Topics on Electrical Engineering

# Electric Drive Fundamentals

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Version: θ.1.0

MCI

B.Sc - Drive Systems

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# Introduction

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# Introduction

- The goal of this lecture is to give you the fundamentals of electric machinery using theory and engineering practice.
- This lecture is a total of 2 SWS with a total of thirty (30) UE.
  - With 28 UE is devoted to lectures.
- There is a written exam at the end of the module worth two (2) UE.
- There is one (1) assignment for this course:

  **1**$^{\text{st}}$ will be a pre-defined work which is individual based.

# Introduction

- The individual assignments focus on understanding electric machinery.
- The assignment is uploaded to SAKAI for you to work on along with what is required of you for submission.
  - The assignments contain electric drive problems to solve.
- The strict deadline is the end day of the **last lecture**.
- Any submission after this date will not be accepted.
- If all submissions are sent early we can do a statistical analysis and go through questions before the final exam.

# Introduction

- Lecture materials and all possible supplements will be present in its Github Repo.
  - You can easily access the link to the web-page from here.

> Github is chosen for easy access to material management and CI/CD capabilities and allowing hosting websites.

- In the lecture some exercises are solved using Python and other examples and can be accessed from the Repo website.

# Introduction

- At the end of the lectures there will be a final examination which you will tested.
- You will be asked three (3) questions related to electric drives.
- The exam will be ninety (90) minutes.
- You are NOT allowed a personal formula sheet or any kind of supporting material.
- You are allowed a calculator.

| Assessment Type | Overall Points | Breakdown | % |
|---|---|---|---|
| Individual Assignment | 40 | | |
| | | Report | 20 |
| | | Solution(s) | 80 |
| | | | 20 |
| Final Exam | 60 | | |
| | | Question 1 | 40 |
| | | Question 2 | 30 |
| | | Question 2 | 30 |

**Table 1:** Assessment Grade breakdown for the lecture.

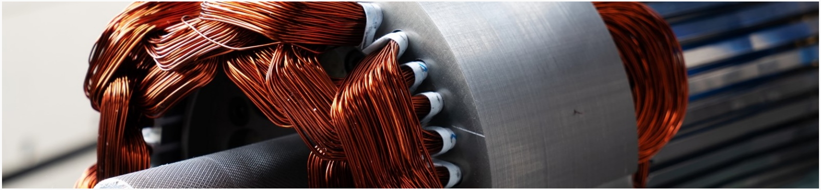
# Introduction

| Covered Topic | Appointment |
|---:|:---:|
| Return to Fundamentals | 1 |
| Magnetic Circuits & Materials | 1 |
| Transformers | 2 |
| Electromechanical Energy Conversion | 2-3 |
| Rotating Magnetic Fields | 3 |
| DC Drives | 4 |
| Poly-phase Induction Drives | 4-5 |
| Single-phase Induction Drives | 5-6 |
| Linear Induction Drives | 6 |
| Poly-phase Synchronous Drives | 6-7 |
| Solid-state Commutation Drives | 7 |

**Table 2:** Distribution of materials across the semester.

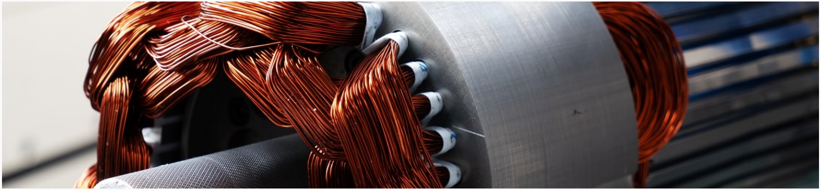
## Return to Fundamentals

- Complex number notation,
- Multi-phase systems,
- Phasors and Wave-forms



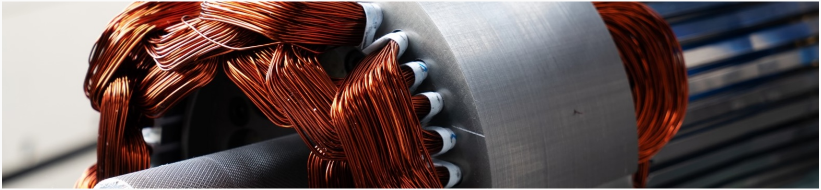
## Transformers

- Construction & Physical Properties
- Modelling
- Parameter Tests
- Connection Types



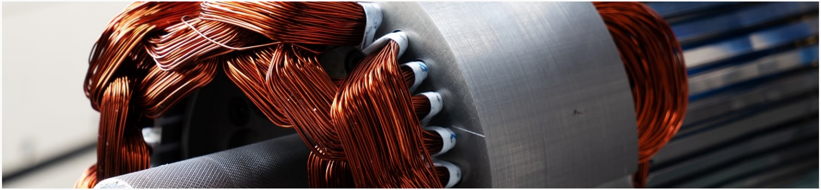
## Rotating Magnetic Fields

- MMF in Winding
- Torque Generation
- Generated Voltage



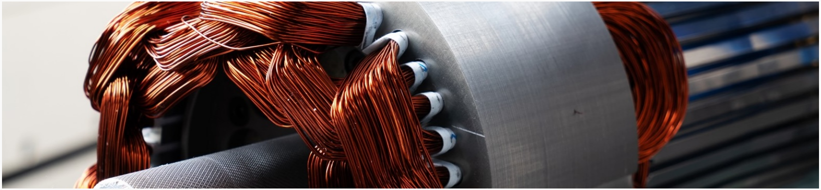
## DC Drives

- Construction
- Physical laws governing its operation
- Types of Connections used in industry and commercial applications.
- Applications in Industrial/Commercial venues.



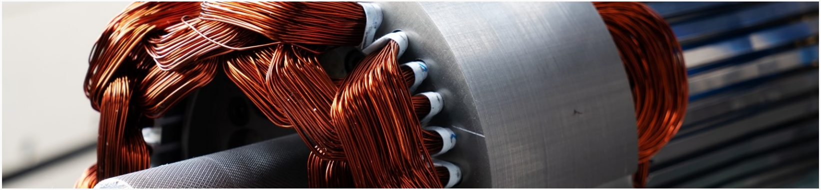
## Poly-phase Induction Drives

- Construction
- Physical laws governing its operation principle.
- Modelling an induction drive mathematically
- Methods of starting an induction drive.



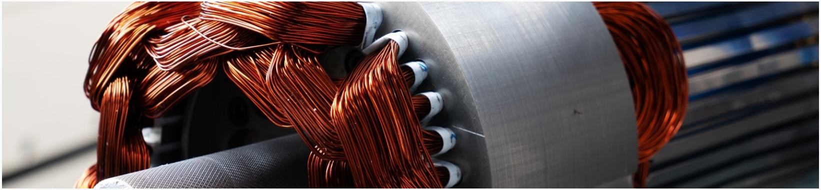
## Single-Phase Induction Drives

- Creating a Rotating Magnetic Field in a single phase
- Types of single-phase types
- Salient-pole



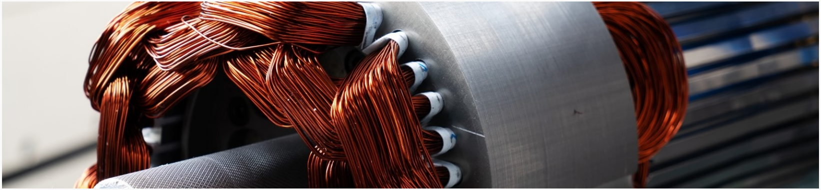
## Poly-phase Synchronous Drives

- Construction and Rotor types,
- Operation principles,
- Regulatory behaviours,
- V-curves,



## Solid-State Commutation Drives

- Solid State Commutation
- BLDC & PMSM Drives
- Switched Reluctance Motor (SRM)
- Stepper Drives



## Books

- Mohan Ned. "*Advanced electric drives: analysis control and modeling using MATLAB/Simulink*" John Wiley & Sons 2014.
- Krause Paul C. et. al. "*Analysis of electric machinery and drive systems*" Vol. 2 IEEE Press 2002.
- Pyrhonen Juha et. al "*Design of rotating electrical machines*" John Wiley & Sons 2013.
- Stephen J. Chapman. "*Electric Machinery Fundamentals (5th Edition)*" (2012).
- Fitzgerald A. E. et. al. "*Electric Machinery*" McGraw Hill (2003).

## Books

- Hughes A. et. al. "*Electric Motors and Drives: Fundamentals Types and Applications*" Newnes 2019.
- Melkebeek A. "*Electrical Machines and Drives: Fundamentals and Advanced Modelling*" Springer 2018.
- Wildi T. "*Electrical machines, drives, and power systems*" Pearson Education 2006.
- Veltman A. et. al. "*Fundamentals of Electrical Drives*" Springer 2007.

# Introduction

## White Papers

- Maddox Transformer "*Guide to transformer cores: types, construction, & purpose*"
- Control Engineering *Springtime for Switched-Reluctance Motors?* .

# Introduction

## Lecture Notes

- Power Transformers "*ESE 470 Energy Distribution Systems*" Oregon State University,
- Principles of Electromechanical Energy Conversion "*Actuators & Sensors in Mechatronics Electromechanical Motion Fundamentals*" NYU,

# Return to Fundamentals

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## Learning Outcomes

(LO1) An Overview of Poly-phase circuits,

(LO2) Definitions on Active-Reactive power,

(LO3) Polar Coordinate System.

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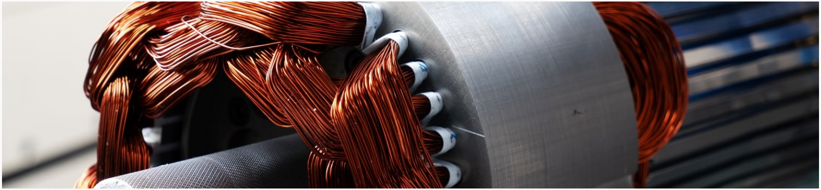

# Return to Fundamentals

- A rotating magnetic field is a magnetic field with moving polarities.
  - Which its opposite poles rotate about a **central point** or **axis**.
- To create a rotating magnetic field (RMF) you need at least a 2-phase system.
- In the industry, RMFs are mostly produced using a 3-phase supply [1].
  - There are also economic reasons, as it is cheaper to design a 3-phase system with minimal cost to wiring [2].
- Before starting with electric drives, it is a good time to look at some fundamental concepts in power engineering.

[1] 3-phase supply produces smoother operation of motors compared to 2-phase. This is due to the power transfer in 3 phase supply being less pulsating than in 2 phase supply.

[2] However, there have been test and feasibility on 6-phase power in 1970s with minimal success.

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- Generation, transmission, and heavy-power utilisation of AC electric energy almost invariably involve a poly-phase circuit.
- In such a system, each voltage source consists of a group of voltages having related magnitudes and phase angles.
- Thus, an $n$-phase system employs voltage sources which typically consist of $n$ voltages substantially equal in magnitude and successively displaced by a phase angle of 360°/n.
- Therefore a 3-phase system would have 3 different voltage sources separated 120 degrees apart.

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- The three individual voltages of a three-phase source may each be connected to its own independent circuit.
- We would then have three separate single-phase systems.
- Alternatively, symmetrical electric connections can be made between the three voltages and the associated circuitry to form a three-phase system.
- It is the latter alternative that we are concerned.
- Note that the word phase now has two distinct meanings.
  - It may refer to a portion of a pol-yphase system or circuit,
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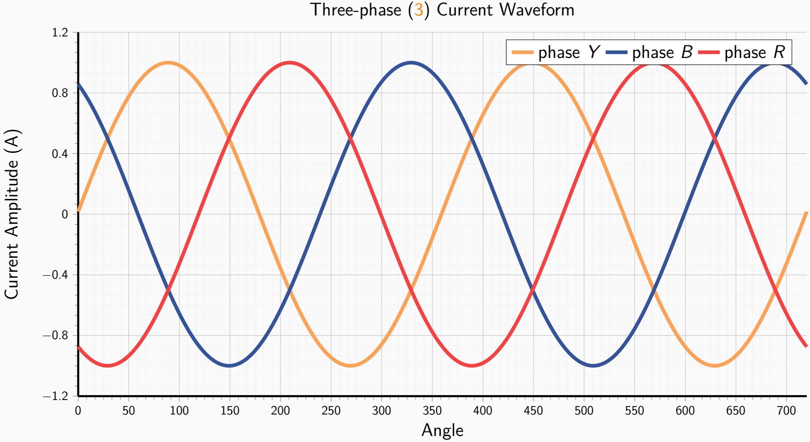


**Figure 1:** A three phase waveform of currents **R**, **B**, and **Y**.

# Return to Fundamentals

- There are **no neutral connection** available.
- Phase voltage appears **across the windings**.
- In a delta connection, Line voltage is equal to Phase voltage:

$$v_{\text{line}} = v_{\text{ph}}$$

- $i_{\text{YB}}$, $i_{\text{YR}}$, $i_{\text{BR}}$ are also called **Line current** (i.e., $i_{\text{line}}$).
- Line current is $\sqrt{3}$ times that of the phase current.

$$i_{\text{line}} = \sqrt{3} i_{\text{subs}}$$

- Power related definitions are:

$$P = \sqrt{3} v_{\text{line}} i_{\text{line}} \cos\varphi, \quad \text{and} \quad P = 3 v_{\text{ph}} i_{\text{ph}} \cos\varphi.$$

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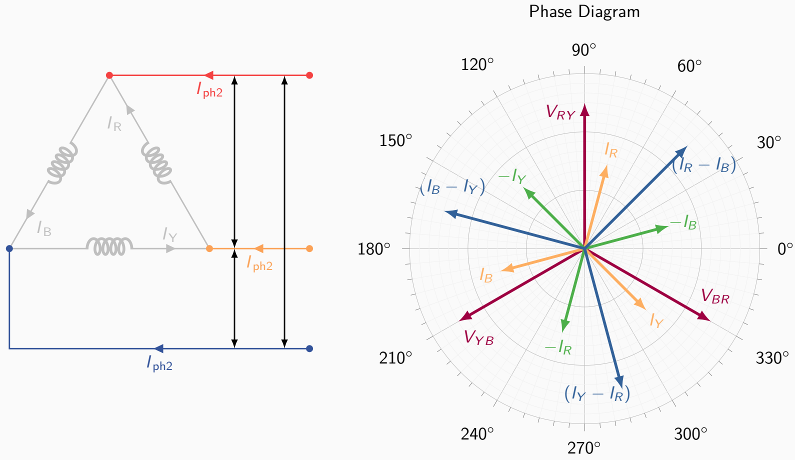


**Figure 2:** The connection and the phasor diagram of a 3-phase delta connection.

**Example**

Three impedance with a value $Z_{\Delta} = 12.00 + \mathbf{j}\,9.00 = 15.00\angle 36.9\ \Omega$ are connected in $\Delta$.

For balanced line-to-line voltages of 208 V, find the line current, the power factor, and the total power.

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- Power related equations for 3-phase:

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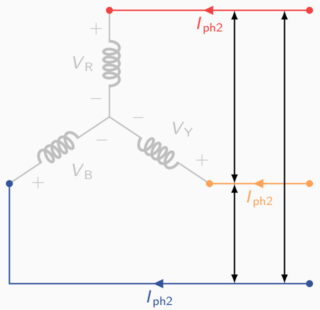
- Power related equations for 3-phase:

$$\begin{aligned} P &= \sqrt{3} v_{\text{line}} i_{\text{line}} \cos\varphi, \\ P &= \sqrt{3}\sqrt{3} \left(v_{\text{ph}}\right) \left(i_{\text{ph}}\right) \cos\varphi, \\ P &= 3 v_{\text{ph}} i_{\text{ph}} \cos\varphi. \end{aligned}$$



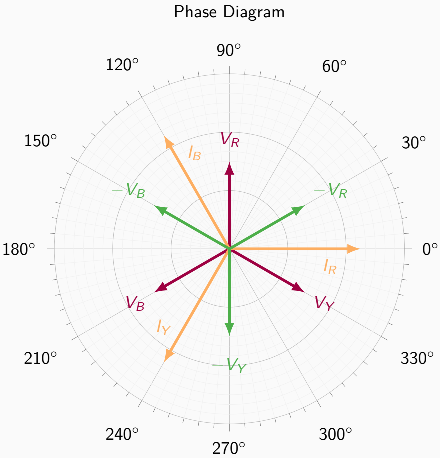




**Figure 3:** The connection and the phasor diagram of a 3-phase wye connection.

**Example**

A Y-connected 120 V source feeds a $\Delta$-connected load through a distribution line having an impedance of $0.3 + \mathbf{j}0.9\ \Omega$. The Y-source impedance is $0.2 + \mathbf{j}0.5\Omega$

The load impedance is $118.5 + \mathbf{j}85.8\ \Omega/o$.

Use the a-phase internal voltage of the generator as the reference.

**(a)** Construct a single-phase equivalent circuit of the three-phase system,

**(b)** Calculate the line currents,

**(c)** Calculate the phase voltages at the load terminals,

**(d)** Calculate the phase currents of the load,

**(e)** Calculate the line voltages at the source terminals.

$$Z = x + \mathbf{j}\, y = A\angle\theta,$$

where:

- $Z$ is the complex vector,
- $A$ is vector magnitude,
- $x$ is real/active part,
- $\mathbf{j}$ is defined as $\sqrt{-1}$.
- $y$ is imag/reactive part,
- $\theta$ is the complex angle.

$$\theta = {}^{y}/_{x}.$$

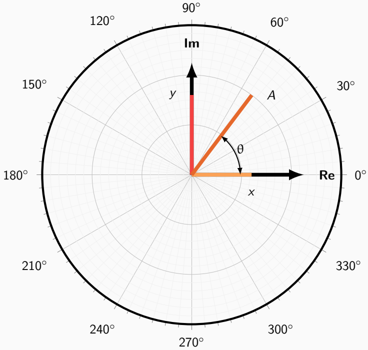



**Figure 4:** A polar coordinate system.

# Return to Fundamentals

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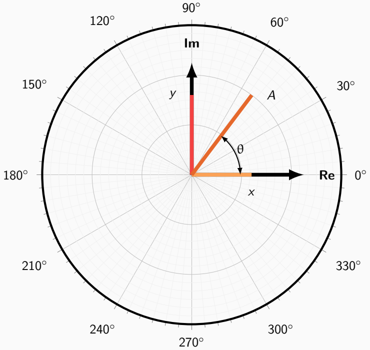


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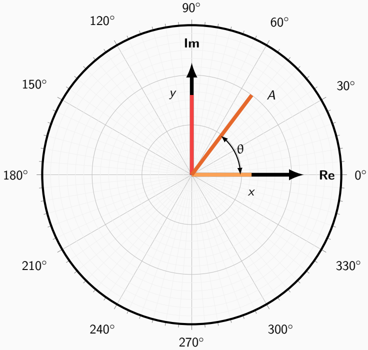


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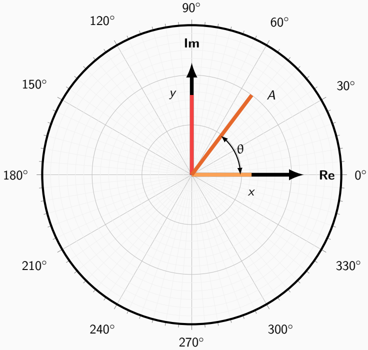


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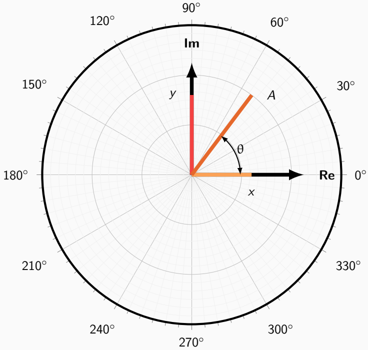


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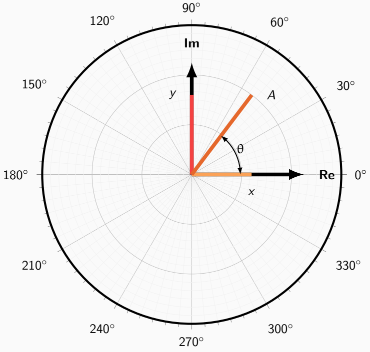


**Figure 4:** A polar coordinate system.

# Return to Fundamentals

When it comes to power in AC, there are three (3) definitions:

- If energy is used/generated by an active element it is called **Active** or **Real** ($P$) power and measured in W.
- If energy is used/generated by an reactive element it is called **Reactive** power ($Q$)and measured in $\mathrm{V} \cdot \mathrm{A}r$.
- The combination of these two values is called **Apparent** power ($S$) and measured in $\mathrm{V} \cdot \mathrm{A}$.

$$S = \sqrt{P^2 + Q^2}$$

- Finally the angle difference between the voltage and current waveform is defined as the **phase** and tells how reactive/active a circuit is.

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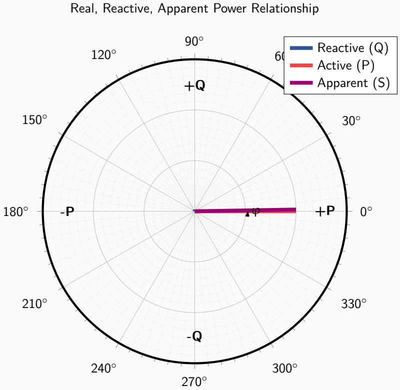


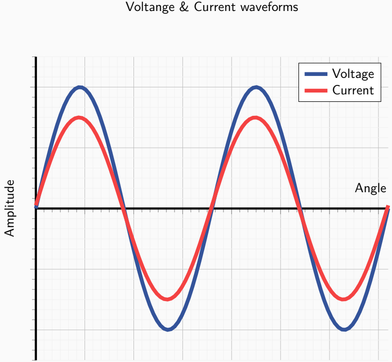




**Figure 5:** An animation showing the relations between Active, Reactive and Apparent power along with phase angle.

**Example**

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8.

The load is fed from a line having an impedance of $0.005 + \mathbf{j}\,0.025\Omega$.

The line voltage at the terminals of the load is 600 V.

(a) Construct a single-phase equivalent circuit of the system.

(b) Calculate the magnitude of the line current.

(c) Calculate the magnitude of the line voltage at the sending end of the line.

(d) Calculate the power factor at the sending end of the line.

# Magnetic Circuits and Materials

# Table of Contents

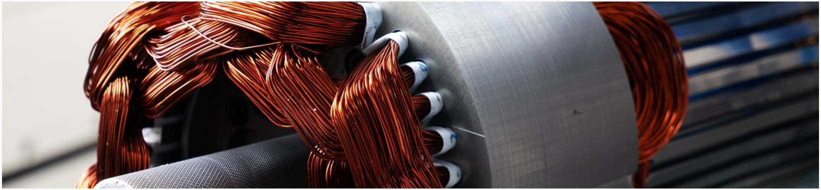
## Learning Outcomes

(LO1) An Overview of Maxwell's Equations,

(LO2) Introduction to Magnetic Circuits,

(LO3) Brief look on Magnetic Materials,

(LO4) Magnetic Material Losses.



- Almost all electric drives use ferromagnetic material for shaping and directing **B**-fields.
  - These fields act as the medium for transferring and converting energy.

> Permanent-magnets are also widely used in drive design.

- Without these materials, practical implementations of most familiar EEC devices would not be possible.
- Analysing and describing systems containing them is essential for designing effective drives.
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# Magnetic Circuits and Materials

- Maxwell's equations are a set of coupled partial differential equations which form the foundation of electric and magnetic circuits.
- In their PDE form, they are written as [5]:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right).$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, $\rho$ is the electric charge density and $\mathbf{J}$ the current density, $\varepsilon_0$ is the vacuum permittivity and $\mu_0$ is the vacuum permeability.

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- First, the displacement-current term ($\varepsilon_0 \partial \mathbf{E}/\partial t$) can be neglected.
  - This term accounts for **B**-fields produced in space by time-varying **E**-fields (i.e., electromagnetic radiation) [10].
- Neglecting $\varepsilon_0 \partial \mathbf{E}/\partial t$ results in the magneto-static form which relate **B**-fields to the currents (**J**) which produce them.
- In integral form [2]:

$$\oint_{\mathcal{C}} \mathbf{H}\, \mathrm{d}\mathbf{l} = \int_{\mathcal{S}} \mathbf{J} \cdot \mathrm{d}\mathbf{a}, \quad \text{and} \quad \oint_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a} = 0.$$

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The line integral ($\oint$) of the tangential component of the magnetic field intensity ($\mathbf{H}$) around a closed contour ($\mathcal{C}$) is equal to the total current ($\mathbf{J}$) passing through any surface ($\mathcal{S}$) linking that contour.

- The second one states the magnetic flux density ($\mathbf{B}$) is conserved, i.e., no net flux enters or leaves a closed surface (i.e, $\nabla \cdot \mathbf{B} = 0$).

These simplifications have allowed us to remove the effect of the $\mathbf{E}$ field on our calculations.

# Magnetic Circuits and Materials

- A second simplifying assumption involves the concept of magnetic circuits.
- The general solution for the **H** and the **B** in a structure of complex geometry is extremely difficult if not practically impossible.
- However, a 3D field problem can often be reduced to what is essentially a circuit equivalent of magnetic elements,
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- The core is assumed to be composed of magnetic material with permeability much greater than of surrounding air (i.e., $\mu_r \gg \mu_0$).
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- Due to high permeability of the material, the magnetic flux is confined almost entirely to the core,
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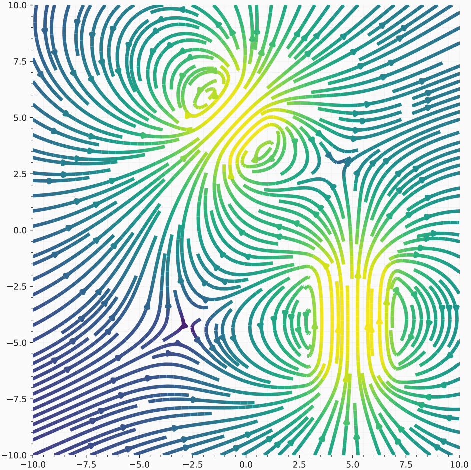


**Figure 6:** An example demonstration of flux paths passing highly permeable material.

- The source of the magnetic field in the core is the ampere-turn $Ni$.
- In magnetic circuit terminology $Ni$ is the magnetomotive force (mmf) acting on the magnetic circuit.
- In systems with more than one winding, $Ni$ must be replaced by the algebraic sum of the ampere-turns of all the windings.
- The magnetic flux $\phi$ crossing a surface $\mathcal{S}$ is the surface integral of the normal component of $\mathbf{B}$:

$$\phi = \int_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$$

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- This equation states that all flux entering the surface enclosing a volume must leave the volume as they form closed loops.
- This is enough to justify the uniformity of the magnetic flux density across the cross section of a magnetic circuit:

$$\phi_{\mathrm{c}} = B_{\mathrm{c}} A_{\mathrm{c}}$$

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- Using the original magneto-static simplifications, we can come up to:

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- The relationship between the **H** and **B** is a property of the material.
- It is common to assume a linear relationship shown as:

$$\mathbf{B} = \mu \mathbf{H},$$

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- Transformers are wound on closed cores.
- However, EEC devices which incorporate a moving element must have air gaps in their magnetic circuits.
- When the air-gap ($g$) is much smaller than the dimensions of the adjacent core faces, the magnetic flux will follow the path defined by the core and the air gap and the techniques of magnetic-circuit analysis can be used.

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- Provided the air-gap length $g$ is sufficiently small, the configuration can be analysed as a magnetic circuit with two (2) series components:
  - a magnetic core of $\mu$, cross-sectional area $A_{\mathrm{c}}$, and mean length $l_{\mathrm{c}}$,
  - an air gap of permeability $\mu_0$, cross-sectional area $A_{\mathrm{g}}$, and length $l_{\mathrm{g}}$.
- In the core the flux density can be assumed uniform; thus

$$B_{\mathrm{c}} = \frac{\phi}{A_{\mathrm{c}}} \quad \text{in the air gap} \quad B_{\mathrm{g}} = \frac{\phi}{A_{\mathrm{g}}}$$

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# Magnetic Circuits and Materials

- Applying this to the magnetic circuit gives:

$$\mathcal{F} = H_c l_c + H_g g,$$

using the **B-H** relationship:

$$\mathcal{F} = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g.$$

- Here, $\mathcal{F}$ is the mmf applied to the magnetic circuit where $\mathcal{F}_c = H_c l_c$ is the magnetic field in the core and $\mathcal{F}_g = H_g l_g$ is the magnetic field in the air-gap.
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$$\mathcal{F} = \phi \left( \frac{l_{\mathrm{c}}}{\mu A_{\mathrm{c}}} + \frac{g}{\mu_0 A_{\mathrm{g}}} \right).$$

- The term which multiplies mmf to flux is called Reluctance.

$$\mathcal{F} = \phi\left(\overbrace{\frac{l_{\mathrm{c}}}{\mu A_{\mathrm{c}}}}^{\mathcal{R}_{\mathrm{c}}} + \overbrace{\frac{g}{\mu_0 A_{\mathrm{g}}}}^{\mathcal{R}_{\mathrm{g}}}\right) \quad \text{or} \quad \mathcal{F} = \phi\left(\mathcal{R}_{\mathrm{c}} + \mathcal{R}_{\mathrm{g}}\right)$$

- Reluctance can be seen as analogous to resistance to electrical circuits.

**Example**

The magnetic circuit shown has dimensions:

$$A_c = A_g = 9\ \text{cm}^2,$$

$$g = 0.050\ \text{cm},$$

$$l_c = 30\ \text{cm},$$

$$N = 500\ \text{Turns}.$$

Assume the value $\mu_0 = 70{,}000\ \text{H} \cdot \text{m}^{-1}$ for core material.

(a) Find the reluctance values $\mathcal{R}_c$ and $\mathcal{R}_g$.

(b) For the value of $B_c = 1$ T, find the flux $\phi$ and the current $i$.

**Example**

The following magnetic structure has infinite permeability ($\mu \to \infty$). Find the air-gap flux $\phi$ and flux density $B_{\mathrm{g}}$.

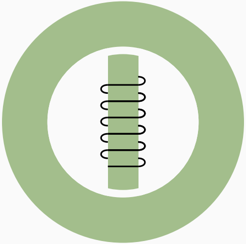

**Figure 7:** A simple synchronous drive.

Parameters: $I = 10$ A, $N = 1000$ Turns, $g = 1$ cm, and $A_{\mathrm{g}} = 2000$ cm$^2$.

- When a **B**-field varies with time, an **E**-field is produced in space as determined by Faraday's law [5]:

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a}.$$

> The line integral of the electric field intensity E around a closed contour C is equal to the time rate of change of the magnetic flux linking (i.e. passing through) that contour.

> For working with electric drives, the effects of **E** field can be neglected.

- This simplification allows us to cancel the LHS of the equation to just the induced voltage (i.e., electromotive force)

$$\underbrace{\oint_C \mathbf{E} \cdot d\mathbf{s}}_{\mathcal{E}} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{a}.$$

- Whereas on the RHS, the flux is dominated by the core flux ($\varphi$).

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- As winding links the core flux $N$ times, our equation is simplified to:

$$\mathcal{E} = N\frac{d\phi}{dt} = \frac{d\lambda}{dt} \quad \text{where} \quad \lambda = N\phi \quad \blacksquare$$

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# Magnetic Circuits and Materials

- For a magnetic circuit composed of magnetic material of constant magnetic permeability or which includes a dominating air gap, the relationship between $\phi$ and $i$ will be linear and we can define the inductance $L$ as:

$$L = \frac{\lambda}{i} \quad \text{which can be written as} \quad L = \frac{N^2}{\mathcal{R}_{\text{tot}}}$$

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**Example**

The magnetic circuit consists of an $N$-turn winding on a magnetic-core of infinite permeability with two parallel air gaps of lengths $g_1$ and $g_2$ and areas $A_1$ and $A_2$ respectively. Find:

(a) The winding inductance,

(b) The flux density $B_1$ in gap 1 when the winding is carrying current $i$.

**Note:** Neglect fringing effects at the air gap.

- The mmf ($\mathcal{F}$) is given by the total ampere-turns.
- The reference directions for the currents ($i_1$, $i_2$) have been chosen to produce flux in the same direction.
- The total mmf is therefore:

$$\mathcal{F} = N_1 i_1 + N_2 i_2.$$

- With the reluctance of the core neglected and assuming that $A_c = A_g$, the core flux $\phi$ is:

$$\phi = (N_1 i_1 + N_2 i_2)\,\frac{\mu_0 A_c}{g},$$

where $\phi$ is the resultant core flux produced by the total mmf of the two windings.

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- This resultant $\phi$ determines the operating point of the core material.
- If $\phi$ is broken up into terms attributable to the individual currents, the resultant flux linkages of coil 1 can be expressed as:

$$\lambda_1 = N_1\phi = \overbrace{N_1^2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{11}} i_1 + \overbrace{N_1 N_2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{12}} i_2,$$

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- It cannot be rigorously applied in situations where the nonlinear characteristics of magnetic materials, dominate the performance of the magnetic system.
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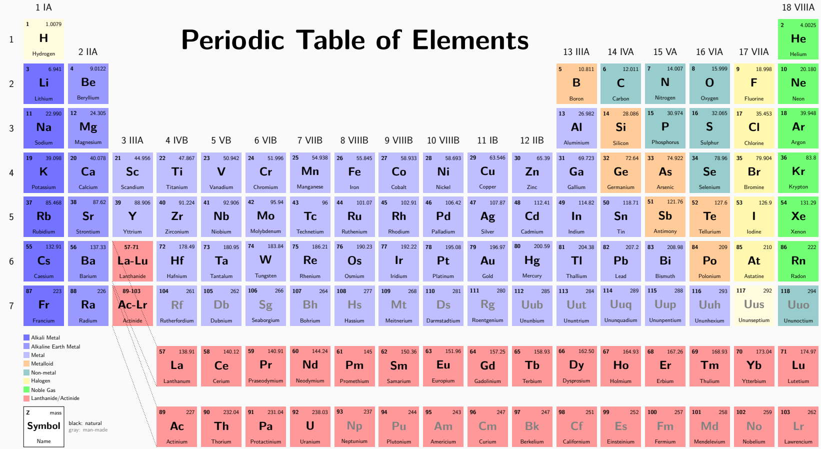


**Figure 8:** The periodic table of elements.

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- These materials are characterised by a wide range of properties,
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- Typically composed of iron (Fe) and alloys of iron with:
  - cobalt, tungsten, nickel, aluminum, and other metals
- These materials are characterised by a wide range of properties,
  - but, the basic phenomena responsible for their properties are common to them all.
- They are found to be composed of a large number of domains, i.e., regions in which the magnetic moments of all the atoms are parallel, giving rise to a net magnetic moment for that domain.
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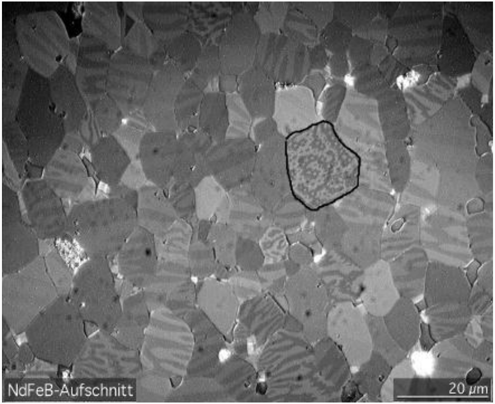


**Figure 9:** Microcrystalline grains within a piece of Nd2Fe14B (the alloy used in neodymium magnets) with magnetic domains made visible with a Kerr microscope. The domains are the light and dark stripes visible within each grain. The outlined grain has its magnetocrystalline axis almost vertical, so the domains are seen end-on [4].

- When **B**-field is applied to the material, the magnetic domain moments tend to align with the applied magnetic field.
- As a result, the magnetic moments add to the applied field, producing a much larger value of flux density than would exist due to the magnetizing force alone.
- Therefore the effective permeability $\mu$, equal to the ratio of the total magnetic flux density to the applied magnetic-field intensity, is large compared with the permeability of free space ($\mu_0$).
- As the magnetising force is increased, this behavior continues until all the magnetic moments are aligned with the applied field; at this point they can no longer contribute to increasing the magnetic flux density
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## Magnetic Circuits and Materials

- Without an external **B**-field, magnetic domains naturally align along certain directions associated with the crystal structure of the domain.
  - This is known as axes of easy magnetisation [6].
- Thus if the applied **B** is reduced, the domain magnetic moments relax to the direction of easy magnetism nearest to the applied field.
- As a result, **B** is reduced to zero (0), although they will tend to relax towards their initial orientation, the magnetic dipole moments will no longer be totally random in their orientation;
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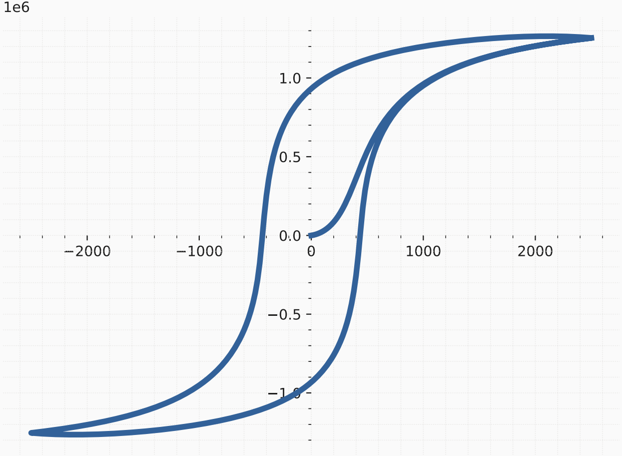


**Figure 10:** A Hysteresis Curve (B-H) Generated using the Jiles-Artherton model.

## 1st Quadrant

- The domains are very small, below the single domain size where there is resistance to demagnetization [1].
- The domains start to increase in size. For a small interval, the magnetization will be reversible.
- As the field increases, magnetization will no longer reverse to zero but move on a minor hysteresis loop.
- Eventually the curve starts to bend over to the right.
  - It will still increase as more magnetic domains reach their full size and their magnetizations become parallel to the external field [9].

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## 1st Quadrant - Saturation

- Eventually the **B**-field will become high enough where no more change in the magnetization occurs.
  - This is called technical saturation.
- technology is sufficient to reach 99+% of “technical saturation.”
- As the field is backed off from “saturation”, the magnetization declines very slightly to the $B_r$ point.
  - This is the remanence, remanent induction, or residual induction [8].

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## 2nd Quadrant - Demagnetisation

- In this quadrant the applied field opposes the materials **B**-field.
- As the external **B**-field increases in magnitude, some domains will reverse.
- At the knee of the demagnetisation curve, this increase has become rapid and the magnetisation will fall to the Hci point.
- At $H_{\text{ci}}$, the number of domains aligned with the original magnetization is the same as the number aligned with the opposing magnetic field.

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## 3rd Quadrant - Re-magnetisation

- The total magnetization of the part will be reversed.
- If we go far enough, magnetization will reach the saturation level in the negative direction.

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## 4th Quadrant - Demagnetisation

- After fully reversing the magnet and removing the field in the third quadrant, magnetization will recoil to a point that is the negative of the $B_r$ observed when in the first and second quadrant.
- If we apply additional field in the positive direction, we duplicate the second quadrant curve.

### 4th Quadrant - Demagnetisation

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- In AC power systems, voltage and flux wave-forms closely approximate sinusoidal functions over time.
- Time to describe the excitation characteristics and losses associated with steady-state ac operation of magnetic materials.
- Assume a closed-core magnetic circuit (i.e., with no air gap),
- And a sinusoidal variation of the core flux ($\phi$) with the following:

$$\phi(t) = \phi_{\text{max}} \sin(wt) = A_{\text{c}} B_{\text{max}} \sin(wt)$$

where:

$\phi_{max}$ amplitude of core flux in Wb,
$B_{max}$ Amplitude of flux density $B_{c}$ in T,
$\omega$ Angular frequency in rad/s,
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- From, the voltage induced in the $N$-turn winding is:

$$e(t) = \omega N \phi_{\max} \cos(wt) = E_{\max} \cos(wt)$$

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$$E_{\max} = \omega N \phi_{\max} = 2\pi f N A_c B_{\max}$$

- In steady-state AC, it is more important to use RMS rather than instant values.
- Generally, the rms value of a periodic function of time $f(t)$, of $T$ is:

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- To produce magnetic flux in the core requires current in the exciting winding known as the exciting current, $i_{\phi}$,
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- There are two (2) mechanisms associated with time varying **B**-fields.

### Ohmic Heating

- Associated with induced currents in the core material.
- From Faraday's law, we see that **B**-fields give rise to **E**-fields.
- In magnetic materials these **E**-fields result in induced currents,
  - These are known as eddy currents, circulating in the core material, opposing the material's internal **B**-field [3].

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- These laminations, which are aligned in the direction of the field lines, are insulated from each other by an oxide layer on their surfaces or by a thin coat of insulating enamel or varnish.
- This greatly reduces the magnitude of the eddy currents since the layers of insulation interrupt the current paths; the thinner the laminations, the lower the losses.
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- Due to the hysteretic nature of magnetic material.
- In a magnetic circuit or the transformer, a time-varying excitation ($i_\phi$) will cause the magnetic material to undergo a cyclic variation described by a hysteresis loop.
- The energy input $W$ to the magnetic core as the material undergoes a single cycle is shown to be:

$$W = \oint i_\phi \, \mathrm{d}\lambda = \oint \left( \frac{H_c l_c}{N} \right) (A_c N \, \mathrm{d}B_c) = A_c l_c \oint H_c \, \mathrm{d}B_c$$

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- Nearly all transformers and certain sections of electric machines use sheet-steel material that has highly favorable directions of magnetization along which the core loss is low and the permeability is high.
- This material is termed grain-oriented steel [12].
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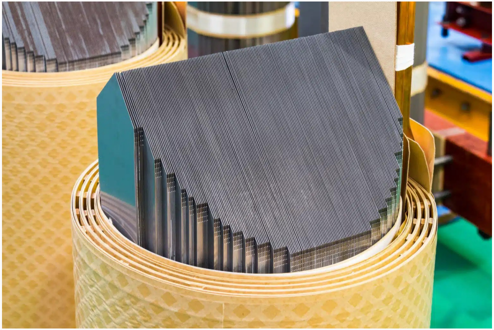

**Figure 11:** An example of Grain Oriented Steel used in industry [7].

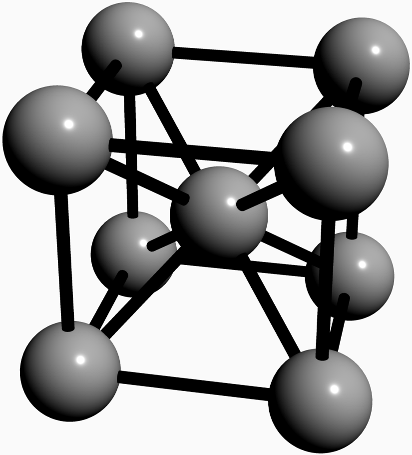

**Figure 12:** The atomic structure of grain oriented steel.

- Each crystalline cube has an atom at each corner as well as one in the center of the cube.
- In the cube, the easiest axis of magnetization is the cube edge.
- The diagonal across the cube face is more difficult
- By suitable manufacturing techniques most of the crystalline cube edges are aligned in the rolling direction to make it the favorable direction of magnetisation.

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- The behaviour in this direction is superior in core loss and permeability to non-oriented steels in which the crystals are randomly oriented to produce a material with characteristics which are uniform in all directions.
- As a result, oriented steels can be operated at higher flux densities than the nonoriented grades.
- Non-oriented electrical steels are used in applications where the flux does not follow a path which can be oriented with the high-Temperature rolling direction or where low cost is of importance.
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# Appendix

- Assume the following simplified induction drive equivalent circuit.

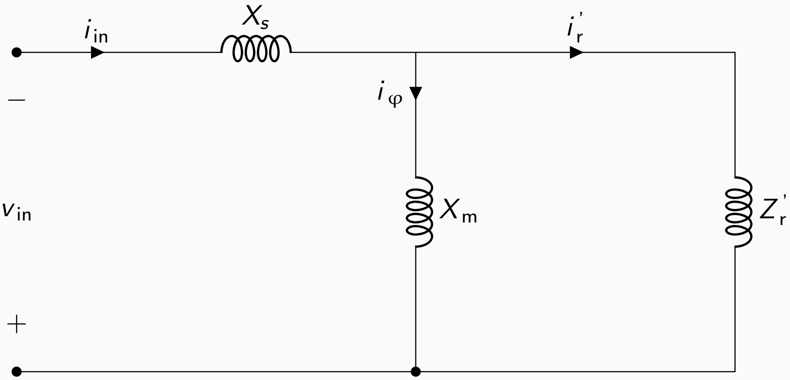


**Figure 13:** An abstract rendition of the simplified induction circuit equivalent circuit.

# Appendix

- where:

$$Z_s = R_s + \mathbf{j} X_s,$$
$$Z_m = \frac{R_c \times \mathbf{j} X_m}{R_c + \mathbf{j} X_m},$$
$$Z_r = \frac{R_r^{'}}{s} + jX_r^{'}.$$

- From Kirchhoff's current law, the following statement holds true:

$$i_{in} = i_{\varphi} + i_r^{'}.$$

- The second identity can be derived from parallel circuit principles;

$$Z_m i_{\varphi} = Z_r^{'} i_r \quad \rightarrow \quad i_m = i_R^{'} \times \left(\frac{Z_R}{Z_m}\right).$$

# Appendix

- We can isolate the magnetizing current ($i_\varphi$).

$$i_\varphi = i_r^{'} \times \left(\frac{Z_r}{Z_m}\right) + i_r^{'}.$$

- Isolating the rotor current ($i_r^{'}$) gives us the final expression:

$$i_r^{'} = \frac{i_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

- Consider the following equivalent circuit of an IM.

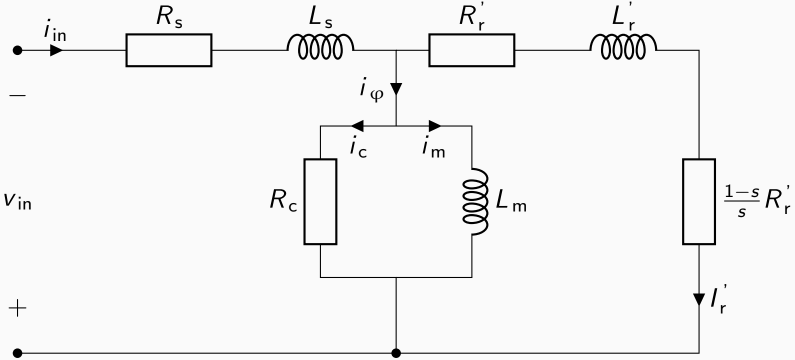


**Figure 14:** An abstract rendition of the induction circuit equivalent circuit.

# Appendix

- We can start by summing up the rotor side.

$$Z_r = \frac{R_r^{'}}{s} + \mathbf{j}\, w_r L_r^{'} \quad \rightarrow \quad Z_r = \frac{R_r^{'}}{s} + X_r^{'}.$$

- The stator side is calculated as:

$$Z_s = R_s + \mathbf{j}\, w_s L_s \quad \rightarrow \quad Z_s = \frac{R_s}{s} + X_s.$$

- The magnetising side is calculated as:

$$Z_m = \frac{R_c \times \mathbf{j}\, w_s L_m}{R_c + \mathbf{j}\, w_s L_m} \quad \rightarrow \quad Z_m = \frac{R_c \times X_m}{R_c + X_m}.$$

# Appendix

- Without substitution, the input impedance is calculate to be

$$Z_{\text{in}} = Z_{\text{s}} + \frac{Z_{\text{m}} \times Z_{\text{r}}}{Z_{\text{m}} + Z_{\text{r}}}.$$

- From here, we can derive the stator current ($I_{\text{in}}$).

$$V_{\text{ph}} = \frac{Z_{\text{in}}}{I_{\text{in}}} \quad \rightarrow \quad I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}}.$$

- Now, the Torque of an IM is related to the air-gap power by:

$$T_{\text{mech}} = \frac{P_{\text{gap}}}{w_{\text{s}}} \qquad \text{and} \qquad P_{\text{gap}} = n_{\text{ph}} \frac{I_{\text{r}}^2}{w_{\text{s}}} \left( \frac{R_2^{'}}{s} \right)$$

# Appendix

- Using the stator-rotor relationship, we can obtain our final result.

$$I_r^{'} = \frac{I_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

# Appendix

Go Back

an empirical relationship or phenomenological relationship is a relationship or correlation that is supported by experiment or observation but not necessarily supported by theory.

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