Topics on Electrical Engineering

Electric Drive Fundamentals

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MCI

B.Sc - Drive Systems

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Introduction

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- Final Examination
- Point Distribution
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- Resources

Introduction

- The goal of this lecture is to give you the fundamentals of electric machinery using theory and engineering practice.
- This lecture is a total of 2 SWS with a total of thirty (30) UE.
  - With 28 UE is devoted to lectures.
- There is a written exam at the end of the module worth two (2) UE.
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Assessment Type	Overall Points	Breakdown	%
Individual Assignment	40		
		Report	20
		Solution(s)	80
			20
Final Exam	60		
		Question 1	40
		Question 2	30
		Question 2	30

Table 1: Assessment Grade breakdown for the lecture.

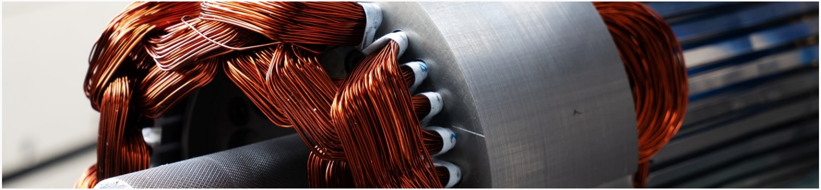
Introduction

Covered Topic	Appointment
Return to Fundamentals	1
Magnetic Circuits & Materials	1
Transformers	2
Electromechanical Energy Conversion	2-3
Rotating Magnetic Fields	3
DC Drives	4
Poly-phase Induction Drives	4-5
Single-phase Induction Drives	5-6
Linear Induction Drives	6
Poly-phase Synchronous Drives	6-7
Solid-state Commutation Drives	7

Table 2: Distribution of materials across the semester.

Return to Fundamentals

- Complex number notation,
- Multi-phase systems,
- Phasors and Wave-forms



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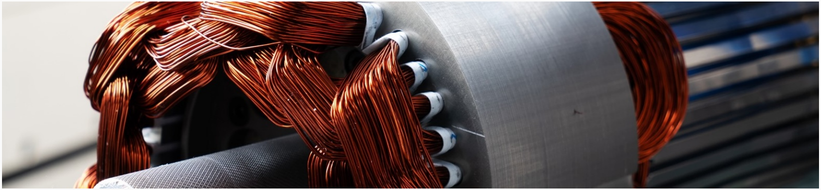

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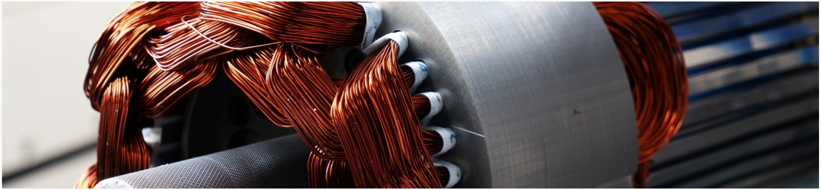

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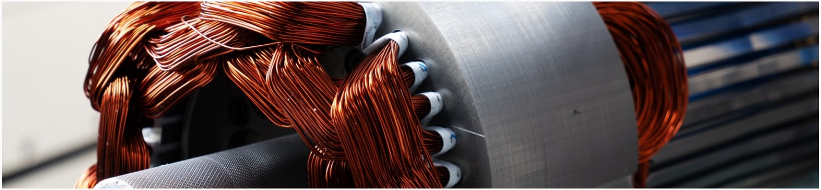
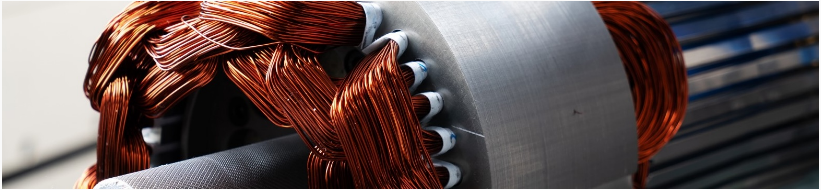


Transformers

- Construction & Physical Properties
- Modelling
- Parameter Tests
- Connection Types



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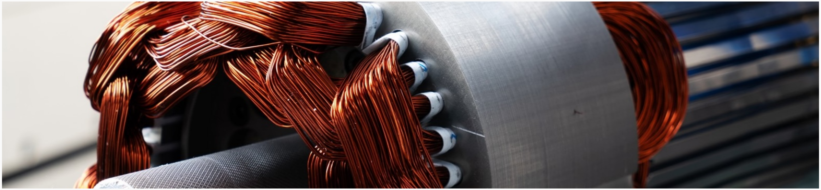

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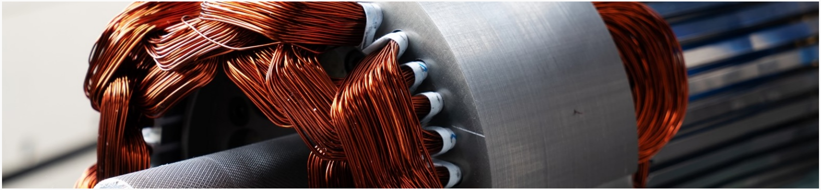

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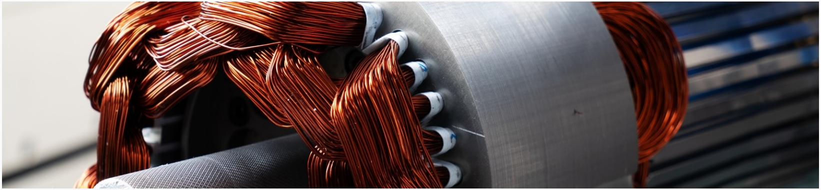
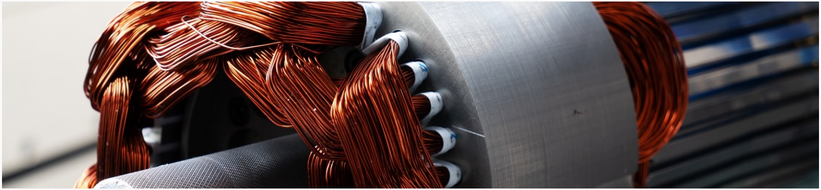


Rotating Magnetic Fields

- MMF in Winding
- Torque Generation
- Generated Voltage



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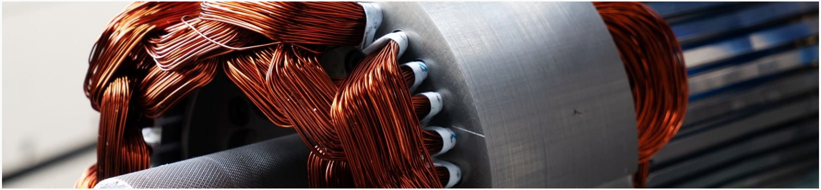

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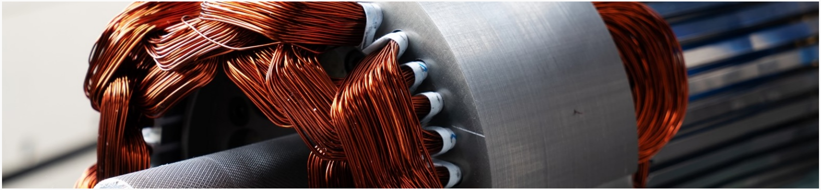

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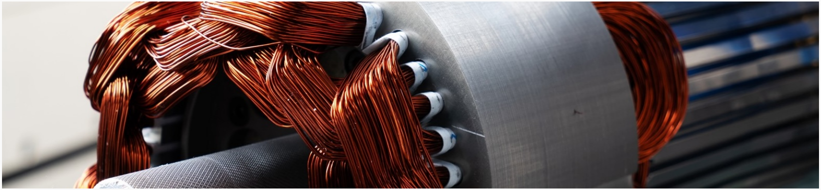
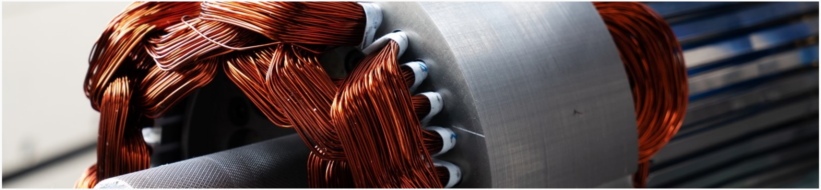


DC Drives

- Construction
- Physical laws governing its operation
- Types of Connections used in industry and commercial applications.
- Applications in Industrial/Commercial venues.



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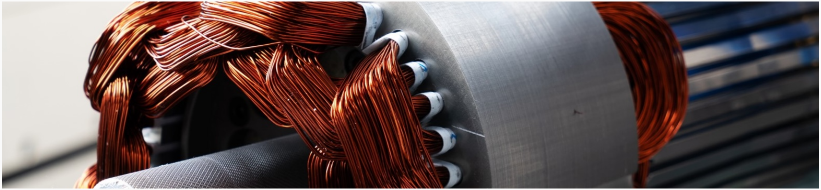

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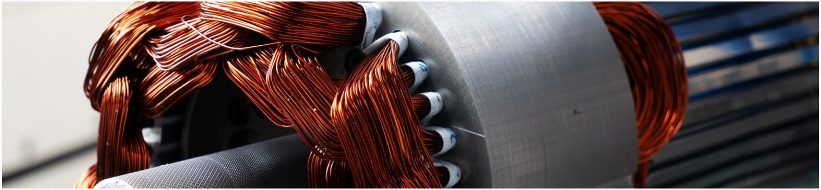

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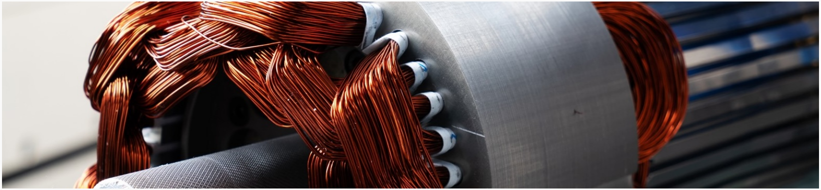

Poly-phase Induction Drives

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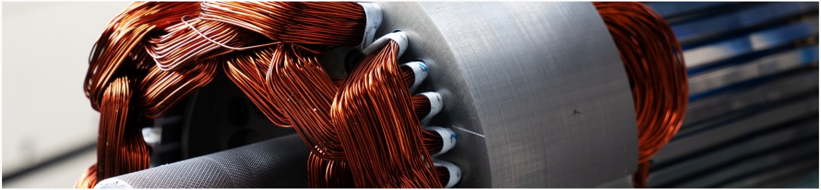
- Modelling an induction drive mathematically
- Methods of starting an induction drive.



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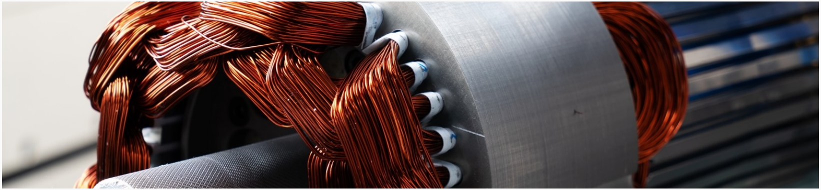

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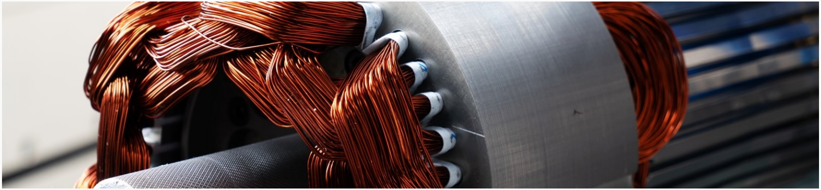

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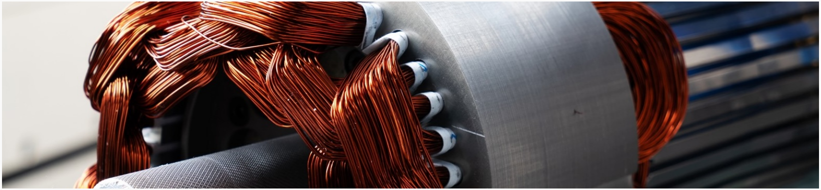
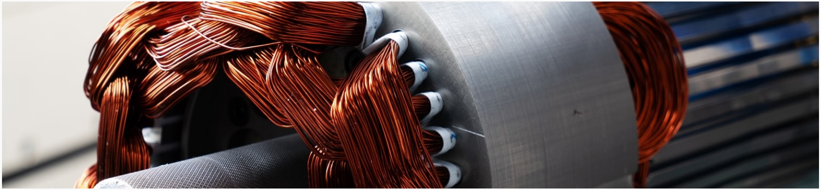


Single-Phase Induction Drives

- Creating a Rotating Magnetic Field in a single phase
- Types of single-phase types
- Salient-pole



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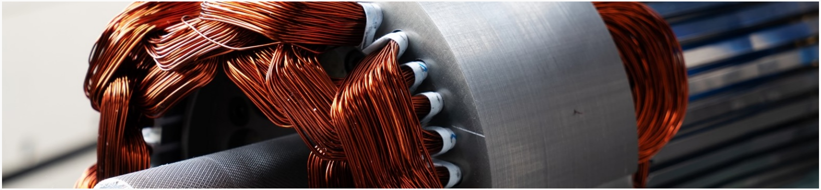

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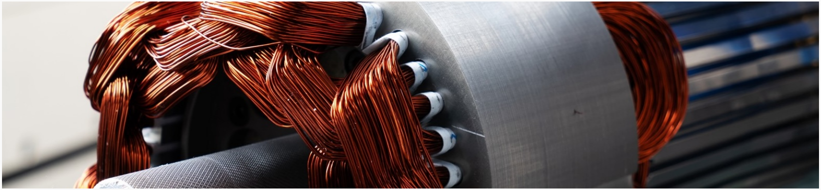

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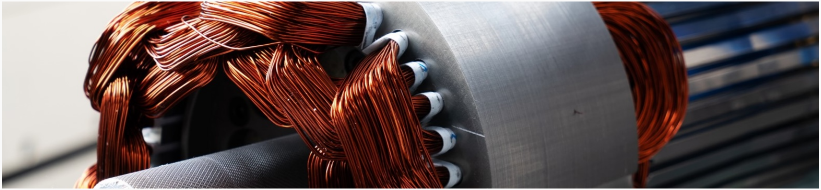

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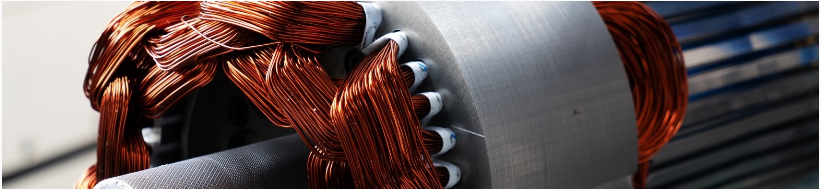

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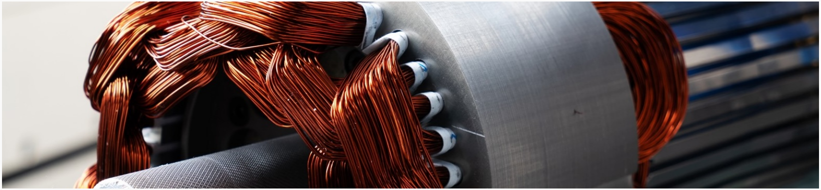

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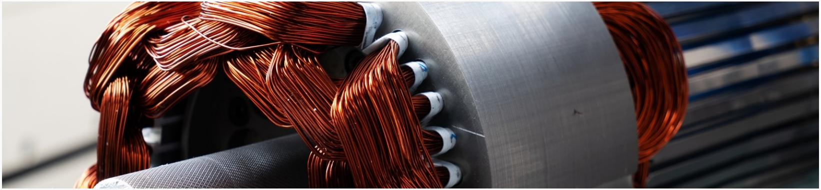

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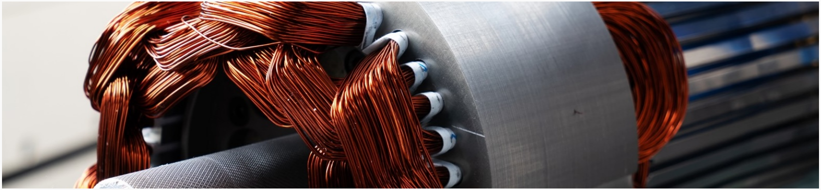
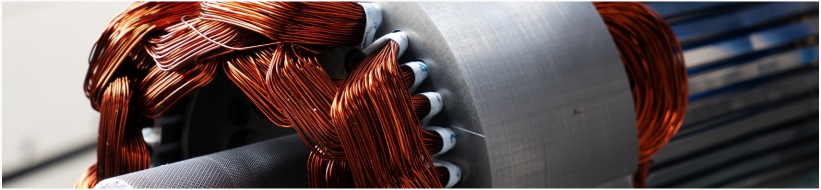


Solid-State Commutation Drives

- Solid State Commutation
- BLDC & PMSM Drives
- Switched Reluctance Motor (SRM)
- Stepper Drives



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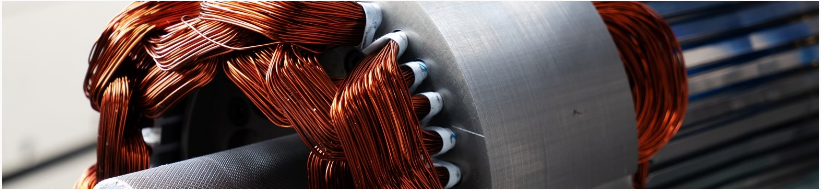

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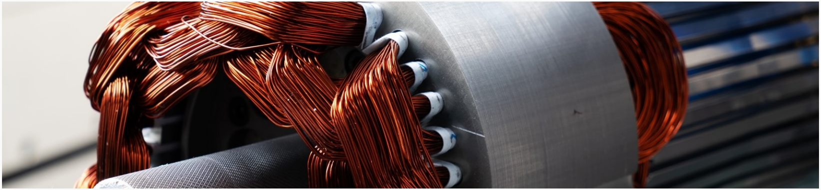

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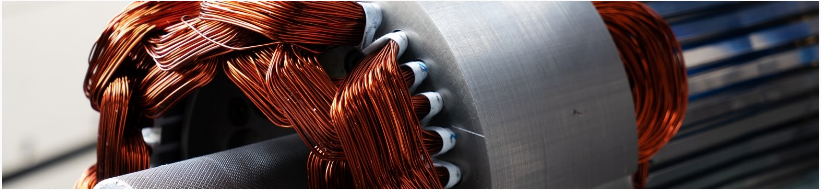

Books

- Mohan Ned. "*Advanced electric drives: analysis control and modeling using MATLAB/Simulink*" John Wiley & Sons 2014.
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White Papers

- Maddox Transformer "*Guide to transformer cores: types, construction, & purpose*"
- Control Engineering *Springtime for Switched-Reluctance Motors?* .

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Learning Outcomes

Introduction

Connection Types

- Three-phase Waveform
- Delta Connection
- Wye Connection

Polar Coordinates

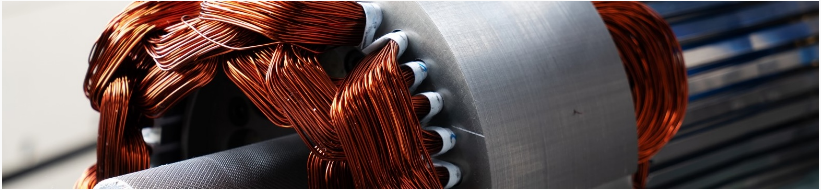
- Power in AC

Learning Outcomes

(LO1) An Overview of Poly-phase circuits,

(LO2) Definitions on Active-Reactive power,

(LO3) Polar Coordinate System.



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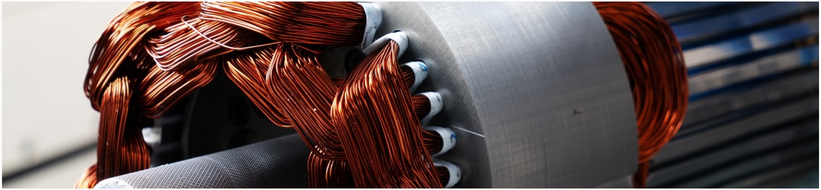

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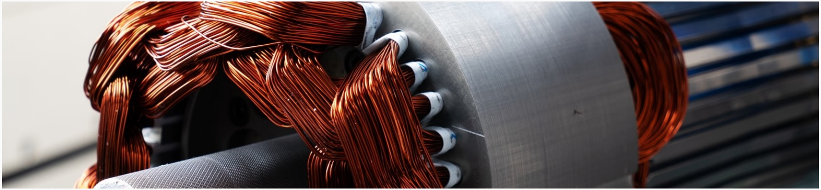

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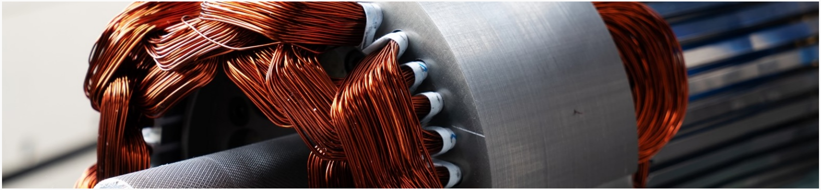

Return to Fundamentals

- A rotating magnetic field is a magnetic field with moving polarities.
  - Which its opposite poles rotate about a **central point** or **axis**.
- To create a rotating magnetic field (RMF) you need at least a 2-phase system.
- In the industry, RMFs are mostly produced using a 3-phase supply [1].
  - There are also economic reasons, as it is cheaper to design a 3-phase system with minimal cost to wiring [2].
- Before starting with electric drives, it is a good time to look at some fundamental concepts in power engineering.

[1] 3-phase supply produces smoother operation of motors compared to 2-phase. This is due to the power transfer in 3 phase supply being less pulsating than in 2 phase supply.

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- Generation, transmission, and heavy-power utilisation of AC electric energy almost invariably involve a poly-phase circuit.
- In such a system, each voltage source consists of a group of voltages having related magnitudes and phase angles.
- Thus, an n-phase system employs voltage sources which typically consist of n voltages substantially equal in magnitude and successively displaced by a phase angle of 360°/n.
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- The three individual voltages of a three-phase source may each be connected to its own independent circuit.
- We would then have three separate single-phase systems.
- Alternatively, symmetrical electric connections can be made between the three voltages and the associated circuitry to form a three-phase system.
- It is the latter alternative that we are concerned.
- Note that the word phase now has two distinct meanings.
  - It may refer to a portion of a pol-yphase system or circuit,
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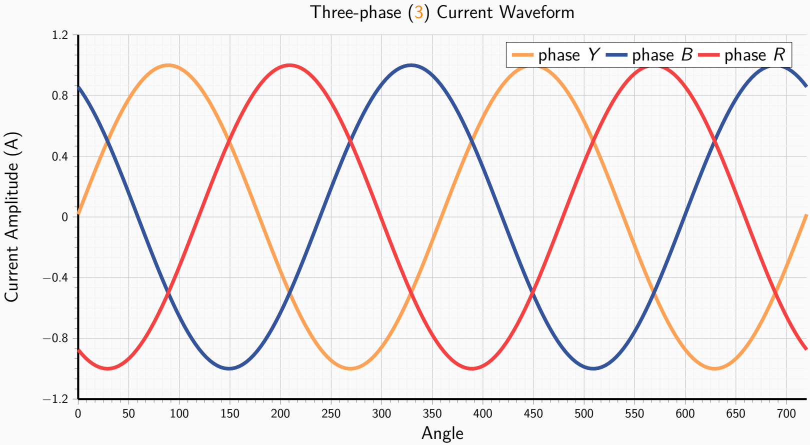

Figure 1: A three phase waveform of currents **R**, **B**, and **Y**.

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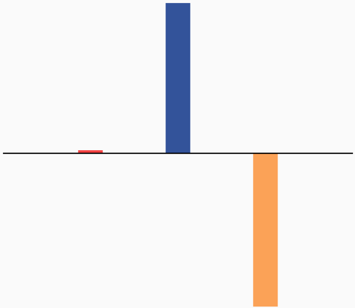

$i_R = 0.01746$

$i_B = 0.85716$

$i_Y =$ -0.87462

$$i_R + i_B + i_Y = 0.0$$

Figure 2: A three phase waveform of currents **R**, **B**, and **Y**.

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- There are **no neutral connection** available.
- Phase voltage appears **across the windings**.
- In a delta connection, Line voltage is equal to Phase voltage:

$$v_{\text{line}} = v_{\text{ph}}$$

- i_{YB}, i_{YR}, i_{BR} are also called **Line current** (i.e., i_{line}).
- Line current is $\sqrt{3}$ times that of the phase current.

$$i_{\text{line}} = \sqrt{3} i_{\text{subs}}$$

- Power related definitions are:

$$P = \sqrt{3} v_{\text{line}} i_{\text{line}} \cos\varphi, \quad \text{and} \quad P = 3 v_{\text{ph}} i_{\text{ph}} \cos\varphi.$$

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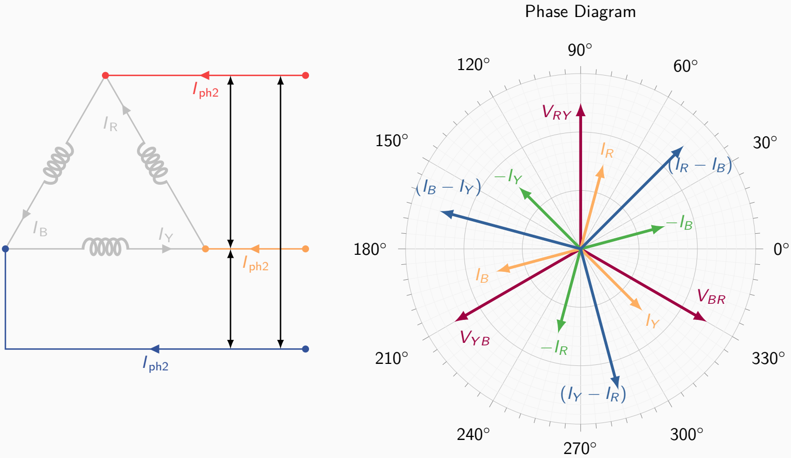

Figure 3: The connection and the phasor diagram of a 3-phase delta connection.

Example

Three impedance with a value $Z_{\Delta} = 12.00 + \mathbf{j}\,9.00 = 15.00\angle 36.9\ \Omega$ are connected in Δ.

For balanced line-to-line voltages of 208 V, find the line current, the power factor, and the total power.

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- Power related equations for 3-phase:

$$P = \sqrt{3} v_{\text{line}} i_{\text{line}} \cos\varphi,$$
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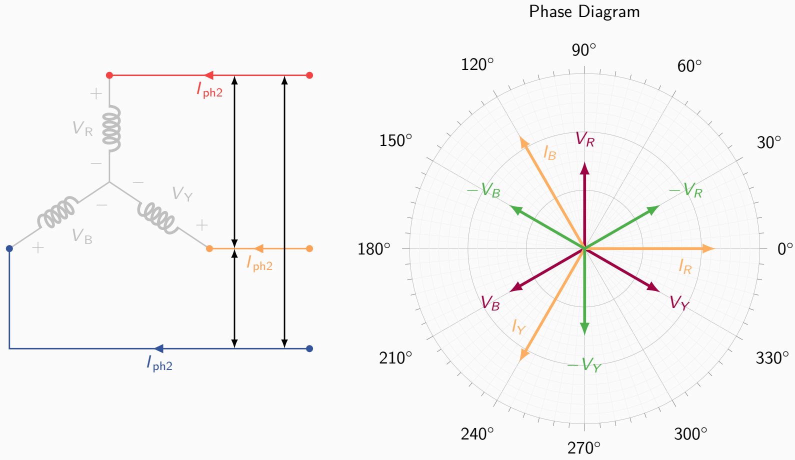

Figure 4: The connection and the phasor diagram of a 3-phase wye connection.

Example

A Y-connected 120 V source feeds a Δ-connected load through a distribution line having an impedance of $0.3 + \mathbf{j}\,0.9\ \Omega$. The Y-source impedance is $0.2 + \mathbf{j}\,0.5\Omega$

The load impedance is $118.5 + \mathbf{j}\,85.8\ \Omega/o$.

Use the a-phase internal voltage of the generator as the reference.

(a) Construct a single-phase equivalent circuit of the three-phase system,

(b) Calculate the line currents,

(c) Calculate the phase voltages at the load terminals,

(d) Calculate the phase currents of the load,

(e) Calculate the line voltages at the source terminals.

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$$Z = x + \mathbf{j}\,y = A\angle\theta,$$

where:

- Z is the complex vector,
- A is vector magnitude,
- x is real/active part,
- $\mathbf{j}$ is defined as $\sqrt{-1}$.
- y is imag/reactive part,
- θ is the complex angle.

$$\theta = {}^{y}\!/_{x}.$$

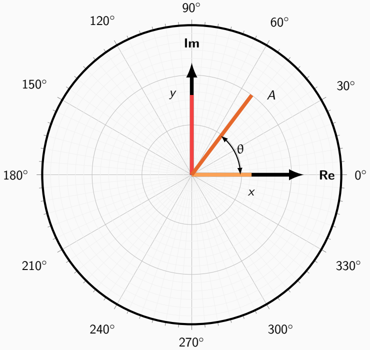

Figure 5: A polar coordinate system.

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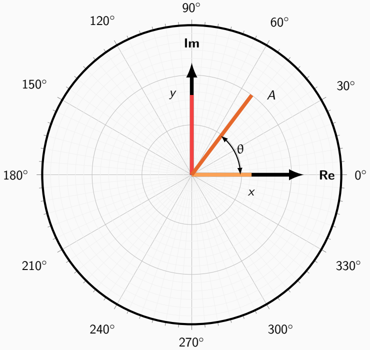

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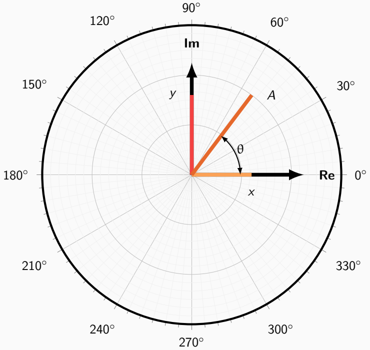

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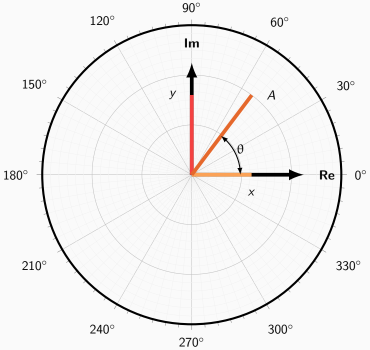

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- x is real/active part,
- $\mathbf{j}$ is defined as $\sqrt{-1}$.
- y is imag/reactive part,
- θ is the complex angle.

$$\theta = {}^{y}\!/_{x}.$$

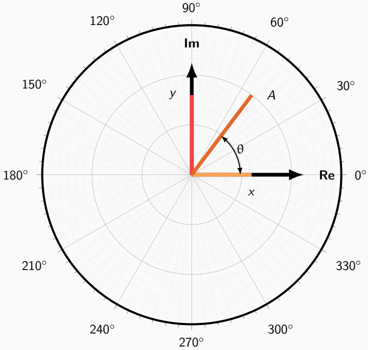

Figure 5: A polar coordinate system.

$$Z = x + \mathbf{j}\,y = A\angle\theta,$$

where:

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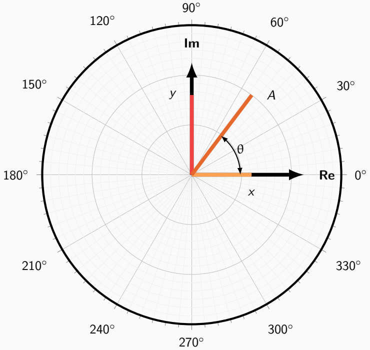

Figure 5: A polar coordinate system.

Example

Express:

$$\frac{3 - \mathbf{j}}{7 - 3\mathbf{j}},$$

in the form of $a + \mathbf{j}\, b$.

Solution

Answer is:

$$-\frac{12}{29} - \mathbf{j}\left(\frac{1}{29}\right) \quad \blacksquare$$

Return to Fundamentals

When it comes to power in AC, there are three (3) definitions:

- If energy is used/generated by an active element it is called **Active** or **Real** (P) power and measured in W.
- If energy is used/generated by an reactive element it is called **Reactive** power (Q)and measured in $\mathrm{V} \cdot \mathrm{A}r$.
- The combination of these two values is called **Apparent** power (S) and measured in $\mathrm{V} \cdot \mathrm{A}$.

$$S = \sqrt{P^2 + Q^2}$$

- Finally the angle difference between the voltage and current waveform is defined as the **phase** and tells how reactive/active a circuit is.

$$\varphi = \varphi_V - \varphi_I.$$

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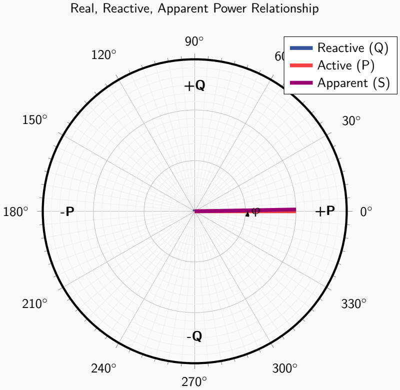

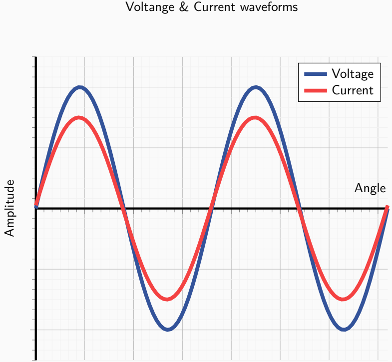

Figure 6: An animation showing the relations between Active, Reactive and Apparent power along with phase angle.

Example

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8.

The load is fed from a line having an impedance of $0.005 + \mathbf{j}\,0.025\Omega$.

The line voltage at the terminals of the load is 600 V.

(a) Construct a single-phase equivalent circuit of the system.

(b) Calculate the magnitude of the line current.

(c) Calculate the magnitude of the line voltage at the sending end of the line.

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Magnetic Circuits and Materials

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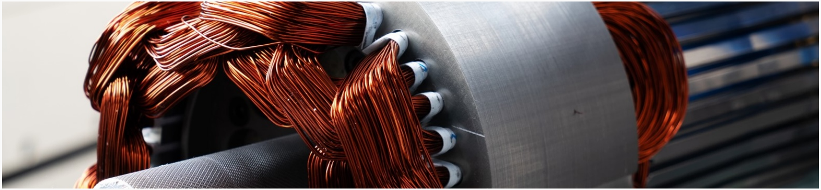

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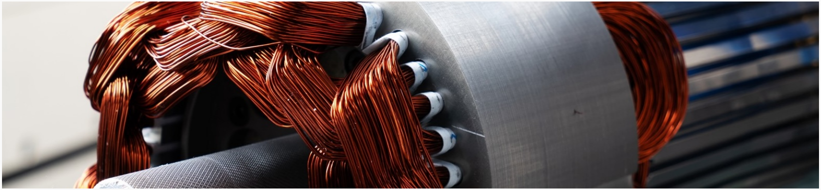

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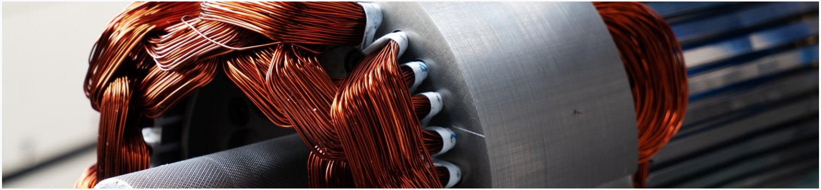

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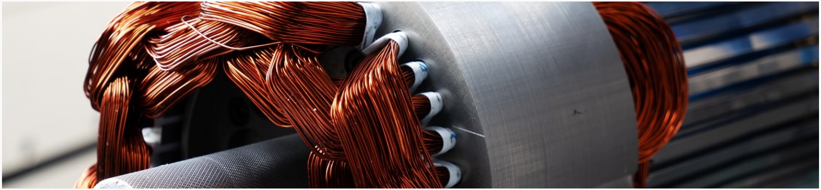

Magnetic Circuits and Materials

- Almost all electric drives use ferromagnetic material for shaping and directing **B**-fields.
  - These fields act as the medium for transferring and converting energy.

> Permanent-magnets are also widely used in drive design.

- Without these materials, practical implementations of most familiar EEC devices would not be possible.
- Analysing and describing systems containing them is essential for designing effective drives.
- We start by looking at Maxwell's equations.

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- Maxwell's equations are a set of coupled partial differential equations which form the foundation of electric and magnetic circuits.
- In their PDE form, they are written as [5]:

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right).
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where **E** is the electric field, **B** is the magnetic field, ρ is the electric charge density and **J** the current density, ε_0 is the vacuum permittivity and μ_0 is the vacuum permeability.

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- Solution to Maxwell's equation can be hard if not attainable and usually simplifying assumptions are made to reach practical solutions.
- First, the displacement-current term ($\varepsilon_0 \partial \mathbf{E}/\partial t$) can be neglected.
  - This term accounts for **B**-fields produced in space by time-varying **E**-fields (i.e., electromagnetic radiation) [10].
- Neglecting $\varepsilon_0 \partial \mathbf{E}/\partial t$ results in the magneto-static form which relates **B**-fields to the currents (**J**) which produce them.
- In integral form [2]:

$$\oint_{\mathcal{C}} \mathbf{H}\, \mathrm{d}\mathbf{l} = \int_{\mathcal{S}} \mathbf{J} \cdot \mathrm{d}\mathbf{a}, \quad \text{and} \quad \oint_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a} = 0.$$

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The line integral ($\oint$) of the tangential component of the magnetic field intensity ($\mathbf{H}$) around a closed contour ($\mathcal{C}$) is equal to the total current ($\mathbf{J}$) passing through any surface ($\mathcal{S}$) linking that contour.

- The second one states the magnetic flux density ($\mathbf{B}$) is conserved, i.e., no net flux enters or leaves a closed surface (i.e, $\nabla \cdot \mathbf{B} = 0$).

These simplifications have allowed us to remove the effect of the $\mathbf{E}$ field on our calculations.

- A second simplifying assumption involves the concept of magnetic circuits.
- The general solution for the **H** and the **B** in a structure of complex geometry is extremely difficult if not practically impossible.
- However, a 3D field problem can often be reduced to what is essentially a circuit equivalent of magnetic elements,
  - with acceptable engineering accuracy [11].

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- A magnetic circuit is a structure composed of highly permeable ($\mu_r \gg 0$) material(s).
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Magnetic Circuits and Materials

- The core is assumed to be composed of magnetic material with permeability much greater than of surrounding air (i.e., $\mu_r \gg \mu_0$).
- The core is of uniform cross section and is excited by a winding of N turns carrying a current of i amperes.
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Magnetic Circuits and Materials

- Due to high permeability of the material (i.e., $\mu_r \gg \mu_0$), the magnetic flux is confined almost entirely to the core,
- The **B**-field lines follow the path defined by the core,
- The flux density is essentially uniform over a cross section as the cross-sectional area is uniform.
- The **B**-field can be visualised in terms of flux lines which form closed loops interlinked with the winding.

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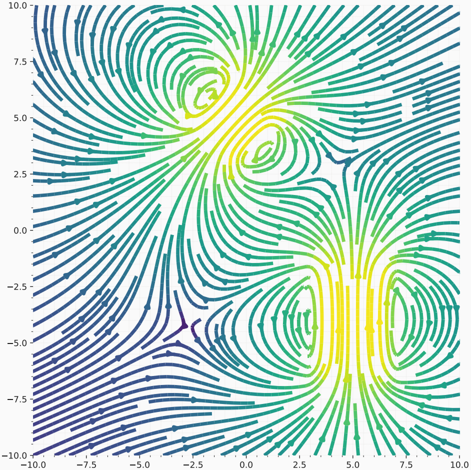

Figure 7: An example demonstration of flux paths passing highly permeable material.

Magnetic Circuits and Materials

- The source of the **B**-field in the core is the ampere-turn Ni.
- In magnetic circuit terminology Ni is the magnetomotive force (mmf) acting on the magnetic circuit.
- In systems with more than one winding, Ni must be replaced by the algebraic sum of the ampere-turns of all the windings:

$$\sum_{n=0}^{\infty} N_i i_i.$$

- The magnetic flux (ϕ) crossing a surface $\mathcal{S}$ is the surface integral of the normal component of **B**:

$$\phi = \int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{a}$$

where the unit of flux (ϕ) is Wb.

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- This equation states that all flux entering the surface enclosing a volume must leave the volume as they form closed loops.
- This is enough to justify the uniformity of the magnetic flux density ($\mathbf{B}$) across the cross section of a magnetic circuit (A_c):

$$\phi_c = B_c A_c,$$

where ϕ_c is the flux, B_c is the magnetic flux density magnitude and A_c is the cross-sectional core area.

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- This is enough to justify the uniformity of the magnetic flux density ($\mathbf{B}$) across the cross section of a magnetic circuit (A_c):

$$\phi_c = B_c A_c,$$

where ϕ_c is the flux, B_c is the magnetic flux density magnitude and A_c is the cross-sectional core area.

Magnetic Circuits and Materials

- Using the original magneto-static simplifications (i.e., no **E**-fields), we can come up to:

$$\mathcal{F} = Ni = \oint \mathbf{H}\,\mathrm{d}\mathbf{l}.$$

- As the path of the flux line is close to the mean length[1] of the core (l_c), this is simplified to:

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- The relationship between the $\mathbf{H}$ and $\mathbf{B}$ is a property of the material.
- It is common to assume a linear relationship shown as:

$$\mathbf{B} = \mu \mathbf{H},$$

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Magnetic Circuits and Materials

- Transformers are wound on closed cores.
- However, EEC devices which incorporate a moving element must have air gaps in their magnetic circuits.
- When the air-gap (g) is much smaller than the dimensions of the adjacent core faces, the magnetic flux will follow the path defined by the core and the air gap and the techniques of magnetic-circuit analysis can be used.

> If g becomes excessively large, the flux will be leak out of the sides of the air gap and the techniques of magnetic-circuit analysis will no longer be strictly applicable.

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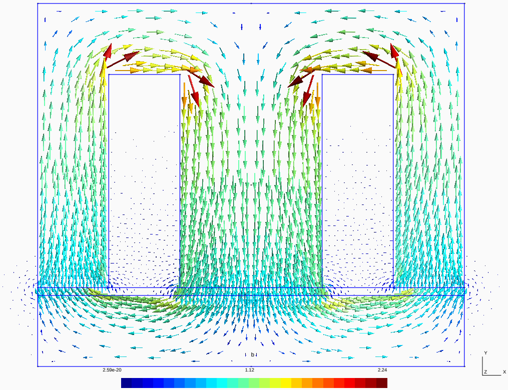

Figure 8: A Magnetic Circuits with two mmf sources.

Magnetic Circuits and Materials

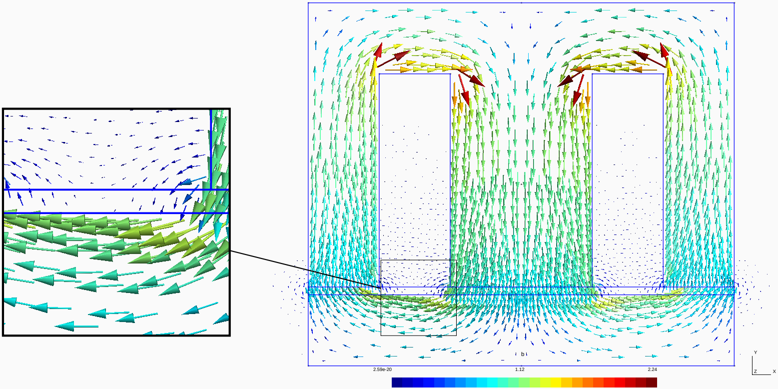

Figure 9: A closeup of the fringe paths caused by the air gap.

- Provided the air-gap length g is sufficiently small, the configuration can be analysed as a magnetic circuit with two (2) series components:
  1. Magnetic core (μ), cross-sectional area A_c, and mean length l_c,
  2. Air gap (μ_0), cross-sectional area A_g, and length l_g.
- In the core the flux density can be assumed uniform and ($\mu \gg \mu_0$):

$$B_\mathrm{c} = \frac{\phi}{A_\mathrm{c}}, \quad \text{and in the air gap,} \quad B_\mathrm{g} = \frac{\phi}{A_\mathrm{g}},$$

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Magnetic Circuits and Materials

- Applying this to the magnetic circuit gives:

$$\mathcal{F} = H_c l_c + H_g g,$$

using the **B-H** relationship:

$$\mathcal{F} = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g.$$

- Here, $\mathcal{F}$ is the mmf applied to the magnetic circuit where $\mathcal{F}_c = H_c l_c$ is the magnetic field in the core and $\mathcal{F}_g = H_g l_g$ is the magnetic field in the air-gap.
- Using ϕ we can re-write this equation as:

$$\mathcal{F} = \phi \left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g} \right).$$

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- The term which multiplies mmf to flux is called Reluctance.

$$\mathcal{F} = \phi \left(\overbrace{\frac{l_\mathrm{c}}{\mu A_\mathrm{c}}}^{\mathcal{R}_\mathrm{c}} + \overbrace{\frac{g}{\mu_0 A_\mathrm{g}}}^{\mathcal{R}_\mathrm{g}} \right) \quad \text{or} \quad \mathcal{F} = \phi \left(\mathcal{R}_\mathrm{c} + \mathcal{R}_\mathrm{g} \right)$$

Reluctance can be seen as analogous to resistance to electrical circuits.

Example

The magnetic circuit shown has dimensions:

$$A_c = A_g = 9 \text{ cm}^2,$$

$$g = 0.05 \text{ cm},$$

$$l_c = 30 \text{ cm},$$

$$N = 500 \text{ Turns.}$$

Assume the value $\mu_0 = 70000 \text{ H} \cdot \text{m}^{-1}$ for core material.

(a) Find the reluctance values $\mathcal{R}_c$ and $\mathcal{R}_g$.

(b) For the value of $B_c = 1$ T, find the flux ϕ and the current i.

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Solution

(a) 3,789.4

442,097.06

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Example

The following magnetic structure has infinite permeability ($\mu \to \infty$). Find the air-gap flux ϕ and flux density B_{g}.

Parameters: $I = 10$ A, $N = 1000$ Turns, $g = 1$ cm, and $A_{\mathrm{g}} = 2000$ cm^2.

Solution

(a) 0.13

- When a **B**-field varies with time, an **E**-field is produced in space as determined by Faraday's law [5]:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a}.$$

> The line integral of the electric field intensity E around a closed contour C is equal to the time rate of change of the magnetic flux linking (i.e. passing through) that contour.

> In electric drives, the effects of **E** field can be neglected.

Magnetic Circuits and Materials

- This simplification allows us to cancel the LHS of the equation to just the induced voltage (i.e., electromotive force)

$$\cancelto{\mathcal{E}}{\oint_{C} \mathbf{E} \cdot} d\mathbf{s} = -\frac{d}{dt}\int_{S} \mathbf{B} \cdot d\mathbf{a}.$$

- Whereas on the RHS, the flux is dominated by the core flux (ϕ).

$$\mathcal{E} = -\frac{d}{dt}\cancelto{\phi}{\int_{S} \mathbf{B} \cdot d\mathbf{a}}.$$

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$$\mathcal{E} = N\frac{d\phi}{dt} = \frac{d\lambda}{dt} \quad \text{where} \quad \lambda = N\phi \quad \blacksquare$$

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- For a magnetic circuit composed of magnetic material of constant magnetic permeability, the relationship between ϕ and i will be linear and we can define the inductance L as:

$$L = \frac{\lambda}{i} \quad \text{which can be written as} \quad L = \frac{N^2}{\mathcal{R}_{\text{tot}}}$$

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- The reference directions for the currents (i_1, i_2) have been chosen to produce flux in the same direction.
- The total mmf is therefore:

$$\mathcal{F} = N_1 i_1 + N_2 i_2.$$

- With the reluctance of the core neglected and assuming that $A_c = A_g$, the core flux ϕ is:

$$\phi = (N_1 i_1 + N_2 i_2) \frac{\mu_0 A_c}{g},$$

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- If ϕ is broken up into terms attributable to the individual currents, the resultant flux linkages of coil 1 can be expressed as:

$$\lambda_1 = N_1\phi = \overbrace{N_1^2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{11}} i_1 + \overbrace{N_1N_2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{12}} i_2,$$

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- It cannot be rigorously applied in situations where the nonlinear characteristics of magnetic materials dominate .
- However, in many situations of practical interest, the reluctance of the system is dominated by that of an air gap (which is of course linear) and the nonlinear effects of the magnetic material can be ignored.
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- EEC[1] devices require two (2) important properties:
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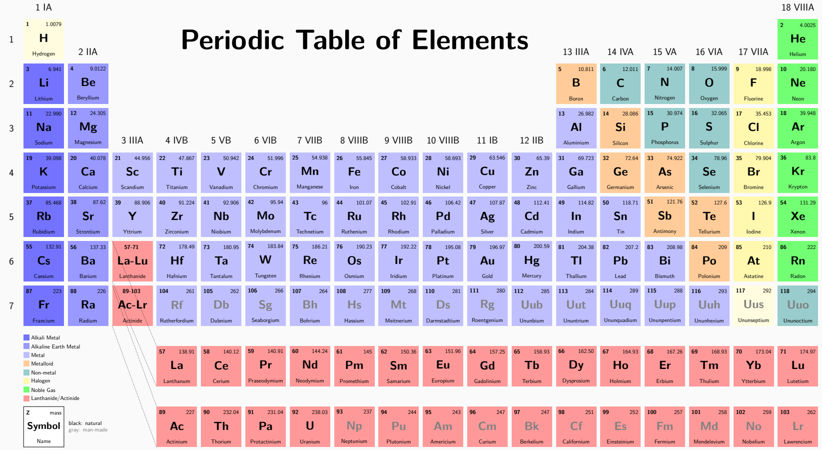

Figure 10: The periodic table of elements.

- Typically composed of iron (**Fe**) and alloys of iron with:
  - Cobalt, Tungsten, Nickel, Aluminium, and other metals
- These materials are characterised by a wide range of properties,
  - Phenomena responsible for their properties are the same.
- Ferro-magnetic materials are found to be composed of a large number of domains.
  - Regions where the magnetic moments of all the atoms are parallel, giving rise to a net magnetic moment for that domain.
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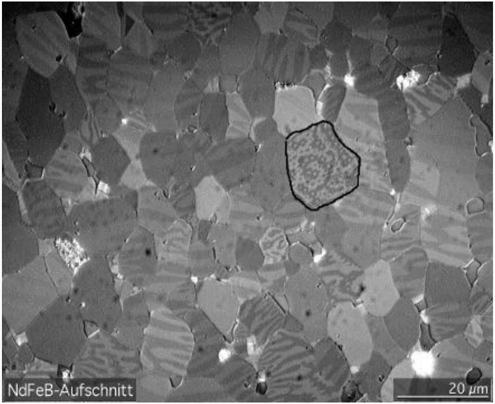

Figure 11: Microcrystalline grains within a piece of Nd2Fe14B (the alloy used in neodymium magnets) with magnetic domains made visible with a Kerr microscope. The domains are the light and dark stripes visible within each grain. The outlined grain has its magnetocrystalline axis almost vertical, so the domains are seen end-on [4].

Magnetic Circuits and Materials

- When **B**-field is applied to the material, the magnetic domain moments tend to align with the applied **B**-field.
- As a result, the magnetic moments add to the **B**-field, producing a higher flux density than would exist due to the magnetizing force alone.
- Therefore the effective permeability μ, equal to the ratio of the total magnetic flux density to the applied magnetic-field intensity, is large compared with the permeability of free space (μ_0).
- As the magnetising force (**H**) is increased, this continues until all the magnetic moments are aligned with the applied field.
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Material becomes fully saturated.

- Without an external **B**-field, magnetic domains naturally align along certain directions associated with the crystal structure of the domain.
  - This is known as axes of easy magnetisation [6].
- If the applied **B** is reduced, the domain magnetic moments relax to the direction of easy magnetism nearest to the applied field.
- As a result, **B** is reduced to zero (0), although they will tend to relax towards their initial orientation, the magnetic dipole moments will no longer be totally random in their orientation;
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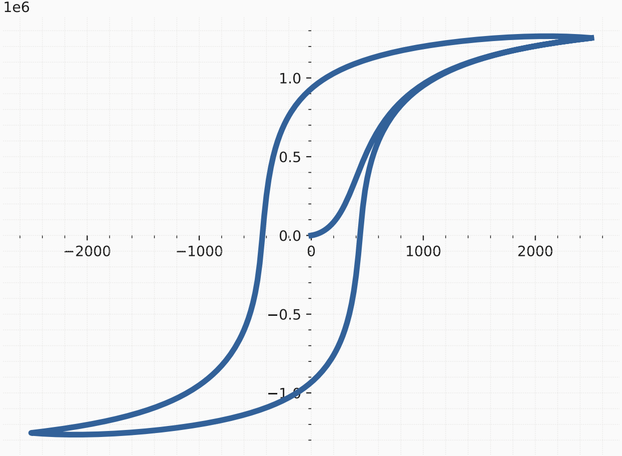

Figure 12: A Hysteresis Curve (B-H) Generated using the Jiles-Artherton model.

1st Quadrant

- The domains are very small, below the single domain size where there is resistance to demagnetization [1].
- The domains start to increase in size.

> For a small interval, the magnetization will be reversible.

- As the field increases, magnetization will no longer reverse to zero but move on a minor hysteresis loop.
- Eventually the curve starts to bend over to the right.
  - It will still increase as more magnetic domains reach their full size and their magnetisation become parallel to the external field [9].

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1st Quadrant - Saturation

- Eventually the **B**-field will become high enough where no more change in the magnetisation occurs.
  - This is called technical saturation.
- It is possible to reach 99+% saturation.
- As the field is backed off from "saturation", the magnetisation declines very slightly to the B_r point.
  - This is the remanence, or residual induction [8].

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2nd Quadrant - Demagnetisation

- In this quadrant the applied field opposes the materials **B**-field.
- As the external **B**-field increases in magnitude, some domains will reverse.
- At the knee of the demagnetisation curve, this increase has become rapid and the magnetisation will fall to the H_{ci} point.
- At H_{ci}, the number of domains aligned with the original magnetization is the same as the number aligned with the opposing magnetic field.

> The net magnetisation is zero (0).

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- The total magnetization of the part will be reversed.
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4th Quadrant - Demagnetisation

- After fully reversing the magnet and removing the field in the third quadrant, magnetization will recoil to a point that is the negative of the B_r observed when in the first and second quadrant.
- If we apply additional field in the positive direction, we duplicate the second quadrant curve.

4th Quadrant - Demagnetisation

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- In AC power systems, voltage and flux wave-forms closely approximate sinusoidal functions over time.
- Time to describe the excitation characteristics and losses associated with steady-state ac operation of magnetic materials.
- Assume a closed-core magnetic circuit (i.e., with no air gap),
- And a sinusoidal variation of the core flux (ϕ) with the following:

$$\phi(t) = \phi_{\mathrm{max}} \sin(wt) = A_{\mathrm{c}} B_{\mathrm{max}} \sin(wt)$$

where:

ϕ_{max} amplitude of core flux in Wb,
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- Generally, the rms value of a periodic function of time $f(t)$, of T is:

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- The rms value of a sine wave can be shown to be $1/\sqrt{2}$ times its peak value. Therefore the induced voltage rms is:

$$E_{\text{rms}} = \frac{2\pi}{\sqrt{2}} f N A_{\text{c}} B_{\text{max}} = \sqrt{2}\pi N A_{\text{c}} B_{\text{max}} = 4.44 N A_{\text{c}} B_{\text{max}} \quad \blacksquare$$

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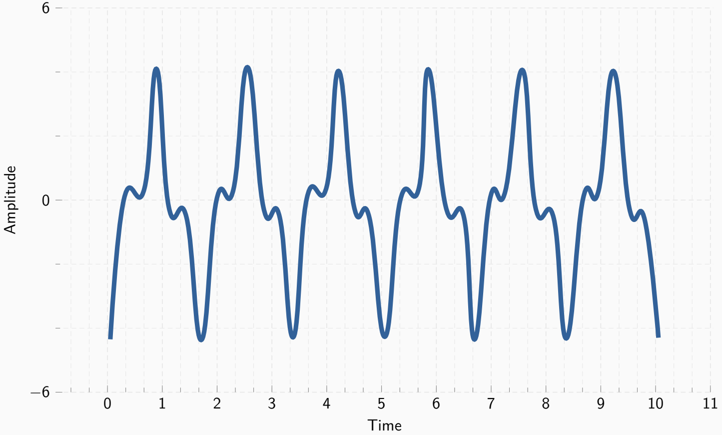

Figure 13: An Example Visualisation of the excitation current experienced by a magnetic circuit.

- The exciting current supplies the mmf required to produce the core flux and the power input associated with the energy in the magnetic field in the core.
- Part of this energy is dissipated as losses and results in heating of the core.
- The rest appears as reactive power associated with energy storage in the magnetic field.

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Magnetic Circuits and Materials

- There are two (2) mechanisms associated with time varying **B**-fields.

Ohmic Heating

- Associated with induced currents in the core material.
- From Faraday's law, we see that **B**-fields give rise to **E**-fields.
- In magnetic materials these **E**-fields result in induced currents,
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- Therefore the resultant "dynamic" B-H loop under ac operation is somewhat "fatter" than the hysteresis loop for slowly varying conditions, and this effect increases as the excitation frequency is increased.
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- To reduce the effects of eddy currents, magnetic structures are usually built of thin sheets of laminations of the magnetic material.
- These laminations, which are aligned in the direction of the field lines, are insulated from each other by an oxide layer on their surfaces or by a thin coat of insulating enamel or varnish.
- This greatly reduces the magnitude of the eddy currents since the layers of insulation interrupt the current paths; the thinner the laminations, the lower the losses.
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- In a magnetic circuit or the transformer, a time-varying excitation (i_ϕ) will cause the magnetic material to undergo a cyclic variation described by a hysteresis loop.
- The energy input W to the magnetic core as the material undergoes a single cycle is shown to be:

$$W = \oint i_\phi \, \mathrm{d}\lambda = \oint \left(\frac{H_\mathrm{c} l_\mathrm{c}}{N} \right) (A_\mathrm{c} N \, \mathrm{d}B_\mathrm{c}) = A_\mathrm{c} l_\mathrm{c} \oint H_\mathrm{c} \, \mathrm{d}B_\mathrm{c}$$

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Magnetic Circuits and Materials

- Nearly all transformers and certain sections of electric machines use sheet-steel material that has highly favorable directions of magnetization along which the core loss is low and the permeability is high.
- This material is termed grain-oriented steel [12].
- The reason lies in the atomic structure of a crystal of the silicon-iron alloy,
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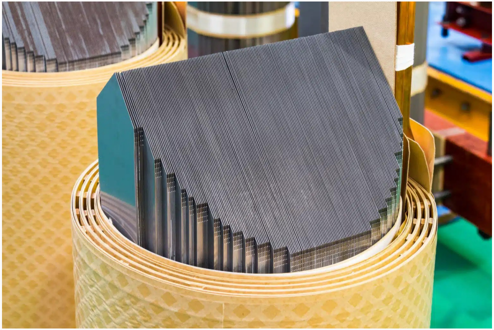

Figure 14: An example of Grain Oriented Steel used in industry [7].

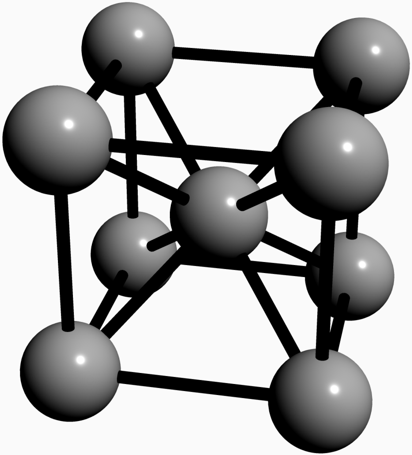

Figure 15: The atomic structure of grain oriented steel.

- Each crystalline cube has an atom at each corner as well as one in the center of the cube.
- In the cube, the easiest axis of magnetization is the cube edge.
- The diagonal across the cube face is more difficult
- By suitable manufacturing techniques most of the crystalline cube edges are aligned in the rolling direction to make it the favorable direction of magnetisation.

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- The behaviour in this direction is superior in core loss and permeability to non-oriented steels in which the crystals are randomly oriented to produce a material with characteristics which are uniform in all directions.
- As a result, oriented steels can be operated at higher flux densities than the nonoriented grades.
- Non-oriented electrical steels are used in applications where the flux does not follow a path which can be oriented with the high-Temperature rolling direction or where low cost is of importance.
- In these steels the losses are somewhat higher and the permeability is very much lower than in grain-oriented steels.

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Appendix

- Assume the following simplified induction drive equivalent circuit.

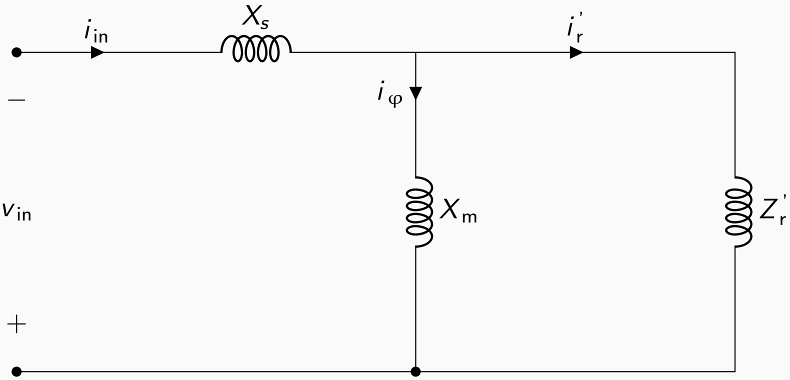

Figure 16: An abstract rendition of the simplified induction circuit equivalent circuit.

Appendix

- where:

$$Z_{\mathrm{s}} = R_{\mathrm{s}} + \mathbf{j}\,X_{\mathrm{s}},$$
$$Z_{\mathrm{m}} = \frac{R_{\mathrm{c}} \times \mathbf{j}\,X_{\mathrm{m}}}{R_{\mathrm{c}} + \mathbf{j}\,X_{\mathrm{m}}},$$
$$Z_{\mathrm{r}} = \frac{R_{\mathrm{r}}^{'}}{s} + jX_{\mathrm{r}}^{'}.$$

- From Kirchhoff's current law, the following statement holds true:

$$i_{\mathrm{in}} = i_{\varphi} + i_{\mathrm{r}}^{'}.$$

- The second identity can be derived from parallel circuit principles;

$$Z_{\mathrm{m}} i_{\varphi} = Z_{\mathrm{r}}^{'} i_{\mathrm{r}} \quad \rightarrow \quad i_{\mathrm{m}} = i_{\mathrm{R}}^{'} \times \left(\frac{Z_{\mathrm{R}}}{Z_{\mathrm{m}}} \right).$$

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Appendix

- We can isolate the magnetizing current (i_φ).

$$i_\varphi = i_r^{'} \times \left(\frac{Z_r}{Z_m}\right) + i_r^{'}.$$

- Isolating the rotor current ($i_r^{'}$) gives us the final expression:

$$i_r^{'} = \frac{i_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

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- Consider the following equivalent circuit of an IM.

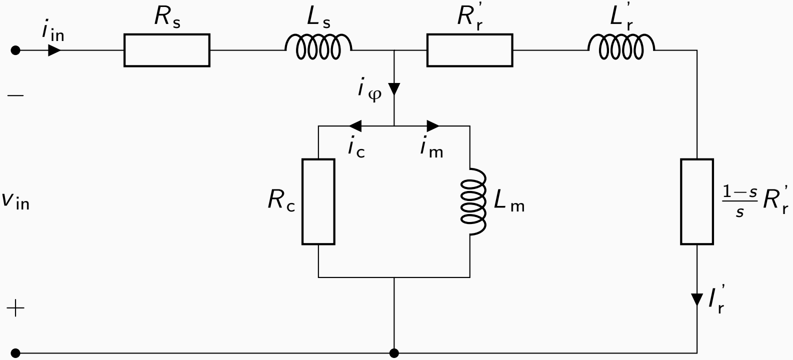

Figure 17: An abstract rendition of the induction circuit equivalent circuit.

Appendix

- We can start by summing up the rotor side.

$$Z_r = \frac{R_r^{'}}{s} + \mathbf{j}\, w_r L_r^{'} \quad \rightarrow \quad Z_r = \frac{R_r^{'}}{s} + X_r^{'}.$$

- The stator side is calculated as:

$$Z_s = R_s + \mathbf{j}\, w_s L_s \quad \rightarrow \quad Z_s = \frac{R_s}{s} + X_s.$$

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Appendix

- Without substitution, the input impedance is calculate to be

$$Z_{\text{in}} = Z_{\text{s}} + \frac{Z_{\text{m}} \times Z_{\text{r}}}{Z_{\text{m}} + Z_{\text{r}}}.$$

- From here, we can derive the stator current (I_{in}).

$$V_{\text{ph}} = \frac{Z_{\text{in}}}{I_{\text{in}}} \quad \rightarrow \quad I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}}.$$

- Now, the Torque of an IM is related to the air-gap power by:

$$T_{\text{mech}} = \frac{P_{\text{gap}}}{w_{\text{s}}} \qquad \text{and} \qquad P_{\text{gap}} = n_{\text{ph}} \frac{I_{\text{r}}^2}{w_{\text{s}}} \left(\frac{R_2^{'}}{s} \right)$$

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Appendix

- Using the stator-rotor relationship, we can obtain our final result.

$$I_r^{'} = \frac{I_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

Go Back

an empirical relationship or phenomenological relationship is a relationship or correlation that is supported by experiment or observation but not necessarily supported by theory.

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