronous rotors:
  - Salient Pole,
  - Round Rotor (i.e., Turbo Rotor)

Synchronous Machines

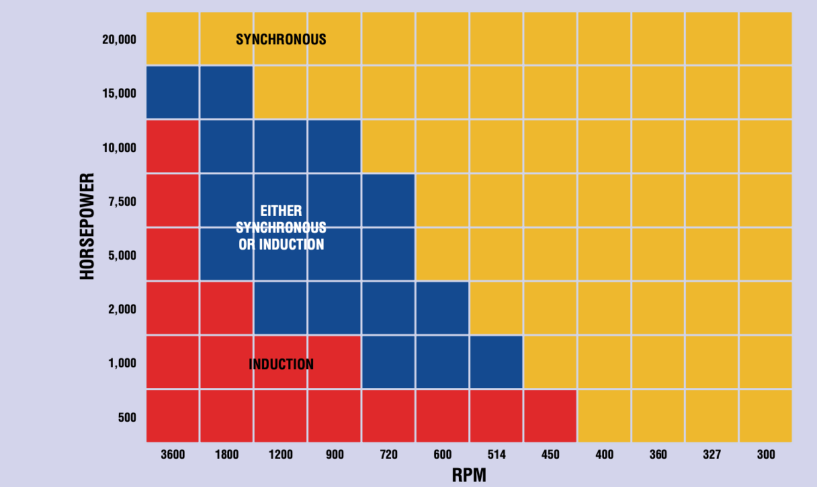


Figure 58: An Approximate guide to choosing machines.

- The rotor consist of large number of projected poles (salient poles).
  - These salient poles are mounted on a magnetic wheel.
  - The projected poles are made up from laminations of steel.
- The rotor winding is provided on these poles and it is supported by pole shoes.

> Salient pole rotors have large diameter and shorter axial length.

- They are generally used in lower speed electrical machines.
  - i.e., $n_s < 1500$ rpm.

- As the rotor speed is lower, more number of poles are required to attain the required frequency. ($p \propto 1/n$)
  - Typically number of salient poles is between 4 to 60.
- Flux distribution is relatively poor compared to round rotor, hence the generated EMF waveform is not as good as cylindrical rotor
- Salient pole rotors generally need damper windings to prevent rotor oscillations during operation.
- They are mostly used in hydro power plants.

Figure 59: The installation of a 532 MVA synchronous generator with a rated speed of 112.5 rpm at the Revelstoke Dam in Canada

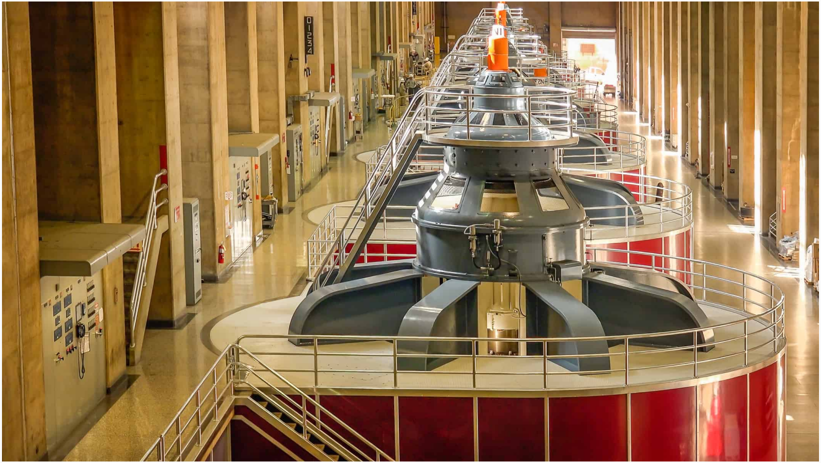


Figure 60: A view of salient pole generators used in a hydro-dam.

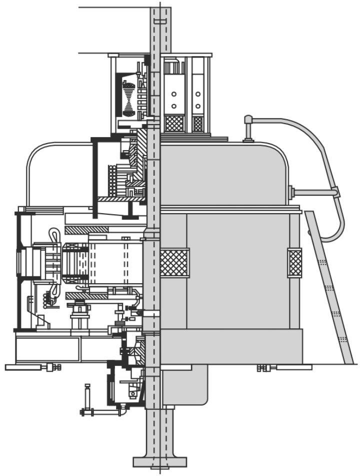


Figure 61: Salient pole type alternator mounted vertically at hydro-electric power plant.

- Round rotors are cylindrical in shape having parallel slots on it to place rotor windings.
- The rotor is made from slid forgings of high-grade Ni-Cr-Mo steel.

> High nickel and chromium contents ensure high strength and toughness, while other components, notably silicon and other impurities, are kept low with the result that magnetic properties remain good [11].

- DC is supplied through slip rings for excitation.
- They are also called as drum rotor.
- They are small in diameter but having long axial length.
- Cylindrical rotors are used in high speed, around 1500 - 3000 rpm.

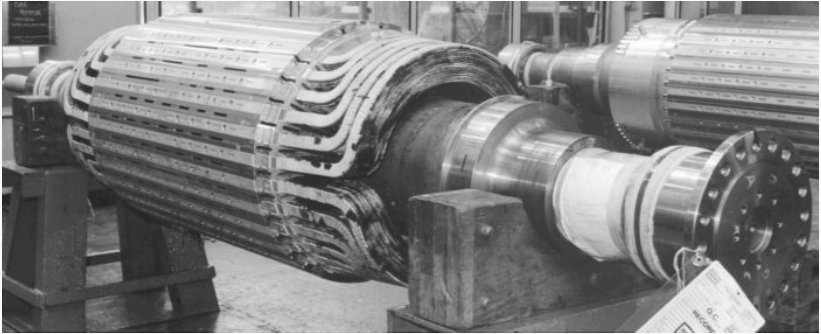


Figure 62: Round rotors are know for their long length and small (relatively) diameter [4].

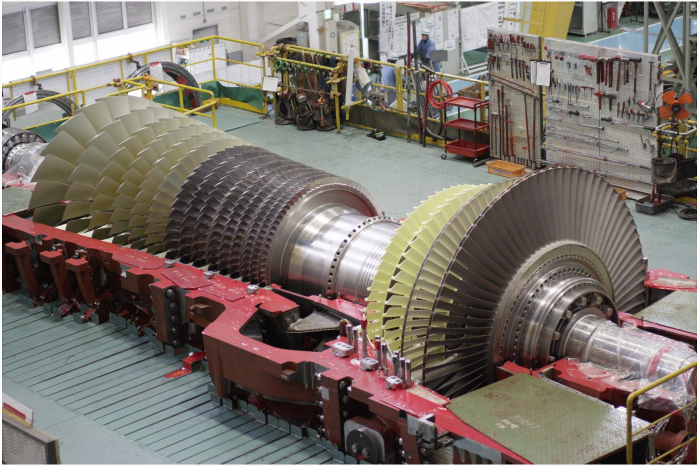


Figure 63: Round rotors are usually used in high-speed applications such as generating electricity which uses gas as its energy [24].

- Windage loss as well as noise is less as compared to salient pole rotors.
- Their construction is robust as compared to salient pole rotors.
- Number of poles is usually 2 or 4.
- Damper windings are not needed in non-salient pole rotors.
- Flux distribution is sinusoidal and hence gives better EMF waveform.
- Non-salient pole rotors are used in nuclear, gas and thermal power plants.

- Can also be used to start the synchronous motors.
- The SynM is **not self-starting**.
- Motor starts initially as an IM through the action of the amortisseur windings.
- When sufficient speed has been attained, the excitation to the rotor of the SynM is switched on.
- The motor then runs at the synchronous speed as a synchronous machine.

- Amortisseur windings additionally play a role in preventing rotor overheating when a synchronous machine is exposed to negative sequence currents.
- Negative sequence currents may be induced when a SynM is subject to a short-circuit.
- Under these conditions, a negative phase sequence currents is induced in the rotor of the generator.
- The amortisseur winding provides a path for the rotor currents and prevents the from flowing through the rotor forging and cause overheating.

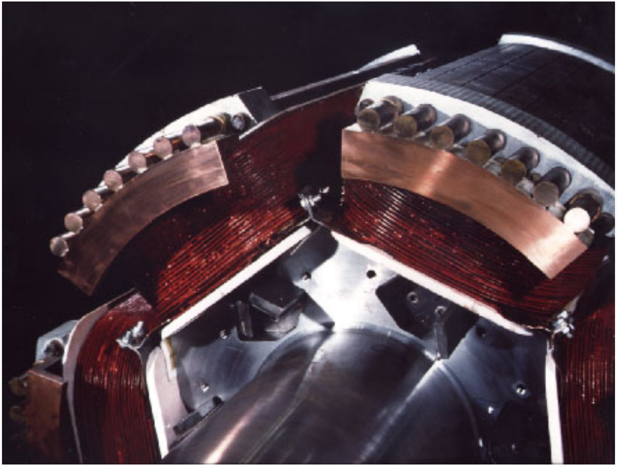


Figure 64: Amortisseur are short circuited similar to the rotor windings in a SCIM and dampen the torsional oscillations in the rotor that may occur as a result of load fluctuations.

- The operation of a SynM is due to the interaction of the magnetic fields of the stator and the rotor.
- The poly-phase stator winding is excited and the rotor is provided with a DC supply.
- The stator provides the rotating magnetic field.
- The rotor locks in with the rotating magnetic field and rotates along with it.

> Once the rotor locks in with the Rotating Magnetic Field (RMF), the motor is in synchronisation.

- A single-phase (or two-phase derived from single phase) stator winding is possible, but in this case the direction of rotation is not defined and the machine may start in either direction unless prevented from doing so by the starting arrangements.
- In steady-state operation, the motor speed is only dependent on the supply frequency.
- When the load is increased above the breakdown load, the motor falls out of synchronisation.

> When the load is higher than the machine could handle, field winding cannot follow the RMF.

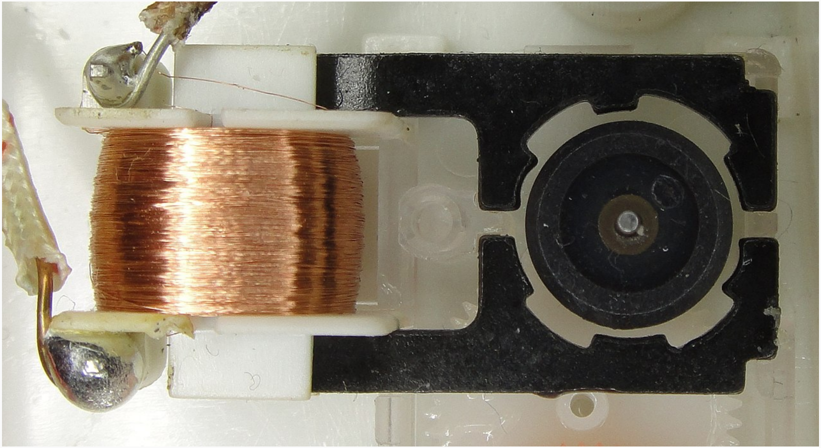


Figure 65: Miniature synchronous motor used in analog clocks. The rotor is made of permanent magnet.

Synchronous Machines

- In large SynM, the field is produced by electromagnets on the rotor.
- Rotor construction can be round-rotor or salient pole.
- To produce a field on the rotor using a winding it must excited using a direct current or have permanent magnets.
- To achieve the field excitation three are (3) ways of doing it:
  1. Use slip rings and carbon brushes to pass direct current to the rotor from a stationary circuit.
  2. Use a PM rotor,
  3. Use a brushless exciter (smaller alternator on same shaft with rectification.

Figure 66: For big generators, slip rings are used to provide the rotor with its excitation current.

- The synchronous speed of a SynM is given in rpm as:

$$n_s = 120\frac{f_e}{p}\,\text{rpm} = 60\frac{f_e}{P}$$

where;

- f_e is the AC supply current frequency in Hz,
- p is the number of magnetic poles.
- P is the number of pole pairs eev

Synchronous Machines

- The maximum power a SynM can deliver is determined by the maximum torque which can be applied without losing synchronous speed.
- Both the external system and the drive itself can be represented as an impedance in series with a voltage source.
- Therefore, study of power limits becomes merely a special case of the more general problem of the limitations on power flow through a series impedance.
- Consider a simple circuit consisting of two AC voltages (E_{af}, V_{a}) connected by an impedance $Z_{\mathrm{s}} = R_{\mathrm{a}} + \mathbf{j} X_{\mathrm{s}}$ through which the current is I.

- The load angle δ is positive while the power angle θ can seen to be negative (i.e., lagging).
- The power (P_2) delivered through the impedance to the load-end voltage source $(\hat{V}_a)$ is;

$$P_2 = V_a I_a \cos \varphi$$

where φ is the phase angle of $(\hat{I}_a)$ with respect to $(\hat{V}_a)$.

- The phasor current is;

$$\hat{I}_a = \left(\hat{E}_{af} - \hat{V}_a \right) / Z$$

- Expressing the phasor voltages (E_{af}, V_a) and the impedance Z in polar form

$$\hat{E}_{af} = E_{af}\angle\delta$$
$$\hat{V}_a = V_a\angle 0$$
$$Z = R_a + \mathbf{j}X_s = Z\angle\varphi Z$$

where,

- δ is the load angle by which E_{af} leads V_a.
- $\varphi_Z = \tan^{-1}(X_s/R_a)$ is the angle of Z.
- Therefore we can write the flowing current I as:

$$I = \frac{E_{af}\angle\delta - V_a\angle 0}{Z\angle\varphi_Z} = \frac{E_{af}}{Z}\angle(\delta - \varphi_Z) - \frac{V_a}{Z}\angle{-\varphi_Z}$$

Synchronous Machines

- As we only care about real power, we will take the real part of the equation.

$$\mathrm{Re}\left(I\right) = \frac{E_{\mathrm{af}}}{Z}\cos\left(\delta - \varphi_{\mathrm{Z}}\right) - \frac{V_{\mathrm{a}}}{Z}\cos\left(-\varphi_{\mathrm{Z}}\right),$$

> Remember; $\cos\left(-\varphi_{\mathrm{Z}}\right) = \cos\left(\varphi_{\mathrm{Z}}\right) = R/Z$

- This makes:

$$P_2 = \frac{E_{\mathrm{af}}V_{\mathrm{a}}}{Z}\cos\left(\delta - \varphi_{\mathrm{Z}}\right) - \frac{V_{\mathrm{a}}^2 R}{Z^2}$$

- If we assume the resistance to be negligible ($R_{\mathrm{a}} \approx 0$), then $Z \approx X_{\mathrm{s}}$.

$$P_1 = P_2 = \frac{E_{\mathrm{af}}V_{\mathrm{a}}}{X_{\mathrm{s}}}\sin\delta$$

Example

A 50-Hz, three-phase synchronous motor is observed to have a terminal voltage ($\hat{V}_{\text{a}}$) of 800 V (line-line) and a terminal current ($\hat{I}_{\text{a}}$) of 120 A at a power factor of 0.9 lagging. The field-current under this operating condition is 53 A. The drive synchronous reactance (X_{s}) is equal to 1.88 Ohm.

Assume the armature resistance to be negligible. Please calculate:

- The generated voltage (E_{af}) in volts,
- The electrical power input (P_{in}) to the motor in kW.
- Based on the calculated power input, to achieve a unity power factor (pf $= 1$), calculate the phase angle (δ).

Example

A 3 MW, 7 kV synchronous drive has a synchronous reactance (X_s) of 8.2 Ω/phase. The stator is connected in wye (Y), and the drive operates at full-load (3 MW) with a leading power factor of pf $= 0.89$. If the efficiency is $\eta = 97$ %, Please calculate the following parameters:

- The apparent power,
- The line current,
- The internal voltage per phase with corresponding phasor diagram,
- The power angle,
- The total reactive power supplied to the system,
- The approximate maximum power the motor can develop without pulling out.

- There are two (2) standard tests that are performed on SynM to determine their steady state parameters and verify the design.
- These are:
  1. The open circuit saturation test,
  2. The steady-state short circuit test

> 2^{nd} test is also known as the synchronous impedance test.

- In the open circuit test, the machine is run at rated speed with the stator winding open circuited.
- The field current is varied, and the terminal voltage is plotted as a function of the field current.
- As the field current rises the stator voltage increases proportionally.
- At about 80% of rated voltage, the curve becomes nonlinear.

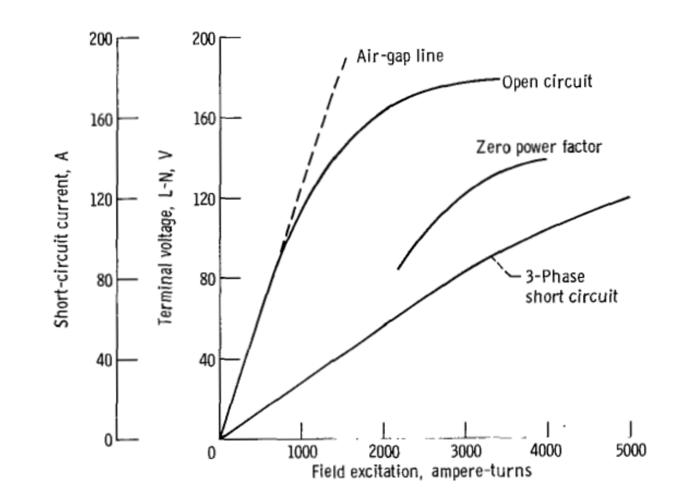


Figure 67: A diagram with multiple SynM curves [21].

- In the short circuit test, the rotor is turned at rated speed with a three-phase short circuit on the stator.
- As before, increasing the field current allows the plotting of the stator current versus the field current.

> The short circuit characteristic is linear.

- The flux density in the machine is very low since the terminal voltage is zero.
- The air-gap flux is sufficient to produce a voltage to overcome the leakage reactance drop.
- From these two curves, the X_s can be found.
- For the open circuit condition, the V_a is equal to the internal voltage E_{af}.
- For the same field current, take I_a from the short circuit test.

- V-curve is a plot of the stator current (I_a) v. field current (I_f) for different constant loads.
- The curve(s) between the I_a v. I_f at no load are known as V Curve.

> The name comes from the shape of the curve.

- SynM $\cos\varphi$ can be controlled by varying I_f.
- It is important to know I_a changes with I_f.
- To study this, assume that motor is running at **NO** load.

- If I_f is increased from this small value, I_a will decrease until the armature current becomes minimum.
- At this minimum point, the motor is operating at a unity power factor.
- The motor operates at a lagging power factor until it reaches up to this point of operation.
- The curve connecting the lowest points of all the V curves for various power levels is called the Unity Power Factor Compounding Curve.
- The compounding curves for 0.8 power factor lagging and 0.8 power factor leading are shown in the figure above by a red dotted line.

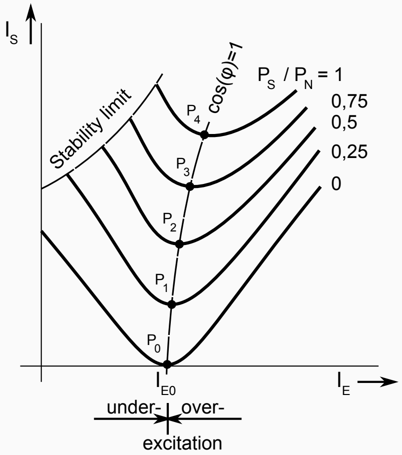


Figure 68: V-curves of a SynM [30].

- A Synchronous Condenser (SynC) is a DC-excited SynM, whose shaft is **not connected to anything but spins freely**.
- Its purpose is not to convert electricity to mechanical power, but to adjust conditions on the electric power transmission grid.
- Its field is controlled by a voltage regulator which:
  1. either generate or absorb reactive power as needed to adjust the grid's voltage,
  2. or improve the power factor.
- The condenser's installation and operation are identical to SynM

> Some generators are actually designed to be able to operate as synchronous condensers with the prime mover disconnected

Synchronous Machines

- Increasing field excitation results in reactive power generation.
- Its principal advantage is the ease of reactive power adjustment.
- SynC are alternative to capacitor banks and static VAR compensators for power-factor correction in power grids
- An advantage is the amount of reactive power from a SynC can be continuously adjusted.
- Reactive power from a capacitor bank decreases when grid voltage decreases while the reactive power from a synchronous condenser inherently increases as voltage decreases.[1]
- Additionally, SynC are more tolerant of power fluctuations and severe drops in voltage.

> However, synchronous machines have higher energy losses than static capacitor banks.

- SynC also help stabilise grids.
- The inertial response of the machine and its inductance can help stabilize a power system during rapid fluctuations of loads such as those created by short circuits or electric arc furnaces.
- For this reason, large installations of SynC are sometimes used in association with high-voltage direct current converter stations to supply reactive power to the alternating current grid.

> Synchronous condensers are also finding use in facilitating the switchover between power grids and providing power grid stabilization as turbine-based power generators are replaced with solar and wind energy.

Figure 69: The application of a synchronous condenser is not to convert electric power to mechanical power or vice versa, but to adjust conditions on the electric power transmission grid [18].

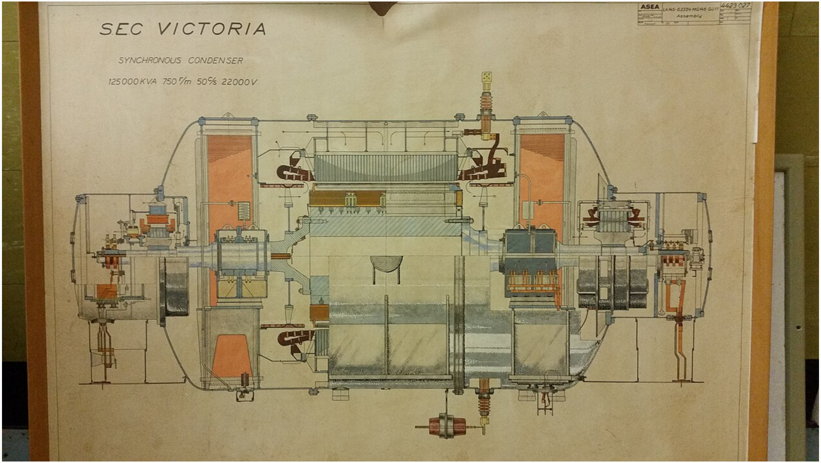


Figure 70: Section view showing interior construction of condenser [17].

- Found on all types of machines, a nameplate provides information about the construction and performance characteristics.
- Whilst motor standards are established on a country by country basis, most motors fall under the two main industry bodies:
  - International Electrotechnical Commission (IEC),
  - National Electric Manufacturers Association (NEMA)

> Understanding how to read the nameplate of a motor can help identify faults more accurately, ensure that the right motor is being used for the job and can result in a more efficient service from a motor repair company if there is a fault.

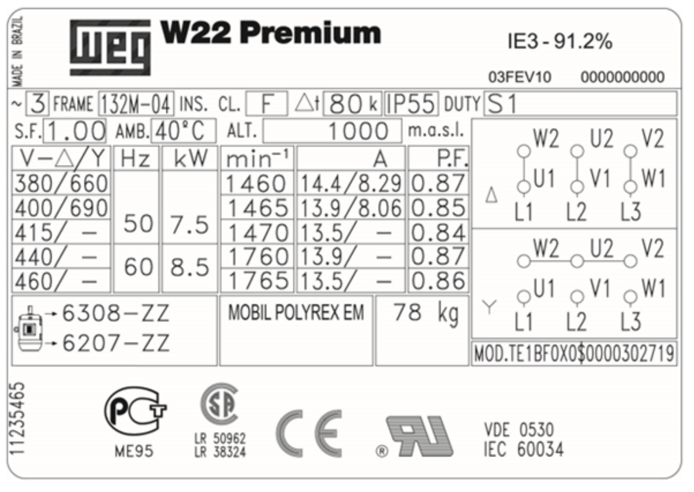
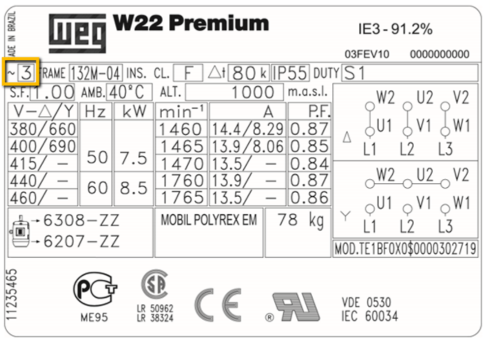


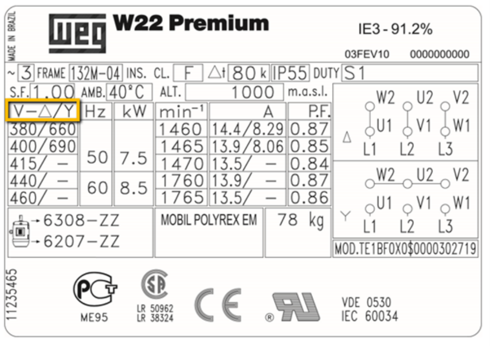
Figure 71: A generic nameplate [14].

Synchronous Machines

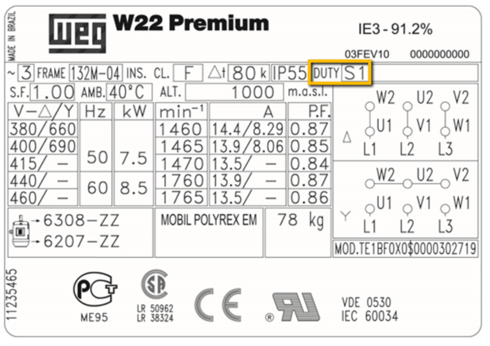


Phases the type of power supply for which the motor is designed. There are single phase and three phase motors. In this example, the number is "3" as the motor is a three phase motor.

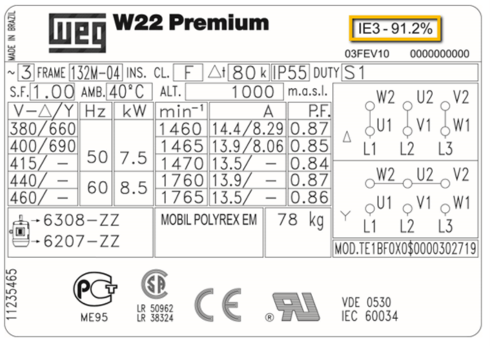
Synchronous Machines



Operating V shows at which voltage is the motor is designed to operate most efficiently. Motors are designed to operate at +/-10% tolerance of this value.



Service Duty S1 shows that this is a continuous duty motor that works at a constant load for enough time to reach temperature equilibrium.



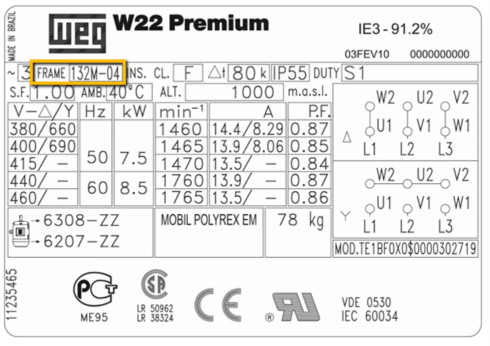
Efficiency Code shows the percentage of the input power that is actually converted to work output from the motor shaft.

Efficiency Code

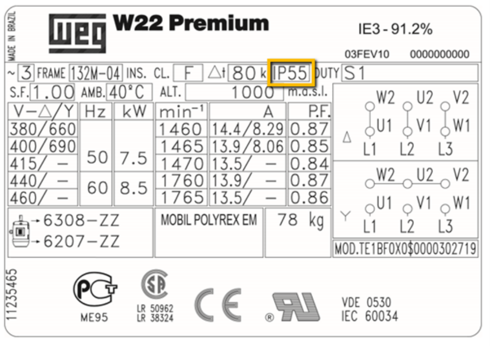
- The motor will have a **nominal** efficiency shown on the plate,
  - This is the average efficiency.
- The closer this value is to 100%, the lower the electricity consumption cost is going to be.
- The four levels of motor efficiency are

  IE1 Standard Efficiency
  IE2 High Efficiency
  IE3 Premium Efficiency
  IE4 Super Premium Efficiency

- In this example, we are shown an IE Code of IE3 which indicating premium efficiency.



Frame Size Shows the frame size and determines the mounting dimensions such as the foot openings pattern and the shaft height.



Protection Shows the frame size and determines the mounting dimensions such as the foot openings pattern and the shaft height.

IEC Making electrotechnology work for you.

Ingress protection (IP) ratings guide

IP ratings are represented by combining the first and second digits of the below columns

1st numeral - solid foreign objects

0	No protection
1	Protected against solid foreign objects of 50 mm Ø and greater
2	Protected against solid foreign objects of 12,5 mm Ø and greater
3	Protected against solid foreign objects of 2,5 mm Ø and greater
4	Protected against solid foreign objects of 1,0 mm Ø and greater
5	Dust-protected
6	Dust-tight

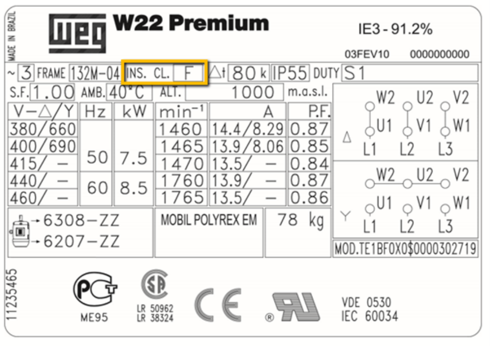
Example: + IP 6 5 → Protected against water jets → Dust-tight

2nd numeral - water

0	No protection	-
1	Protected against vertically falling water drops	Vertically falling drops shall have no harmful effects
2	Protected against vertically falling water drops when enclosure tilted up to 15°	Vertically falling drops shall have no harmful effects when the enclosure is tilted at any angle up to15° on either side of the vertical
3	Protected against spraying water	Water sprayed at an angle up to 60° on either side of the vertical shall have no harmful effects
4	Protected against splashing water	Water splashed against the enclosure from any direction shall have no harmful effects
5	Protected against water jets	Water projected in jets against the enclosure from any directions shall have no harmful effects
6	Protected against powerful water jets	Water projected in powerful jets against the enclosure from any direction shall have no harmful effects
7	Protected against the effects of temporary immersion in water	Ingress of water in quantities causing harmful effects shall not be possible when the enclosure is temporarily immersed in water under standardized conditions of pressure and time
8	Protected against the effects of continuous immersion in water	Ingress of water in quantities causing harmful effects shall not be possible when the enclosure is continuously immersed in water under conditions which shall be agreed between manufacturer and user but which are more severe than for numeral 7
9	Protected against high pressure and temperature water jets	Water projected at high pressure and high temperature against the enclosure from any direction shall not have harmful effects

Figure 72: IEC IP Ratings guide [9].

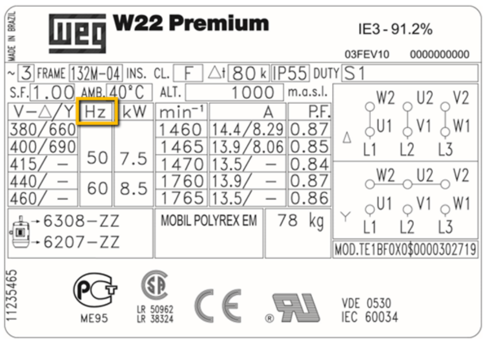
Synchronous Machines



Insulation Class Shows the motors maximum tolerance to heat.

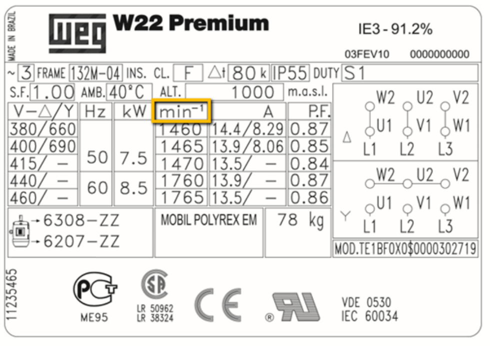
- The motors have different insulation capabilities.
- The insulation codes show their thermal tolerance or ability to survive at a specified temperature for a period of time.
- The higher the designated code letter, the greater the heat capability.
- It is based on the highest temperature the material can withstand continuously without degrading or reducing motor life.
- IES specify five (5) different types of insulation classes:

  Class A 105 C,
  Class E 120 C,
  Class B 130 C,
  Class F 155 C,
  Class H 180 C.



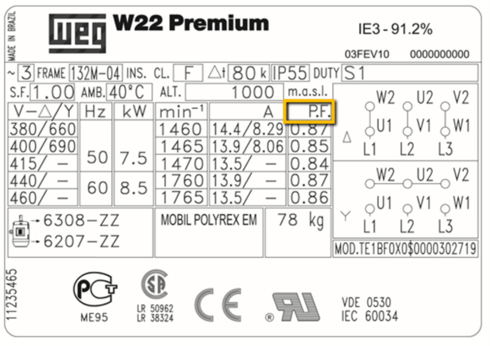
Frequency Show the machine's operation frequency. If more than one frequency is marked, other parameters that will differ at different input frequencies have to be indicated on the nameplate as well.

Synchronous Machines

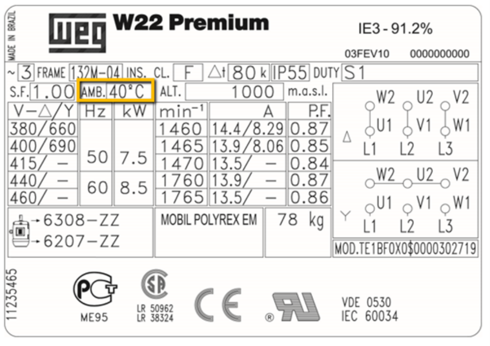


Full Load RPM Full-load speed is the speed at which rated full-load torque is delivered at rated power output, this speed is sometimes called slip-speed or actual rotor speed.

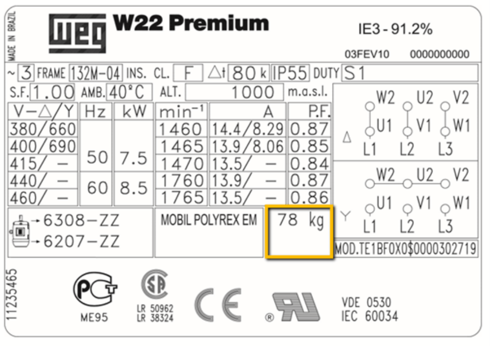
Synchronous Machines



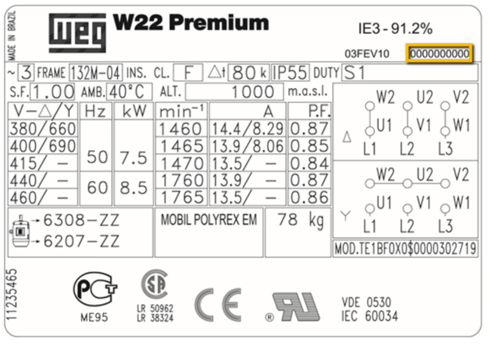
Power Factor Gives the power factor for the respective speed/ampere value.



Ambient Temperature lists the temperature at which the motor can operate and still be within the tolerance of the insulation class at the maximum temperature rise.



Weight This shows the weight of the motor.



Weight Shows the serial number of the motor. As it is unique to the motor, knowing this number can help when liaising with manufacturers about the type of motor you have.

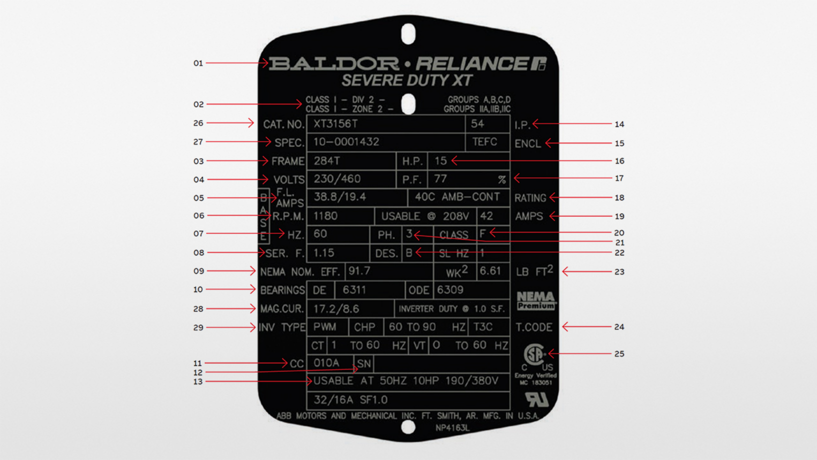


Figure 73: An alternative nameplate used in industry [1].

Appendix

- Assume the following simplified induction drive equivalent circuit.

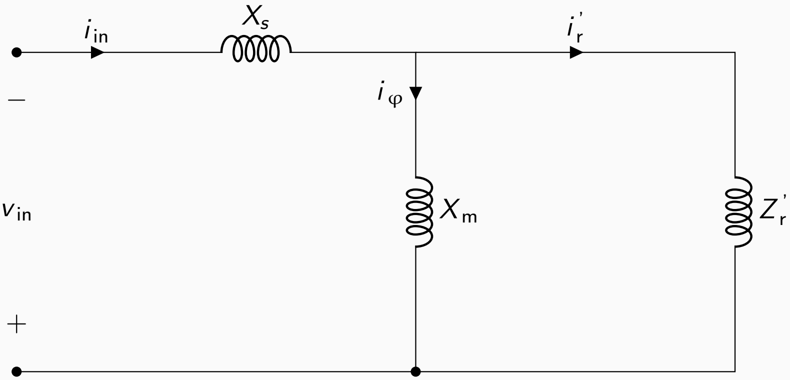


Figure 74: An abstract rendition of the simplified induction circuit equivalent circuit.

Appendix

- where:

$$Z_s = R_s + \mathbf{j}X_s,$$
$$Z_m = \frac{R_c \times \mathbf{j}X_m}{R_c + \mathbf{j}X_m},$$
$$Z_r = \frac{R_r^{'}}{s} + jX_r^{'}.$$

- From Kirchhoff's current law, the following statement holds true:

$$i_{in} = i_{\varphi} + i_r^{'}.$$

- The second identity can be derived from parallel circuit principles;

$$Z_m i_{\varphi} = Z_r^{'} i_r \quad \rightarrow \quad i_m = i_R^{'} \times \left(\frac{Z_R}{Z_m}\right).$$

- We can isolate the magnetizing current (i_φ).

$$i_\varphi = i_r^{'} \times \left(\frac{Z_r}{Z_m}\right) + i_r^{'}.$$

- Isolating the rotor current ($i_r^{'}$) gives us the final expression:

$$i_r^{'} = \frac{i_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

- Consider the following equivalent circuit of an IM.

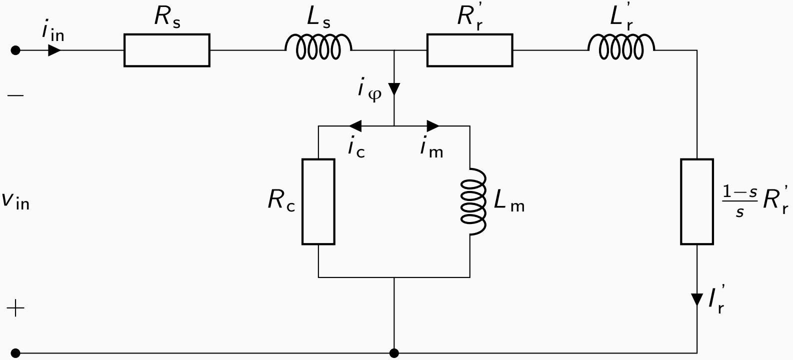


Figure 75: An abstract rendition of the induction circuit equivalent circuit.

Appendix

- We can start by summing up the rotor side.

$$Z_r = \frac{R_r^{'}}{s} + \mathbf{j}\, w_r L_r^{'} \quad \rightarrow \quad Z_r = \frac{R_r^{'}}{s} + X_r^{'}.$$

- The stator side is calculated as:

$$Z_s = R_s + \mathbf{j}\, w_s L_s \quad \rightarrow \quad Z_s = \frac{R_s}{s} + X_s.$$

- The magnetising side is calculated as:

$$Z_m = \frac{R_c \times \mathbf{j}\, w_s L_m}{R_c + \mathbf{j}\, w_s L_m} \quad \rightarrow \quad Z_m = \frac{R_c \times X_m}{R_c + X_m}.$$

Appendix

- Without substitution, the input impedance is calculate to be

$$Z_{\text{in}} = Z_{\text{s}} + \frac{Z_{\text{m}} \times Z_{\text{r}}}{Z_{\text{m}} + Z_{\text{r}}}.$$

- From here, we can derive the stator current (I_{in}).

$$V_{\text{ph}} = \frac{Z_{\text{in}}}{I_{\text{in}}} \quad \rightarrow \quad I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}}.$$

- Now, the Torque of an IM is related to the air-gap power by:

$$T_{\text{mech}} = \frac{P_{\text{gap}}}{w_{\text{s}}} \qquad \text{and} \qquad P_{\text{gap}} = n_{\text{ph}} \frac{I_{\text{r}}^2}{w_{\text{s}}} \left(\frac{R_2^{'}}{s} \right)$$

Appendix

- Using the stator-rotor relationship, we can obtain our final result.

$$I_r^{'} = \frac{I_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

Go Back

an empirical relationship or phenomenological relationship is a relationship or correlation that is supported by experiment or observation but not necessarily supported by theory.

Go Back

Faraday's law is a law of electromagnetism predicting how a **B**-field will interact with an electric circuit to produce Electro-motive Force (EMF).

This phenomenon, known as electromagnetic induction, is the fundamental operating principle of transformers, inductors, and many types of electric motors, generators and solenoids.

> *The EMF around a closed path is equal to the negative of the time rate of change of the magnetic flux enclosed by the path.*

AC Alternating Current. 2, 6, 8, 22, 31, 43

DC Direct Current. 2, 6, 9, 10, 13, 22, 44

IM Induction Machine. 2, 22–24

MMF Magneto-Motive Force. 2, 31, 32, 44, 46

PM Permanent Magnet. 2, 10

SynM Synchronous Machine. 2, 10, 22–24

Symbol	Description	Unit
C	total number of conductors in armature winding	-
B_{peak}	Magnetic Flux Density Magnitude	Tesla
B	Magnetic Flux Density Magnitude	Tesla
H	Magnetic Field Intensity Magnitude	A/m
I_{m}	peak value of current	A
$L_{\text{s}}, L_{\text{r}}$	Stator & Rotor Inductance	H
N_{ph}	Number of Turns per phase	-
N	Number of Turns	-
$R_{\text{s}}, R_{\text{r}}$	Stator & Rotor resistance	Ohm
R	Resistance	Ohm
T	Torque	$\text{N} \cdot \text{m}$
$W_{\text{fld}}, W'_{\text{fld}}$	Field energy & co-energy	J
Z	Impedance	Ohm

$\mathcal{E}$	Electro-motive force	V
$\mathcal{F}$	Magneto-motive Force	Ampere-Turns
$\mathcal{L}$	Inductance function	H
$\mathcal{R}_{\mathrm{c}}, \mathcal{R}_{\mathrm{ag}}, \mathcal{R}_{\mathrm{tot}}$	Core, air-gap & Total Reluctance	1/H
$\mathcal{R}$	Reluctance	1/H
f_{e}	Electrical Frequency	Hz
f	Frequency	Hz
g	Air-gap	m
λ	Flux Linkage	Weber (Wb)
ω_{e}	electrical angular frequency	rad/s
ϕ	Magnetic Flux	Wb
$\theta_{\mathrm{a}}, \theta_{\mathrm{e}}$	Spatial & Electrical angle	deg
θ_{e}	Electrical Angle	deg
i_{a}	armature current	A

i_r, i_b, i_y	poly-phase currents	A
i	Current	A
k_w	Winding Factor	-
l	length	m
μ_0	Permeability of free-space	H/m
n_s	synchronous speed	rpm
p	Poles	-
t	Time	s
$\mathbf{B}$	Magnetic Flux Density	Tesla
$\mathbf{H}$	Magnetic Field Intensity	A/m
m	number of parallel paths through armature winding	-

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