Lecture Book
B.Sc Electric Drives

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This document is based on the books on the topics of Magnetic Circuits, Magnetic Materials, Induction and Synchonous Drives, DC and Solid State Drives which includes *Advanced electric drives: analysis control and modeling using MATLAB/Simulink* by N. Mohan, *Analysis of Electric Machinery and Drive Systems* by Krause, Paul C. et. al., *Design of rotating electrical machines* by Pyrhonen Juha et. al , *Electric Machinery Fundamentals* by Stephen J. Chapman., *Electric Motors and Drives: Fundamentals Types and Applications* by Hughes A. et. al., *Electrical Machines and Drives: Fundamentals and Advanced Modelling* by Melkebeek A, *Electrical machines, drives, and power systems* by Wildi T., *Fundamentals of Electrical Drives* by Veltman A. et. al., and *Electric Machinery* by A. E. Fitzgerald. et. al.

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Contents

Chapter 1

Magnetic Circuits and Materials

Example A Simple Magnetic Circuit 1

A magnetic circuit is formed using an iron core with relative permeability (μ_r = 1000) and single air gap, and has the following parameters:

- Cross-sectional area ($A_C = 0.0016\, m^2$),
- Mean core length ($l_C = 0.8\, m$),
- Air gap length ($g = 0.002\, m$).

A coil of 100 turns (N = 100) is wound on the iron core.

1. Calculate the reluctance of the core $\mathcal{R}_C$ and that of the gap $\mathcal{R}_G$ neglecting the effects of fringing fields at the air gap and leakage flux from the coil. ($\mu_0 \approx 4\pi \times 10^{-7} H/m$)
2. A current of $i = 2$ A now flows in the coil. Calculate the total flux, the flux linkage and the coil inductance L.
3. Design a new coil and choose an operating current to double ($\pm$0.1) the flux linkage while keeping the coil inductance below 12 mH.

Solution A Simple Magnetic Circuit

1. Reluctance of a magnetic circuit is calculated by using the following formula;

$$\mathcal{R} = \frac{l}{\mu_0 \mu_r A},$$

where $\mathcal{R}$ is reluctance, μ_0 is the permeability of free-space, μ_r is the relative permeability and A is the cross-sectional area of the material.

Plugging the values of the question to this equation produces the following result for the air gap for the air gap ($\mathcal{R}_g$) and the core ($\mathcal{R}_c$):

$$\mathcal{R}_g = \frac{0.002}{(4\pi \times 10^{-7}) \times (1.6 \times 10^{-3})} \quad \mathrm{H}^{-1}, \tag{1.1}$$

$$\mathcal{R}_g = 9.95 \times 10^5 \quad \mathrm{H}^{-1} \text{ or } \mathrm{A/Wb}, \tag{1.2}$$

$$\mathcal{R}_c = \frac{0.8}{(4\pi \times 10^{-7}) \times (1000) \times (1.6 \times 10^{-3})} \quad \mathrm{H}^{-1}, \tag{1.3}$$

$$\mathcal{R}_c = 3.98 \times 10^5 \text{ or } \mathrm{A/Wb} \quad \blacksquare \tag{1.4}$$

2. The relation for flux (ϕ) and reluctance ($\mathcal{R}$) is shown as:

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}},$$

$$\Phi = \frac{Ni}{\mathcal{R}_c + \mathcal{R}_g} = \frac{100 \times 2}{1.4 \times 10^6} = 1.44 \times 10^{-4}\,\mathrm{Wb}.$$

The flux-linkage (λ) of the magnetic circuit is:

$$\lambda = N\Phi = 100 \times 1.43 \times 10^{-4} = 1.44 \times 10^{-2} \ \mathrm{Wb-turns}. \tag{1.5}$$

Using these relations, inductance (L) is calculated to be:

$$L = \frac{\lambda}{i} = \frac{1.43 \times 10^{-2}}{2} = 7.18 \times 10^{-3} \ \mathrm{H} \quad \blacksquare \tag{1.6}$$

3. In this design question we need to double the flux linkage (λ), but we need to keep the inductance (L) below 12 mH.

Therefore we need to increase N along with i. Let's choose N=128 turns and i = 2.5 A. Using these values the new flux value is:

$$\Phi = \frac{Ni}{\mathcal{R}} = \frac{128 \times 2.5}{1.4 \times 10^6} = 2.29 \times 10^{-4} \ \mathrm{Wb} \tag{1.7}$$

$$\lambda = 128 \times (2.29 \times 10^{-4}) = 0.0292 \ \mathrm{Wb-turns} \tag{1.8}$$

This is a 2.05 times increase which is acceptable within the error margins.

$$L = \frac{\lambda}{i} = \frac{0.0292}{2.5} = 11.7 \ \mathrm{mH} < 12 \ \mathrm{mH} \quad \blacksquare \tag{1.9}$$

Chapter 2

Rotating Magnetic Fields

Example Six-phase Rotating Magnetic Field 2

We have seen a three-phase rotating magnetic field (RMF) and how it was generated. For this exercise, here is a six-phase current waveform input. Please calculate the generated magnetic flux density ($\mathbf{B}_{\text{net}}$).

The currents flowing through each phases (A, B, C, D, E, F) are as follows:

$$i_{\mathbf{AA'}} = I_{\text{max}} \sin(wt), \qquad i_{\mathbf{BB'}} = I_{\text{max}} \sin(w\,t - 60^\circ),$$
$$i_{\mathbf{CC'}} = I_{\text{max}} \sin(w\,t - 120^\circ), \qquad i_{\mathbf{DD'}} = I_{\text{max}} \sin(w\,t - 180^\circ),$$
$$i_{\mathbf{EE'}} = I_{\text{max}} \sin(w\,t - 240^\circ), \qquad i_{\mathbf{FF'}} = I_{\text{max}} \sin(w\,t - 300^\circ).$$

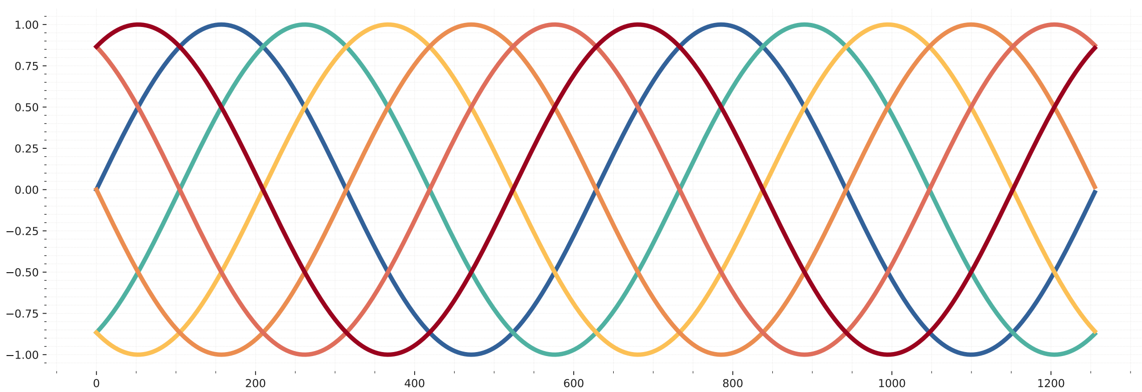

Figure 2.1: A balanced 6-phase waveform.

Solution Six-phase Rotating Magnetic Field

Let us first consider, what happens at the time instant of the voltage waveforms. Unlike a three-phase system, at the instant $\theta = 90^\circ$, the voltage in phase **A** is positive maximum (1.0), phases **B** and **F** are half amplitude (0.5), **E** and **C** are also half amplitude but opposite

sign (-0.5) and phase **D** is negative maximum (-1.0).
These six (6) wave-forms are represented by the following equations:

$$v_{AA'} = V_{max} \sin(wt), \qquad v_{BB'} = V_{max} \sin(w\,t - 60°), \tag{2.1a}$$
$$v_{CC'} = V_{max} \sin(w\,t - 120°), \qquad v_{DD'} = V_{max} \sin(w\,t - 180°), \tag{2.1b}$$
$$v_{EE'} = V_{max} \sin(w\,t - 240°), \qquad v_{FF'} = V_{max} \sin(w\,t - 300°). \tag{2.1c}$$

where $\omega = 2\pi \quad \mathrm{rad \cdot s^{-1}}$, V_{max} is the maximum value of the voltage or induced EMF in each phase and f is the supply frequency (Hz).

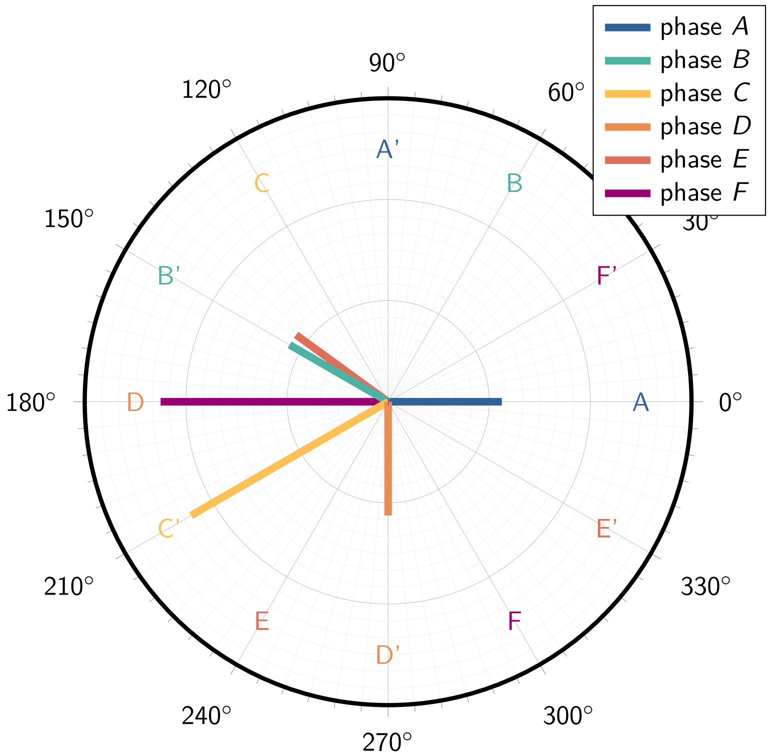

Figure 2.2: Polar coordinate form of the acting magnetic flux densities for the six phases at the electrical angle of $\theta_e = 30°$.

If the flux densities (or fluxes (ϕ) as both are valid approaches) are added to find the resultant magnetic flux density in phasor form the magnitude is found as;

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= B_{\mathbf{AA'}} \sin(wt) \, \underline{/0^\circ} + B_{\mathbf{BB'}} \sin(w\,t - 60^\circ) \, \underline{/60^\circ} \\
&+ B_{\mathbf{CC'}} \sin(w\,t - 120^\circ) \, \underline{/120^\circ} + B_{\mathbf{DD'}} \sin(w\,t - 180^\circ) \, \underline{/180^\circ} \\
&+ B_{\mathbf{EE'}} \sin(w\,t - 240^\circ) \, \underline{/240^\circ} + B_{\mathbf{FF'}} \sin(w\,t - 300^\circ) \, \underline{/300^\circ},
\end{aligned} \tag{2.2a}
$$

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= B_{\text{max}} \sin(wt) \left[\hat{\mathbf{x}} \cos(0^\circ) + \hat{\mathbf{y}} \sin(0) \right] \\
&+ B_{\text{max}} \sin(w\,t - 60^\circ) \left[\hat{\mathbf{x}} \cos(60^\circ) + \hat{\mathbf{y}} \sin(60^\circ) \right] \\
&+ B_{\text{max}} \sin(w\,t - 120^\circ) \left[\hat{\mathbf{x}} \cos(120^\circ) + \hat{\mathbf{y}} \sin(120^\circ) \right] \\
&+ B_{\text{max}} \sin(w\,t - 180^\circ) \left[\hat{\mathbf{x}} \cos(180^\circ) + \hat{\mathbf{y}} \sin(180^\circ) \right] \\
&+ B_{\text{max}} \sin(w\,t - 240^\circ) \left[\hat{\mathbf{x}} \cos(240^\circ) + \hat{\mathbf{y}} \sin(240^\circ) \right] \\
&+ B_{\text{max}} \sin(w\,t - 300^\circ) \left[\hat{\mathbf{x}} \cos(300^\circ) + \hat{\mathbf{y}} \sin(300^\circ) \right]
\end{aligned} \tag{2.2b}
$$

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= B_{\text{max}} \times \sin(wt) \times \left[1\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}} \right] \\
&+ B_{\text{max}} \times \sin(w\,t - 60^\circ) \times \left[\frac{1}{2}\,\hat{\mathbf{x}} + \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}} \right] \\
&+ B_{\text{max}} \times \sin(w\,t - 120^\circ) \times \left[-\frac{1}{2}\,\hat{\mathbf{x}} + \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}} \right] \\
&+ B_{\text{max}} \times \sin(w\,t - 180^\circ) \times \left[-1\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}} \right] \\
&+ B_{\text{max}} \times \sin(w\,t - 240^\circ) \times \left[-\frac{1}{2}\,\hat{\mathbf{x}} - \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}} \right] \\
&+ B_{\text{max}} \times \sin(w\,t - 300^\circ) \times \left[\frac{1}{2}\,\hat{\mathbf{x}} - \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}} \right]
\end{aligned} \tag{2.2c}
$$

Let's rearrange the equation by their cardinal components, namely $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$.

$$
\begin{aligned}
\mathbf{B}_{\text{net}} = &\Big[B_{\text{max}} \sin(wt) + \frac{1}{2} B_{\text{max}} \sin(w\,t - 60^\circ) \\
&- \frac{1}{2} B_{\text{max}} \sin(w\,t - 120^\circ) - 1 B_{\text{max}} \sin(w\,t - 180^\circ) \\
&- \frac{1}{2} B_{\text{max}} \sin(w\,t - 240^\circ) + \frac{1}{2} B_{\text{max}} \sin(w\,t - 300^\circ) \Big] \,\hat{\mathbf{x}} \\
&+ \Big[\frac{\sqrt{3}}{2} B_{\text{max}} \sin(w\,t - 60^\circ) + \frac{\sqrt{3}}{2} B_{\text{max}} \sin(w\,t - 120^\circ) \\
&- \frac{\sqrt{3}}{2} B_{\text{max}} \sin(w\,t - 240^\circ) - \frac{\sqrt{3}}{2} B_{\text{max}} \sin(w\,t - 300^\circ) \Big] \,\hat{\mathbf{y}}
\end{aligned} \tag{2.3}
$$

We need to do the following trigonometric conversions if we want to simplify the aforementioned equation of a mess

These conversions are based on the following trigonometric identities:

$$\sin(x + 180^\circ) = -\sin(x), \tag{2.4a}$$

$$\sin(x + 360^\circ) = +\sin(x). \tag{2.4b}$$

$$\sin(w\,t + 300^\circ + 360^\circ) = +\sin(w\,t + 60^\circ)\,, \tag{2.5a}$$

$$\sin(w\,t - 240^\circ + 360^\circ) = +\sin(w\,t + 120^\circ)\,, \tag{2.5b}$$

$$\sin(w\,t - 180^\circ + 360^\circ) = -\sin(wt)\,. \tag{2.5c}$$

Inserting these equations to Eq. (2.3),

$$\begin{aligned}
\mathbf{B}_{\text{net}} = &\Big[B_{\text{max}}\sin(wt) + \frac{1}{2}B_{\text{max}}\sin(w\,t - 60^\circ) \\
&- \frac{1}{2}B_{\text{max}}\sin(w\,t - 120^\circ) + B_{\text{max}}\sin(wt) \\
&- \frac{1}{2}B_{\text{max}}\sin(\omega t + 120^\circ) + \frac{1}{2}B_{\text{max}}\sin(w\,t + 60^\circ)\Big]\,\hat{\mathbf{x}} \\
&+ \Big[\frac{\sqrt{3}}{2}B_{\text{max}}\sin(w\,t - 60^\circ) + \frac{\sqrt{3}}{2}B_{\text{max}}\sin(w\,t - 120^\circ) \\
&- \frac{\sqrt{3}}{2}B_{\text{max}}\sin(\omega t + 120^\circ) - \frac{\sqrt{3}}{2}B_{\text{max}}(\sin(w\,t + 60^\circ)\Big]\,\hat{\mathbf{y}}
\end{aligned} \tag{2.6}$$

Now we can do some tidying up and do some arrangements of these sine values;

$$\begin{aligned}
\mathbf{B}_{\text{net}} = &\Big[2B_{\text{max}}\sin(wt) + \frac{1}{2}B_{\text{max}}\Big(\sin(w\,t - 60^\circ) + \sin(w\,t + 60^\circ)\Big) \\
&- \frac{1}{2}B_{\text{max}}\Big(\sin(w\,t - 120^\circ) + \sin(\omega t + 120^\circ)\Big)\Big]\,\hat{\mathbf{x}} \\
&+ \Big[-\frac{\sqrt{3}}{2}B_{\text{max}}\Big(\sin(w\,t - 60^\circ) - \sin(\omega t + 60^\circ)\Big) \\
&- \frac{\sqrt{3}}{2}B_{\text{max}}\Big(\sin(\omega t + 120^\circ) - \sin(w\,t - 120^\circ)\Big)\Big]\,\hat{\mathbf{y}}
\end{aligned} \tag{2.7}$$

Now it is time to do some trigonometric manipulation

These conversions are based on the following trigonometric identities:

$$\sin(A + B) + \sin(A - B) = 2\sin(A)\cos(B), \tag{2.8}$$

$$\sin(A + B) - \sin(A - B) = 2\cos(A)\sin(B). \tag{2.9}$$

and come up with our final equation to explain this rotating magnetic field for a six-phase system.

$$\begin{aligned}\mathbf{B}_{\text{net}} = &\Big[2B_{\text{max}} \sin(wt) + \frac{1}{2}B_{\text{max}}\Big(2\sin(wt)\cos(60^\circ)\Big) \\ &- \frac{1}{2}B_{\text{max}}\Big(2\sin(wt)\cos(120^\circ)\Big)\Big]\,\hat{\mathbf{x}} \\ &+ \Big[-\frac{\sqrt{3}}{2}B_{\text{max}}\Big(2\cos(\omega t)\sin(60^\circ)\Big) \\ &- \frac{\sqrt{3}}{2}B_{\text{max}}\Big(2\sin(wt)\cos(120^\circ)\Big)\Big]\,\hat{\mathbf{y}}\end{aligned} \tag{2.10a}$$

$$\begin{aligned}\mathbf{B}_{\text{net}} = &B_{\text{max}}\Big[2B_{\text{max}} \sin(wt) + 0.5\sin(wt) + 0.5\sin(wt)\Big]\,\hat{\mathbf{x}} \\ &+ B_{\text{max}}\Big[-\frac{3}{2}\cos(\omega t) - \frac{3}{2}\cos(\omega t)\Big]\,\hat{\mathbf{y}}\end{aligned} \tag{2.10b}$$

$$\mathbf{B}_{\text{net}} = 3B_{\text{max}}\sin(wt)\,\hat{\mathbf{x}} - 3B_{\text{max}}\cos(wt)\,\hat{\mathbf{y}}, \tag{2.11}$$

$$\mathbf{B}_{\text{net}} = \left(\frac{6}{2}\right) B_{\text{max}}\left[\sin(wt)\,\hat{\mathbf{x}} - \cos(wt)\,\hat{\mathbf{y}}\right] \quad \blacksquare \tag{2.12}$$

Chapter 3

Induction Machine

3.1 Introduction

An Induction Machine (IM) is type of electro-mechanical device which Alternating Current (AC) is supplied to the stator **directly** and to the rotor by induction from the stator. When excited from a balanced poly-phase source (i.e., star or delta), it will produce a magnetic field in the air gap rotating at **synchronous speed** as determined by the number of stator poles and the applied stator frequency f_e. A standard diagram for use in IM is shown below.

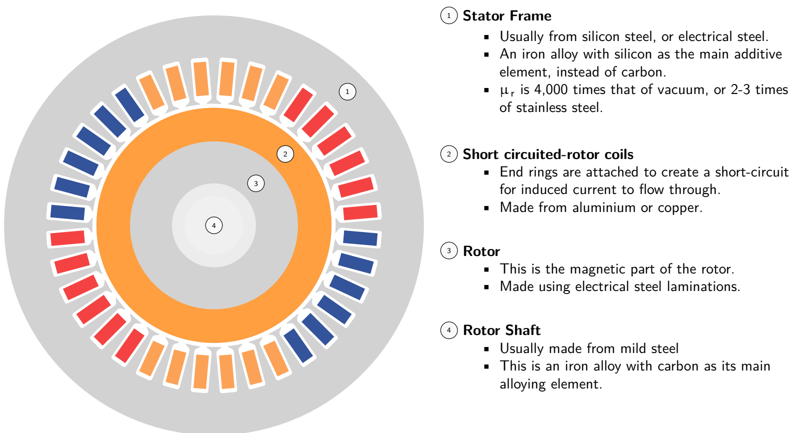

Figure 3.1: A cross-sectional view of a poly-phase IM with descriptions on which material can be used in its construction.

3.1.1 Rotor Types

The rotor of a poly-phase IM may be one of two (2) types.

Wound Rotor built with a polyphase winding similar to, and wound with the same number of poles as, the stator. The terminals of the rotor winding are connected to insulated slip rings mounted on the shaft. Carbon bruhes bearing on these rings make the rotor terminals available external to the machine. Wound-rotor induction machines are relatively uncommon, being found only in a limited number of specialized applications.

Squirrel Cage rotor windings consist of conducting bars embedded in slots in the rotor iron and short-circuited at each end by conducting end rings. The extreme simplicity and ruggedness of the squirrel-cage construction are outstanding advantages of this type of IM and make it by far the most commonly used type of machine in sizes ranging from a less than a kW to a MW.

3.1.2 Speed of Operation

Let us assume that the rotor is turning at the steady speed of n r/min in the same direction as the rotating stator field.
Let the synchronous speed of the stator field be n_s r/min. This difference between synchronous speed and the rotor speed is commonly referred to as the **slip** of the rotor.
The rotor slip is $n_s - n$, as measured in r/min.

> Slip is more usually expressed as a fraction of synchronous speed.

The **fractional slip** (s) is:

$$s = \frac{n_s - n}{n_s} \tag{3.1}$$

The slip is often expressed in percent (%), simply equal to 100 percent times the fractional slip of Eq. (3.1) whereas the rotor speed in r/min can be expressed in terms of the slip and the synchronous speed as:

$$n = (1 - s)n_s \tag{3.2}$$

Similarly, the mechanical angular velocity ω_m can be expressed in terms of the synchronous angular velocity ω_s and the slip as:

$$\omega_m = (1 - s)\omega_s \tag{3.3}$$

The relative motion of the stator flux and the rate of frequency f_r

$$f_r = s f_e \tag{3.4}$$

called the slip frequency of the rotor.

> The electrical behavior of an IM is similar to a transformer but with the additional feature of **frequency transformation** produced by the relative motion of the stator and rotor windings.

> wound-rotor IM can be used as a frequency change.

The terminals of an squirrel-cage IM rotors are internally **short circuited** whereas in a wound-rotor IM it is short circuited externally.
The rotating air-gap flux induces slip-frequency voltages in the rotor windings. The rotor currents are then determined by the magnitudes of the induced voltages and the rotor impedance at slip frequency.
During startup, the rotor is stationary ($n = 0$), the slip is unity ($s = 1$), and the rotor frequency equals the stator frequency f_e.
The field produced by the rotor currents therefore revolves at the same speed as the stator field, and a starting torque results, ending to turn the rotor **in the direction of the stator field rotation**.
If this torque is sufficient to overcome the opposition to rotation created by the shaft load, the machine will come up to its operating speed.

> The operating speed can never equal the synchronous speed as the rotor conductors would then be stationary with respect to the stator field; no current would be induced in them, and hence no torque would be produced.

With rotor revolving in the same direction of the stator field, the frequency of the rotor currents is sf_e and they will produce a rotating flux wave which will rotate at sn_n r/min with respect to the rotor in the forward direction.
But superimposed on this rotation is the mechanical rotation of the rotor at n r/min.
Thus, with respect to the stator, the speed of the flux wave produced by the rotor currents is the sum of these two speeds and equals

$$sn_s + n = sn_s + n_s(1 - s) = n_s \tag{3.5}$$

From Eq. (3.5) we see the rotor currents produce an air-gap flux wave which rotates at synchronous speed and hence in synchronism with that produced by the stator currents. Because the stator and rotor fields each rotate synchronously, they are stationary with respect to each other and produce a steady torque, thus maintaining rotation of the rotor. Such torque, which exists for any mechanical rotor speed n other than synchronous speed, is called an **asynchronous torque**.

3.1.3 Normal Operation

Under normal running conditions the slip (s is small: 2 to 10 percent at full load in most squirrel-case IMs. The rotor frequency ($f_r = s/\epsilon_r$) is very low (of the order of 1 to 5 Hz in

50-Hz machines). In this range the rotor impedance is **largely resistive and independent of slip.**

Approximate proportionality of torque with slip is therefore to be expected in the range where the slip is small. As slip increases, the rotor impedance increases because of the increasing contribution of the rotor leakage inductance. Therefore, the rotor current is less than proportional to slip.

The result is that the torque increases with increasing slip up to a maximum value and then decreases. The maximum torque, or **breakdown torque**, which is typically a factor of two larger than the rated machine torque, limits the short-time overload capability of the machine.

> The slip at which the peak torque occurs is proportional to the rotor resistance

For squirrel-cage machines this peak-torque slip is relatively small. Thus, the squirrel-cage machine is substantially a constant-speed machine having a few percent drop in speed from no load to full load. In the case of a wound-rotor machine, the rotor resistance can be increased by inserting external resistance, hence increasing the slip at peak-torque, and thus decreasing the machine speed for a specified value of torque.

Wound-rotor IM are generally made to be larger, and therefore are more expensive and require significantly more maintenance than squirrel-cage IMs, this method of speed control is rarely used, and IMs driven from constant-frequency sources tend to be limited to essentially constant-speed applications.

3.2 Analysis of the Equivalent Circuit

The single-phase equivalent circuit can be used to determine a wide variety of steady-state performance characteristics of poly-phase induction machines. These include variations of current, speed, and losses as the load-torque requirements change, as well as the starting torque, and the maximum torque.

The equivalent circuit shows that the total power P_{gap} transferred across the air gap from the stator is:

$$P_{\text{gap}} = n_{\text{ph}} \, I_2^2 \left(\frac{R_2}{s} \right)$$

where n_{ph} is the number of stator phases.

The total rotor I^2R loss, P_{rotor}, can be calculated from the I^2R loss in the equivalent rotor as

$$P_{\text{rotor}} = n_{\text{ph}} \, I_{2h}^2 \, R_2$$

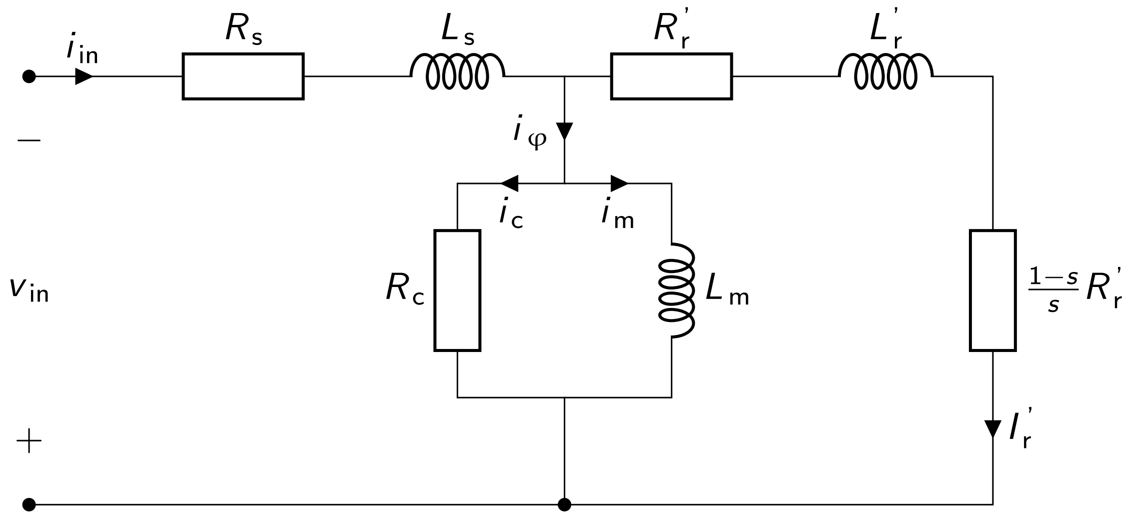

Figure 3.2: The equivalent circuit of an IM.

Since $I_{2h} = I_2$, we can write Eq. 6.18 as

$$P_{\text{rotor}} = n_{\text{ph}}\ I_2^2\ R_2$$

The electromagnetic power P_{mech} developed by the machine can now be determined by subtracting the rotor power dissipation of Eq. 6.19 from the air-gap power of Eq. 6.17.

$$P_{\text{mech}} = P_{\text{gap}} - P_{\text{rotor}} = n_{\text{ph}}\ I_2^2 \left(\frac{R_2}{s}\right) - n_{\text{ph}}\ I_2^2\ R_2$$

or equivalently

$$P_{\text{mech}} = n_{\text{ph}}\ I_2^2\ R_2 \left(\frac{1-s}{s}\right)$$

Comparing Eq. 6.17 with Eq. 6.21 gives

$$P_{\text{mech}} = (1-s)P_{\text{gap}}$$

and

$$P_{\text{rotor}} = sP_{\text{gap}}$$

We see then that, of the total power delivered across the air gap to the rotor, the fraction $1-s$ is converted to mechanical power and the fraction s is dissipated as I^2R loss in the rotor conductors. From this it is evident that an IM operating at high slip is an inefficient device. The electromechanical power per stator phase is equal to the power delivered to the resistance $R_2(1-s)/s$.

Example A Simple Analysis 3

A three-phase, two-pole, 60-Hz IM is observed to be operating at a speed of 3502 r/min with an input power of 15.7 kW and a terminal current of 22.6 A. The stator-winding resistance is 0.20 Ω/phase.

Calculate the I^2R power dissipated in rotor.

Solution A Simple Analysis

The power dissipated in the stator winding is given by:

$$P_{\text{stator}} = 3I_1^2R_1 = 3(22.6)^2 0.2 = 306 \text{ W}$$

Hence the air-gap power is:

$$P_{\text{gap}} = P_{\text{input}} - P_{\text{stator}} = 15.7 - 0.3 = 15.4 \text{ kW}$$

The synchronous speed of this machine can be found from Eq. 4.41

$$n_s = \left(\frac{120}{\text{poles}}\right) f_e = \left(\frac{120}{2}\right) 60 = 3600 \text{ r/min}$$

and hence from Eq. 6.1, the slip is $s = (3600 - 3502)/3600 = 0.0272$. Thus, from Eq. 6.23,

$$P_{\text{rotor}} = s\,P_{\text{trap}} = 0.0272 \times 15.4 \text{ kW} = 419 \text{ W} \quad \blacksquare$$

The electromechanical T_{mech} corresponding to the power P_{mech} can be obtained by recalling that mechanical power equals torque times angular velocity. Thus,

$$P_{\text{mech}} = \omega_m T_{\text{mech}} = (1 - s)\omega_s T_{\text{mech}}$$

For P_{mech} in watts and ω_s in rad/sec, T_{mech} will be in newton-meters. Use of Eqs. 6.21 and 6.22 leads to

$$T_{\text{mech}} = \frac{P_{\text{mech}}}{\omega_m} = \frac{P_{\text{ggp}}}{\omega_s} = \frac{n_{\text{ph}} I_2^2 (R_2/s)}{\omega_s}$$

with the synchronous mechanical angular velocity ω_s being given by

$$\omega_s = \frac{4\pi f_e}{\text{poles}} = \left(\frac{2}{\text{poles}}\right) \omega_e$$

The mechanical torque T_{mech} and power P_{mech} are not the output values available at the shaft because friction, windage, and stray-load losses remain to be accounted for. It is obviously correct to subtract friction, windage, and other rotational losses from T_{mech} or

P_{mech} and it is generally assumed that stray load effects can be subtracted in the same manner. The remainder is available as output power from the shaft for useful work. Thus

$$P_{\text{shaft}} = P_{\text{mech}} - P_{\text{rot}}$$
$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_{\text{m}}} = T_{\text{mech}} - T_{\text{rot}}$$

where P_{rot} and T_{rot} are the power and torque associated with the friction, windage, and remaining rotational losses.

Example Poly-phase Equivalent Circuit Analysis 4

A three-phase Y-connected 110-V (line-to-line) 7.5-kW 50-Hz six-pole IM has the following parameter values in Ω/phase referred to the stator.

$$R_1 = 0.294, \quad R_2 = 0.144, \quad X_1 = 0.503, \quad X_2 = 0.209, \quad X_m = 13.25.$$

The total friction, windage, and core losses may be assumed to be constant at 403 W, independent of load.
For a slip of 0.02 percent, compute the speed, output torque and power, stator current, power factor, and efficiency when the machine is operated at rated voltage and frequency.

Solution Poly-phase Equivalent Circuit Analysis

Let the impedance Z_{f} represent the per-phase impedance presented to the stator by the magnetising reactance and the rotor. Therefore we can bulk collect the reactance on the rotor size as:

$$Z_{\text{f}} = R_{\text{f}} + jX_{\text{t}} = \left(\frac{R_2}{s} + jX_2\right) \text{ in parallel with } jX_{\text{m}}$$

Substitution of numerical values gives, for $s = 0.02$,

$$R_{\text{f}} + jX_{\text{t}} = (5.43 + 3.11j)\ \Omega$$

The stator input impedance can now be calculated as:

$$Z_{\text{in}} = R_1 + jX_1 + Z_{\text{f}} = (5.72 + 3.61j)\ \Omega$$

The line-to-neutral terminal voltage is equal to:

$$V_1 = \frac{110}{\sqrt{3}} = 63.51 \text{ V}$$

and hence the stator current can be calculated as

$$\hat{I}_1 = \frac{V_1}{Z_{\text{u}}} = \frac{63.51}{(5.72 + 3.61j)} = (7.94 - 5.01j)\text{A}$$

The stator current is thus 9.39 A and the power factor is equal to $\cos(-32.27) = 0.85$ lagging. The synchronous speed can be found as:

$$n_s = \left(\frac{120}{\text{poles}}\right) f_e = \left(\frac{120}{6}\right) 50 = 1000.0 \text{ r/min}$$

or the angular speed can be calculated to be:

$$\omega_s = \frac{4\pi f_e}{\text{poles}} = 104.72 \text{ rad/sec}$$

The rotor speed is

$$n = (1-s)n_s = (0.98)1000.0 = 980.0 \text{ r/min}$$

or

$$\omega_m = (1-s)\omega_s = (0.98)104.72 = 102.63 \text{ rad/sec}$$

The air-gap power is calculated to be:

$$P_{\text{gap}} = n_{\text{ph}} I_2^2 \left(\frac{R_2}{s}\right)$$

Note however that because the only resistance included in Z_f is R_2/s, the power dissipated in Z_f is equal to the power dissipated in R_2/s and hence we can write

$$P_{\text{gap}} = n_{\text{ph}} I_1^2 R_f = 3\,(9.39)^2\,(5) = 1434.85 \quad W$$

We can now calculate P_{mech} and the shaft output power as:

$$P_{\text{shaft}} = P_{\text{max}} - P_{\text{rot}} = (1-s)\,P_{\text{sep}} - P_{\text{rot}}$$
$$= (0.98)1434.85 - 403 = 1406.15 \text{ W}$$

and the shaft output torque can be found as

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_t} = \frac{1003.15}{102.63} = 9.77\text{N} \cdot \text{m}$$

The efficiency is calculated as the ratio of shaft output power to stator input power. The input power is given by

$$P_n = n_{\text{pt}}\text{Re}[\hat{V}_1 \hat{I}_1^*] = 3\text{Re}[63.51(9.39\rangle]$$
$$= 3 \times 127 \times 18.8 \cos(32.2^\circ) = 1512.6 \text{ W}$$

Thus the efficiency (η) is equal to:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{1003.15}{1512.6} = 0.66 \approx 66.0\%$$

The complete performance characteristics of the machine can be determined by repeating these calculations for other assumed values of slip.

3.3 Parameters Tests

The equivalent-circuit parameters needed for determining the performance of a polyphase IM under load can be obtained from the results of:

1. no-load test,
2. blocked-rotor test,
3. Direct Current (DC) injection test.

. Stray-load losses, which must be taken into account when accurate values of efficiency are to be calculated, can also be measured by tests which do not require loading the machine.

> In this document the calculation of the stray-load losses will **NOT** be discussed.

3.3.1 No-Load Test

Similar to the open-circuit test of a transformer, the no-load test on an IM gives information with respect to exciting current and no-load losses. This test is ordinarily performed at **rated frequency** and with balanced poly-phase voltages applied to the stator terminals.

> Readings are taken at rated voltage, after the machine has reached steady-state.

We will assume that the no-load test is made with the machine operating at its rated electrical frequency f_r and that the following measurements are available from the no-load test:

> In poly-phase IM line-to-line ($V_{\text{L-L}}$) voltage is generally measured., To calculate phase-to-neutral (V_{ph}) voltage, divide by $\sqrt{3}$ for a 3-phase system.

Term	Definition
$V_{\text{l, st}}$	The line-to-neutral voltage [V]
$I_{\text{l, st}}$	The line current [V]
P_{st}	The total polyphase electrical input power [W]

Table 3.1: Measurable parameters in an Open-Circuit test.

At no load, the rotor current is only the very small value needed to produce sufficient torque to overcome the friction and windage losses associated with rotation. The no-load rotor I^2R loss is, therefore, negligibly small. Unlike the continuous magnetic core in a transformer, the magnetising path in an IM includes an air gap which significantly increases the required exciting current. Thus, in contrast to the case of a transformer, whose no-load primary I^2R loss is negligible, the no-load stator I^2R loss of an IM may be appreciable because of this larger exciting current.

Neglecting rotor I^2R losses, the rotational loss (P_{rot}) for normal running conditions can be found by subtracting the stator I^2R losses from the no-load input power:

$$P_{\text{rot}} = P_{\text{in}} - n_{\text{ph}} I_{1,\text{ph}}^2 R_1 \tag{6.33}$$

The total rotational loss at rated voltage and frequency under load usually is considered to be constant and equal to its no-load value. Note that the stator resistance R_1 varies with stator-winding temperature.

Hence, when applying Eq. 6.37, care should be taken to use the value corresponding to the temperature of the no-load test.

Example Wound Rotor Torque-Resistance Calculation 5

A three-phase, 230-V, 60-Hz, 12-kW, four-pole wound-rotor induction motor has the following parameters expressed in Ω/phase.

$$R_1 = 0.095 \quad X_1 = 0.680 \quad X_2 = 0.672 \quad X_m = 18.7$$

Using `Python` plot the electromechanical mechanical torque T_{mech} as a function of rotor speed in r/min for rotor resistances of $R_2 = 0.1$, 0.2, 0.5, 1.0 and $1.5\ \Omega$.

Solution Wound Rotor Torque-Resistance Calculation

The code along with the produced plot is as follows:

```python
import matplotlib.pyplot as plt
import numpy as np

# Given Parameters
Vin =  230     # (V)     Input Voltage (3-phase)
ph  =  3       # (-)     Number of phases
p   =  4       # (-)     Number of poles
f   =  60      # (Hz)    Grid frequency
Rs  =  0.095   # (Ohm)   Stator Resistance
Xs  =  0.680   # (Ohm)   Stator Reactance
Xr  =  0.672   # (Ohm)   Rotor Reactance
Xm  =  18.70   # (Ohm)   Magnetising Reactance
Rrt = [0.1, 0.2, 0.5, 1.0, 1.5] # (Ohm) Rotor Resistance


# Calculate the per-phase voltage
Vin_p = Vin / np.sqrt(3)

# Calculating the snychronous speed
ws = 4 * np.pi * f / p
ns = 120 * f / p

# Calculating the stator Thevenin Equivalent
Z1_eq = 1j * Xm * (Rs + 1j * Xs) / (Rs + 1j * (Xs + Xm))
R1_eq = Z1_eq.real; X1_eq = Z1_eq.imag
V1_eq = np.abs(Vin_p * (1j * Xm) / (Rs + 1j * (Xs + Xm)))

slip = np.linspace(0.001,1, 200) # set an empty array for
Tmech = np.zeros([200, 5])

j = 0
for Rr in Rrt:
    i = 0
    for s in slip:
        rpm = ns * (1 - s)
        Ir = np.abs(V1_eq / (Z1_eq + 1j * Xr + Rr / s))
        Tmech[i, j] = ph * Ir ** 2 * Rr / (s * ws)

        i = i + 1

    j = j + 1

Result_Array = np.c_[np.transpose(slip), Tmech]
plt.style.use("dawn")
```

```python
plt.rcParams['axes.facecolor'] = 'white'
plt.rcParams['savefig.facecolor'] = 'white'

plt.figure(figsize=(18, 6), dpi=80, facecolor='white')
plt.plot(Result_Array[:,1])
plt.plot(Result_Array[:,2])
plt.plot(Result_Array[:,3])
plt.plot(Result_Array[:,4])
plt.plot(Result_Array[:,5])
plt.savefig("images/Induction-Motors/wound-rotor-example-result.pdf",
↪ bbox_inches='tight')
plt.show()
plt.close()
```

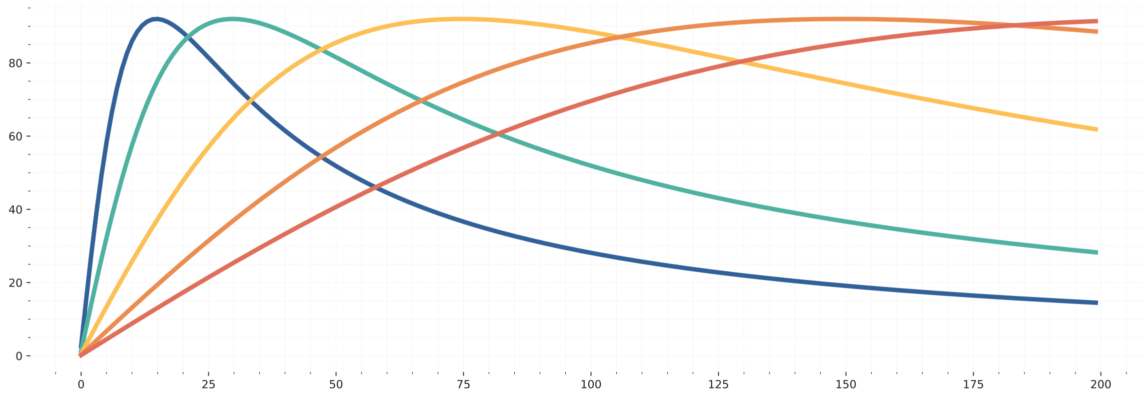

Figure 3.3: Different types of resistance connected to the rotor will produce different torque v. speed plots.

3.4 Starting Methods

IM up to roughly 5 kW can be considered self-starting whereby it is easy as plugging them to a power outlet without the need of an external device. However, as with every device with high levels of inductive properties the control of current during startup needs to be controlled to avoid the problem of **inrush current**.

Inrush Current the maximal instantaneous input current drawn by an electrical device when first turned on. AC machines and transformers may draw several times their normal full-load current when first energised, for a few cycles of the input waveform.

3.4.1 Direct On-Line Starter

As the name suggests, the IM is started by connecting it directly to three-phase supply. In this method, the machine still draws a high starting current (about 4 to 7 times of the rated current) and therefore operates at a low power factor during startup.

> DOL starting is suitable for relatively small machines (up to 10 kW)

3.4.2 Stator Resistance Starting

External resistance is connected in series with each phase of the stator winding during starting. The external resistance causes voltage to drop across it so that reduced voltage available across the machine terminals.

> This addition of resistances to the stator causes the starting current to reduce.

The starting external resistances are **gradually** cut out in steps from the stator circuit, as the machine accelerates.
When the machine attains the rated speed, the starting resistances are completely cut out and full line voltage is applied across the machine terminals.
This method has two (2) major drawbacks:

1. Reduced voltage during starting reduces the starting torque, increasing acceleration time
2. Energy is lost in the starting resistances as heat.

3.4.3 Auto-transformer Starting

An autotransformer is used to reduce the starting voltage of the IM. The tapings of the autotransformer is so set that when it is in the circuit, 60 to 80 % of the line voltage is applied to machine during starting and then connecting it to the full-line voltage as the machine attains a sufficient speed.

Auto-Transformer A transformer with only one winding. The auto prefix refers to the single coil acting alone. Portions of the same winding act as both the primary winding and secondary winding sides of the transformer. This allows one to change the output of the transformer by adjusting the taps on the coil.

During startup, the change-over switch is connected to **Start**. This supplies the reduced voltage to the machine through the autotransformer. Consequently, the starting current is limited to safe value. When the machine attains about 80% of rated-speed, the change-over switch is now thrown to **Run**. This removes the autotransformer from the circuit and full line voltage is applied across the machine terminals.
The autotransformer starting has many advantages such as low power loss, low starting current etc. This method is used for large machines over 20 kW.

3.4.4 Star-Delta Starter

In star-delta ($Y - \Delta$) starting method of squirrel cage IM, the machine starts in star and runs in delta, i.e. the stator winding of the machine is designed for delta operation and is connected in **star during starting**.
When the machine attains sufficient speed, the connections are changed to **delta**.
The six leads of stator winding of the machine are connected to a change-over switch. At the time of starting, the change-over switch is switched to Star which connects the stator windings in star. Thus, each phase gets $V/\sqrt{3}$ volts, where V is the three phase line voltage. This reduces the starting current.
When the machine attains 80% of rated speed, the changeover switch is switched to Delta which connects the stator windings in delta. Now, each phase gets full line voltage.

> A major disadvantage of this method is large reduction in stating torque due to reduced voltage in the star connection at the instant of starting

The star-delta starting is used for medium size machines up to 20 kW.

3.4.5 Rotor Resistance Starter

> This method only applies to wound rotor (i.e., slip-ring)

A star connected variable resistance is connected in the rotor circuit through slip-rings. The full voltage is applied to the stator windings.
During startup, the handle of variable resistance (rheostat) is set to OFF position. This inserts maximum resistance in series with the each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased due to external rotor resistance.
As the machine accelerates, the external resistance is gradually removed from the rotor circuit. When the machine attains rated speed, the handle is switched in the ON position, this removes the whole external resistance from the rotor circuit.

Chapter 4

Single-Phase Induction Machines

4.1 Introduction

While industry relies on the availability of a 3-phase system, the residential and commercial sections only have access to single-phases. Therefore machines capable of running from a single phase needs to be worked on. However, most single-phase induction motors are actually two-phase motors with unsymmetrical windings; the two windings are typically quite different, with different numbers of turns and/or winding distributions.

4.2 Structure

The most common types of single-phase IM resemble polyphase squirrel-cage motors except for the arrangement of the stator windings.
Instead of being a concentrated coil, the actual stator winding is distributed in slots to produce an approximately sinusoidal space distribution of mmf.

> A single-phase winding produces equal forward- and backward-rotating mmf waves.

By symmetry, it is clear that such a motor inherently will produce no starting torque since at standstill, it will produce equal torque in both directions.
If, however, it is started by auxiliary means, the result will be a net torque in the direction in which it is started, and hence the motor will continue to run.

4.3 Classification

Single-phase IMs are classified in accordance with their starting methods and are usually referred to by names descriptive of these methods. Selection of the appropriate motor is based on:

- the starting torque,

- running torque requirements of the load,
- the duty cycle of the load
- limitations on starting and running current from the supply line for the motor

The cost of single-phase motors increases with their rating and with their performance characteristics such as starting-torque-to-current ratio. Typically, in order to minimize cost, an application engineer will select the motor with the lowest rating and performance that can meet the specifications of the application.
Where a large number of motors are to be used for a specific purpose, a special motor may be designed in order to ensure the least cost. In the fractional-kilowatt motor business, small differences in cost are important.

4.3.1 Split-phase

Split-phase motors have two stator windings, a **main winding** (i.e., run winding) which we will refer to with the subscript 'main' and an **auxiliary winding** (i.e., start winding) which we will refer to with the subscript 'aux'. As in a two-phase motor, the axes of these windings are displaced 90 electrical degrees in space. The auxiliary winding has a higher resistance-to-reactance ratio than the main winding, with the result that the two currents will be out of phase, which is representative of conditions at starting.
Since the auxiliary-winding current I_{aux} leads the main-winding current $\hat{I}_{main}$, the stator field first reaches a maximum along the axis of the auxiliary winding and then somewhat later in time reaches a maximum along the axis of the main winding.
The winding currents are equivalent to unbalanced two-phase currents, and the motor is equivalent to an unbalanced two-phase motor. The result is a rotating stator field which causes the motor to start. After the motor starts, the auxiliary winding is disconnected, usually by means of a centrifugal switch that operates at about 75 percent of synchronous speed. The simple way to obtain the high resistance-to-reactance ratio for the auxiliary winding is to wind it with smaller wire than the main winding, a permissible procedure because this winding operates only during starting. Its reactance can be reduced somewhat by placing it in the tops of the slots.
Split-phase motors have moderate starting torque with low starting current. Typical applications include fans, blowers, centrifugal pumps, and office equipment. Typical ratings are 50 to 500 watts; in this range they are the lowest-cost motors available.

4.3.2 Capacitor Types

Capacitors can be used to improve motor starting performance, running performance, or both, depending on the size and connection of the capacitor.

Capacitor Start

It can also be classified as a split-phase motor, but the time-phase displacement between the two currents is obtained by means of a capacitor in series with the auxiliary winding. Again the auxiliary winding is disconnected after the motor has started, and consequently the auxiliary winding and capacitor can be designed at minimum cost for intermittent service.
These connections allow high starting torque which are useful for compressions, pumps, refrigeration and air-conditioning equipment, and other hard-to-start loads.

Permanent-Split

The capacitor and auxiliary winding are not cut out after starting. The construction can be simplified by omission of the switch, and the power factor, efficiency, and torque pulsations improved.
For example, the capacitor and auxiliary winding could be designed for perfect two-phase operation. The losses due to the backward field at this operating point would then be eliminated, with resulting **improvement in efficiency**. The double-star-frequency torque pulsations would also be eliminated, with the capacitor setting as an energy storage reservoir for smoothing out the pulsations in power input from the single-phase line, resulting in quiet operation.
Starting torque must be searched because the choice of capacitance is necessarily a compromise between the best starting and running value.

Capacitor Run

If two capacitors are used, one for starting and one for running, theoretically optimum starting and running performance can both be obtained. One way of accomplishing is to have the small value of capacitance required for optimum running conditions permanently connected in series with the auxiliary winding, and the much larger value required for starting is obtained by a capacitor connected in parallel with the running capacitor via a switch with opens as the motor comes up to speed.
The capacitor for a capacitor-start motor has a typical value of 300 μF for a 500-W motor. Since it must carry current for just the starting time, the capacitor is a special compact AC electolytic type made for motor-starting duty. The capacitor for the same motor permanently connected has a typical rating of 40 μF, and since it operates continuously, the capacitor is an ac paper, foil, and oil type.
The cost of the various motor types is related to performance:

- the capacitor-start motor has the lowest cost,

- the permanent-split-capacitor motor next
- capacitor-run motor

Example **Capacitor Start Capacitor Calculation** 6

A 2.5-kW 120-V 60-Hz capacitor-start motor has the following impedances for the main and auxiliary windings (at starting):

$$Z_{\text{min}} = 4.5 + j3.7\ \Omega \qquad \text{main winding}$$

$$Z_{\text{max}} = 9.5 + j3.5\ \Omega \qquad \text{auxiliary winding}$$

Find the value of starting capacitance that will place the main and auxiliary winding currents in quadrature at starting.

Solution **Capacitor Start Capacitor Calculation**

The currents $\hat{I}_{\text{min}}$ and $\hat{I}_{\text{max}}$ are shown in Fig. 9.4a and b. The impedance angle of the main winding is

$$\phi_{\text{min}} = \tan^{-1}\left(\frac{3.7}{4.5}\right) = 39.6^{\circ}$$

To produce currents in time quadrature with the main winding, the impedance angle of the auxiliary winding circuit (including the starting capacitor) must be

$$\phi = 39.6^{\circ} - 90.0^{\circ} = -50.4^{\circ}$$

The combined impedance of the auxiliary winding and starting capacitor is equal to

$$Z_{\text{total}} = Z_{\text{tan}} + j\,X_{\text{c}} = 9.5 + j\,(3.5 + X_{\text{c}})\ \Omega$$

where $X_{\text{c}} = -\frac{1}{\omega c}$ is the reactance of the capacitor and $\omega = 2\pi\,60 \approx 377$ rad/sec. Thus

$$\tan^{-1}\left(\frac{3.5 + X_{\text{c}}}{9.5}\right) = -50.4^{\circ}$$

$$\frac{3.5 + X_{\text{c}}}{9.5} = \tan\left(-50.4^{\circ}\right) = -1.21$$

and hence

$$X_{\text{c}} = -1.21 \times 9.5 - 3.5 = -15.0\ \Omega$$

The capacitance C is then

$$C = \frac{-1}{\omega X_{\epsilon}} = \frac{-1}{377 \times (-15,0)} = 177\ \mu\text{F} \quad \blacksquare$$

4.4 Shaded-Pole

Shaded-pole IM has **salient poles** with one portion of each pole surrounded by a short-circuited turn of copper called a shading coil. Induced currents in the shading coil cause the flux in the shaded portion of the pole to lag the flux in the other portion. The result is similar to a rotating field moving in the direction from the unshaded to the shaded portion of the pole. This motion of field causes currents to induce in the squirrel-case rotor and a low starting torque is produced.

4.4.1 Performance

The starting torque of this motor is very small about 50% of full load torque. Efficiency is low because of continuous power loss in shading coil. These motors are used for small fans, electric clocks, gramophones which are in around 50 W range.

> They are known to have low efficiency and cost of production.

Its direction of rotation depends upon the position of the shading coil, i.e., which portion of the pole is wrapped with shading coil. The direction of rotation is from un-shaded portion of the pole to the shaded portion.

> Its direction of rotation cannot be reversed unless the position of the poles is reversed.