Topics on Electrical Engineering

Electric Drive Fundamentals

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MCI

B.Sc - Drive Systems

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Introduction

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Introduction

- The goal of this lecture is to give you the fundamentals of electric machinery using theory and engineering practice.
- This lecture is a total of 2 SWS with a total of thirty (30) UE.
  - With 28 UE is devoted to lectures.
- There is a written exam at the end of the module worth two (2) UE.
- There is one (1) assignment for this course:

  1st will be a pre-defined work which is individual based.

Introduction

- The individual assignments focus on understanding electric machinery.
- The assignment is uploaded to SAKAI for you to work on along with what is required of you for submission.
    - The assignments contain electric drive problems to solve.
- The strict deadline is the end day of the **last lecture**.
- Any submission after this date will not be accepted.
- If all submissions are sent early we can do a statistical analysis and go through questions before the final exam.

Introduction

- Lecture materials and all possible supplements will be present in its Github Repo.
  - You can easily access the link to the web-page from here.

> Github is chosen for easy access to material management and CI/CD capabilities and allowing hosting websites.

- In the lecture some exercises are solved using Python and other examples and can be accessed from the Repo website.

Introduction

- At the end of the lectures there will be a final examination which you will tested.
- You will be asked three (3) questions related to electric drives.
- The exam will be ninety (90) minutes.
- You are NOT allowed a personal formula sheet or any kind of supporting material.
- You are allowed a calculator.

Assessment Type	Overall Points	Breakdown	%
Individual Assignment	40		
		Report	20
		Solution(s)	80
			20
Final Exam	60		
		Question 1	40
		Question 2	30
		Question 2	30

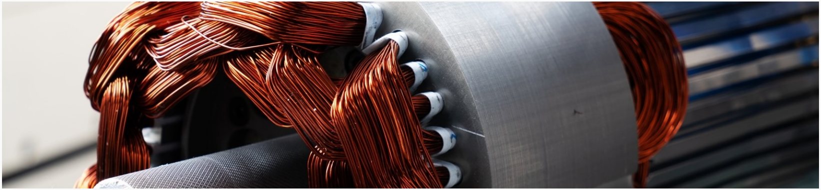
Table 1: Assessment Grade breakdown for the lecture.

Covered Topic	Appointment
Return to Fundamentals	1
Magnetic Circuits & Materials	1
Transformers	2
Electromechanical Energy Conversion	2-3
Rotating Magnetic Fields	3
DC Drives	4
Poly-phase Induction Drives	4-5
Single-phase Induction Drives	5-6
Linear Induction Drives	6
Poly-phase Synchronous Drives	6-7
Solid-state Commutation Drives	7

Table 2: Distribution of materials across the semester.

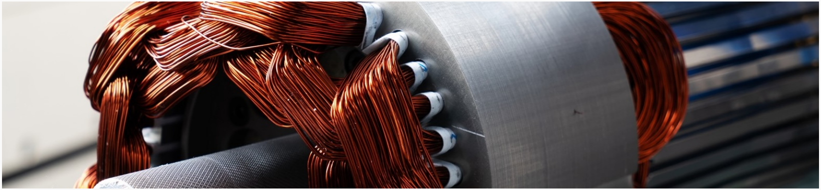
Return to Fundamentals

- Complex number notation,
- Multi-phase systems,
- Phasors and Wave-forms



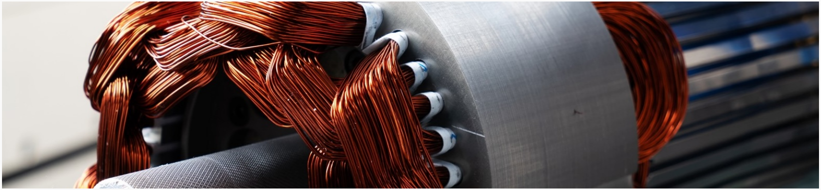
Transformers

- Construction & Physical Properties
- Modelling
- Parameter Tests
- Connection Types



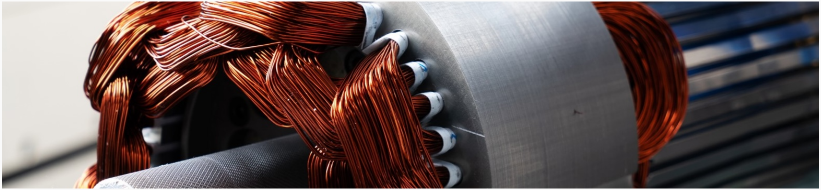
Rotating Magnetic Fields

- MMF in Winding
- Torque Generation
- Generated Voltage



DC Drives

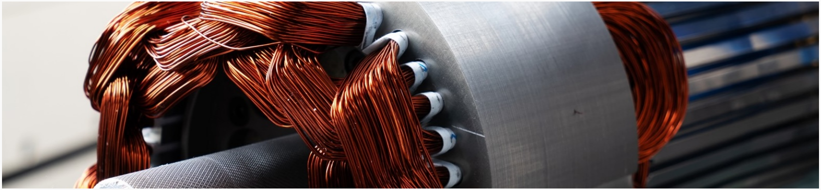
- Construction
- Physical laws governing its operation
- Types of Connections used in industry and commercial applications.
- Applications in Industrial/Commercial venues.



Introduction

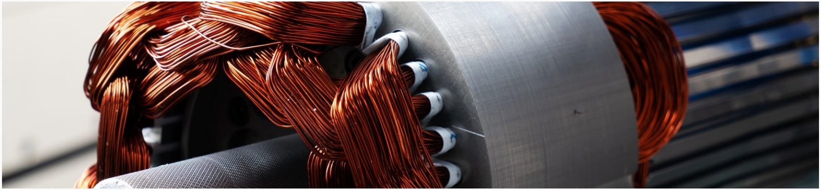
Poly-phase Induction Drives

- Construction
- Physical laws governing its operation principle.
- Modelling an induction drive mathematically
- Methods of starting an induction drive.



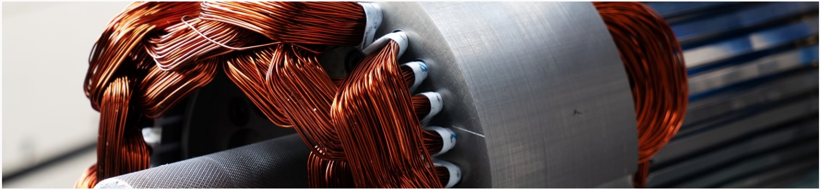
Single-Phase Induction Drives

- Creating a Rotating Magnetic Field in a single phase
- Types of single-phase types
- Salient-pole



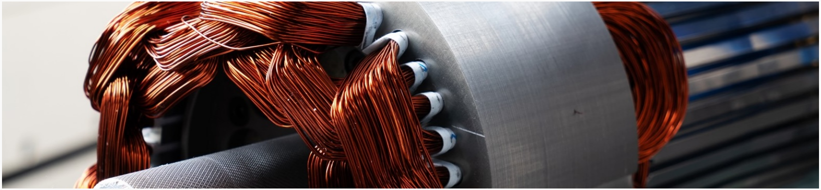
Poly-phase Synchronous Drives

- Construction and Rotor types,
- Operation principles,
- Regulatory behaviours,
- V-curves,



Solid-State Commutation Drives

- Solid State Commutation
- BLDC & PMSM Drives
- Switched Reluctance Motor (SRM)
- Stepper Drives



Books

- Mohan Ned. "*Advanced electric drives: analysis control and modeling using MATLAB/Simulink*" John Wiley & Sons 2014.
- Krause Paul C. et. al. "*Analysis of electric machinery and drive systems*" Vol. 2 IEEE Press 2002.
- Pyrhonen Juha et. al "*Design of rotating electrical machines*" John Wiley & Sons 2013.
- Stephen J. Chapman. "*Electric Machinery Fundamentals (5th Edition)*" (2012).
- Fitzgerald A. E. et. al. "*Electric Machinery*" McGraw Hill (2003).

Books

- Hughes A. et. al. "*Electric Motors and Drives: Fundamentals Types and Applications*" Newnes 2019.
- Melkebeek A. "*Electrical Machines and Drives: Fundamentals and Advanced Modelling*" Springer 2018.
- Wildi T. "*Electrical machines, drives, and power systems*" Pearson Education 2006.
- Veltman A. et. al. "*Fundamentals of Electrical Drives*" Springer 2007.

White Papers

- Maddox Transformer "*Guide to transformer cores: types, construction, & purpose*"
- Control Engineering *Springtime for Switched-Reluctance Motors?* .

Lecture Notes

- Power Transformers "*ESE 470 Energy Distribution Systems*" Oregon State University,
- Principles of Electromechanical Energy Conversion "*Actuators & Sensors in Mechatronics Electromechanical Motion Fundamentals*" NYU,

Return to Fundamentals

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Learning Outcomes

(LO1) An Overview of Poly-phase circuits,

(LO2) Definitions on Active-Reactive power,

(LO3) Polar Coordinate System.

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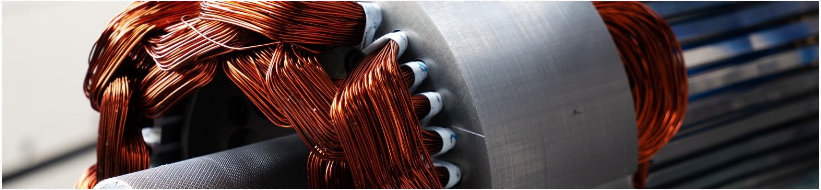


Return to Fundamentals

- A rotating magnetic field is a magnetic field with moving polarities.
  - Which its opposite poles rotate about a **central point** or **axis**.
- To create a rotating magnetic field (RMF) you need at least a 2-phase system.
- In the industry, RMFs are mostly produced using a 3-phase supply [1].
  - There are also economic reasons, as it is cheaper to design a 3-phase system with minimal cost to wiring [2].
- Before starting with electric drives, it is a good time to look at some fundamental concepts in power engineering.

[1] 3-phase supply produces smoother operation of motors compared to 2-phase. This is due to the power transfer in 3 phase supply being less pulsating than in 2 phase supply.

[2] However, there have been test and feasibility on 6-phase power in 1970s with minimal success.

Return to Fundamentals

- Generation, transmission, and heavy-power utilisation of AC electric energy almost invariably involve a poly-phase circuit.
- In such a system, each voltage source consists of a group of voltages having related magnitudes and phase angles.
- Thus, an n-phase system employs voltage sources which typically consist of n voltages substantially equal in magnitude and successively displaced by a phase angle of $360°/n$.
- Therefore a 3-phase system would have 3 different voltage sources separated 120 degrees apart.

Return to Fundamentals

- The three individual voltages of a three-phase source may each be connected to its own independent circuit.
- We would then have three separate single-phase systems.
- Alternatively, symmetrical electric connections can be made between the three voltages and the associated circuitry to form a three-phase system.
- It is the latter alternative that we are concerned.
- Note that the word phase now has two distinct meanings.
  - It may refer to a portion of a pol-yphase system or circuit,
  - It may be used in reference to the angular displacement between voltage or current phasors.

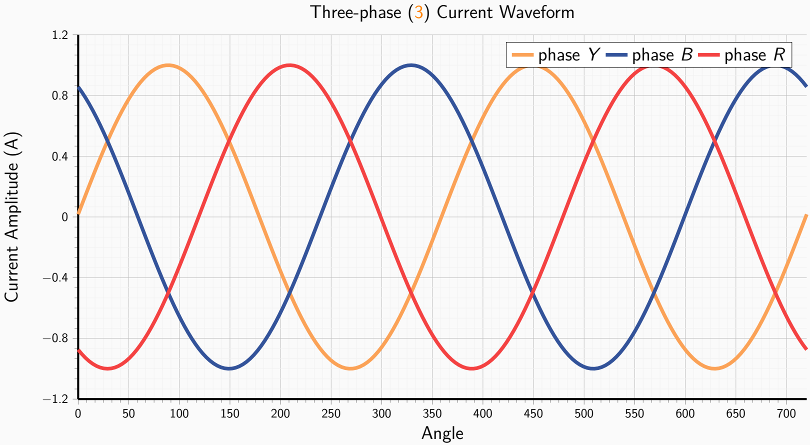
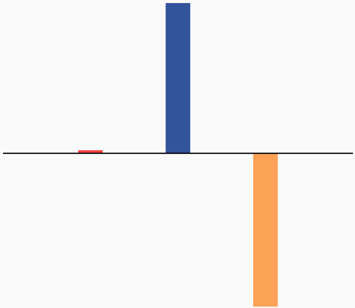


Figure 1: A three phase waveform of currents **R**, **B**, and **Y**.

Return to Fundamentals



$i_R = 0.01746$

$i_B = 0.85716$

$i_Y = -0.87462$

$$i_R + i_B + i_Y = 0.0$$

Figure 2: A three phase waveform of currents **R**, **B**, and **Y**.

Return to Fundamentals

- There are **no neutral connection** available.
- Phase voltage appears **across the windings**.
- In a delta connection, Line voltage is equal to Phase voltage:

$$v_{\text{line}} = v_{\text{ph}}$$

- i_{YB}, i_{YR}, i_{BR} are also called **Line current** (i.e., i_{line}).
- Line current is $\sqrt{3}$ times that of the phase current.

$$i_{\text{line}} = \sqrt{3} i_{\text{subs}}$$

- Power related definitions are:

$$P = \sqrt{3} v_{\text{line}} i_{\text{line}} \cos\varphi, \quad \text{and} \quad P = 3 v_{\text{ph}} i_{\text{ph}} \cos\varphi.$$

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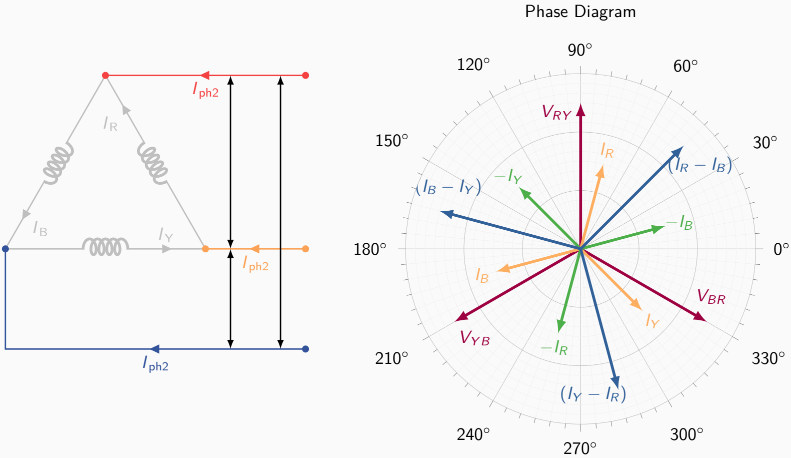


Figure 3: The connection and the phasor diagram of a 3-phase delta connection.

Example

Three impedance with a value $Z_{\Delta} = 12.00 + \mathbf{j}\,9.00 = 15.00\angle 36.9\ \Omega$ are connected in Δ.

For balanced line-to-line voltages of 208 V, find the line current, the power factor, and the total power.

Return to Fundamentals

- There is a neutral connection available.
- Phase voltage appear across windings.

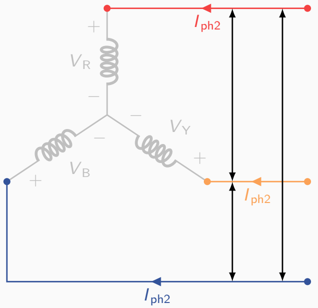
$$v_{\text{line}} = \sqrt{3} v_{\text{ph}},$$

- i_{YB}, i_{YR}, i_{BR} are also known as line current (i_{line}).
- In a star connection, line current is equal to phase current.

$$i_{\text{line}} = i_{\text{ph}}.$$

- Power related equations for 3-phase:

$$\begin{aligned} P &= \sqrt{3} v_{\text{line}} i_{\text{line}} \cos \varphi, \\ P &= \sqrt{3}\sqrt{3} \left(v_{\text{ph}} \right) \left(i_{\text{ph}} \right) \cos \varphi, \\ P &= 3 v_{\text{ph}} i_{\text{ph}} \cos \varphi. \end{aligned}$$



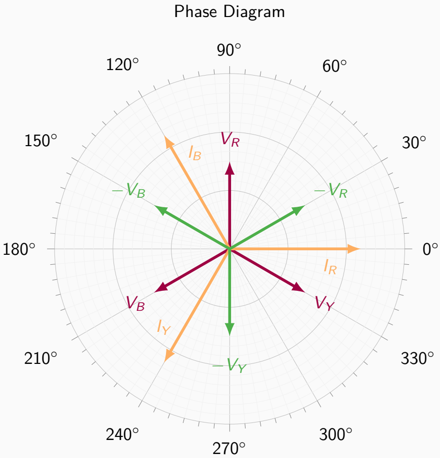


Figure 4: The connection and the phasor diagram of a 3-phase wye connection.

Example

A Y-connected 120 V source feeds a Δ-connected load through a distribution line having an impedance of $0.3 + \mathbf{j}0.9\ \Omega$. The Y-source impedance is $0.2 + \mathbf{j}0.5\Omega$

The load impedance is $118.5 + \mathbf{j}85.8\ \Omega/o$.

Use the a-phase internal voltage of the generator as the reference.

(a) Construct a single-phase equivalent circuit of the three-phase system,

(b) Calculate the line currents,

(c) Calculate the phase voltages at the load terminals,

(d) Calculate the phase currents of the load,

(e) Calculate the line voltages at the source terminals.

Return to Fundamentals

$$Z = x + \mathbf{j}\, y = A\underline{/\theta},$$

where:

- Z is the complex vector,
- A is vector magnitude,
- x is real/active part,
- $\mathbf{j}$ is defined as $\sqrt{-1}$.
- y is imag/reactive part,
- θ is the complex angle.

$$\theta = {}^{y}/_{x}.$$

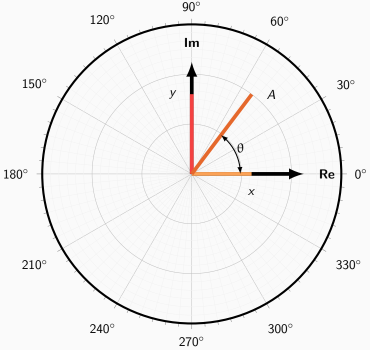


Figure 5: A polar coordinate system.

Example

Express:

$$\frac{3 - \mathbf{j}}{7 - 3\mathbf{j}},$$

in the form of $a + \mathbf{j}\, b$.

Solution

Answer is:

$$-\frac{12}{29} - \mathbf{j}\left(\frac{1}{29}\right) \quad \blacksquare$$

Return to Fundamentals

When it comes to power in AC, there are three (3) definitions:

- If energy is used/generated by an active element it is called **Active** or **Real** (P) power and measured in W.
- If energy is used/generated by an reactive element it is called **Reactive** power (Q)and measured in $\mathrm{V} \cdot \mathrm{A}r$.
- The combination of these two values is called **Apparent** power (S) and measured in $\mathrm{V} \cdot \mathrm{A}$.

$$S = \sqrt{P^2 + Q^2}$$

- Finally the angle difference between the voltage and current waveform is defined as the **phase** and tells how reactive/active a circuit is.

$$\varphi = \varphi_V - \varphi_I.$$

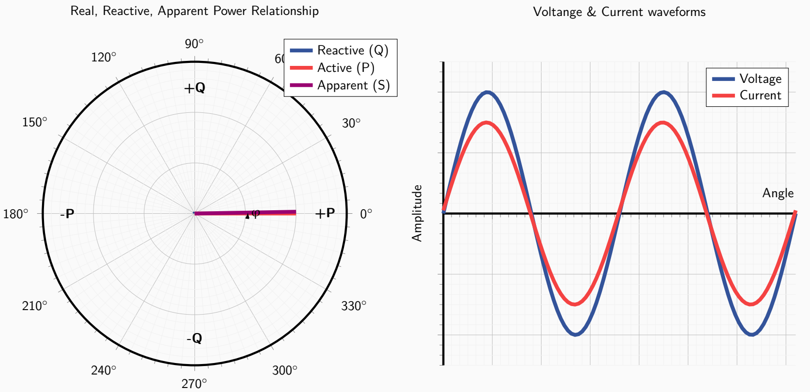


Figure 6: An animation showing the relations between Active, Reactive and Apparent power along with phase angle.

Example

A balanced three-phase load requires 480 kW at a lagging power factor of 0.8.

The load is fed from a line having an impedance of $0.005 + \mathbf{j}\,0.025\Omega$.

The line voltage at the terminals of the load is 600 V.

(a) Construct a single-phase equivalent circuit of the system.

(b) Calculate the magnitude of the line current.

(c) Calculate the magnitude of the line voltage at the sending end of the line.

(d) Calculate the power factor at the sending end of the line.

Magnetic Circuits and Materials

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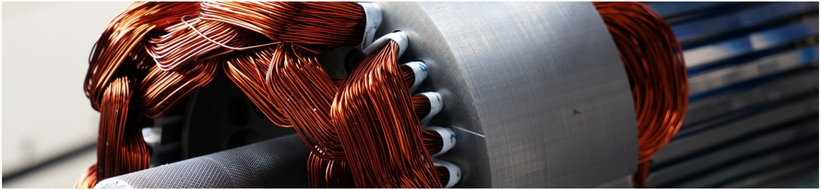
Learning Outcomes

(LO1) An Overview of Maxwell's Equations,

(LO2) Introduction to Magnetic Circuits,

(LO3) Brief look on Magnetic Materials,

(LO4) Magnetic Material Losses.



Magnetic Circuits and Materials

- Almost all electric drives use ferromagnetic material for shaping and directing **B**-fields.
  - These fields act as the medium for transferring and converting energy.

> Permanent-magnets are also widely used in drive design.

- Without these materials, practical implementations of most familiar EEC devices would not be possible.
- Analysing and describing systems containing them is essential for designing effective drives.
- We start by looking at Maxwell's equations.

Magnetic Circuits and Materials

- Maxwell's equations are a set of coupled partial differential equations which form the foundation of electric and magnetic circuits.
- In their PDE form, they are written as [6]:

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0}, \\
\nabla \cdot \mathbf{B} &= 0, \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \\
\nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right).
\end{aligned}
$$

where **E** is the electric field, **B** is the magnetic field, ρ is the electric charge density and **J** the current density, ε_0 is the vacuum permittivity and μ_0 is the vacuum permeability.

Magnetic Circuits and Materials

- Solution to Maxwell's equation can be hard if not attainable and usually simplifying assumptions are made to reach practical solutions.
- First, the displacement-current term ($\varepsilon_0 \partial \mathbf{E}/\partial t$) can be neglected.
  - This term accounts for **B**-fields produced in space by time-varying **E**-fields (i.e., electromagnetic radiation) [12].
- Neglecting $\varepsilon_0 \partial \mathbf{E}/\partial t$ results in the magneto-static form which relates **B**-fields to the currents (**J**) which produce them.
- In integral form [3]:

$$\oint_{\mathcal{C}} \mathbf{H}\, \mathrm{d}\mathbf{l} = \int_{\mathcal{S}} \mathbf{J} \cdot \mathrm{d}\mathbf{a}, \quad \text{and} \quad \oint_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a} = 0.$$

$$\oint_{\mathcal{C}} \mathbf{H}\,\mathrm{d}\mathbf{l} = \int_{\mathcal{S}} \mathbf{J} \cdot \mathrm{d}\mathbf{a}, \quad \text{and} \quad \oint_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a} = 0.$$

The line integral ($\oint$) of the tangential component of the magnetic field intensity ($\mathbf{H}$) around a closed contour ($\mathcal{C}$) is equal to the total current ($\mathbf{J}$) passing through any surface ($\mathcal{S}$) linking that contour.

- The second one states the magnetic flux density ($\mathbf{B}$) is conserved, i.e., no net flux enters or leaves a closed surface (i.e, $\nabla \cdot \mathbf{B} = 0$).

These simplifications have allowed us to remove the effect of the **E** field on our calculations.

- A second simplifying assumption involves the concept of magnetic circuits.
- The general solution for the **H** and the **B** in a structure of complex geometry is extremely difficult if not practically impossible.
- However, a 3D field problem can often be reduced to what is essentially a circuit equivalent of magnetic elements,
  - with acceptable engineering accuracy [14].

- A magnetic circuit is a structure composed of highly permeable ($\mu_r \gg 0$) material(s).
- The presence of high-permeability causes magnetic flux to be confined to the paths defined by the structure.

> This is similar to how currents are confined to the conductors of an electric circuit.

Magnetic Circuits and Materials

- The core is assumed to be composed of magnetic material with permeability much greater than of surrounding air (i.e., $\mu_{\mathrm{r}} \gg \mu_0$).
- The core is of uniform cross section and is excited by a winding of N turns carrying a current of i amperes.

- This winding produces a **B**-field in the core.

Magnetic Circuits and Materials

- Due to high permeability of the material (i.e., $\mu_r \gg \mu_0$), the magnetic flux is confined almost entirely to the core,
- The **B**-field lines follow the path defined by the core,
- The flux density is essentially uniform over a cross section as the cross-sectional area is uniform.
- The **B**-field can be visualised in terms of flux lines which form closed loops interlinked with the winding.

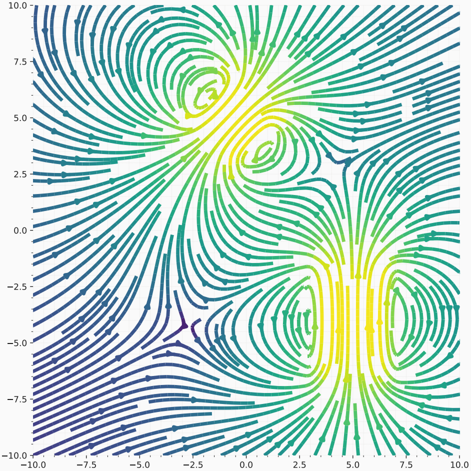


Figure 7: An example demonstration of flux paths passing highly permeable material.

Magnetic Circuits and Materials

- The source of the **B**-field in the core is the ampere-turn Ni.
- In magnetic circuit terminology Ni is the magnetomotive force (mmf) acting on the magnetic circuit.
- In systems with more than one winding, Ni must be replaced by the algebraic sum of the ampere-turns of all the windings:

$$\sum_{n=0}^{\infty} N_i i_i.$$

- The magnetic flux (ϕ) crossing a surface $\mathcal{S}$ is the surface integral of the normal component of **B**:

$$\phi = \int_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$$

where the unit of flux (ϕ) is Wb.

$$\phi = \int_{\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$$

- This equation states that all flux entering the surface enclosing a volume must leave the volume as they form closed loops.
- This is enough to justify the uniformity of the magnetic flux density ($\mathbf{B}$) across the cross section of a magnetic circuit (A_c):

$$\phi_\mathrm{c} = B_\mathrm{c} A_\mathrm{c},$$

where ϕ_c is the flux, B_c is the magnetic flux density magnitude and A_c is the cross-sectional core area.

- Using the original magneto-static simplifications (i.e., no **E**-fields), we can come up to:

$$\mathcal{F} = Ni = \oint \mathbf{H}\,\mathrm{d}\mathbf{l}.$$

- As the path of the flux line is close to the mean length[1] of the core (l_c), this is simplified to:

$$\mathcal{F} = Ni = H_\mathrm{c} l_\mathrm{c},$$

where H_c is the average magnitude of **H**.

[1] The centre path going through the magnetic circuit.

Magnetic Circuits and Materials

- The relationship between the **H** and **B** is a property of the material.
- It is common to assume a linear relationship shown as:

$$\mathbf{B} = \mu \mathbf{H},$$

  where μ is the magnetic permeability.
- In SI units, **H** is measured in $\mathrm{A} \cdot \mathrm{m}^{-1}$ and **B** is in $\mathrm{Wb} \cdot \mathrm{m}^{-2}$ or T.

Magnetic Circuits and Materials

- Transformers are wound on closed cores.
- However, EEC devices which incorporate a moving element must have air gaps in their magnetic circuits.
- When the air-gap (g) is much smaller than the dimensions of the adjacent core faces, the magnetic flux will follow the path defined by the core and the air gap and the techniques of magnetic-circuit analysis can be used.

> If g becomes excessively large, the flux will be leak out of the sides of the air gap and the techniques of magnetic-circuit analysis will no longer be strictly applicable.

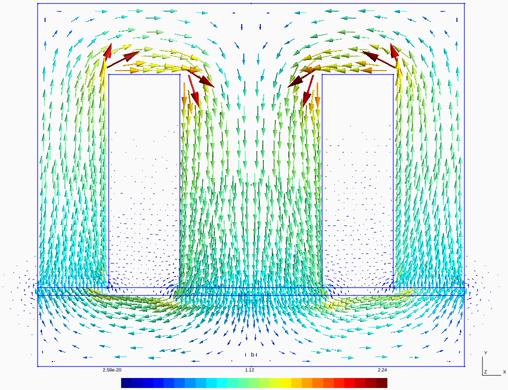


Figure 8: A Magnetic Circuits with two mmf sources.

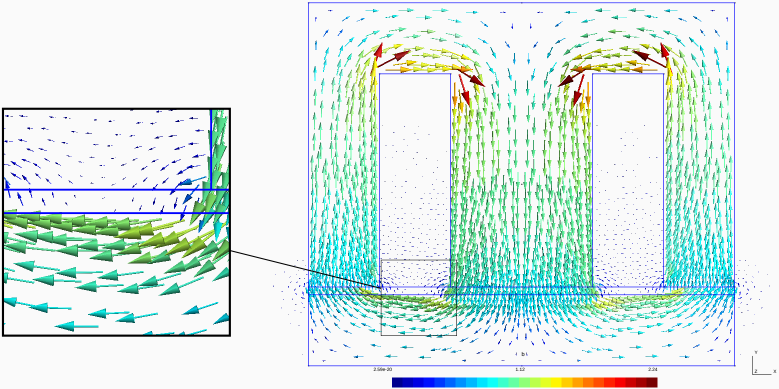


Figure 9: A closeup of the fringe paths caused by the air gap.

Magnetic Circuits and Materials

- Provided the air-gap length g is sufficiently small, the configuration can be analysed as a magnetic circuit with two (2) series components:
  1. Magnetic core (μ), cross-sectional area A_{c}, and mean length l_{c},
  2. Air gap (μ_0), cross-sectional area A_{g}, and length l_{g}.
- In the core the flux density can be assumed uniform and ($\mu \gg \mu_0$):

$$B_{\text{c}} = \frac{\phi}{A_{\text{c}}}, \quad \text{and in the air gap,} \quad B_{\text{g}} = \frac{\phi}{A_{\text{g}}},$$

- Applying this to the magnetic circuit gives:

$$\mathcal{F} = H_{\mathrm{c}} l_{\mathrm{c}} + H_{\mathrm{g}} g,$$

using the **B-H** relationship:

$$\mathcal{F} = \frac{B_{\mathrm{c}}}{\mu} l_{\mathrm{c}} + \frac{B_{\mathrm{g}}}{\mu_0} g.$$

- Here, $\mathcal{F}$ is the mmf applied to the magnetic circuit where $\mathcal{F}_{\mathrm{c}} = H_{\mathrm{c}} l_{\mathrm{c}}$ is the magnetic field in the core and $\mathcal{F}_{\mathrm{g}} = H_{\mathrm{g}} l_{\mathrm{g}}$ is the magnetic field in the air-gap.
- Using ϕ we can re-write this equation as:

$$\mathcal{F} = \phi \left(\frac{l_{\mathrm{c}}}{\mu A_{\mathrm{c}}} + \frac{g}{\mu_0 A_{\mathrm{g}}} \right).$$

- The term which multiplies mmf to flux is called Reluctance.

$$\mathcal{F} = \phi \left(\overbrace{\frac{l_c}{\mu A_c}}^{\mathcal{R}_c} + \overbrace{\frac{g}{\mu_0 A_g}}^{\mathcal{R}_g} \right) \quad \text{or} \quad \mathcal{F} = \phi \left(\mathcal{R}_c + \mathcal{R}_g \right)$$

> Reluctance can be seen as analogous to resistance to electrical circuits.

Example

The magnetic circuit shown has dimensions:

$$A_c = A_g = 9 \ \text{cm}^2,$$
$$g = 0.05 \ \text{cm},$$
$$l_c = 30 \ \text{cm},$$
$$N = 500 \text{ Turns.}$$

Assume the value $\mu_0 = 70000 \ \text{H} \cdot \text{m}^{-1}$ for core material.

(a) Find the reluctance values $\mathcal{R}_c$ and $\mathcal{R}_g$.

(b) For the value of $B_c = 1$ T, find the flux ϕ and the current i.

Solution

(a) 3,789.4

442,097.06

(b) 0.0009

(c)

Example

The following magnetic structure has infinite permeability ($\mu \to \infty$). Find the air-gap flux ϕ and flux density B_{g}.

Parameters: $I = 10$ A, $N = 1000$ Turns, $g = 1$ cm, and $A_{\mathrm{g}} = 2000$ cm^2.

Solution

(a) 0.13

- When a **B**-field varies with time, an **E**-field is produced in space as determined by Faraday's law [6]:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a}.$$

> The line integral of the electric field intensity E around a closed contour C is equal to the time rate of change of the magnetic flux linking (i.e. passing through) that contour.

> In electric drives, the effects of **E** field can be neglected.

- This simplification allows us to cancel the LHS of the equation to just the induced voltage (i.e., electromotive force)

$$\cancelto{\mathcal{E}}{\oint_{\mathcal{C}} \mathbf{E} \cdot \mathrm{d}\mathbf{s}} = -\frac{d}{dt} \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{a}.$$

- Whereas on the RHS, the flux is dominated by the core flux (ϕ).

$$\mathcal{E} = -\frac{d}{dt} \cancelto{\phi}{\int_S \mathbf{B} \cdot \mathrm{d}\mathbf{a}}.$$

- As winding links the core flux N times, our equation is simplified to:

$$\mathcal{E} = N\frac{d\phi}{dt} = \frac{d\lambda}{dt} \quad \text{where} \quad \lambda = N\phi \quad \blacksquare$$

where λ is flux linkage, measured in Weber-turns,

- For a magnetic circuit composed of magnetic material of constant magnetic permeability, the relationship between ϕ and i will be linear and we can define the inductance L as:

$$L = \frac{\lambda}{i} \quad \text{which can be written as} \quad L = \frac{N^2}{\mathcal{R}_{\text{tot}}}$$

where L is the inductance measured in Henry (H).

> Inductance requires a linear relationship between flux and mmf.

Example

The magnetic circuit consists of an N-turn winding on a magnetic-core of infinite permeability with two parallel air gaps of lengths g_1 and g_2 and areas A_1 and A_2 respectively. Find:

(a) The winding inductance,

(b) The flux density B_1 in gap 1 when the winding is carrying current i.

Note: Neglect fringing effects at the air gap.

- The mmf ($\mathcal{F}$) is given by the total ampere-turns.
- The reference directions for the currents (i_1, i_2) have been chosen to produce flux in the same direction.
- The total mmf is therefore:

$$\mathcal{F} = N_1 i_1 + N_2 i_2.$$

- With the reluctance of the core neglected and assuming that $A_c = A_g$, the core flux ϕ is:

$$\phi = (N_1 i_1 + N_2 i_2) \frac{\mu_0 A_c}{g},$$

where ϕ is the resultant core flux produced by the total mmf of the two windings.

- This resultant ϕ determines the operating point of the core material.
- If ϕ is broken up into terms attributable to the individual currents, the resultant flux linkages of coil 1 can be expressed as:

$$\lambda_1 = N_1\phi = \overbrace{N_1^2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{11}} i_1 + \overbrace{N_1 N_2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{12}} i_2,$$

and this is simplified to:

$$\lambda_1 = L_{11} i_1 + L_{12} i_2,$$

where L_{11} is the self inductance and L_{12} is the mutual inductance of the coil.

- Similarly, the flux linkage on coil 2 is:

$$\lambda_2 = N_2\phi = \overbrace{N_1 N_2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{21}} i_1 + \overbrace{N_1^2\left(\frac{\mu_0 A_c}{g}\right)}^{L_{22}} i_2$$

and this is simplified to:

$$\lambda_1 = L_{21} i_1 + L_{22} i_2$$

where $L_{21} = L_{12}$ is the mutual inductance and L_{22} is the self inductance.

> It is important to note the ϕ is calculated based on the superposition of i_1, i_2 which implies a linear relationship between flux and mmf.

- It cannot be rigorously applied in situations where the nonlinear characteristics of magnetic materials dominate .
- However, in many situations of practical interest, the reluctance of the system is dominated by that of an air gap (which is of course linear) and the nonlinear effects of the magnetic material can be ignored.
- In other cases it may be perfectly acceptable to assume an average value of magnetic permeability for the core material.
- Using this it is possible to calculate a corresponding average inductance which can be used for calculations of reasonable engineering accuracy.

- EEC[1] devices require two (2) important properties:
  - Obtain large magnetic flux densities ($\uparrow \mathbf{B}$)
  - Requiring relatively low levels of magnetizing force ($\downarrow \mathbf{H}$).

> Since magnetic forces and energy density increase with increasing flux density, this effect plays a large role in the performance of energy-conversion devices.

[1] Electro-mechanical Energy Conversion

Magnetic Circuits and Materials

- Magnetic materials can be used to constrain and direct **B**-fields in well-defined paths.
- In a transformer, they are used to maximize the coupling between the windings as well as to lower the excitation current required for transformer operation.
- In electric drives, they are used to shape the fields to obtain desired torque-production and electrical terminal characteristics.

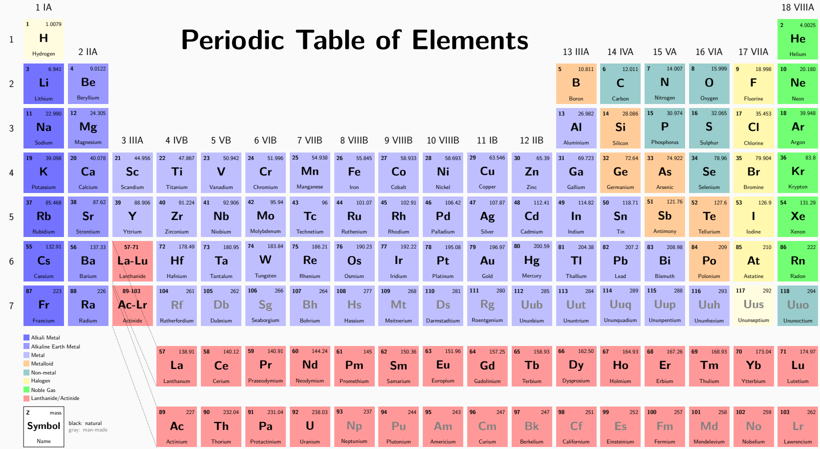


Figure 10: The periodic table of elements.

Magnetic Circuits and Materials

- Typically composed of iron (**Fe**) and alloys of iron with:
  - Cobalt, Tungsten, Nickel, Aluminium, and other metals
- These materials are characterised by a wide range of properties,
  - Phenomena responsible for their properties are the same.
- Ferro-magnetic materials are found to be composed of a large number of domains.
  - Regions where the magnetic moments of all the atoms are parallel, giving rise to a net magnetic moment for that domain.
- In an un-magnetised sample of material, the domain magnetic moments are randomly oriented, and the net resulting magnetic flux in the material is zero (0).

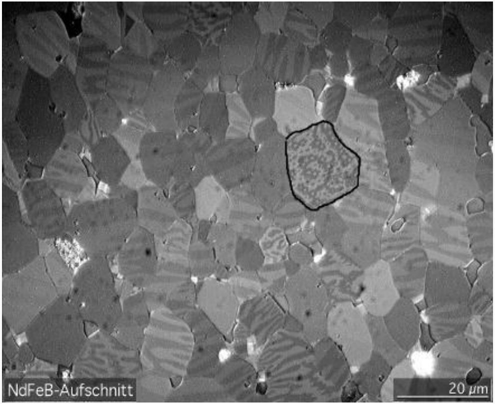


Figure 11: Microcrystalline grains within a piece of Nd2Fe14B (the alloy used in neodymium magnets) with magnetic domains made visible with a Kerr microscope. The domains are the light and dark stripes visible within each grain. The outlined grain has its magnetocrystalline axis almost vertical, so the domains are seen end-on [5].

- When **B**-field is applied to the material, the magnetic domain moments tend to align with the applied **B**-field.
- As a result, the magnetic moments add to the **B**-field, producing a higher flux density than would exist due to the magnetizing force alone.
- Therefore the effective permeability μ, equal to the ratio of the total magnetic flux density to the applied magnetic-field intensity, is large compared with the permeability of free space (μ_0).
- As the magnetising force (**H**) is increased, this continues until all the magnetic moments are aligned with the applied field.
  - at this point they can no longer contribute to increasing the magnetic flux density

Material becomes fully saturated.

- Without an external **B**-field, magnetic domains naturally align along certain directions associated with the crystal structure of the domain.
  - This is known as axes of easy magnetisation [8].
- If the applied **B** is reduced, the domain magnetic moments relax to the direction of easy magnetism nearest to the applied field.
- As a result, **B** is reduced to zero (0), although they will tend to relax towards their initial orientation, the magnetic dipole moments will no longer be totally random in their orientation;
- They will retain a net magnetisation along the applied field.
- It is this effect which is responsible for the phenomenon known as magnetic hysteresis [18].

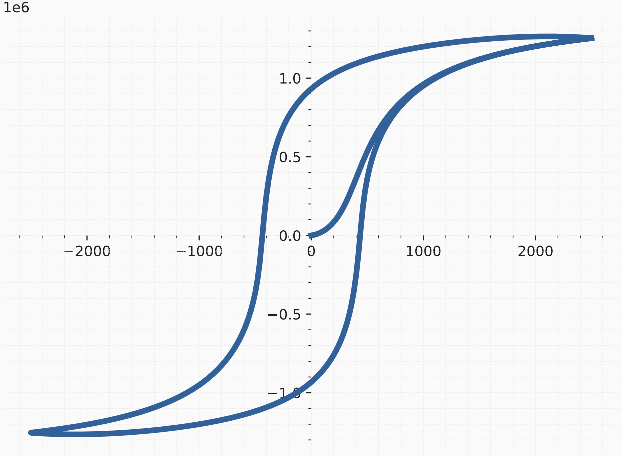


Figure 12: A Hysteresis Curve (B-H) Generated using the Jiles-Artherton model.

1st Quadrant

- The domains are very small, below the single domain size where there is resistance to demagnetization [2].
- The domains start to increase in size.

> For a small interval, the magnetization will be reversible.

- As the field increases, magnetization will no longer reverse to zero but move on a minor hysteresis loop.
- Eventually the curve starts to bend over to the right.
  - It will still increase as more magnetic domains reach their full size and their magnetisation become parallel to the external field [11].

1st Quadrant - Saturation

- Eventually the **B**-field will become high enough where no more change in the magnetisation occurs.
  - This is called technical saturation.
- It is possible to reach 99+% saturation.
- As the field is backed off from "saturation", the magnetisation declines very slightly to the B_r point.
  - This is the remanence, or residual induction [10].

> All of the magnetic energy is now in the magnet and its field.

2^{nd} Quadrant - Demagnetisation

- In this quadrant the applied field opposes the materials **B**-field.
- As the external **B**-field increases in magnitude, some domains will reverse.
- At the knee of the demagnetisation curve, this increase has become rapid and the magnetisation will fall to the H_{ci} point.
- At H_{ci}, the number of domains aligned with the original magnetization is the same as the number aligned with the opposing magnetic field.

> The net magnetisation is zero (0).

3rd Quadrant - Re-magnetisation

- The total magnetization of the part will be reversed.
- If we go far enough, magnetization will reach the saturation level in the negative direction.

4^{th} Quadrant - Demagnetisation

- After fully reversing the magnet and removing the field in the third quadrant, magnetization will recoil to a point that is the negative of the B_r observed when in the first and second quadrant.
- If we apply additional field in the positive direction, we duplicate the second quadrant curve.

- In AC power systems, voltage and flux wave-forms closely approximate sinusoidal functions over time.
- Time to describe the excitation characteristics and losses associated with steady-state ac operation of magnetic materials.
- Assume a closed-core magnetic circuit (i.e., with no air gap),
- And a sinusoidal variation of the core flux (ϕ) with the following:

$$\phi(t) = \phi_{\mathrm{max}} \sin(wt) = A_{\mathrm{c}} B_{\mathrm{max}} \sin(wt)$$

where:

ϕ_{max} amplitude of core flux in Wb,

B_{max} Amplitude of flux density B_{c} in T,

ω Angular frequency in rad/s,

f Frequency in Hz.

- From, the voltage induced in the N-turn winding is:

$$e(t) = \omega N \phi_{\text{max}} \cos(wt) = E_{\text{max}} \cos(wt)$$

where:

$$E_{\text{max}} = \omega N \phi_{\text{max}} = 2\pi f N A_{\text{c}} B_{\text{max}}$$

- In steady-state AC, it is more important to use rms rather than instant values.
- Generally, the rms value of a periodic function of time $f(t)$, of T is:

$$F_{\text{rms}} = \sqrt{\left(\frac{1}{T}\int_0^T f^2(t)\ \mathrm{d}t\right)}$$

- The rms value of a sine wave can be shown to be $1/\sqrt{2}$ times its peak value. Therefore the induced voltage rms is:

$$E_{\mathrm{rms}} = \frac{2\pi}{\sqrt{2}} f N A_{\mathrm{c}} B_{\mathrm{max}} = \sqrt{2}\pi N A_{\mathrm{c}} B_{\mathrm{max}} = 4.44 N A_{\mathrm{c}} B_{\mathrm{max}} \quad \blacksquare$$

- To produce magnetic flux in the core requires current in the exciting winding known as the exciting current, i_{ϕ},

> The nonlinear magnetic properties of the core require that the waveform of the exciting current differs from the sinusoidal waveform of the flux.

- When the hysteresis curve saturates the excitation current spikes.

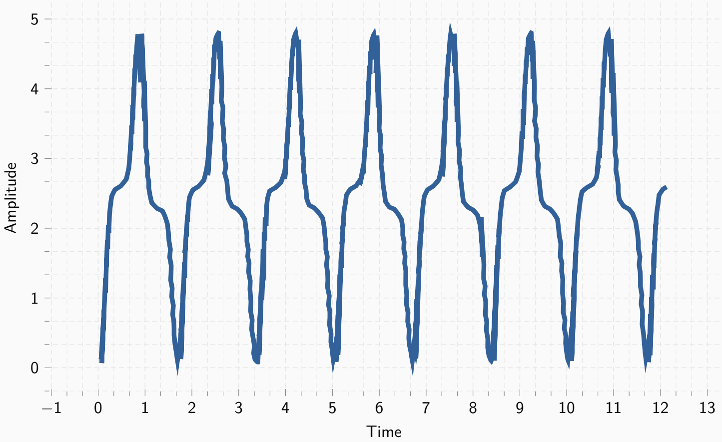


Figure 13: An Example Visualisation of the excitation current experienced by a magnetic circuit.

- The exciting current supplies the mmf required to produce the core flux and the power input associated with the energy in the magnetic field in the core.
- Part of this energy is dissipated as losses and results in heating of the core.
- The rest appears as reactive power associated with energy storage in the magnetic field.

> This reactive power is not dissipated in the core as it is cyclically supplied and absorbed by the excitation source.

- There are two (2) mechanisms associated with time varying **B**-fields.

Ohmic Heating

- Associated with induced currents in the core material.
- From Faraday's law, we see that **B**-fields give rise to **E**-fields.
- In magnetic materials these **E**-fields result in induced currents,
  - These are known as eddy currents, circulating in the core material, opposing the material's internal **B**-field [4].

- To counteract the corresponding demagnetizing effect by the eddy currents, the current in the exciting winding must increase.
- Therefore the resultant ”dynamic” B-H loop under ac operation is somewhat ”fatter” than the hysteresis loop for slowly varying conditions, and this effect increases as the excitation frequency is increased.
- It is for this reason that the characteristics of electrical steels vary with frequency and hence manufacturers typically supply characteristics over the expected operating frequency range of a particular electrical steel.

Magnetic Circuits and Materials

- To reduce the effects of eddy currents, magnetic structures are usually built of thin sheets of laminations of the magnetic material.
- These laminations, which are aligned in the direction of the field lines, are insulated from each other by an oxide layer on their surfaces or by a thin coat of insulating enamel or varnish.
- This greatly reduces the magnitude of the eddy currents since the layers of insulation interrupt the current paths; the thinner the laminations, the lower the losses.
- In general, eddy-current loss tends to increase as the square of the excitation frequency and also as the square of the peak flux density.

Magnetic Circuits and Materials

- Due to the hysteretic nature of magnetic material.
- In a magnetic circuit or the transformer, a time-varying excitation (i_ϕ) will cause the magnetic material to undergo a cyclic variation described by a hysteresis loop.
- The energy input W to the magnetic core as the material undergoes a single cycle is shown to be:

$$W = \oint i_\phi \, \mathrm{d}\lambda = \oint \left(\frac{H_\mathrm{c} l_\mathrm{c}}{N}\right)(A_\mathrm{c} N \, \mathrm{d}B_\mathrm{c}) = A_\mathrm{c} l_\mathrm{c} \oint H_\mathrm{c} \, \mathrm{d}B_\mathrm{c}$$

- Notice $A_\mathrm{c} l_\mathrm{c}$ is the core volume and the integral is the area of the ac hysteresis loop, we see that each time the magnetic material undergoes a cycle, there is a net energy input into the material.

- This is the required energy to move around the magnetic dipoles in the material and is dissipated as heat in the material.
- Therefore for a given flux level, the corresponding hysteresis losses are proportional to the area of the hysteresis loop and to the total volume of material.
- As there is an energy loss per cycle, hysteresis power loss is proportional to the frequency of the applied excitation. (Data Sheet)

- In general, both losses depend on the metallurgy of the material as well as the flux density and frequency.
- Information on core loss is typically presented in graphical form.
- It is plotted in terms of watts per unit weight as a function of flux density;
    - Often a family of curves for different frequencies are given.
    - Generally it is either 50 or 60 Hz.

Magnetic Circuits and Materials

- Nearly all transformers and certain sections of electric machines use sheet-steel material that has highly favorable directions of magnetization along which the core loss is low and the permeability is high.
- This material is termed grain-oriented steel [15].
- The reason lies in the atomic structure of a crystal of the silicon-iron alloy,
    - which is a body-centred cube.

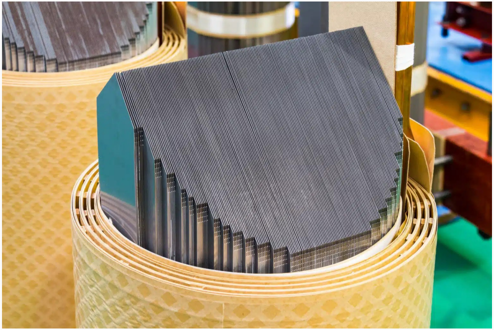


Figure 14: An example of Grain Oriented Steel used in industry [9].

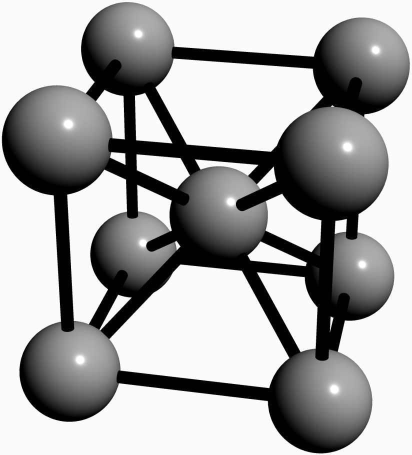


Figure 15: The atomic structure of grain oriented steel.

- Each crystalline cube has an atom at each corner as well as one in the center of the cube.
- In the cube, the easiest axis of magnetization is the cube edge.
- The diagonal across the cube face is more difficult
- By suitable manufacturing techniques most of the crystalline cube edges are aligned in the rolling direction to make it the favorable direction of magnetisation.

- The behaviour in this direction is superior in core loss and permeability to non-oriented steels in which the crystals are randomly oriented to produce a material with characteristics which are uniform in all directions.
- As a result, oriented steels can be operated at higher flux densities than the nonoriented grades.
- Non-oriented electrical steels are used in applications where the flux does not follow a path which can be oriented with the high-Temperature rolling direction or where low cost is of importance.
- In these steels the losses are somewhat higher and the permeability is very much lower than in grain-oriented steels.

Transformers

Table of Contents

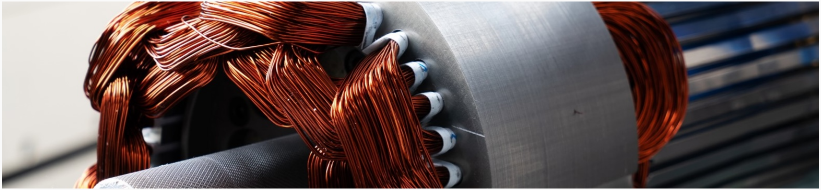
Learning Outcomes

(LO1) Modelling an Ideal Transformer,

(LO2) Construction methods,

(LO3) Physical properties,

(LO4) Types of transformers.



Transformers

- Although transformers are not an energy conversion device, it is an indispensable component in many energy conversion systems.
- In AC power systems, it makes it possible:
  1. Electric generation at the most economical generator voltage,
  2. Power transfer at the most economical transmission voltage,
  3. Power utilisation at the most suitable voltage.
- The transformer is also widely used in:
  - Impedance matching of a source and its load for maximum power transfer,
  - Isolating one circuit from another,
  - Isolating DC while maintaining ac continuity between two circuits[3]

[3] Galvanic isolation.

Figure 16: An example of a arc furnace reactor. They are used to improve operating regime of furnace transformers in iron and steel plants [16].

Transformers

Figure 17: Delta Transformers specializes in the design and manufacture of dry-type power transformers up to 15,000 kVA, 34.5 kV class [17].

Transformers

- Transformer is one of the simpler devices comprising two or more electric circuits coupled by a common magnetic circuit.
- Its analysis involves many of the principles essential to the study of electric machinery.

- Thus, our study of the transformer will serve as a bridge between the introduction to magnetic-circuit analysis of Chapter 1 and the more detailed study of electric machinery to follow.

Transformers

- Essentially, a transformer consists of two (2) or more windings coupled by mutual magnetic flux.
- If primary is connected to an AC-voltage source, an alternating flux will be produced whose amplitude will depend on:
  - The primary voltage (v_{in}),
  - The primary voltage frequency (f_1),
  - Number of turns (N_1),
- The mutual flux will link the secondary, and will induce a voltage in it whose value will depend on the number of secondary turns as well as the magnitude of the mutual flux and the frequency.

> By changing the number of primary and secondary turns, almost any voltage ratio can be obtained.

- The essence of transformer action requires only the existence of time-varying mutual flux linking two windings.
- Such action can occur for two windings coupled through air.
- This would be highly inefficient, therefore an iron-core or other ferromagnetic materials are used.

> This forces the flux path to be confined to the material.

- Such a transformer is commonly called an iron-core transformer.

> Most transformers are of this type.

- We will focus almost wholly with iron-core transformers.

Transformers

An ideal transformer is a lossless device with:

- An input winding (N_1)
- An output winding (N_2)

It has the following properties:

- No iron and copper losses.
- No leakage fluxes.
- A core of infinite permeability and infinite electrical resistivity.
- Flux is confined to the core and winding resistance are negligible.

Transformers

- The ideal transformer may be defined from Faraday's law:

> Change in magnetic field will create emf ($\mathcal{E}$)

$$v_1 = \mathcal{E}_1 = -\frac{d\lambda}{dt} = -\frac{d(N_1\phi)}{dt} = -N_1\frac{d\phi}{dt}.$$

- The core flux also links the secondary and produces an induced emf $\mathcal{E}_2$, and an equal secondary terminal voltage v_2, given by:

$$v_2 = \mathcal{E}_2 = -\frac{d\lambda}{dt} = -\frac{d(N_2\phi)}{dt} = -N_2\frac{d\phi}{dt}.$$

Transformers

- As flux is present in both v_1, v_2 we can derive the following:

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \quad \blacksquare$$

> An ideal transformer transforms voltages in the direct ratio of the turns in its windings.

Transformers

- Let us add a load to the transformer.
- This creates current i_2 and mmf $N_2 i_2$ on the secondary.
- As it is ideal, it has infinite permeability ($\mu \to \infty$) which means the flux is unchanged between primary and secondary.

$$N_1 i_1 - N_2 i_2 = 0$$

- Rearranging it we derive:

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} \quad \blacksquare$$

> This means in an ideal transformer $P_{in} = P_{out}$.

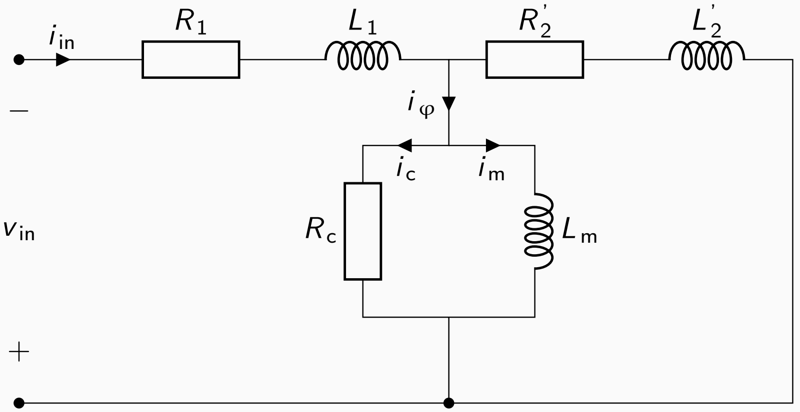


Figure 18: The equivalent circuit for a transformer.

- The open-circuit test (no-load test), is one of the methods used to determine the no-load impedance in the excitation branch of a transformer.
- The test is done on the primary side of the transformer.
- The wattmeter, ammeter and the voltage are connected to their primary winding.
- The nominal rated voltage is supplied to their primary winding.

> As the no-load current is very small, the copper losses experienced are negligible.

- The secondary winding of the transformer is kept open, and the voltmeter is connected to their terminal.

> This voltmeter measures the secondary induced voltage.

- As the secondary of the transformer is open, thus no-load current flows through the primary winding.
- The value of no-load current is very small as compared to the full rated current.
- The copper loss occurs only on the primary winding of the transformer because the secondary winding is open.
- Wattmeter reading only represents the core and iron losses.
- The core loss of the transformer is the same for all types of loads.

Calculation

- Let:

$$\begin{aligned} W_0 &\ \text{wattmeter reading,} \\ V_1 &\ \text{voltmeter reading,} \\ I_0 &\ \text{ammeter reading.} \end{aligned}$$

- We can state, the iron loss of the transformer (P_C) as:

$$W_0 = P_C = V_1 I_0 \cos\varphi_0$$

The no-load power factor $(\cos\varphi_0)$ is then:

$$\cos\varphi_0 = \frac{W_0}{V_1 I_0}$$

Transformers

- The real part of the current $\hat{I}_0$ is:

$$\hat{I}_0 = I_0 \cos \varphi_0, \qquad \text{and} \qquad \hat{I}_0 = I_0 \sin \varphi_0,$$

- Using these derivations, the core resistance and magnetising inductances are:

$$R_\mathrm{C} = \frac{V_1}{I_\mathrm{m}} \qquad \text{and} \qquad X_\mathrm{m} = \frac{V_1}{I_\mathrm{C}} \quad \blacksquare$$

- Determines the copper loss occur on the full load.
- The copper loss is used for finding the efficiency of the transformer.
- The equivalent resistance, impedance, and leakage reactance are known by the short circuit test.

> Performed on the secondary or the high voltage winding.

- Wattmeter, voltmeter and ammeter are connected to the high voltage winding.
- Using a variac, the applied voltage is slowly increased until the ammeter gives a reading equal to the rated current of the HV side.
- The full load current is measured by the ammeter connected across their secondary winding.

Transformers

- The low voltage source is applied across the secondary winding, which is approximately 5 to 10% of the normal rated voltage.
- The flux is set up in the core of the transformer.

> The magnitude of the flux is small as compared to the normal flux.

- The iron loss of the transformer depends on the flux.
- This is negligible as the generated flux is very low.
- Wattmeter only determines the copper loss occurred, in their windings.

Example

A single phase, 100 kVA, 480 / 120 V transformer is subjected to short-circuit and open-circuit test to determine model parameters:

Open Circuit	Short Circuit
$I_{1,\mathrm{OC}} = 0.05$ A	$P_{\mathrm{OC}} = 0.1$ W
$V_{1,\mathrm{SC}} = 80$ V	$P_{\mathrm{SC}} = 10$ kW

Table 3: Results of the Tests

Using this information, please determine the model parameters: R_{s}, X_{s}, R_{c}, X_{m}.

Solution

A code solution is as follows:

```
import numpy as np

I_1OC, V_1OC = 0.05, 480
P_OC = 0.1

Z_1OC =  V_1OC / I_1OC
R_C = V_1OC**2 / P_OC
X_m = round(np.sqrt(1 / Z_1OC ** 2 - 1 / R_C ** 2) ** (-1),2)

print(f"{Z_1OC = } Ohm, {R_C = } Ohm, {X_m = } Ohm")
```

```
Z_1OC = 9600.0 Ohm, R_C = 2304000.0 Ohm, X_m = 9600.08 Ohm
```

- During its operation, a transformer experiences unavoidable losses.
- These can be categorised as:
  - Iron (Core) Loss,
  - Copper Loss,
  - Stray Loss,
  - Dielectric Loss.
- These losses appear in the form of heat and causes two (2) problems:
  - Increases transformer temperature,
  - Reduces overall efficiency.

Transformers

- Heat is generated in the transformer while running and is produced by the excitation of the windings and core.
- If the temperature of the transformer continues to increase rapidly, it results in the degradation of the various parts, specifically the insulation materials, and could lead to the failure of the equipment.
- Depending on the cooling methods used, transformers can be divided into two (2) types:
  1. dry,
  2. oil.

Dry-Type Cooling

- Dry-type transformers, are normally cooled by air.
- The following two (2) methods adopted in dry-type transformers.

Air Natural Cooled by surrounding air via convection.

Air Force Forced air circulation using fans and blowers.

Oil-Type Cooling

- Uses either:
    1. Oil-air cooling,
    2. Oil-water cooling.
- Looking at them in more detail:

Oil Natural Air Natural

- The core and coils are cooled by surrounding in oil.
- Heat transfer of oil by natural air convection.

(Non-Mineral) Oil Natural Air Natural

- The core and coils are cooled by surrounding in synthetic oil.
- Heat transfer of oil by natural air convection.

Oil-Type Cooling

- Uses either:
    1. Oil-air cooling,
    2. Oil-water cooling.
- Looking at them in more detail:

Oil Natural Air Forced

- Cooled by surrounding in oil.
- Forced air circulation using pumps, fans and blowers.

Oild Forced Air Forced

- Forced oil and air circulation using fans and blowers.

Oil-Type Cooling

- Uses either:
  1. Oil-air cooling,
  2. Oil-water cooling.
- Looking at them in more detail:

Oil Natural Water Forced

- Cooled by surrounding in oil.
- Forced water circulation using heat exchanges.

Oil Forced Water Forced

- Forced oil and water circulation using oil-to-water heat exchanges.

Transformers

Figure 19: An example of oil cooled transformer which uses radiators to exchange heat [13].

- A transformer core is composed of limbs and yokes joined together to form a single structure around which the coils are placed.

> The manner in which the respective yoke and limbs join together will depend on the type and design of the core.

Limb Vertical sections which the coils are formed around. The limbs can also be located on the exterior of the outermost coils in the case of some core designs. The limbs on a transformer core can also be referred to as legs.

Yoke The yoke is the horizontal section of the core which joins the limbs together. The yoke and limbs form a pathway for magnetic flux to flow freely.

Transformers

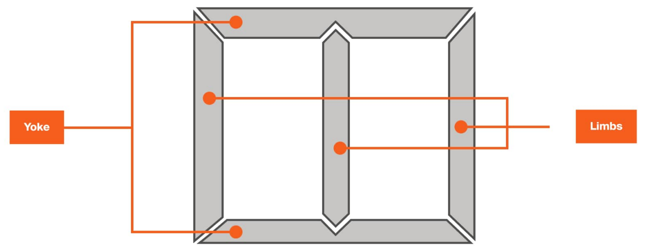


Figure 20: Diagram of the transformer geometry and its parts [7].

Transformers

- The earliest transformer cores utilized solid iron.
- Methods developed over the years to refine raw iron ore into more permeable materials such as silicon steel, which is used today for transformer core designs due to its higher permeability.
- Also, the use of many densely packed laminated sheets reduces issues of circulating currents and overheating caused by solid iron core designs.
- Further increases in core design are made through cold rolling, annealing, and using grain oriented steel.

Cold Rolling

- Silicon steel is a softer metal.
- They also have small hysteresis area, low core loss and high μ.
- They are used in making sheets less than 2mm thick.
- Cold rolling silicon steel will increase its strength–making it more durable when assembling the core and coils together.
- Permeability can be increased even further by orienting the grain of the steel in the same direction.

Annealing

- Involves heating the core steel up to a high temperature to remove impurities.
- This process will increase the softness and ductility of the metal by decreasing the internal stress of the material.

Shell-Type

- The core surrounds the windings.
- Creates a closed pathway surrounding the windings for magnetic flux.
- This design also typically yields less energy loss than a core type design.
- A shell type design is the classification for most distribution class padmounts and substations with a wrapped 5-legged core.

Core-Type

- A core type design is where the windings surround the core steel.
- In this design, there is no return path (or closed loop) for the magnetic flux around the coils.
- This design typically yields more energy losses, and it requires more copper or aluminum winding material than a shell-type configuration.

Transformers

- For low-frequency, high-power applications, laminated cores are an economical way of reducing eddy current loss.
- Note that the laminations are in parallel with the magnetic flux, while the eddy currents are perpendicular.
- This arrangement restricts the eddy currents to the width of the laminations

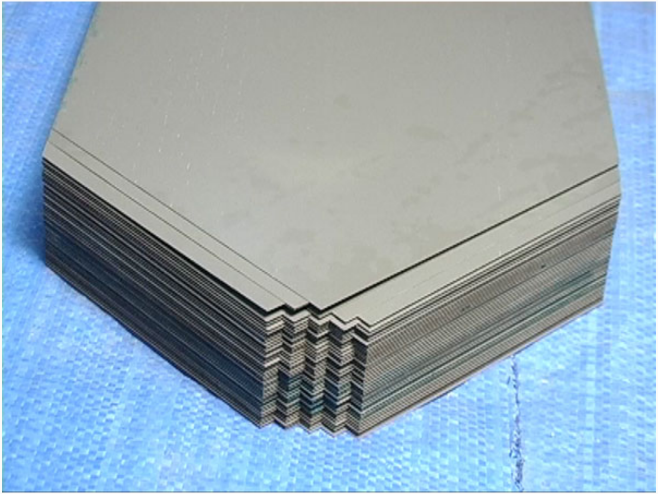


Figure 21: Lamination is done to restrict the eddy current flow.

Auto Transfomer

- It is a transformer with only one (1) winding.
- portions of the same winding act as both the primary winding and secondary winding sides of the transformer.
- Has higher efficiency than two winding transformer.
  - Less ohmic loss and core loss due to reduction of transformer material.

> They are often smaller, lighter, and cheaper than typical dual-winding transformers, but the disadvantage of not providing electrical isolation between primary and secondary circuits.

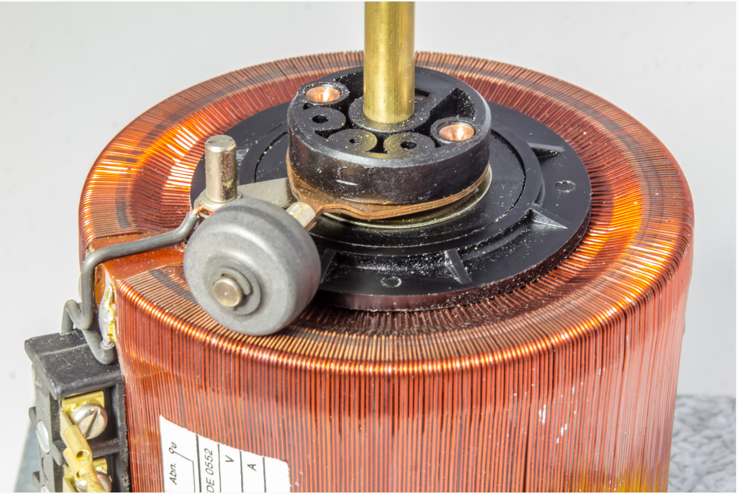


Figure 22: Variable autotransformer 0-220 V, 4 A, 880 VA.

Isolation (Galvanic) Transformer

- Used in transferring power from an AC source to an equipment while isolating the powered device from the power source.
  - Usually for safety reasons or to reduce transients and harmonics.

> They provide galvanic isolation
> no conductive path between source and load.

- This isolation is used to protect against electric shock, to suppress electrical noise in sensitive devices, or to transfer power between two circuits which must not be connected.

Transformers

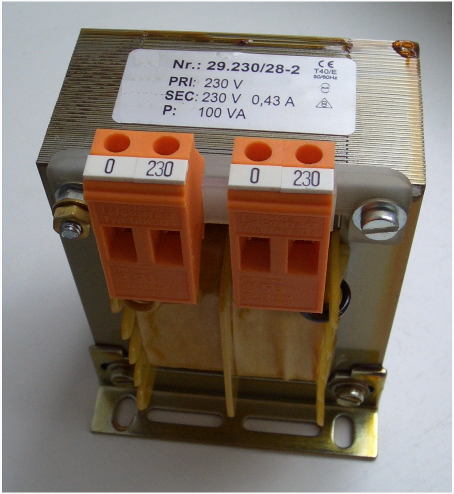


Figure 23: A 230 V isolation transformer.

- Three single-phase transformers can be connected to form a three-phase transformer bank in any of the four (4) ways:
  - Y-Δ Connection,
  - Δ-Y Connection,
  - Δ-Δ Connection,
  - Y-Y Connection.
- The windings at the left are the primaries,
- The ones at the right are the secondaries,

Rated V and I at the primary and secondary of the three-phase transformer bank depends upon the connection used but that the rated kVA of the three-phase bank is three times that of the individual single-phase transformers, regardless of the connection.

Star-Delta Connection

- Y-Δ connection is commonly used in stepping down from a high voltage to a medium or low voltage due to grounding on the high-voltage side.
- This is due to the neutral connection being available on the high-voltage side for safety reasons.

Delta-Star Connection

- Δ-Y connection is commonly used for stepping up to a high voltage.

> Having a neutral connection on the high-voltage side is always a benefit.

Delta-Delta Connection

- The $\Delta - \Delta$ connection has the advantage where one transformer can be removed for repair or maintenance while the remaining two continue to function as a three-phase bank with the rating reduced to 58 percent of that of the original bank
- This is known as the open-delta, or V, connection.

> An open delta transformer is a three phase transformer that only has two primary and secondary windings, with one side of the delta phase diagram "open".

Star-Star Connection

- Seldom used due to significant disadvantages.

Disadvantages

- If the neutral connection is not provided and an unbalanced load is connected, the phase voltages tend to become severely unbalanced.
- Magnetising current varies non-sinusoidal having a third harmonic.
  - In balance, the 3rd harmonic primary winding magnetising currents are equal in magnitude and in phase with each other.
  - These will be directly additive at the neutral point.
- These components will distort the magnetic flux which induces a voltage having a third harmonic component in both windings.
- This third harmonic component of the induced voltage may be as large as the fundamental voltage.

Transformers

- Instead of three single-phase transformers, a three-phase bank may consist of one three-phase transformer having all six windings on a common multi-legged core and contained in a single tank. Advantages of three-phase transformers over connections of three single-phase transformers are that they cost less, weigh less, require less floor space, and have somewhat higher efficiency.

Electromechanical Energy Conversion

Table of Contents

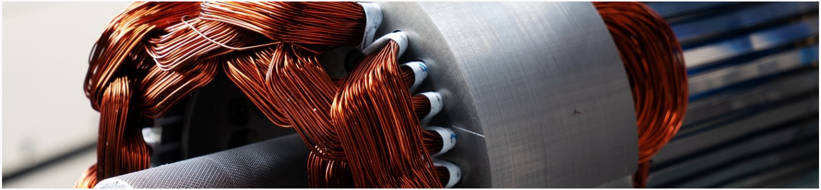
Learning Outcomes

(LO1) Definition of Electromechanical devices,

(LO2) Understanding the conversion process,

(LO3) Describing the energy co-energy relationship,

(LO4) Looking at multiply-excited system..



Electromechanical Energy Conversion

- The electromechanical energy conversion (EEC) process, takes place through the medium of the electric or magnetic field of the conversion device.
- We can classify these items into three (3) categories:

1. Devices for measurement and control are frequently referred to as transducers.
   - Generally operate under linear input-output conditions and with relatively small signals.
   - i.e., microphones, pickups, sensors, and loudspeakers
2. Devices encompasses force-producing motions: solenoids, relays, and electromagnets.
3. Continuous energy-conversion equipment: motors and generators.

- The purpose of studying EEC is threefold:

1. To understand how energy conversion takes place,
2. To provide techniques for designing and optimising the devices for specific requirements,
3. To develop models of EEC devices that can be used in analysing their performance as components in engineering systems.

- The Lorentz force law states:

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

where $\mathbf{F}$ is the force on a particle, q is the charge. $\mathbf{B}$ is the magnetic field, $\mathbf{E}$ is the electric field and $\mathbf{v}$ is the speed in which the particle is moving.

- In a pure $\mathbf{E}$ field, the force is simply:

$$\mathbf{F} = q\mathbf{E}$$

- In a $\mathbf{B}$ field the force is slightly more complex.

$$\mathbf{F} = q\left(\mathbf{v} \times \mathbf{B}\right)$$

Example

A nonmagnetic rotor containing a single-turn coil is placed in a uniform magnetic field of magnitude B_0. The coil sides are at radius R and the wire carries current I.

Find the θ-directed torque as a function of rotor position α when $I = 10$ A, $B_0 = 0.02$ T and $R = 0.05$ m.

Assume the rotor is of length $l = 0.3$m.

Example

The force per unit length on a wire carrying current I can be found by multiplying Eq. 3.6 by the cross-sectional area of the wire. When we recognize that the product of the cross-sectional area and the current density is simply the current I, the force per unit length acting on the wire is given by

$$\mathbf{F} = \mathbf{I} \times \mathbf{B}$$

Thus, for wire ℓ carrying current I into the paper, the θ-directed force is given by:

$$F_{1_\theta} = -IB_0\ell \sin\alpha$$

Solution

And for wire 2 (which carries current in the opposite direction and is located 180° away from wire 1)

$$F_{2_\theta} = -IB_0\ell \sin\alpha$$

where l is the length of the rotor. The torque T acting on the rotor is given by the sum of the force-moment-arm products for each wire

$$T = -2IB_0R\ell \sin\alpha = 2(10)(0.02)(0.05)(0.3)\sin\alpha = -0.006\sin\alpha$$

Electromechanical Energy Conversion

- The principle of conservation of energy states that energy is neither created nor destroyed.
  - It is only a change in form.
- For isolated systems with clear boundaries, this allows to keep track of energy:

> the net flow of energy into system across its boundary is equal to the sum of the time rate of change of energy stored in the system.

- For the case of electro-mechanical systems we can write:

$$\begin{pmatrix} \text{Energy input} \\ \text{from electric} \\ \text{sources} \end{pmatrix} = \begin{pmatrix} \text{Mechanical} \\ \text{energy} \\ \text{output} \end{pmatrix} + \begin{pmatrix} \text{increase in energy} \\ \text{stored in} \\ \text{magnetic field} \end{pmatrix} + \begin{pmatrix} \text{energy} \\ \text{converted} \\ \text{into heat} \end{pmatrix}$$

- For the lossless magnetic energy storage system:

$$\mathrm{d}W_{\text{elec}} = \mathrm{d}W_{\text{mech}} + \mathrm{d}W_{\text{fld}} \tag{1}$$

- In time $\mathrm{d}t$, W_{elec} is given as :

$$\mathrm{d}W_{\text{elec}} = ei\,\mathrm{d}t$$

  where e is the voltage induced in the electric terminals by the changing magnetic stored energy.

- It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals.

- Let's rewrite this equation:

- Consider the electromagnetic relay shown schematically

- The excitation coil resistance is shown as R,
- The mechanical terminal variables are shown as a force f_{fld} produced by the B-field directed from the relay to the external mechanical system and a displacement x.
- mechanical losses can be included as external elements connected to the mechanical terminal. Similarly, the moving armature is shown as being massless; its mass represents me- chanical energy storage and can be included as an external mass connected to the mechanical terminal. As a result, the magnetic core and armature constitute a lossless magnetic-energy-storage system,

- As you know, and inductance L is a function of the geometry magnetic structure and material.
- EEC devices contain air-gaps in their circuits for moving parts.

> The reluctance of the air gap is significantly higher than that of the magnetic material which forces most of the energy storage to occur in the air gap.

- This allows us to disregard the non-linear relations in the system for practical devices.
- Our second assumption is flux and MMF are directly proportional to the entire magnetic circuit.

- The flux linkages λ and current i are considered linearly dependent by a geometrically defined **inductance**:

$$\lambda = L(x)\, i \tag{2}$$

- As magnetic force (f_{fld}) has been defined as acting from the relay upon the external mechanical system and $\mathrm{d}W_{\text{mech}}$ is defined as the mechanical energy output of the relay, we can write

$$\mathrm{d}W_{\text{mech}} = f_{\text{fld}}\, \mathrm{d}x \tag{3}$$

- Using Eq. (3), and substituting $\mathrm{d}W_{\text{elec}} = i\,\mathrm{d}\lambda$, we can write:

$$\begin{aligned} \mathrm{d}W_{\text{fld}} &= \mathrm{d}W_{\text{elec}} - \mathrm{d}W_{\text{mech}}, \\ \mathrm{d}W_{\text{fld}} &= i\,\mathrm{d}\lambda - f_{\text{fld}}\,\mathrm{d}x. \end{aligned}$$

- As the magnetic energy storage system is considered lossless, it is a **conservative system**.
- The value of W_{fld} uniquely specified by the values of λ and x.
- λ and x are referred to as state variables since their values uniquely determine the state of the system.

- We see that W_{fld} is uniquely defined by the value of λ and x.

> This means the value of W_{fld} is the same regardless of how λ and x are brought to their final values.

- Consider the following paths:

Electromechanical Energy Conversion

- These two separate paths are shown to be integrated to find W_{fld} at the point (λ_0, x_0).
- The curvy path is the general case and it is difficult to integrate unless both i and f_{fld} are known as a function of λ and x.
- As integration is path independent Path 3 gives the same result and is much easier to integrate:

$$W_{\text{fld}}(\lambda_0, x_0) = \int_{\text{path 2a}} \mathrm{d}W_{\text{fld}} + \int_{\text{path 2b}} \mathrm{d}W_{\text{fld}}$$

Example

The relay drawn is made infinitely-permeable magnetic material with a movable plunger, also of infinitely-permeable material.

The height of the plunger is much greater than the air-gap length (h » g).

Calculate the magnetic stored energy W_{fld} as a function of plunger position ($0 < x < d$) for $N = 1000$ turns, $g = 2.0$ mm, $d = 0.15$ m, $l = 0.1$ m, and $i = 10$ A.

Solution

We can solve for W_{fld} when λ is known.

For this situation, i is held constant, and thus it would be useful to have an expression for W_{fld} as a function of i and x.

This can be obtained quite simply by:

$$W_{\text{fld}} = \frac{1}{2} L(x)\, i^2 \qquad \text{where} \qquad L(x) = \frac{\mu_0 N^2 A_{\text{gap}}}{2g}$$

where Agapis the gap cross-sectional area. From Fig. 3.6b, Agapcan be seen to be

$$A_{\text{gap}} = l(d-x) = ld\left(1 - \frac{x}{d}\right)$$

Solution

Therefore:

$$L\left(x\right) = \frac{\mu_0 N^2 ld\left(1 - x/d\right)}{2g}$$

and:

$$\begin{aligned} W_{\text{fld}} &= \frac{1}{2}\frac{N^2 \mu_0 \ell d\left(1 - x/d\right)}{2g} i^2 \\ &= \frac{1}{2}\frac{\left(1000^2\right)}{2(0.002)} \times 10^2 \left(1 - x/d\right) \\ 236 & \left(1 - \frac{x}{d}\right) \quad \blacksquare \end{aligned}$$

Practice

The relay of from the previous question is modified, making the air gaps surrounding the plunger are no longer uniform. The top air gap length is increased to $g_{\text{top}} = 3.5$ mm and that of the bottom gap is increased to $g_{\text{bot}} = 2.5$ mm.

The number of turns is increased to $N = 1500$.

Calculate the stored energy as a function of plunger position $(0 < x < d)$ for a current of $i = 5$ A.

$$W_{\text{fld}} = 88.5\,(1 - x/d)$$

- For a lossless magnetic-energy-storage system, the magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x.
- Rewriting Eq. (165) in the following form gives us:

$$\mathrm{d}W_{\text{fld}}(\lambda\,,\,x) = i\,\mathrm{d}\lambda - f_{\text{fld}}\,\mathrm{d}x. \tag{4}$$

- For any state function of two (2) independent variables, W_{fld} can be written as:

$$\mathrm{d}W_{\text{fld}}(\lambda\,,\,x) = \left.\frac{d\,W_{\text{fld}}}{d\lambda}\right|_{x}\mathrm{d}\lambda + \left.\frac{d\,W_{\text{fld}}}{dx}\right|_{\lambda}\mathrm{d}x, \tag{5}$$

- As λ and x are independent variables, Eq. (4) and Eq. (5) **must** be equal for all values of $\mathrm{d}\lambda$ and $\mathrm{d}x$.
- Taking the partial derivative while holding x constant:

$$i = \left.\frac{d\,W_{\text{fld}}\,(\lambda\,,\,x)}{d\lambda}\right|_{x} \tag{6}$$

- Taking the partial derivative while holding λ constant:

$$f_{\text{fld}} = \left.\frac{d\,W_{\text{fld}}\,(\lambda\,,\,x)}{dx}\right|_{\lambda}$$

- This is the results we needed.
- Once we know W_{fld} as a function of λ and x, we can solve Eq. (6) as a function for $i(\lambda, x)$

Example

The magnetic circuit consists of a single-coil stator and an oval motor. As the air-gap is nonuniform, the coil inductance varies with rotor angular position, measured between the magnetic axis of the stator coil and the major axis of the rotor as:

$$L(\theta) = L_0 + L_2 \cos 2\theta$$

where $L_0 = 10.6$ mH and $L_2 = 2.7$ mH.

> Note the 2^{nd} harmonic variation of inductance with rotor angle θ. This is consistent with inductance being unchanged if the rotor is rotated through an angle of 180°.

Find the torque as a function of θ for a coil current of 2 A.

Solution

$$T_{\text{fld}}(\theta) = \frac{i^2}{2}\frac{dL(\theta)}{d\theta} = \frac{i^2}{2}\left(-2L(2)\sin(2\theta)\right)$$

Notes on Solution

```python
import sympy as sy

# Define symbols for symbolic calculation
i, theta, L0, L2 = sy.symbols('i theta L0 L2')

L = L0 + L2 * sy.cos(2*theta)
T = i ** 2 / 2 * sy.diff(L, theta)

T.subs([(L2, 2.7e-3), (i, 2)])

print(T) # Print the output

del i, theta, L0, L2  # Remove the used variables from memory
```

```
#+RESULTS: ELECTROMECHANICAL-OVAL-ROTOR
```

Example

The coil inductance coil on a magnetic circuit is found to vary with rotor position as:

$$L(\theta) = L_0 + L_2 \cos(2\theta) + L_4 \sin(4\theta)$$

where L_0= 25.4 mH, L_2 = 8.3 mH and L_4 = 1.8 mH. Using these values:

1. Find the torque as a function of θ (i.e., $T(\theta)$) for a winding current of 3.5 A. 2. Find a rotor position θ_{max} that produces the largest negative torque.

Solution

```python
import sympy as sy

i, theta, L0, L2, L4 = sy.symbols('i theta L0 L2 L4')  # Define
↪  symbols for symbolic calculation

L = L0 + L2 * sy.cos(2*theta) + L4 * sy.sin(4*theta)
T = i ** 2 / 2 * sy.diff(L, theta)

T.subs([(L2, 8.3e-3), (i, 3.5), (L4, 1.8e-3)])

print(T)

del i, theta, L0, L2, L4  # Remove the used variables from memory
```

```
#+RESULTS: COIL-INDUCTANCE
```

Example

Consider a plunger whose inductance varies as with position as:

$$L(x) = L_0(1 - (x/d)^2)$$

Find the force on the plunger as a function of x when the coil is driven by a controller which produces a current as a function of x of the form

$$i(x) = I_0 \left(\frac{x}{d}\right)^2 A$$

Solution

```python
import sympy as sy

i, L0, I0, x, d, mu0, N, l, g = sy.symbols('i L0 I0 x d mu0 N l g')

i = I0 * (x / d)

L = L0*( 1 - (x/d)**2)

f = i ** 2 / 2 * sy.diff(L, x)

W_fld = i ** 2 / 2 * L

print(f)
```

```
#+RESULTS: PLUNGER
```

Example

In a drive with a rotow with Hlight(non-uniform air gap), the inductances in henrys are given as:

$$L_{11} = (3 + \cos 2\theta) \times 10^{-3}, L_{12} = 0.3 \cos \theta, L_{22} = 3\theta + 10cos2\theta.$$

Using this information, please find and plot the torque $T_{fld}(\theta)$ for current $I_1 = 0.8$ A and $I_2 = 0.01$ A.

Solution

```
import sympy as sy
import numpy as np

theta = sy.symbols('theta')

L = np.array([[(3 + sy.cos(2 * theta))*1e-3, 0.3 *
↪  sy.cos(theta)],[0, (30 + 10 * sy.cos(2 * theta))]])

i = [0.8, 0.01]

T_fld = i[0]**2 / 2 * sy.diff(L[0,0], theta) + i[1]**2 / 2 *
↪  sy.diff(L[1,1], theta) + i[0] * i[1] * sy.diff(L[0,1], theta)

sy.plot(T_fld, (theta, -10, 10), adaptive=True, depth =50 )
```

- To obtain force, directly as a function of current, we can use co-energy.

> Co-energy does not exist in reality. It is a mathematical convenience [1].

- The selection of energy or coenergy as the state function is purely a matter of convenience.
  - They both give the same result, but one or the other may be simpler analytically, depending on the desired result and the characteristics of the system being analysed.

- The coenergy (W'_{fld}) is defined as a function of λ and x:

$$W'_{\text{fld}}(\lambda, x) = i\lambda - W_{\text{fld}}(\lambda, x)$$

- The desired derivation is carried out by using the differential of $i\lambda$:

$$\mathrm{d}(i\lambda) = i\,\mathrm{d}\lambda + \lambda\,\mathrm{d}i$$

Electromechanical Energy Conversion

- The analysis of a system with multiple excitation is similar.
- Assume a system having:
  - a mechanical terminal with the values T_{fld} and
  - and two (2) electrical terminals (i.e., λ_1, λ_2, i_1, i_2)

> This could be a system where rotor and stator are **both** excited.

- Using the fluxes, the differential energy function $\mathrm{d}W_{\mathrm{fld}}(\lambda_1, \lambda_2, \theta)$ can be written as:

$$\mathrm{d}W_{\mathrm{fld}}(\lambda_1, \lambda_2, \theta) = i_1\,\mathrm{d}\lambda_1 + i_2\lambda_2 - T_{\mathrm{fld}}\,\mathrm{d}\theta$$

- Using our previous analogies:

$$i_1 = \left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1} \right|_{\lambda_2, \theta}$$

$$i_2 = \left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2} \right|_{\lambda_1, \theta}$$

$$T_{\text{fld}} = -\left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \theta} \right|_{\lambda_1, \lambda_2} \quad \blacksquare$$

> The partial derivative with respect to each independent variable must be taken holding the other variables **constant**.

Electromechanical Energy Conversion

- Using co-energy definition, we can re-write our equation as:

$$W'_{\text{fld}}(i_1, i_2, \theta) = \lambda_1 i_1 + \lambda_2 i_2 - W_{\text{fld}}.$$

- Taking the derivative of this statement gives us:

$$\mathrm{d}W'_{\text{fld}}(i_1, i_2, \theta) = \lambda_1 \,\mathrm{d}i_1 + \lambda_2 \,\mathrm{d}i_2 - T_{\text{fld}} \,\mathrm{d}\theta.$$

- Through significant simplification we arrive at:

$$T_{\text{fld}} = \frac{i_1^2}{2} \frac{d^{L_{11}()}}{d^{L_{11}()}}$$

Example

In the following system, the inductance in H are given as:

- $L_{11} = (3 + \cos 2\theta) \times 10^{-3}$,
- $L_{12} = 0.3 \cos \theta$,
- $L_{22} = 30 + 10 \cos 2\theta$.

Find the torque equation for currents $i_1 = 0.8$ A and $i_2 = 0.01$ A.

Solution

The torque is simply determined by:

$$T_{\text{fld}} = \frac{i_1^2}{2}\frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta}$$

$$= \frac{i_1^2}{2}\left(-2 \times 10^{-3}\right)\sin 2\theta + \frac{i_2^2}{2}(-20 \sin 2\theta) - i_1 i_2 (0.3) \sin\theta\,.$$

For $i_1 = 0.8$ A and $i_2 = 0.01$ A, the torque is:

$$T_{\text{fld}} = -1.64 \times 10^{-3} \sin 2\theta - 2.4 \times 10^{-3} \sin\theta \quad \blacksquare$$

Notes on the Solution

The torque expression consists of terms worth discussing.

- The term proportional to $i_1 i_2 \sin\theta$, is due to the mutual interaction between the rotor and stator currents.
- This acts in a direction to align the rotor and stator so as to maximize their mutual inductance.

> It can be thought of as being due to the tendency of two magnetic fields (in this case those of the rotor and stator) to align.

Notes on the Solution

The torque expression consists of terms worth discussing.

- Two terms each proportional to $\sin 2\theta$ and to the square of one of the coil currents $(i_1^2,\ i_2^2)$.
- Due to the action of the individual winding currents alone and correspond to the torques one sees in singly-excited systems.
- The torque is due to self inductances being a function of rotor position and the corresponding torque acts in a direction to maximize each inductance so as to maximize the co-energy.
- The $\sin 2\theta$ variation is due to the corresponding variation in the self inductances caused by the variation of the air-gap reluctance.

Appendix

- Assume the following simplified induction drive equivalent circuit.

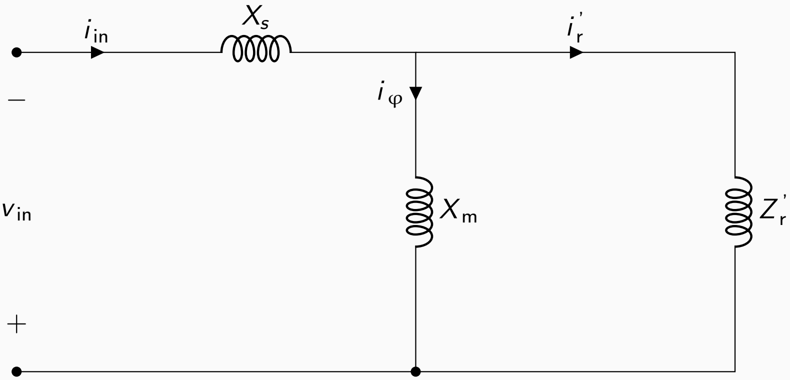


Figure 24: An abstract rendition of the simplified induction circuit equivalent circuit.

Appendix

- where:

$$Z_s = R_s + \mathbf{j} X_s,$$
$$Z_m = \frac{R_c \times \mathbf{j} X_m}{R_c + \mathbf{j} X_m},$$
$$Z_r = \frac{R_r^{'}}{s} + jX_r^{'}.$$

- From Kirchhoff's current law, the following statement holds true:

$$i_{in} = i_{\varphi} + i_r^{'}.$$

- The second identity can be derived from parallel circuit principles;

$$Z_m i_{\varphi} = Z_r^{'} i_r \quad \rightarrow \quad i_m = i_R^{'} \times \left(\frac{Z_R}{Z_m}\right).$$

- We can isolate the magnetizing current (i_φ).

$$i_\varphi = i_r^{'} \times \left(\frac{Z_r}{Z_m}\right) + i_r^{'}.$$

- Isolating the rotor current ($i_r^{'}$) gives us the final expression:

$$i_r^{'} = \frac{i_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

- Consider the following equivalent circuit of an IM.

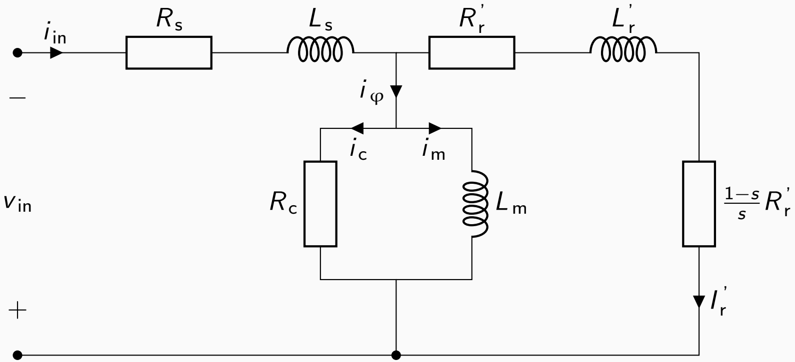


Figure 25: An abstract rendition of the induction circuit equivalent circuit.

Appendix

- We can start by summing up the rotor side.

$$Z_r = \frac{R_r^{'}}{s} + \mathbf{j}\, w_r L_r^{'} \quad \rightarrow \quad Z_r = \frac{R_r^{'}}{s} + X_r^{'}.$$

- The stator side is calculated as:

$$Z_s = R_s + \mathbf{j}\, w_s L_s \quad \rightarrow \quad Z_s = \frac{R_s}{s} + X_s.$$

- The magnetising side is calculated as:

$$Z_m = \frac{R_c \times \mathbf{j}\, w_s L_m}{R_c + \mathbf{j}\, w_s L_m} \quad \rightarrow \quad Z_m = \frac{R_c \times X_m}{R_c + X_m}.$$

- Without substitution, the input impedance is calculate to be

$$Z_{\text{in}} = Z_{\text{s}} + \frac{Z_{\text{m}} \times Z_{\text{r}}}{Z_{\text{m}} + Z_{\text{r}}}.$$

- From here, we can derive the stator current (I_{in}).

$$V_{\text{ph}} = \frac{Z_{\text{in}}}{I_{\text{in}}} \quad \rightarrow \quad I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}}.$$

- Now, the Torque of an IM is related to the air-gap power by:

$$T_{\text{mech}} = \frac{P_{\text{gap}}}{w_{\text{s}}} \qquad \text{and} \qquad P_{\text{gap}} = n_{\text{ph}} \frac{I_{\text{r}}^2}{w_{\text{s}}} \left(\frac{R_2^{'}}{s} \right)$$

Appendix

- Using the stator-rotor relationship, we can obtain our final result.

$$I_r^{'} = \frac{I_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

Appendix

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an empirical relationship or phenomenological relationship is a relationship or correlation that is supported by experiment or observation but not necessarily supported by theory.

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