Electric Drive Fundamentals

Daniel T. McGuiness, PhD

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MCI

B.Sc - Drive Systems

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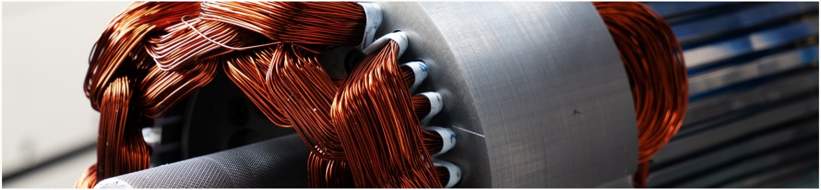
Rotating Magnetic Fields

- The topic is to introduce and discuss some of the principles underlying the performance of electric machinery.

> Following principles are common to both Alternating Current (AC) and Direct Current (DC) machines.

- Various techniques and approximations involved in reducing the machine to simple mathematical models will be discussed.



Rotating Magnetic Fields

- To determine the voltages induced by time-varying magnetic fields we can use Faraday's law:

$$\mathcal{E} = \frac{d\lambda}{dt},$$

- Electromagnetic Energy Conversion (EEC) occurs when changes in λ results from mechanical motion.
- $\mathcal{E}$ are generated in windings or groups of coils by rotating these windings mechanically through a **B**-field, either:
  1. mechanically rotating a magnetic field past the winding,
  2. designing the magnetic circuit so that the reluctance varies with rotation of the rotor (i.e., $\mathcal{R}(\theta)$),

> A coil's λ is changed cyclically generating time-varying voltage.

Rotating Magnetic Fields

- Coils connected together is referred to as an **armature winding**.
- The term is used to refer to a winding or a set of windings on a rotating machine which carry AC.
- In AC machines, the armature winding is typically on the stationary (i.e., stator).
    - Which case these windings may also be referred to as stator windings.

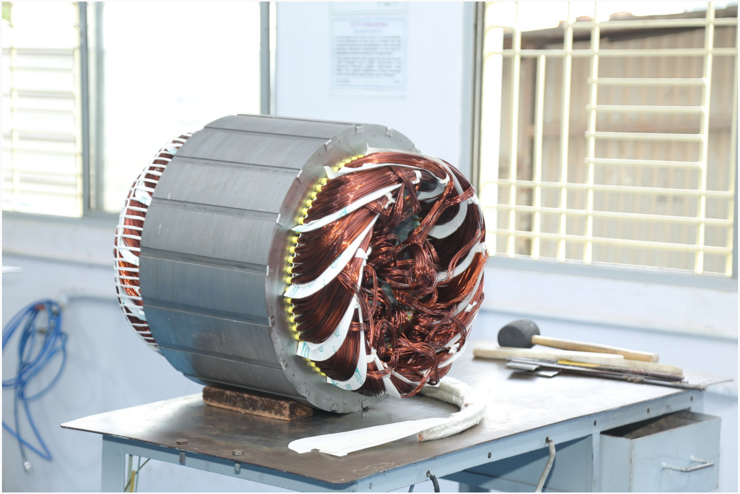


Figure 1: The armature of a AC is the stator as the stator has AC flowing during operation [1].

Rotating Magnetic Fields

- In DC machines, armature winding is found on the rotating member (i.e., rotor).
- The armature winding of a DC machine consists of numerous coils connected together to form a closed loop.
- A rotating mechanical contact is used to supply current to the armature winding as the rotor rotates.
  - This is known as a **commutator**.

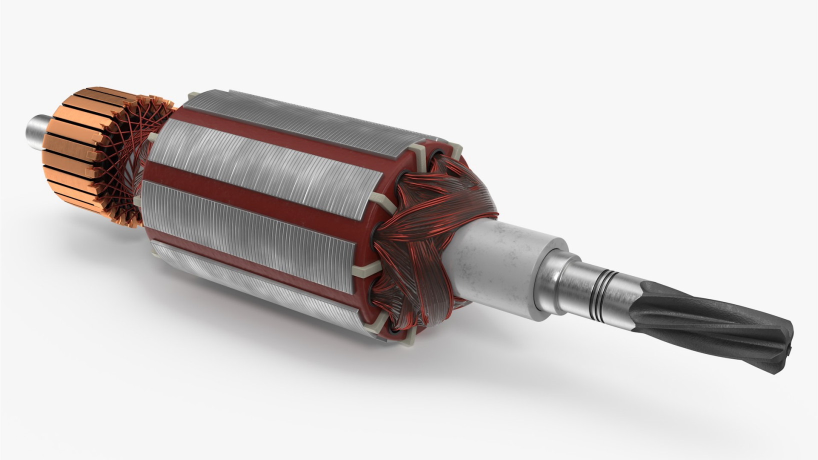


Figure 2: The armature of a DC is the rotor as the rotor has AC flowing during operation [2].

- Synchronous Machine (SynM) and DC machines usually include a second winding(s) which carry DC and which are used to produce main operating flux in the machine.
- Such a winding is typically referred to as **field winding**.
  - On DC machines, it is found on the stator,
  - On SynM machines, it is found on the rotor.

> Permanent Magnet (PM)s also produce DC magnetic flux and are used in the place of field windings in some machines.

Rotating Magnetic Fields

- Some machines, such as variable reluctance machines and stepper motors, have no windings on the rotor.
- Operation of these depends on the non-uniform air-gap reluctance associated with variations in rotor position in conjunction with time-varying currents applied to their stator windings.
- For these, both the stator and rotor structures are subjected to time-varying magnetic flux.
  - Both may require lamination to reduce eddy-current losses.

Rotating Magnetic Fields

Figure 3: An example of a switched reluctance drive with its rotor (left) and stator (right).

- Start with the voltage induced in the armature of the simplified salient-pole ac synchronous generator.

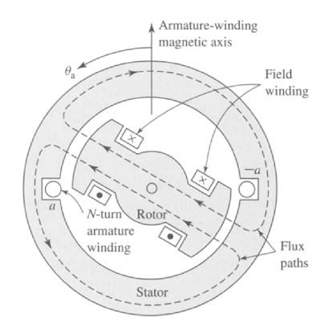


Figure 4: A simple diagram of a 2-pole salient generator.

Figure 5: A unique characteristics of salient-pole machine is its protrusions from the core.

Rotating Magnetic Fields

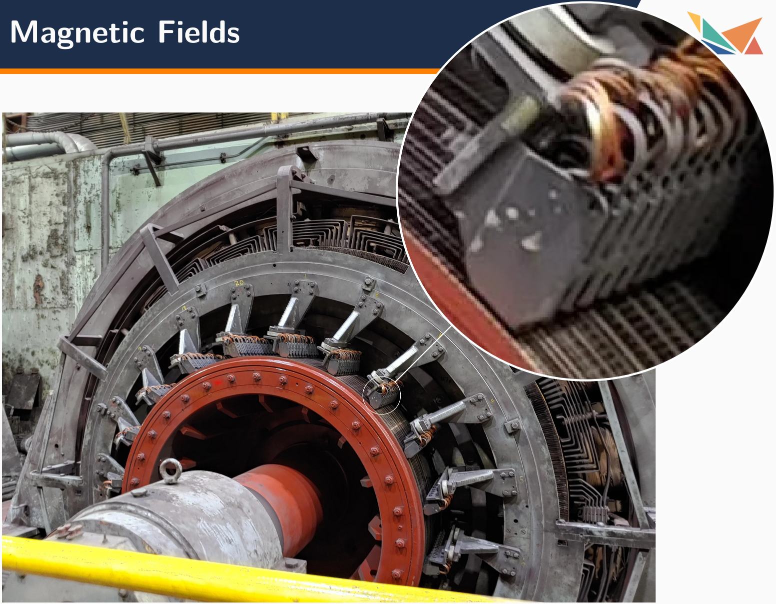
- The field-winding of this machine produces one pair of magnetic pole.
  - Referred to as a two-pole machine.
- With rare exceptions, the armature winding of a SynM is on the stator, and the field winding is on the rotor.
- The field winding is excited by DC conducted to it by means of stationary carbon brushes which contact rotatating slip rings or collector rings.
- Practical factors usually dictate this orientation of the two windings:

> It is advantageous to have the single, low-power field winding on the rotor while having the high-power, typically multiple-phase, armature winding on the stator.

Rotating Magnetic Fields

Figure 6: A 10,000 HP SynM.

Rotating Magnetic Fields



Rotating Magnetic Fields

- The armature winding, consisting of only a single coil of N turns placed in slots on the inner periphery of the stator.
- The conductors forming these coil sides are parallel to the shaft of the machine and are connected in series by end connections.
- The rotor is turned at a constant speed by a source of mechanical power connected to its shaft.
- The armature winding is assumed to be open-circuited and hence the flux in this machine is produced by the field winding alone.
- An idealised analysis of this machine would assume a sinusoidal distribution of magnetic flux in the air gap.

> The resultant radial distribution of B is a function of the spatial angle θ_a (respect to the magnetic axis of the armature winding) around the rotor periphery.

Rotating Magnetic Fields

- In practice, the air-gap flux-density of practical salient-pole machines can be made to approximate a sinusoidal distribution by properly shaping the pole faces.
- As the rotor rotates, the λ of the armature winding change with time.
- Under the assumption of a sinusoidal flux distribution and constant rotor speed, the resulting coil voltage will be sinusoidal in time.
- The coil voltage passes through a complete cycle for each revolution of the two-pole machine.
- Its frequency (f) in cycles per second (Hz) is the same as the speed of the rotor in revolutions per second: the electric frequency of the generated voltage is synchronized with the mechanical speed, and this is the reason for the designation "synchronous" machine. video
- Thus a two-pole synchronous machine must revolve at 3600 revolutions per minute to produce a 60-Hz voltage.

Rotating Magnetic Fields

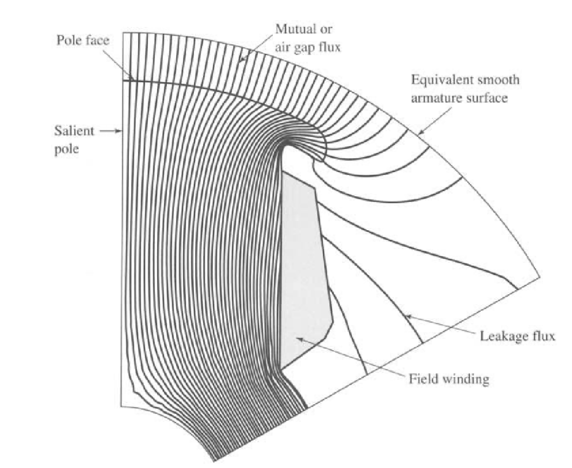


Figure 7: Finite-element solution for the flux distribution around a salient pole.

Rotating Magnetic Fields

- Many SynM have more than two poles.
- As a specific example, a schematic form a four-pole single-phase generator.

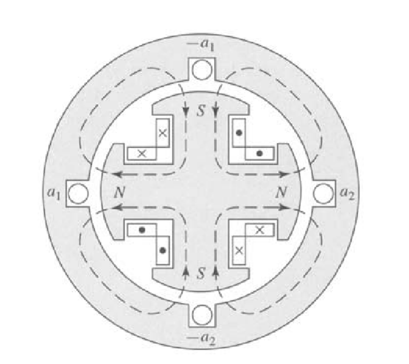


Figure 8

Rotating Magnetic Fields

- The field coils are connected so that the poles are of alternate polarity.
- There are two complete wavelengths, or cycles, in the flux distribution around the periphery.
- The armature winding now consists of two coils connected in series by their end connections.
- The span of each coil is one wavelength of flux.
- The generated voltage now goes through two complete cycles per revolution of the rotor.
- The frequency in hertz will thus be twice the speed in revolutions per second.

Rotating Magnetic Fields

- When a machine has more than two poles, it is convenient to concentrate on a single pair of poles and to recognize that the electric, magnetic, and mechanical conditions associated with every other pole pair are repetitions of those for the pair under consideration.
- For this reason it is convenient to express angles in electrical degrees or electrical radians rather than in physical units.
- One pair of poles in a multipole machine or one cycle of flux distribution equals 360 electrical degrees.
- Since there are poles/2 complete wavelengths, or cycles, in one complete revolution, it follows, for example, that

$$\theta_{\mathrm{e}} = \left(\frac{p}{2}\right)\theta_{\mathrm{a}}$$

Rotating Magnetic Fields

- Armature currents rotates at synchronous speed.
- To produce a steady electromechanical torque, the **B**-fields of the stator and rotor must be constant in amplitude and stationary with respect to each other.
- In a SynM, the steady-state speed is determined by the number of poles and the frequency of the armature current.

> a SynM operated from a constant-frequency AC source will operate at a constant steady-state speed.

Rotating Magnetic Fields

- Similar to SynM, stator winding of an Induction Machine (IM) is excited with AC.
- Unlike a SynM, rotor field windings are excited with AC.
- In IM, AC is applied directly to the stator windings.
- Rotor currents are produced by induction.
- IM may be regarded as a transformer where electric power is transformed between rotor and stator with a change of frequency and flow of mechanical power.

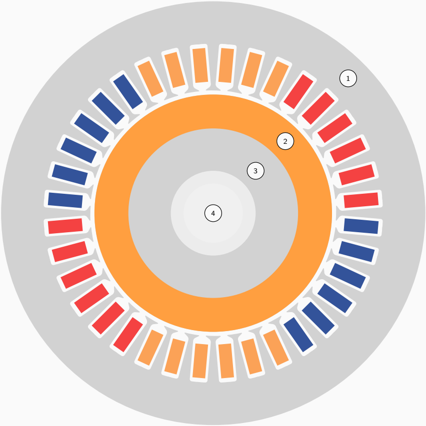
> Most common of all motors, is seldom used as a generator.

- Performance characteristics as a generator are unsatisfactory, although it has been found to be well suited for wind-power applications.

Rotating Magnetic Fields

- In IMs, stator windings are essentially same as those of a SynM.
- The rotor windings are electrically short-circuited and frequently have no external connections.
- currents are induced by transformer action from the stator winding.
- Here the rotor "windings" are actually solid aluminum bars which are cast into the slots in the rotor and which are shorted together by cast aluminum rings at each end of the rotor.
- This type of rotor construction results in IMs which are relatively inexpensive and highly reliable, factors contributing to their immense popularity and widespread application

Rotating Magnetic Fields



① **Stator Frame**
- Usually from silicon steel, or electrical steel.
- An iron alloy with silicon as the main additive element, instead of carbon.
- μ_r is 4,000 times that of vacuum, or 2-3 times of stainless steel.

② **Short circuited-rotor coils**
- End rings are attached to create a short-circuit for induced current to flow through.
- Made from aluminium or copper.

③ **Rotor**
- This is the magnetic part of the rotor.
- Made using electrical steel laminations.

④ **Rotor Shaft**
- Usually made from mild steel
- This is an iron alloy with carbon as its main alloying element.

Figure 9: A Schematic view of a Stator with a squirrel cage rotor.

Rotating Magnetic Fields

- Similar to SynM, the armature flux in the IM leads the rotor and produces torque.
- just as in a SynM, the rotor and stator fluxes rotate in synchronism with each other and that torque is related to the relative displacement between them.
- However, unlike a SynM, the rotor of an IM does not itself rotate synchronously.
- it is the ”slipping” of the rotor with respect to the synchronous armature flux that gives rise to the induced rotor currents and hence the torque.
- Induction motors operate at speeds less than the synchronous mechanical speed.

Rotating Magnetic Fields

- The armature winding of a DC generator is on the rotor with current conducted from it by means of **carbon brushes**.

Figure 10: Brushes are needed for conventional DC Machines to transfer electricity to the rotor.

Rotating Magnetic Fields

- The field winding is on the stator and is excited by direct current.
- The rotor is normally turned at constant speed by a source of mechanical power connected to the shaft.
- The air-gap flux distribution usually approximates a flat-topped wave, rather than the sine wave found in AC machines.
- Rotation of the coil generates a coil voltage which is a time function having the same waveform as the spatial flux-density distribution.

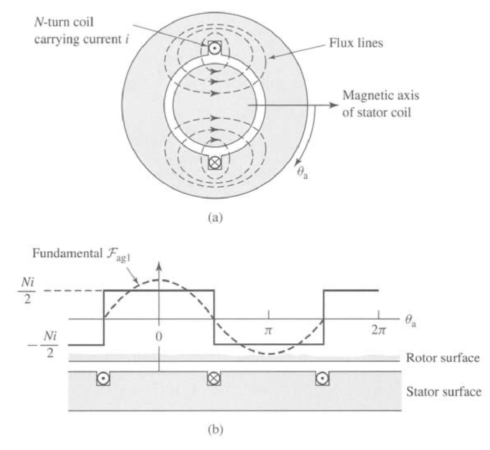
Rotating Magnetic Fields

- While the purpose is the generation of a direct voltage, the voltage induced in an individual armature coil is an alternating voltage, which must therefore be rectified.
- The output voltage of an ac machine can be rectified using external semiconductor rectifiers.
- This is in contrast to the conventional dc machine in which rectification is produced mechanically by means of a commutator, which is a cylinder formed of copper segments insulated from each other by mica or some other highly insulating material and mounted on, but insulated from, the rotor shaft.
- Stationary carbon brushes held against the commutator surface connect the winding to the external armature terminals.

> The need for commutation is the reason why the armature windings of dc machines are placed on the rotor.

Rotating Magnetic Fields

- Most armatures have distributed windings, i.e., windings which are spread over a number of slots around the air-gap periphery.
- The individual coils are interconnected so that the result is a magnetic field having the same number of poles as the field winding.
- The study of the magnetic fields of distributed windings can be approached by examining the magnetic field produced by a winding consisting of a single N-turn coil which spans 180 electrical degrees called a full-pitch coil



N-turn coil carrying current i
Flux lines
Magnetic axis of stator coil
θ_a
(a)
Fundamental $\mathcal{F}_{ag1}$
$\frac{Ni}{2}$
$-\frac{Ni}{2}$
0
π
2π
θ_a
Rotor surface
Stator surface
(b)

Figure 11

Rotating Magnetic Fields

- Fourier analysis shows the air-gap Magneto-Motive Force (MMF) produced by a single coil such as the full-pitch coil of Fig. 4.19 consists of a fundamental space-harmonic component as well as a series of higher-order harmonic components.
- In AC machine design, serious efforts are made to distribute the coils making up the windings so as to minimise higher-order harmonic components and to produce an air-gap MMF wave which consists predominantly of the space-fundamental sinusoidal component.
- It is thus appropriate here to assume that this has been done and to focus our attention on the fundamental component.

Rotating Magnetic Fields

- The rectangular air-gap MMF wave of the concentrated two-pole, full-pitch coil can be resolved into a Fourier series comprising a fundamental component and a series of odd harmonics.
- The fundamental component ($\mathcal{F}_{\text{ag1}}$) is:

$$\mathcal{F}_{\text{ag1}} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta_{\text{a}}$$

  where θ_{a} is measured from the **B**-axis of the stator coil.
- It is a sinusoidal space wave of amplitude:

$$\left(\mathcal{F}_{\text{ag1}}\right)_{\text{peak}} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)$$

- with its peak aligned with the magnetic axis of the coil.

- Electric machinery often contains systems of multiple windings, requiting careful bookkeeping to account for the flux contributions of the various windings. Although the details of such analysis are beyond the scope of this book, it is useful to discuss these effects in a qualitative fashion and to describe how they affect the basic machine inductances.

Air-Gap Space-Harmonic Flux

- although sin- gle distributed coils create air-gap flux with a significant amount of space-harmonic content, it is possible to distribute these windings so that the space-fundamental component is emphasized while the harmonic effects are greatly reduced.

> we can neglect harmonic effects and consider only space-fundamental fluxes in calculat- ing the self and mutual-inductance expressions

- often small, the space-harmonic components of air-gap flux do exist.
- Dc machines they are useful torque-producing fluxes and therefore can be counted as mutual flux between the rotor and stator windings.
- ac machines, however, they may generate time-harmonic voltages or asynchronously rotating flux waves.

Slot Leakage Flux

- there are flux components which cross the slot.
- t also forms a component of the leakage inductance of the winding producing it.

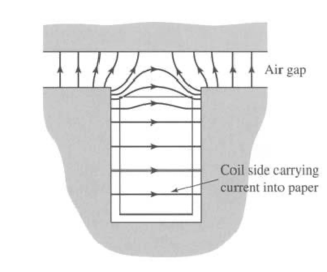


Figure 12: Flux created by a single coil side in a slot.

End-Turn Flux

- The magnetic field distribution created by end turns is extremely complex.
- In general these fluxes do not contribute to useful rotor-to-stator mutual flux, and thus they, too, contribute to leakage inductance.
- This self-inductance expression of Eq. B.26 must, in general, be modified by an additional term Ll, which represents the winding leakage inductance. This leakage inductance corresponds directly to the leakage inductance of a transformer winding as discussed in Chapter 1. Although the leakage inductance is usually difficult to calculate analytically and must be determined by approximate or empirical techniques, it plays an important role in machine performance.

Rotating Magnetic Fields

- A set of current are applied to the 3-phase stator coils.

$$
\begin{aligned}
i_{\text{RR'}} &= I_{\max} \sin(wt)\,, \\
i_{\text{YY'}} &= I_{\max} \sin(w\,t - 120^\circ)\,, \\
i_{\text{BB'}} &= I_{\max} \sin(w\,t - 240^\circ)\,.
\end{aligned}
$$

- The magnetic flux densities generated from these currents are;

$$
\begin{aligned}
B_{\text{RR'}} &= B_{\max} \sin(wt)\,, \\
B_{\text{YY'}} &= B_{\max} \sin(w\,t - 120^\circ)\,, \\
B_{\text{BB'}} &= B_{\max} \sin(w\,t - 240^\circ)\,.
\end{aligned}
$$

Rotating Magnetic Fields

- Let's add up all these generated magnetic flux densities.

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) &= \mathbf{B}_{\mathbf{RR'}} + \mathbf{B}_{\mathbf{YY'}} + \mathbf{B}_{\mathbf{BB'}},\\ &= B_{\max}\sin(wt) + B_{\max}\sin(w\,t - 120^\circ) + B_{\max}\sin(w\,t - 240^\circ)\,.\end{aligned}$$

- Each of magnetic field can now be broken to its $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ components.

$$\begin{aligned}\mathbf{B}_{\text{net}}(t) = {} & B_{\max}\sin(wt)\,\hat{\mathbf{x}} - \left[\frac{1}{2}B_{\max}\sin(w\,t - 120^\circ)\right]\hat{\mathbf{x}}\\ & + \left[\frac{\sqrt{3}}{2}B_{\max}\sin(w\,t - 240^\circ)\right]\hat{\mathbf{y}} + \left[\frac{\sqrt{3}}{2}B_{\max}\sin(w\,t - 240^\circ)\right]\hat{\mathbf{y}}\\ & + \left[\frac{\sqrt{3}}{2}B_{\max}\sin(w\,t - 240^\circ)\right]\hat{\mathbf{y}}.\end{aligned}$$

Rotating Magnetic Fields

- Simplification presents us with the following:

$$\mathbf{B}_{\text{net}}(t) = \frac{3}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} - \cos(wt)\,\hat{\mathbf{y}}\right] \quad \blacksquare$$

For an AC drive with n phases, the net magnetic vector would be:

$$\mathbf{B}_{\text{net}}(t) = \frac{n}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} - \cos(wt)\,\hat{\mathbf{y}}\right] \quad \blacksquare$$

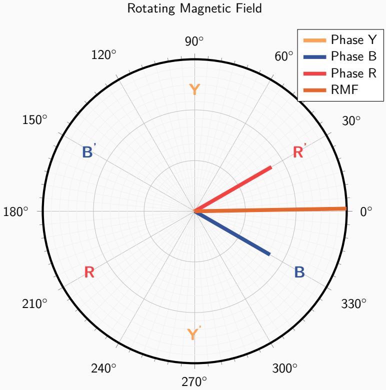
Rotating Magnetic Fields

- If the current in any two of the three coils is swapped, the direction of the magnetic field's rotation will be reversed.
- This means that it is possible to reverse the direction of rotation of an AC motor just by switching the supply connections on any of two of the three coils.
- For a 3-phase motor, the rotation of $\mathbf{B}_{\text{net}}(t)$ is:
  - Clockwise:

$$\mathbf{B}_{\text{net}}(t) = \frac{3}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} - \cos(wt)\,\hat{\mathbf{y}}\right].$$

  - Counter-clockwise:

$$\mathbf{B}_{\text{net}}(t) = \frac{3}{2} B_{\text{m}} \left[\sin(wt)\,\hat{\mathbf{x}} + \cos(wt)\,\hat{\mathbf{y}}\right].$$



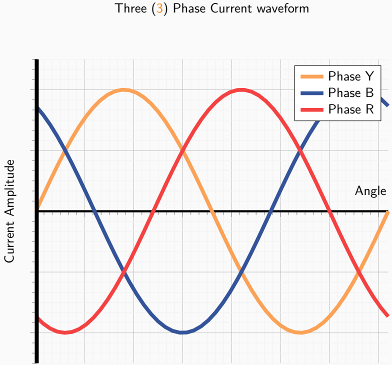


Figure 13: A three phase rotating magnetic field.

Appendix

- Assume the following simplified induction drive equivalent circuit.

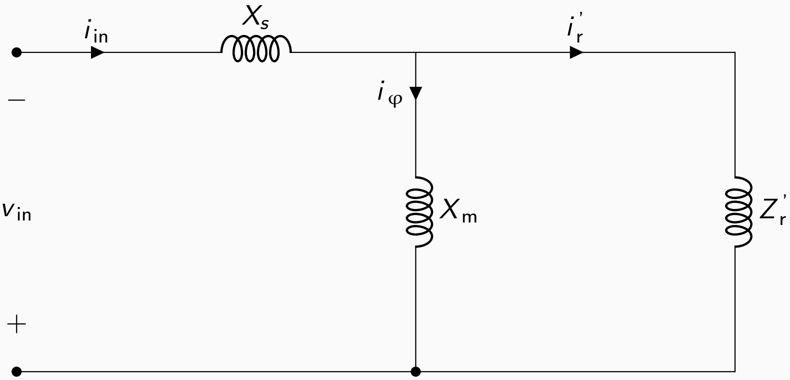


Figure 14: An abstract rendition of the simplified induction circuit equivalent circuit.

Appendix

- where:

$$Z_{\mathrm{s}} = R_{\mathrm{s}} + \mathbf{j}\,X_{\mathrm{s}},$$
$$Z_{\mathrm{m}} = \frac{R_{\mathrm{c}} \times \mathbf{j}\,X_{\mathrm{m}}}{R_{\mathrm{c}} + \mathbf{j}\,X_{\mathrm{m}}},$$
$$Z_{\mathrm{r}} = \frac{R_{\mathrm{r}}^{'}}{s} + jX_{\mathrm{r}}^{'}.$$

- From Kirchhoff's current law, the following statement holds true:

$$i_{\mathrm{in}} = i_{\varphi} + i_{\mathrm{r}}^{'}.$$

- The second identity can be derived from parallel circuit principles;

$$Z_{\mathrm{m}} i_{\varphi} = Z_{\mathrm{r}}^{'} i_{\mathrm{r}} \quad \rightarrow \quad i_{\mathrm{m}} = i_{\mathrm{R}}^{'} \times \left(\frac{Z_{\mathrm{R}}}{Z_{\mathrm{m}}}\right).$$

Appendix

- We can isolate the magnetizing current (i_φ).

$$i_\varphi = i_r^{'} \times \left(\frac{Z_r}{Z_m}\right) + i_r^{'}.$$

- Isolating the rotor current ($i_r^{'}$) gives us the final expression:

$$i_r^{'} = \frac{i_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

- Consider the following equivalent circuit of an IM.

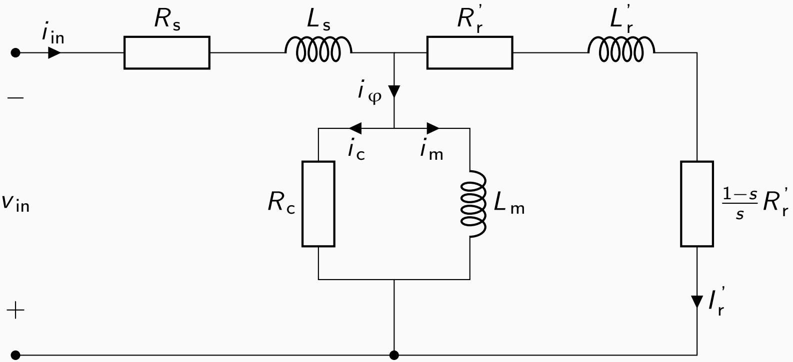


Figure 15: An abstract rendition of the induction circuit equivalent circuit.

Appendix

- We can start by summing up the rotor side.

$$Z_r = \frac{R_r^{'}}{s} + \mathbf{j}\, w_r L_r^{'} \quad \rightarrow \quad Z_r = \frac{R_r^{'}}{s} + X_r^{'}.$$

- The stator side is calculated as:

$$Z_s = R_s + \mathbf{j}\, w_s L_s \quad \rightarrow \quad Z_s = \frac{R_s}{s} + X_s.$$

- The magnetising side is calculated as:

$$Z_m = \frac{R_c \times \mathbf{j}\, w_s L_m}{R_c + \mathbf{j}\, w_s L_m} \quad \rightarrow \quad Z_m = \frac{R_c \times X_m}{R_c + X_m}.$$

Appendix

- Without substitution, the input impedance is calculate to be

$$Z_{\text{in}} = Z_{\text{s}} + \frac{Z_{\text{m}} \times Z_{\text{r}}}{Z_{\text{m}} + Z_{\text{r}}}.$$

- From here, we can derive the stator current (I_{in}).

$$V_{\text{ph}} = \frac{Z_{\text{in}}}{I_{\text{in}}} \quad \rightarrow \quad I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}}.$$

- Now, the Torque of an IM is related to the air-gap power by:

$$T_{\text{mech}} = \frac{P_{\text{gap}}}{w_{\text{s}}} \qquad \text{and} \qquad P_{\text{gap}} = n_{\text{ph}} \frac{I_{\text{r}}^2}{w_{\text{s}}} \left(\frac{R_2^{'}}{s} \right)$$

Appendix

- Using the stator-rotor relationship, we can obtain our final result.

$$I_r^{'} = \frac{I_s}{\left(1 + \frac{Z_r}{Z_m}\right)} \quad \blacksquare$$

Appendix

Go Back

an empirical relationship or phenomenological relationship is a relationship or correlation that is supported by experiment or observation but not necessarily supported by theory.

Go Back

Faraday's law is a law of electromagnetism predicting how a **B**-field will interact with an electric circuit to produce Electro-motive Force (EMF).

This phenomenon, known as electromagnetic induction, is the fundamental operating principle of transformers, inductors, and many types of electric motors, generators and solenoids.

> *The EMF around a closed path is equal to the negative of the time rate of change of the magnetic flux enclosed by the path.*

AC Alternating Current. 2, 6, 8, 22, 31, 43

DC Direct Current. 2, 6, 9, 10, 13, 22, 44

IM Induction Machine. 2, 22–24

MMF Magneto-Motive Force. 2, 31, 32, 44, 46

PM Permanent Magnet. 2, 10

SynM Synchronous Machine. 2, 10, 22–24

Symbol	Description	Unit
C	total number of conductors in armature winding	-
B_{peak}	Magnetic Flux Density Magnitude	Tesla
B	Magnetic Flux Density Magnitude	Tesla
H	Magnetic Field Intensity Magnitude	A/m
I_m	peak value of current	A
L_s, L_r	Stator & Rotor Inductance	H
N_{ph}	Number of Turns per phase	-
N	Number of Turns	-
R_s, R_r	Stator & Rotor resistance	Ohm
R	Resistance	Ohm
T	Torque	$N \cdot m$
W_{fld}, W'_{fld}	Field energy & co-energy	J
Z	Impedance	Ohm

$\mathcal{E}$	Electro-motive force	V
$\mathcal{F}$	Magneto-motive Force	Ampere-Turns
$\mathcal{L}$	Inductance function	H
$\mathcal{R}_{\text{c}}, \mathcal{R}_{\text{ag}}, \mathcal{R}_{\text{tot}}$	Core, air-gap & Total Reluctance	1/H
$\mathcal{R}$	Reluctance	1/H
f_{e}	Electrical Frequency	Hz
f	Frequency	Hz
g	Air-gap	m
λ	Flux Linkage	Weber (Wb)
ω_{e}	electrical angular frequency	rad/s
ϕ	Magnetic Flux	Wb
$\theta_{\text{a}}, \theta_{\text{e}}$	Spatial & Electrical angle	deg
θ_{e}	Electrical Angle	deg
i_{a}	armature current	A

i_r, i_b, i_y	poly-phase currents	A
i	Current	A
k_w	Winding Factor	-
l	length	m
μ_0	Permeability of free-space	H/m
n_s	synchronous speed	rpm
p	Poles	-
t	Time	s
$\mathbf{B}$	Magnetic Flux Density	Tesla
$\mathbf{H}$	Magnetic Field Intensity	A/m
m	number of parallel paths through armature winding	-

[1] indiamart. *AC Stator Coils*. 2024. URL: https://www.indiamart.com/proddetail/ac-stator-coils-2849109472212.html?mTd=1.

[2] turbosquid. *Electric Motor Armature*. 2024. URL: https://www.turbosquid.com/3d-models/electric-motor-armature-3d-model-1974354.