Lecture Book
B.Sc Drive Technology

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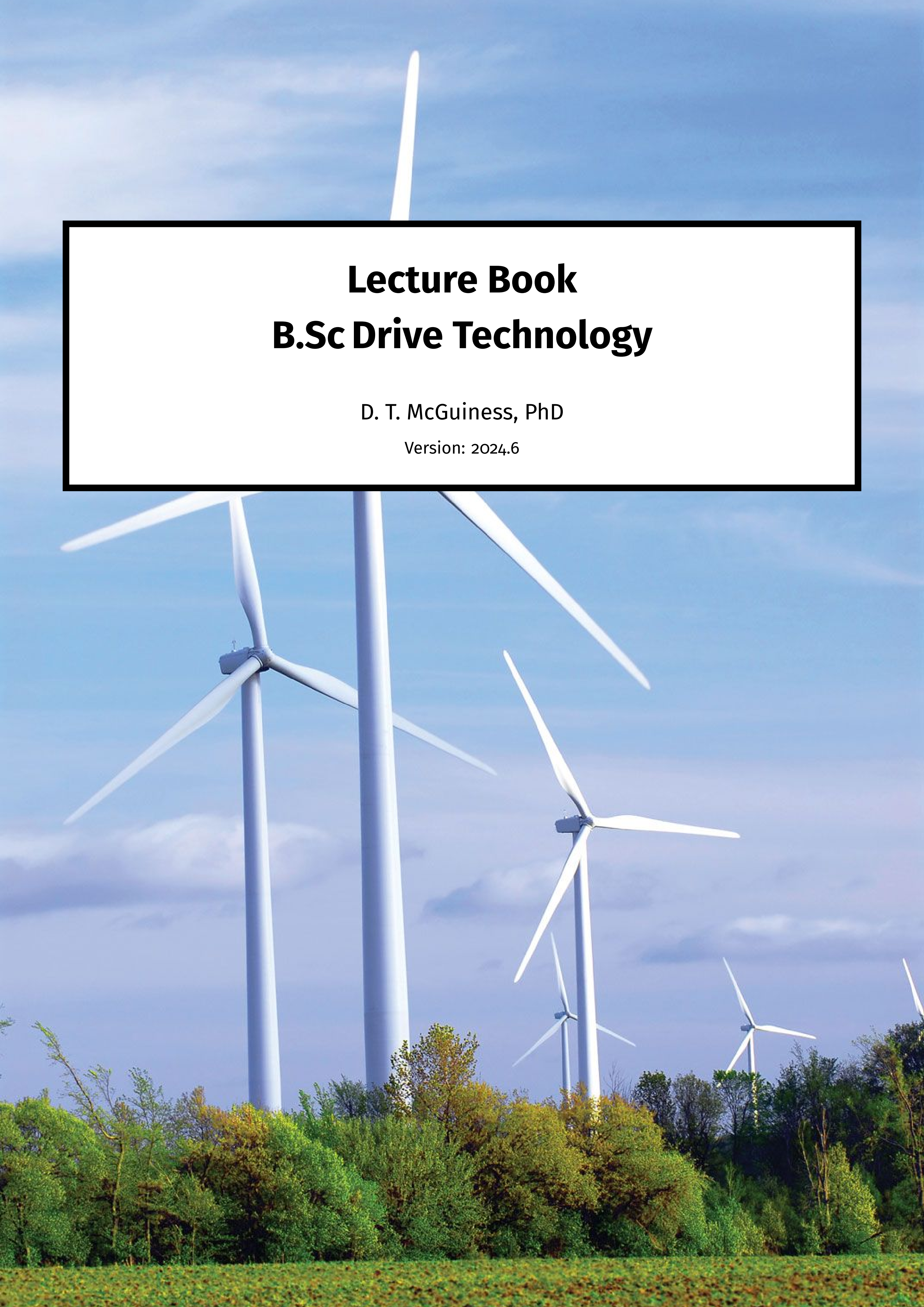

Contents

1 Preliminary Fundamentals **5**

1.1 Poly-Phase Circuits 5

2 Magnetic Circuits and Materials **7**

3 Induction Machine **9**

3.1 Introduction 9

3.1.1 Rotor Types 9

3.1.2 Speed of Operation 10

3.1.3 Normal Operation 11

3.2 Power Flow of an Induction Machine (IM) 12

3.3 Parameters Tests 17

3.3.1 No-Load Test 17

3.4 Starting Methods 20

3.4.1 Direct On-Line Starter 21

3.4.2 Stator Resistance Starting 21

3.4.3 Auto-transformer Starting 21

3.4.4 Star-Delta Starter 22

3.4.5 Rotor Resistance Starter 22

4 Single-Phase Induction Machines **24**

4.1 Introduction 24

4.2 Structure 24

4.3 Classification 24

4.3.1 Split-phase 25

4.3.2 Capacitor Types 25

4.4 Shaded-Pole 27

4.4.1 Performance 28

5 Synchronous Machines **29**

5.1 Introduction 29

5.1.1 Construction 31

5.2 Equivalent Circuit Model 31

5.3 Open and Short Circuit Analysis 32
5.4 Power Angle Characteristics of Salient-Pole Machines 33

6 Solid State Commutation Machines **37**
6.1 Brushless DC Motors 37
6.1.1 Advantages v. Disadvantages 44
6.2 Switched Reluctance Motors 45
6.2.1 Operation Principle 45
6.2.2 Modeling and Torque 46
6.2.3 Advantages v. Disadvantages 47
6.2.4 Applications 48

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This document is based on the books on the topics of Magnetic Circuits, Magnetic Materials, Induction and Synchonous Drives, DC and Solid State Drives which includes *Advanced electric drives: analysis control and modeling using MATLAB/Simulink* by N. Mohan, *Analysis of Electric Machinery and Drive Systems* by Krause, Paul C. et. al., *Design of rotating electrical machines* by Pyrhonen Juha et. al , *Electric Machinery Fundamentals* by Stephen J. Chapman., *Electric Motors and Drives: Fundamentals Types and Applications* by Hughes A. et. al., *Electrical Machines and Drives: Fundamentals and Advanced Modelling* by Melkebeek A, *Electrical machines, drives, and power systems* by Wildi T., *Fundamentals of Electrical Drives* by Veltman A. et. al., and *Electric Machinery* by A. E. Fitzgerald. et. al.

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Chapter 1

Preliminary Fundamentals

1.1 Poly-Phase Circuits

Example Poly-phase Circuit - Delta Configuration 1

Three impedances of value $Z_\Delta = 12.00 + \mathbf{j}\,9.00\,\Omega$ are connected in Δ. For balanced line-to-line voltages of 208 V, find the line current, the power factor, and the total power, reactive power, and voltamperes.

Solution Poly-phase Circuit - Delta Configuration

The voltage across any one leg of the Δ, V_Δ is equal to the line-to-line voltage V_{LL}, which is equal to $\sqrt{3}$ times the line-to-neutral voltage V_{LN}.

Consequently,

$$V_{\text{LN}} = \frac{V_{\text{LL}}}{\sqrt{3}} = \frac{208}{\sqrt{3}} \approx 208\,\mathrm{V}$$

and the current in the Δ is given by the line-to-line voltage divided by the Δ impedance

$$\hat{I}_\Delta = \frac{V_{\text{LL}}}{Z_\Delta} = \frac{208}{12 + \mathbf{j}\,9} = 13.87\angle{-36.9}$$

Power factor $= \cos\theta = \cos(-36.9) = 0.80$ lagging The phase current is equal to

$$I = \sqrt{3} I_\Delta = \sqrt{3}\,(13.87) \approx 24\,\mathrm{A}$$

Also:

$$P = 3\,P_A = 3I_A^2\,R_A = 3(13.87)^2(12.00) = 6910\ W$$
$$Q = 3\,Q_A = 3I_A^2\,X_A = 3(13.87)^2(9.00) = 5180\ VA\ reactive$$

and

$$VA = 3(VA)_A = 3V_{L-1}I_A = 3(208)(13.87) = 8640\ VA \quad \blacksquare$$

Example Analysing a Wye-Delta Circuit 2

The Y-connected source feeds a Δ-connected load through a distribution line having an impedance of $0.3 + \mathbf{j}\,0.9\,\Omega$ per phase. The load impedance is $118.5 + \mathbf{j}\,85.8\,\Omega$ per phase.

> Use the a-phase internal voltage of the generator as the reference.

a. Construct a single-phase equivalent circuit of the three-phase system.

b. Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{b0}$, and $\mathbf{I}_{cC}$.

c. Calculate the phase voltages at the load terminals.

d. Calculate the phase currents of the load.

e. Calculate the line voltages at the source terminals.

Solution Analysing a Wye-Delta Circuit

Chapter 2

Magnetic Circuits and Materials

Example A Simple Magnetic Circuit 3

A magnetic circuit is formed using an iron core with relative permeability (μ_r = 1000) and single air gap, and has the following parameters:

- Cross-sectional area ($A_C = 0.0016\ m^2$),
- Mean core length ($l_C = 0.8\ m$),
- Air gap length ($g = 0.002\ m$).

A coil of 100 turns (N = 100) is wound on the iron core.

1. Calculate the reluctance of the core $\mathcal{R}_C$ and that of the gap $\mathcal{R}_G$ neglecting the effects of fringing fields at the air gap and leakage flux from the coil. ($\mu_0 \approx 4\pi \times 10^{-7} H/m$)
2. A current of $i = 2$ A now flows in the coil. Calculate the total flux, the flux linkage and the coil inductance L.
3. Design a new coil and choose an operating current to double ($\pm$0.1) the flux linkage while keeping the coil inductance below 12 mH.

Solution A Simple Magnetic Circuit

1. Reluctance of a magnetic circuit is calculated by using the following formula;

$$\mathcal{R} = \frac{l}{\mu_0 \mu_r A},$$

where $\mathcal{R}$ is reluctance, μ_0 is the permeability of free-space, μ_r is the relative permeability and A is the cross-sectional area of the material.
Plugging the values of the question to this equation produces the following result for the air gap for the air gap ($\mathcal{R}_g$) and the core ($\mathcal{R}_c$):

$$\mathcal{R}_g = \frac{0.002}{(4\pi \times 10^{-7}) \times (1.6 \times 10^{-3})} \quad \mathrm{H}^{-1}, \tag{2.1}$$

$$\mathcal{R}_g = 9.95 \times 10^5 \quad \mathrm{H}^{-1} \text{ or } \mathrm{A/Wb}, \tag{2.2}$$

$$\mathcal{R}_c = \frac{0.8}{(4\pi \times 10^{-7}) \times (1000) \times (1.6 \times 10^{-3})} \quad \mathrm{H}^{-1}, \tag{2.3}$$

$$\mathcal{R}_c = 3.98 \times 10^5 \text{ or } \mathrm{A/Wb} \quad \blacksquare \tag{2.4}$$

2. The relation for flux (ϕ) and reluctance ($\mathcal{R}$) is shown as:

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}},$$
$$\Phi = \frac{Ni}{\mathcal{R}_\mathrm{c} + \mathcal{R}_\mathrm{g}} = \frac{100 \times 2}{1.4 \times 10^6} = 1.44 \times 10^{-4}\,\mathrm{Wb}.$$

The flux-linkage (λ) of the magnetic circuit is:

$$\lambda = N\Phi = 100 \times 1.43 \times 10^{-4} = 1.44 \times 10^{-2}\ \mathrm{Wb-turns}. \tag{2.5}$$

Using these relations, inductance (L) is calculated to be:

$$L = \frac{\lambda}{i} = \frac{1.43 \times 10^{-2}}{2} = 7.18 \times 10^{-3}\ \mathrm{H} \quad \blacksquare \tag{2.6}$$

3. In this design question we need to double the flux linkage (λ), but we need to keep the inductance (L) below 12 mH.

Therefore we need to increase N along with i. Let's choose N=128 turns and i = 2.5 A. Using these values the new flux value is:

$$\Phi = \frac{Ni}{\mathcal{R}} = \frac{128 \times 2.5}{1.4 \times 10^6} = 2.29 \times 10^{-4}\ \mathrm{Wb} \tag{2.7}$$

$$\lambda = 128 \times (2.29 \times 10^{-4}) = 0.0292\ \mathrm{Wb-turns} \tag{2.8}$$

This is a 2.05 times increase which is acceptable within the error margins.

$$L = \frac{\lambda}{i} = \frac{0.0292}{2.5} = 11.7\ \mathrm{mH} < 12\ \mathrm{mH} \quad \blacksquare \tag{2.9}$$

Chapter 3

Induction Machine

3.1 Introduction

An IM is type of electro-mechanical device which Alternating Current (AC) is supplied to the stator **directly** and to the rotor by induction from the stator. When excited from a balanced poly-phase source (i.e., star or delta), it will produce a magnetic field in the air gap rotating at **synchronous speed** as determined by the number of stator poles and the applied stator frequency f_e. A standard diagram for use in IM is shown below.

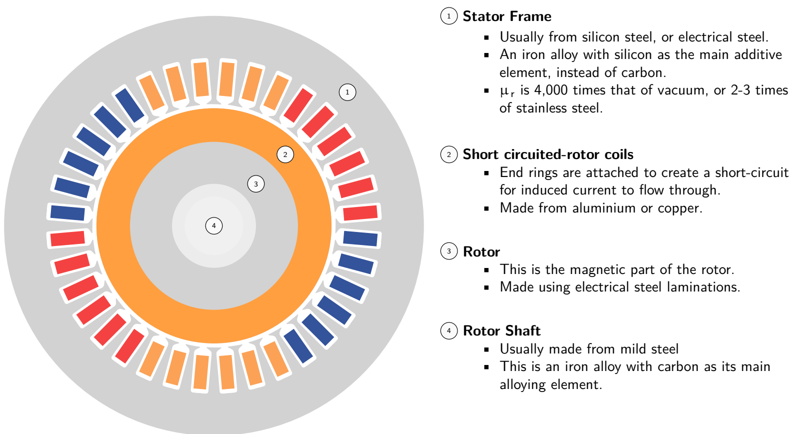


Figure 3.1: A cross-sectional view of a poly-phase IM with descriptions on which material can be used in its construction.

3.1.1 Rotor Types

The rotor of a poly-phase IM may be one of two (2) types.

Wound Rotor built with a polyphase winding similar to, and wound with the same number of poles as, the stator. The terminals of the rotor winding are connected to insulated slip rings mounted on the shaft. Carbon bruhes bearing on these rings make the rotor terminals available external to the machine. Wound-rotor induction machines are relatively uncommon, being found only in a limited number of specialized applications.

Squirrel Cage rotor windings consist of conducting bars embedded in slots in the rotor iron and short-circuited at each end by conducting end rings. The extreme simplicity and ruggedness of the squirrel-cage construction are outstanding advantages of this type of IM and make it by far the most commonly used type of machine in sizes ranging from a less than a kW to a MW.

3.1.2 Speed of Operation

Let us assume that the rotor is turning at the steady speed of n r/min in the same direction as the rotating stator field.
Let the synchronous speed of the stator field be n_{s} r/min. This difference between synchronous speed and the rotor speed is commonly referred to as the **slip** of the rotor.

> Slip is more usually expressed as a fraction of synchronous speed.

The **fractional slip** (s) is:

$$s = \frac{n_{\mathrm{s}} - n_{\mathrm{r}}}{n_{\mathrm{s}}} \tag{3.1}$$

The slip is often expressed in percent (%), simply equal to 100 percent times the fractional slip of Eq. (3.1) whereas the rotor speed in r/min can be expressed in terms of the slip and the synchronous speed as:

$$n_{\mathrm{r}} = (1 - s)\, n_{\mathrm{s}} \tag{3.2}$$

Similarly, the mechanical angular velocity ω_{r} can be expressed in terms of the synchronous angular velocity ω_{s} and the slip as:

$$\omega_{\mathrm{r}} = (1 - s)\, \omega_{\mathrm{s}} \tag{3.3}$$

The relative motion of the stator flux and the rate of frequency f_{r}

$$f_{\mathrm{r}} = s f_{\mathrm{e}} \tag{3.4}$$

called the slip frequency of the rotor.

> The electrical behavior of an IM is similar to a transformer but with the additional feature of **frequency transformation** produced by the relative motion of the stator and rotor windings.

> wound-rotor IM can be used as a frequency changer as the frequency of the rotor and the stator is **different**.

The terminals of an squirrel-cage IM rotors are internally **short circuited** whereas in a wound-rotor IM it is short circuited externally.
The rotating air-gap flux induces slip-frequency voltages in the rotor windings. The rotor currents are then determined by the magnitudes of the induced voltages and the rotor impedance at slip frequency.
During startup, the rotor is stationary ($n = 0$), the slip is unity ($s = 1$), and the rotor frequency equals the stator frequency f_e.
The field produced by the rotor currents therefore revolves at the same speed as the stator field, and a starting torque results, ending to turn the rotor **in the direction of the stator field rotation**.
If this torque is sufficient to overcome the opposition to rotation created by the shaft load, the machine will come up to its operating speed.

> The operating speed can never equal the synchronous speed as the rotor conductors would then be stationary with respect to the stator field; no current would be induced in them, and hence no torque would be produced.

With rotor revolving in the same direction of the stator field, the frequency of the rotor currents is sf_e and they will produce a rotating flux wave which will rotate at sn_n r/min with respect to the rotor in the forward direction.
But superimposed on this rotation is the mechanical rotation of the rotor at n r/min.
Thus, with respect to the stator, the speed of the flux wave produced by the rotor currents is the sum of these two speeds and equals

$$sn_s + n = sn_s + n_s(1 - s) = n_s \tag{3.5}$$

From Eq. (3.5) we see the rotor currents produce an air-gap flux wave which rotates at synchronous speed and hence in synchronism with that produced by the stator currents. Because the stator and rotor fields each rotate synchronously, they are stationary with respect to each other and produce a steady torque, thus maintaining rotation of the rotor. Such torque, which exists for any mechanical rotor speed n other than synchronous speed, is called an **asynchronous torque**.

3.1.3 Normal Operation

Under normal running conditions the slip (s is small: 2 to 10 percent at full load in most squirrel-case IMs. The rotor frequency ($f_r = s/\epsilon_r$) is very low (of the order of 1 to 5 Hz in

50-Hz machines). In this range the rotor impedance is **largely resistive and independent of slip.**
Approximate proportionality of torque with slip is therefore to be expected in the range where the slip is small. As slip increases, the rotor impedance increases because of the increasing contribution of the rotor leakage inductance. Therefore, the rotor current is less than proportional to slip.
The result is that the torque increases with increasing slip up to a maximum value and then decreases. The maximum torque, or **breakdown torque**, which is typically a factor of two larger than the rated machine torque, limits the short-time overload capability of the machine.

> The slip at which the peak torque occurs is proportional to the rotor resistance

For squirrel-cage machines this peak-torque slip is relatively small. Thus, the squirrel-cage machine is substantially a constant-speed machine having a few percent drop in speed from no load to full load. In the case of a wound-rotor machine, the rotor resistance can be increased by inserting external resistance, hence increasing the slip at peak-torque, and thus decreasing the machine speed for a specified value of torque.
Wound-rotor IM are generally made to be larger, and therefore are more expensive and require significantly more maintenance than squirrel-cage IMs, this method of speed control is rarely used, and IMs driven from constant-frequency sources tend to be limited to essentially constant-speed applications.

3.2 Power Flow of an IM

The single-phase equivalent circuit can be used to determine a wide variety of steady-state performance characteristics of poly-phase IM which include variations of current, speed, and losses as the load-torque requirements change, as well as the starting torque, and the maximum torque.
The equivalent circuit shows that the total power P_{gap} transferred across the air gap from the stator is:

$$P_{\text{gap}} = n_{\text{ph}} \left| I_r' \right|^2 \left(\frac{R_r'}{s} \right) \tag{3.6}$$

where n_{ph} is the *number of stator phases*. So for a standard 3-phase IM it would be $n_{\text{ph}} = 3$. The total rotor I^2R loss, P_{rotor}, can be calculated from the I^2R loss in the equivalent rotor as:

$$P_{\text{rotor}} = n_{\text{ph}} \left| I_r' \right|^2 R_r' \tag{3.7}$$

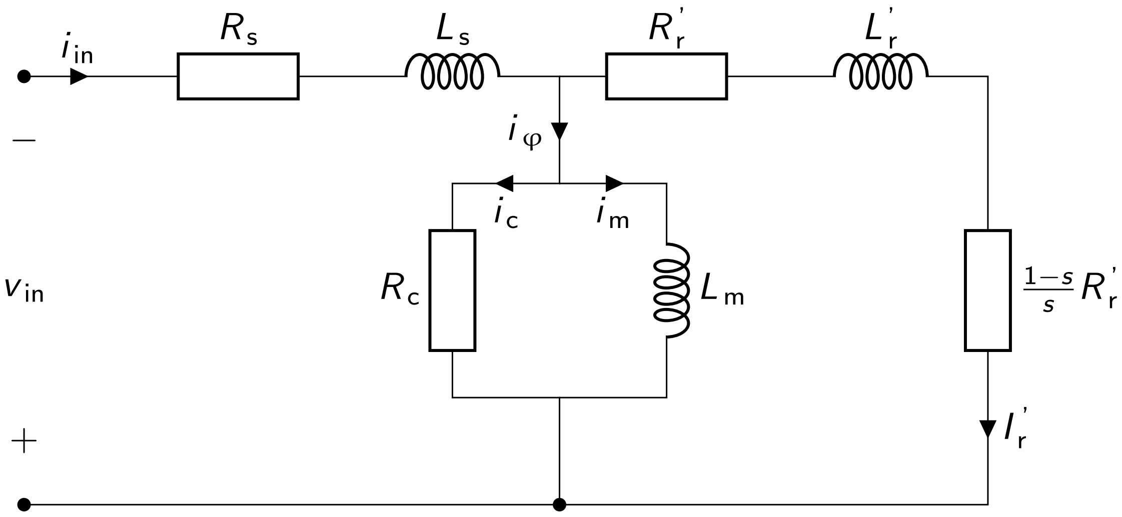


Figure 3.2: The equivalent circuit of an IM.

The electromagnetic power P_{mech} developed by the machine can now be determined by subtracting the rotor power dissipation from the air-gap power:

$$P_{mech} = P_{gap} - P_{rotor} = n_{ph} \left| I'_r \right|^2 \left(\frac{R'_r}{s} \right) - n_{ph} \left| I'_r \right|^2 R'_r \tag{3.8}$$

or equivalently

$$P_{mech} = n_{ph} \left| I'_r \right|^2 R'_r \left(\frac{1-s}{s} \right) \tag{3.9}$$

Comparing Eq. (3.6) with Eq. (3.9) gives:

$$P_{mech} = (1-s)\, P_{gap} \tag{3.10}$$

and

$$P_{rotor} = s P_{gap}$$

We see then that, of the total power delivered across the air gap to the rotor, the fraction $1-s$ is converted to mechanical power and the fraction s is dissipated as $I^2 R$ loss in the rotor conductors.

From this it is evident that an IM operating at high slip is an inefficient device. The electromechanical power per stator phase is equal to the power delivered to the resistance $R'_r (1-s)/s$.

Example A Simple IM Analysis 4

A three-phase ($n_{\text{ph}} = 3$), two-pole ($p = 2$), 60-Hz IM is observed to be operating at a speed of $n_{\text{r}} = 3502\,\mathrm{rpm}$ with an input power of 15.7 kW and a terminal current of 22.6 A.

The stator-winding (R_{s}) resistance is 0.20 Ω/phase.

Calculate the I^2R power dissipated in rotor.

Solution A Simple IM Analysis

The power dissipated in the stator winding is given by:

$$\begin{aligned} P_{\text{stator}} &= n_{\text{ph}} I_{\text{in}}^2 R_{\text{s}} \\ &= 3 \times (22.6)^2 \times 0.2 = 306\,\mathrm{W}. \end{aligned}$$

Hence the air-gap power (P_{gap}) is:

$$\begin{aligned} P_{\text{gap}} &= P_{\text{in}} - P_{\text{stator}} \\ &= 15.7 - 0.3 = 15.4\,\mathrm{W} \end{aligned}$$

The synchronous speed (n_{s}) of this machine can be found as:

$$\begin{aligned} n_{\text{s}} &= \left(\frac{120}{\text{poles}}\right) f_{\text{e}} \\ &= \left(\frac{120}{2}\right) \times 60 = 3600\,\mathrm{rpm} \end{aligned}$$

The slip is $s = (3600 - 3502)\,/3600 = 0.0272$.

Therefore the real losses experienced by the rotor (P_{rotor}) is:

$$P_{\text{rotor}} = sP_{\text{mech}} = 0.0272 \times 15.4 = 419\,\mathrm{W} \quad \blacksquare$$

The electromechanical (T_{mech}) corresponding to the power (P_{mech}) can be obtained by recalling that mechanical power equals torque times angular velocity.

Thus,

$$P_{\text{mech}} = \omega_{\text{r}} T_{\text{mech}} = (1 - s)\,\omega_{\text{s}} T_{\text{mech}}$$

> For P_{mech} in watts and ω_{s} in rad/sec, T_{mech} will be in newton-meters.

Use of Eq. (3.9) and Eq. (3.10) leads to:

$$T_{\text{mech}} = \frac{P_{\text{mech}}}{\omega_{\text{r}}} = \frac{P_{\text{gap}}}{\omega_{\text{s}}} = \frac{n_{\text{ph}} \left|I_{\text{r}}'\right|^2 \left(R_{\text{r}}'/s\right)}{\omega_{\text{s}}}$$

with the synchronous mechanical angular velocity ω_s being given by

$$\omega_s = \frac{4\pi f_e}{\text{poles}} = \left(\frac{2}{\text{poles}}\right)\omega_e$$

The mechanical torque T_{mech} and power P_{mech} are not the output values available at the shaft because friction, windage, and stray-load losses remain to be accounted for.

It is obviously correct to subtract friction, windage, and other rotational losses from T_{mech} or P_{mech} and it is generally assumed that stray load effects can be subtracted in the same manner.

The remainder is available as output power from the shaft for useful work. Thus

$$P_{\text{shaft}} = P_{\text{mech}} - P_{\text{rot}}$$
$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_r} = T_{\text{mech}} - T_{\text{rot}}$$

where P_{rot} and T_{rot} are the power and torque associated with the friction, windage, and remaining rotational losses.

Example Poly-phase IM Equivalent Circuit Analysis 5

A three-phase ($n_{\text{ph}} = 3$) Y-connected 110-V (line-to-line) 7.5-kW 50-Hz six-pole ($p = 6$) IM has the following parameter values in Ω/phase *referred to the stator*.

$$R_s = 0.294, \quad R_r' = 0.144, \quad X_s = 0.503, \quad X_r' = 0.209, \quad X_m = 13.25.$$

The total friction, windage, and core losses (P_{rot}) may be **assumed to be constant** at 403 W, independent of load.

For a slip value of 0.02, compute the speed, output torque and power, stator current, power factor, and efficiency when the machine is operated at rated voltage and frequency.

Solution Poly-phase IM Equivalent Circuit Analysis

Let the impedance Z_f represent the per-phase impedance presented to the stator by the magnetising reactance and the rotor. Therefore we can bulk collect the reactance on the rotor size as:

$$Z_f = R_f + \mathbf{j}X_f = \left(\frac{R_r'}{s} + \mathbf{j}X_r'\right) \quad \text{in parallel with} \quad \mathbf{j}X_m$$
$$= \frac{\left(\frac{R_r'}{s} + \mathbf{j}X_r'\right) \times (\mathbf{j}X_m)}{\left(\frac{R_r'}{s} + \mathbf{j}X_r'\right) + (\mathbf{j}X_m)}$$

Substitution of numerical values gives, for $s = 0.02$,

$$Z_\mathrm{f} = R_\mathrm{f} + \mathbf{j} X_\mathrm{f} = (5.43 + 3.11j)\,\Omega$$

The stator input impedance can now be calculated as:

$$Z_\mathrm{in} = R_\mathrm{s} + \mathbf{j} X_\mathrm{s} + Z_\mathrm{f} = (5.72 + 3.61j)\,\Omega$$

The line-to-neutral terminal voltage is equal to:

$$V_\mathrm{in} = \frac{110}{\sqrt{3}} = 63.51\,\mathrm{V}$$

and hence the stator current can be calculated as:

$$\hat{I}_\mathrm{in} = \frac{V_\mathrm{in}}{Z_\mathrm{in}} = \frac{63.51}{(5.72 + 3.61j)} = (7.94 - 5.01j)\,\mathrm{A}$$

The stator current (I_in) is thus 9.39 A and the power factor is equal to $\cos(-32.27) = 0.85$ **lagging**.

The synchronous speed (n_s) can be found as:

$$n_\mathrm{s} = \left(\frac{120}{\text{poles}}\right) f_\mathrm{e} = \left(\frac{120}{6}\right) \times 50 = 1000.0\,\mathrm{rpm}$$

or the angular stator speed (ω_s) can be calculated to be:

$$\omega_\mathrm{s} = \frac{4\pi f_\mathrm{e}}{\text{poles}} = 104.72\,\mathrm{rad \cdot s^{-1}}$$

The rotor speed (n_r) is:

$$n_\mathrm{r} = (1 - s)\, n_\mathrm{s} = (0.98) \times 1000.0 = 980.0\,\mathrm{rpm}$$

or in the angular speed form:

$$\omega_\mathrm{r} = (1 - s)\, \omega_\mathrm{s} = (0.98)) \times 104.72 = 102.63\,\mathrm{rad \cdot s^{-1}}$$

The air-gap power (P_gap) is calculated to be:

$$P_\mathrm{gap} = n_\mathrm{ph} I_2^2 \left(\frac{R_\mathrm{r}'}{s}\right)\,\mathrm{W}$$

> The only resistance included in Z_f is R_r'/s.

The power dissipated in Z_f is equal to the power dissipated in R_r'/s and hence we can write:

$$P_\mathrm{gap} = n_\mathrm{ph} I_1^2 R_\mathrm{f} = 3\,(9.39)^2\,(5) = 1434.85\,\mathrm{W}$$

We can now calculate P_mech and the shaft output (P_shaft) power as:

$$\begin{aligned} P_\mathrm{shaft} &= P_\mathrm{max} - P_\mathrm{rot} = (1 - s)\, P_\mathrm{gap} - P_\mathrm{rot}, \\ &= (0.98) \times 1434.85 - 403 = 1406.15\,\mathrm{W} \end{aligned}$$

and the shaft output torque (T_{shaft}) can be found as:

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_{\text{r}}} = \frac{1003.15}{102.63} = 9.77\,\mathrm{N}\cdot\mathrm{m}^{-1}$$

The efficiency (η) is calculated as the ratio of shaft output power to stator input power.

The input power is given by

$$\begin{aligned} P_{\text{in}} &= n_{\text{ph}}\text{Re}\left[V_{\text{in}} I_{\text{in}}\right] \\ &= 1512.6\,\mathrm{W} \end{aligned}$$

Thus the efficiency (η) is equal to:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{1003.15}{1512.6} = 0.66 \approx 66.0\% \quad \blacksquare$$

The complete performance characteristics of the machine can be determined by repeating these calculations for *other assumed values of slip*.

3.3 Parameters Tests

The equivalent-circuit parameters needed for determining the performance of a poly-phase IM under load can be obtained from the results of:

1. no-load test,
2. blocked-rotor test,
3. Direct Current (DC) injection test.

Stray-load losses, which must be taken into account when accurate values of efficiency are to be calculated, can also be measured by tests which do not require loading the machine.

> In this document the calculation of the stray-load losses will **NOT** be discussed.

3.3.1 No-Load Test

Similar to the open-circuit test of a transformer, the no-load test on an IM gives information with respect to exciting current and no-load losses. This test is ordinarily performed at **rated frequency** and with balanced poly-phase voltages applied to the stator terminals.

Readings are taken at rated voltage, after the machine has reached steady-state.

We will assume that the no-load test is made with the machine operating at its rated electrical frequency f_r and that the following measurements are available from the no-load test:

Term	Definition
$V_{in,LN}$	The line-to-neutral voltage [V]
$I_{in,LN}$	The line current [V]
P_{in}	The total polyphase electrical input power [W]

Table 3.1: Measurable parameters in an Open-Circuit test.

In poly-phase IM line-to-line (V_{L-L}) voltage is generally measured., To calculate phase-to-neutral (V_{ph}) voltage, divide by $\sqrt{3}$ for a 3-phase system.

At no load, the rotor current is only the very small value needed to produce sufficient torque to overcome the friction and windage losses associated with rotation. The no-load rotor I^2R loss is, therefore, negligibly small. Unlike the continuous magnetic core in a transformer, the magnetising path in an IM includes an air gap which significantly increases the required exciting current. Thus, in contrast to the case of a transformer, whose no-load primary I^2R loss is **negligible**, the no-load stator I^2R loss of an IM may be appreciable because of this larger exciting current.
Neglecting rotor I^2R losses, the rotational loss (P_{rot}) for normal running conditions can be found by subtracting the stator I^2R losses from the no-load input power:

$$P_{rot} = P_{in} - n_{ph} I_{1,LN}^2 R_s$$

The total rotational loss at rated voltage and frequency under load usually is considered to be constant and equal to its no-load value.

Note that the stator resistance (R_s) varies with stator-winding temperature.

Example **Wound Rotor Torque-Resistance Calculation** 6

A three-phase, 230-V, 60-Hz, 12-kW, four-pole wound-rotor induction motor has the following parameters expressed in Ω/phase.

$$R_s = 0.095 \quad X_s = 0.680 \quad X_r' = 0.672 \quad X_m = 18.7$$

Using Python, plot the electromechanical mechanical torque T_{mech} as a function of rotor speed in r/min for rotor resistances of $R_r' = 0.1, 0.2, 0.5, 1.0$ and $1.5\ \Omega$.

Solution Wound Rotor Torque-Resistance Calculation

The code along with the produced plot is as follows:

```python
import matplotlib.pyplot as plt
import numpy as np

# Given Parameters
Vin =  230     # (V)     Input Voltage (3-phase)
ph  =  3       # (-)     Number of phases
p   =  4       # (-)     Number of poles
f   =  60      # (Hz)    Grid frequency
Rs  =  0.095   # (Ohm)   Stator Resistance
Xs  =  0.680   # (Ohm)   Stator Reactance
Xr  =  0.672   # (Ohm)   Rotor Reactance
Xm  =  18.70   # (Ohm)   Magnetising Reactance
Rrt = [0.1, 0.2, 0.5, 1.0, 1.5] # (Ohm) Rotor Resistance


# Calculate the per-phase voltage
Vin_p = Vin / np.sqrt(3)

# Calculating the snychronous speed
ws = 4 * np.pi * f / p
ns = 120 * f / p

# Calculating the stator Thevenin Equivalent
Z1_eq = 1j * Xm * (Rs + 1j * Xs) / (Rs + 1j * (Xs + Xm))
R1_eq = Z1_eq.real; X1_eq = Z1_eq.imag
V1_eq = np.abs(Vin_p * (1j * Xm) / (Rs + 1j * (Xs + Xm)))

slip = np.linspace(0.001,1, 200) # set an empty array for
Tmech = np.zeros([200, 5])

j = 0
for Rr in Rrt:
    i = 0
    for s in slip:
        rpm = ns * (1 - s)
        Ir = np.abs(V1_eq / (Z1_eq + 1j * Xr + Rr / s))
        Tmech[i, j] = ph * Ir ** 2 * Rr / (s * ws)

        i = i + 1
```

```python

    j = j + 1

Result_Array = np.c_[np.transpose(slip), Tmech]
plt.style.use("dawn")

plt.rcParams['axes.facecolor'] = 'white'
plt.rcParams['savefig.facecolor'] = 'white'

plt.figure(figsize=(18, 6), dpi=80, facecolor='white')
plt.plot(Result_Array[:,1])
plt.plot(Result_Array[:,2])
plt.plot(Result_Array[:,3])
plt.plot(Result_Array[:,4])
plt.plot(Result_Array[:,5])
plt.savefig("images/Induction-Motors/wound-rotor-example-result.pdf", bbox_inches='tight')
plt.show()
plt.close()
```

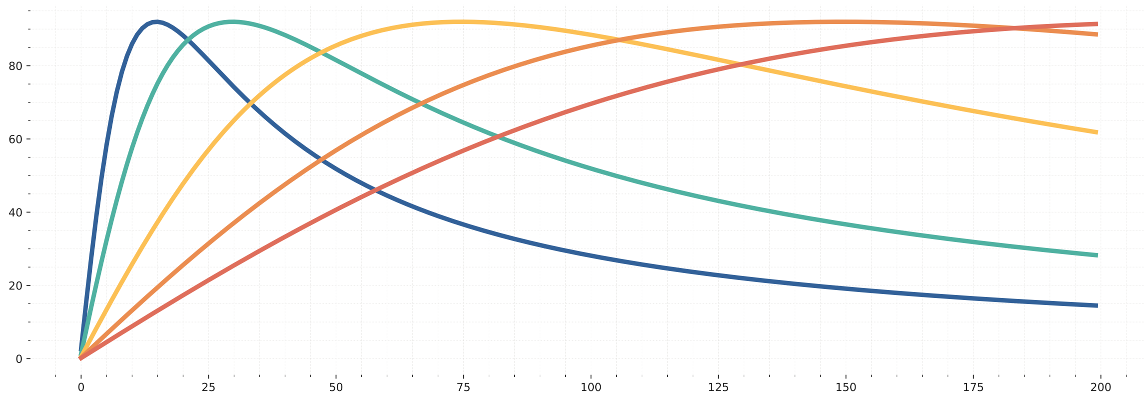


Figure 3.3: Different types of resistance connected to the rotor will produce different torque v. speed plots.

3.4 Starting Methods

IM up to roughly 5 kW can be considered self-starting whereby it is easy as plugging them to a power outlet without the need of an external device. However, as with every device with high levels of inductive properties the control of current during startup needs to be controlled to avoid the problem of **inrush current**.

Inrush Current

the maximal instantaneous input current drawn by an electrical device when first turned on. AC machines and transformers may draw several times their normal full-load current when first energised, for a few cycles of the input waveform.

3.4.1 Direct On-Line Starter

As the name suggests, the IM is started by connecting it directly to three-phase supply. In this method, the machine still draws a high starting current (about 4 to 7 times of the rated current) and therefore operates at a low power factor during startup.

> DOL starting is suitable for relatively small machines (up to 10 kW)

3.4.2 Stator Resistance Starting

External resistance is connected in series with each phase of the stator winding during starting. The external resistance causes voltage to drop across it so that reduced voltage available across the machine terminals.

> This addition of resistances to the stator causes the starting current to reduce.

The starting external resistances are **gradually** cut out in steps from the stator circuit, as the machine accelerates.
When the machine attains the rated speed, the starting resistances are completely cut out and full line voltage is applied across the machine terminals.
This method has two (2) major drawbacks:

1. Reduced voltage during starting reduces the starting torque, increasing acceleration time
2. Energy is lost in the starting resistances as heat.

3.4.3 Auto-transformer Starting

An autotransformer is used to reduce the starting voltage of the IM. The tapings of the autotransformer is so set that when it is in the circuit, 60 to 80 % of the line voltage is applied to machine during starting and then connecting it to the full-line voltage as the machine attains a sufficient speed.

Autotransformer

A transformer with only one winding. The auto prefix refers to the single coil acting alone. Portions of the same winding act as both the primary winding and secondary winding sides of the transformer. This allows one to change the output of the transformer by adjusting the taps on the coil.

During startup, the change-over switch is connected to **Start**. This supplies the reduced voltage to the machine through the autotransformer. Consequently, the starting current is limited to safe value. When the machine attains about 80% of rated-speed, the change-over switch is now thrown to **Run**. This removes the autotransformer from the circuit and full line voltage is applied across the machine terminals.
The autotransformer starting has many advantages such as low power loss, low starting current etc. This method is used for large machines over 20 kW.

3.4.4 Star-Delta Starter

In star-delta ($Y - \Delta$) starting method of squirrel cage IM, the machine starts in star and runs in delta, i.e. the stator winding of the machine is designed for delta operation and is connected in **star during starting**.
When the machine attains sufficient speed, the connections are changed to **delta**.
The six leads of stator winding of the machine are connected to a change-over switch. At the time of starting, the change-over switch is switched to Star which connects the stator windings in star. Thus, each phase gets $V/\sqrt{3}$ volts, where V is the three phase line voltage. This reduces the starting current.
When the machine attains 80% of rated speed, the changeover switch is switched to Delta which connects the stator windings in delta. Now, each phase gets full line voltage.

> A major disadvantage of this method is large reduction in stating torque due to reduced voltage in the star connection at the instant of starting

The star-delta starting is used for medium size machines up to 20 kW.

3.4.5 Rotor Resistance Starter

> This method only applies to wound rotor (i.e., slip-ring)

A star connected variable resistance is connected in the rotor circuit through slip-rings. The full voltage is applied to the stator windings.

During startup, the handle of variable resistance (rheostat) is set to OFF position. This inserts maximum resistance in series with the each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased due to external rotor resistance.
As the machine accelerates, the external resistance is gradually removed from the rotor circuit. When the machine attains rated speed, the handle is switched in the ON position, this removes the whole external resistance from the rotor circuit.

Chapter 4

Single-Phase Induction Machines

4.1 Introduction

While industry relies on the availability of a 3-phase system, the residential and commercial sections only have access to single-phases. Therefore machines capable of running from a single phase needs to be worked on. However, most single-phase induction motors are actually two-phase motors with unsymmetrical windings; the two windings are typically quite different, with different numbers of turns and/or winding distributions.

4.2 Structure

The most common types of single-phase IM resemble polyphase squirrel-cage motors except for the arrangement of the stator windings.
Instead of being a concentrated coil, the actual stator winding is distributed in slots to produce an approximately sinusoidal space distribution of mmf.

> A single-phase winding produces equal forward- and backward-rotating mmf waves.

By symmetry, it is clear that such a motor inherently will produce no starting torque since at standstill, it will produce equal torque in both directions.
If, however, it is started by auxiliary means, the result will be a net torque in the direction in which it is started, and hence the motor will continue to run.

4.3 Classification

Single-phase IMs are classified in accordance with their starting methods and are usually referred to by names descriptive of these methods. Selection of the appropriate motor is based on:

- the starting torque,

- running torque requirements of the load,
- the duty cycle of the load
- limitations on starting and running current from the supply line for the motor

The cost of single-phase motors increases with their rating and with their performance characteristics such as starting-torque-to-current ratio. Typically, in order to minimize cost, an application engineer will select the motor with the lowest rating and performance that can meet the specifications of the application.
Where a large number of motors are to be used for a specific purpose, a special motor may be designed in order to ensure the least cost. In the fractional-kilowatt motor business, small differences in cost are important.

4.3.1 Split-phase

Split-phase motors have two stator windings, a **main winding** (i.e., run winding) which we will refer to with the subscript 'main' and an **auxiliary winding** (i.e., start winding) which we will refer to with the subscript 'aux'. As in a two-phase motor, the axes of these windings are displaced 90 electrical degrees in space. The auxiliary winding has a higher resistance-to-reactance ratio than the main winding, with the result that the two currents will be out of phase, which is representative of conditions at starting.
Since the auxiliary-winding current I_{aux} leads the main-winding current $\hat{I}_{\text{main}}$, the stator field first reaches a maximum along the axis of the auxiliary winding and then somewhat later in time reaches a maximum along the axis of the main winding.
The winding currents are equivalent to unbalanced two-phase currents, and the motor is equivalent to an unbalanced two-phase motor. The result is a rotating stator field which causes the motor to start. After the motor starts, the auxiliary winding is disconnected, usually by means of a centrifugal switch that operates at about 75 percent of synchronous speed. The simple way to obtain the high resistance-to-reactance ratio for the auxiliary winding is to wind it with smaller wire than the main winding, a permissible procedure because this winding operates only during starting. Its reactance can be reduced somewhat by placing it in the tops of the slots.
Split-phase motors have moderate starting torque with low starting current. Typical applications include fans, blowers, centrifugal pumps, and office equipment. Typical ratings are 50 to 500 watts; in this range they are the lowest-cost motors available.

4.3.2 Capacitor Types

Capacitors can be used to improve motor starting performance, running performance, or both, depending on the size and connection of the capacitor.

Capacitor Start

It can also be classified as a split-phase motor, but the time-phase displacement between the two currents is obtained by means of a capacitor in series with the auxiliary winding. Again the auxiliary winding is disconnected after the motor has started, and consequently the auxiliary winding and capacitor can be designed at minimum cost for intermittent service.
These connections allow high starting torque which are useful for compressions, pumps, refrigeration and air-conditioning equipment, and other hard-to-start loads.

Permanent-Split

The capacitor and auxiliary winding are not cut out after starting. The construction can be simplified by omission of the switch, and the power factor, efficiency, and torque pulsations improved.
For example, the capacitor and auxiliary winding could be designed for perfect two-phase operation. The losses due to the backward field at this operating point would then be eliminated, with resulting **improvement in efficiency**. The double-star-frequency torque pulsations would also be eliminated, with the capacitor setting as an energy storage reservoir for smoothing out the pulsations in power input from the single-phase line, resulting in quiet operation.
Starting torque must be searched because the choice of capacitance is necessarily a compromise between the best starting and running value.

Capacitor Run

If two capacitors are used, one for starting and one for running, theoretically optimum starting and running performance can both be obtained. One way of accomplishing is to have the small value of capacitance required for optimum running conditions permanently connected in series with the auxiliary winding, and the much larger value required for starting is obtained by a capacitor connected in parallel with the running capacitor via a switch with opens as the motor comes up to speed.
The capacitor for a capacitor-start motor has a typical value of 300 μF for a 500-W motor. Since it must carry current for just the starting time, the capacitor is a special compact AC electolytic type made for motor-starting duty. The capacitor for the same motor permanently connected has a typical rating of 40 μF, and since it operates continuously, the capacitor is an ac paper, foil, and oil type.
The cost of the various motor types is related to performance:

- the capacitor-start motor has the lowest cost,

- the permanent-split-capacitor motor next
- capacitor-run motor

Example **Capacitor Start Capacitor Calculation** 7

A 2.5-kW 120-V 60-Hz capacitor-start motor has the following impedances for the main (Z_{main}) and auxiliary (Z_{aux}) windings (at starting):

$$Z_{\text{main}} = 4.5 + \mathbf{j}\,3.7\,\Omega,$$
$$Z_{\text{aux}} = 9.5 + \mathbf{j}\,3.5\,\Omega.$$

Find the value of starting capacitance that will place the main and auxiliary winding currents in quadrature at starting.

Solution **Capacitor Start Capacitor Calculation**

The currents $\hat{I}_{\text{main}}$ and $\hat{I}_{\text{aux}}$ are the currents which flow into Z_{main} and Z_{aux} respectively. The impedance angle of the main winding (ϕ_{main}) is:

$$\phi_{\text{main}} = \tan^{-1}\left(\frac{3.7}{4.5}\right) = 39.6^{\circ}$$

To produce currents in time quadrature with the main winding (i.e., to create the necessary phase difference), the impedance angle of the auxiliary winding circuit (including the starting capacitor) must be:

$$\phi = 39.6^{\circ} - 90.0^{\circ} = -50.4^{\circ}$$

The combined impedance of the auxiliary winding and starting capacitor is equal to

$$Z_{\text{total}} = Z_{\text{aux}} + \mathbf{j}\,X_{\text{c}} = 9.5 + \mathbf{j}\,(3.5 + X_{\text{c}})\;\Omega$$

where $X_{\text{c}} = -\frac{1}{\omega_{\text{c}}}$ is the reactance of the capacitor and $\omega = 2\pi\,60 \approx 377$ rad/sec. Thus

$$\tan^{-1}\left(\frac{3.5 + X_{\text{c}}}{9.5}\right) = -50.4^{\circ}$$
$$\frac{3.5 + X_{\text{c}}}{9.5} = \tan\left(-50.4^{\circ}\right) = -1.21.$$

and hence

$$X_{\text{c}} = -1.21 \times 9.5 - 3.5 = -15.0\,\Omega$$

The capacitance C is then

$$C = \frac{-1}{\omega X_{\text{c}}} = \frac{-1}{377 \times (-15.0)} = 177\,\mu\text{F} \quad \blacksquare$$

4.4 Shaded-Pole

Shaded-pole IM has **salient poles** with one portion of each pole surrounded by a short-circuited turn of copper called a shading coil. Induced currents in the shading coil cause the flux in the shaded portion of the pole to lag the flux in the other portion. The result is similar to a rotating field moving in the direction from the unshaded to the shaded portion of the pole. This motion of field causes currents to induce in the squirrel-case rotor and a low starting torque is produced.

4.4.1 Performance

The starting torque of this motor is very small about 50% of full load torque. Efficiency is low because of continuous power loss in shading coil. These motors are used for small fans, electric clocks, gramophones which are in around 50 W range.

> They are known to have low efficiency and cost of production.

Its direction of rotation depends upon the position of the shading coil, i.e., which portion of the pole is wrapped with shading coil. The direction of rotation is from un-shaded portion of the pole to the shaded portion.

> Its direction of rotation cannot be reversed unless the position of the poles is reversed.

Chapter 5

Synchronous Machines

5.1 Introduction

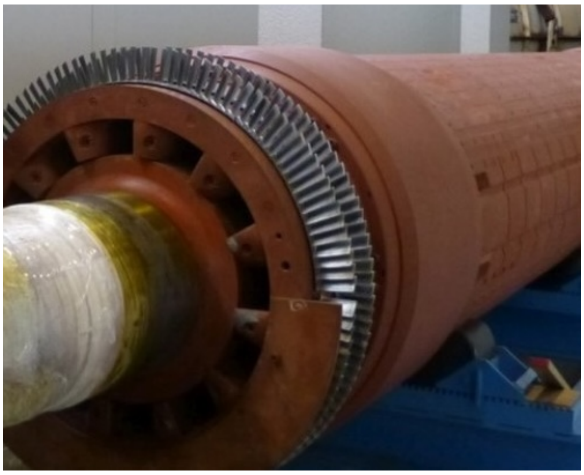
A Synchronous Machine (SynM) is an AC machine whose speed under steady-state conditions is **proportional to the frequency of the current in its armature**.

Armature

In the context of AC machinery, any part of the motor which houses AC flow is considered to be an armature. In this case, the stator part of the SynM is the armature.

The rotor, along with the magnetic field created by the DC field current on the rotor, rotates at the same speed as the Rotating Magnetic Field (RMF) produced by the armature currents, and a produces a steady torque output as a result.
The armature winding is almost always on the stator and is usually a three-phase winding with the **field winding is on the rotor**.
The cylindrical-rotor construction is used for two- and four-pole turbine generators. The salient-pole construction is best adapted to multi-polar, slow-speed, hydropower generators and to most systems. The power required for excitation–approximately one to a few notes of the rating of the

(a) Lorem ipsum



(b) Lorem ipsum, lorem ipsum,Lorem ipsum, lorem ipsum,Lorem ipsum

Figure 5.1: Caption place holder

synchronous machine–is supplied by the excitation system.
In older machines, the excitation current was typically supplied through slip rings from a DC machine, referred to as the excite, which was often mounted on the same shaft as the synchronous machine. In more modern systems, the excitation is supplied from ac ceiters and solid-state rectifiers (either simple diode bridges or phase-controlled rectifiers).
In some cases, the rectification occurs in the stationary frame, and the rectified excitation current is fed to the rotor via slip rings. In other systems, referred to as brushless excitation systems, the alternator of the ac exciter is on the rotor, as is the rectification system, and the current is supplied directly to the field-winding without the need for slip rings.
A synchronous generator supplying power to an impedance load acts as a voltage source whose frequency is determined by the speed of its mechanical drive (or prime power),and the amplitude of the generated voltage is proportional to the frequency and the field current. The current and power factor are then determined by the generator field excitation and the impedance of the generator and load.

Figure 5.2: Synchronous Machines are mostly used as generators in industry, such as hydro-dams.

Synchromous generators can be readily operated in parallel, and, in fact, the electricity supply systems of industrialized countries typically have scores or even hundreds of them operating in parallel, interconnected by thousands of miles of transmission lines, and supplying electric energy to loads scattered over areas of many thousands of square miles. These huge systems have grown in spite of the necessity for designing the system so that synchronism is maintained following disturbances and the problems, both technical and administrative, which must be solved to coordinate the operation of such a complex system of machines and personnel. The principal reasons for these interconnected systems are reliability of service and economies in plant investment and operating costs.

5.1.1 Construction

From construction point of view, there are two (2) types of rotors named as:

1. Salient Pole,
2. Non-salient (i.e., turbo, round) Pole.

Salient Pole

Projected poles are provided on the rotor. The cost of construction of salient pole type rotors is low, moreover sufficient space is available to accommodate field winding but these cannot bear high mechanical stresses at high speeds.

> Salient poles are suited for medium and low speeds and are usually employed at hydro-electric and diesel power plants as synchronous generators.

As the speed of these machines (generators) is quite low, to obtain the required frequency, the machines have large number of poles. To accommodate such a large number of poles, these machines have larger diameter and small length.

> For a speed of 200 rpm (alternators coupled with water turbines) the diameter of the machines is as large as 14 metre and length is only 1 metre.

Non-Salient (i.e., Round) Rotor

No projected poles but the poles are formed by the current flowing through the rotor (exciting) winding.

> Non-salient pole type construction is suited for the high speeds.

The steam turbines rotate at a high speed (3000 rpm). When these turbines are used as prime-mover for this machine working as a generator, a small number of poles are required for given frequency.

> These machines have smaller diameter and larger length.

5.2 Equivalent Circuit Model

In Section 5.1, synchronous-machine torque-angle characteristics are described in terms of the interacting air-gap flux and mmf waves. Our purpose now is to derive an equivalent circuit which represents the steady-state terminal volt-ampere characteristics.
A cross-sectional sketch of a three-phase cylindrical-rotor synchronous machine is shown schematically in Fig. 5.2. The figure shows a two-pole machine; alternatively, this can be considered as two poles of a multipole machine. The three-phase armature winding on the stator is of the same

type used in the discussion of rotating magnetic fields in Section 4.5. Coils aa', bb', and cc' represent distributed windings producing sinusoidal mmf and flux-density waves in the air gap. The reference directions for

Example Equivalent Circuit Analysis - A 8

A 60-Hz, three-phase SynM is observed to have a terminal voltage of 460 V (line-line) and a terminal current of 120 A at a power factor of 0.95 lagging.

The field-current under this operating condition is 47 A. The machine synchronous reactance is equal to 1.68 Ω (0.794 per unit on a 460-V, 100-kVA, 3-phase base).

> Assume the armature resistance to be negligible.

Calculate:

a. the generated voltage E_{af} in volts,
b. the magnitude of the field-to-armature mutual inductance L_{el},
c. the electrical power input to the motor in kW and in horsepower.

Solution Equivalent Circuit Analysis - A

Example Equivalent Circuit Analysis - B 9

Assuming the input power and terminal voltage for the motor of Example 5.1 remain constant, calculate:

a. the phase angle δ of the generated voltage,
b. the field current required to achieve unity power factor at the motor terminals.

5.3 Open and Short Circuit Analysis

The fundamental characteristics of a SynM can be determined by a pair of tests:

1. the armature terminals open-circuited,
2. the armature terminals short-circuited.

> Except for a few remarks on the degree of validity of certain assumptions, the discussions apply to both cylindrical-rotor and salient-pole machines.

Example Open Test Analysis 10

An open-circuit test performed on a three-phase, 60-Hz synchronous generator shows that the rated open-circuit voltage of 13.8 kV is produced by a field current of 318 A. Extrapolation of the air-gap line from a complete set of measurements on the machine shows that the field-current corresponding to 13.8 kV on the air-gap line is 263 A. Calculate the saturated and unsaturated values of L_{sf}.

Solution **Open Test Analysis**

The reactances X_d and X_q of a salient-pole SynM are 1.00 and 0.60 per unit, respectively.

> The armature resistance (R_a) is considered to be negligible.

Calculate the generated voltage when the generator delivers its rated kVA at 0.80 lagging power factor and rated terminal voltage.

5.4 Power Angle Characteristics of Salient-Pole Machines

For the purposes of this discussion, it is sufficient to limit our discussion to the simple system shown in the schematic diagram below, consisting of a salient-pole SynM SM connected to an **infinite bus** of voltage $\hat{V}_\text{EQ}$ through a series impedance of reactance X_EQ.

Resistance will be neglected because it is usually small.

Infinite Bus

The bus whose voltage and frequency remain constant even after the variation in the load is known as the infinite bus.

Consider the SynM is acting as a **generator**. The phasor diagram is shown by the solid-line phasors. The dashed phasors show the external reactance drop resolved into components due to $\hat{I}_d$ and $\hat{I}_q$.

> The effect of the external impedance is merely to add its reactance to the reactances of the machine.

The total values of the reactance between the excitation voltage (E_af) and the bus voltage ($\hat{V}_\text{EQ}$) is therefore:

$$X_{dT} = X_d + X_{EQ} \qquad \text{and} \qquad X_{qT} = X_q + X_{EQ}$$

If the bus voltage ($\hat{V}_\text{EQ}$) is resolved into components;

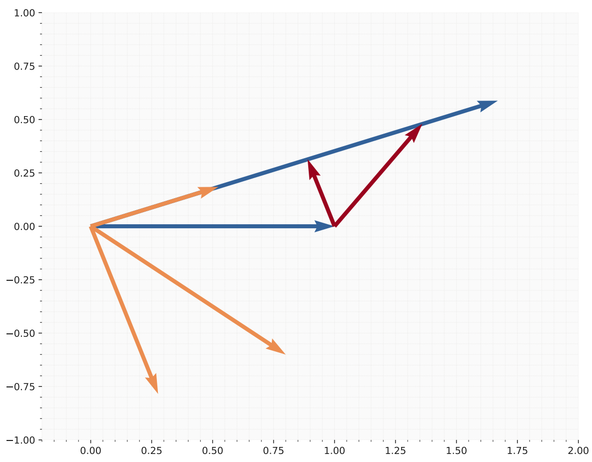


Figure 5.3: Generator phasor diagram for the example.

- direct-axis component $V_{\mathrm{d}} = V_{\mathrm{EQ}} \sin\delta$,
- quadrature-axis component $V_{\mathrm{q}} = V_{\mathrm{EQ}} \cos\delta$,

in-phase with $\hat{I}_{\mathrm{d}}$ and $\hat{I}_{\mathrm{q}}$, respectively, the power P delivered to the bus per phase is:

$$P = I_d V_d + I_q V_q = I_d V_{EQ} \sin\delta + I_q V_{EQ} \cos\delta$$

Also, from Fig. 5.26b,

$$I_{\mathrm{d}} = \frac{E_{\mathrm{af}} - V_{\mathrm{EQ}} \cos\delta}{X_{\mathrm{dT}}}.$$

$$I_{\mathrm{q}} = \frac{V_{\mathrm{EQ}} \sin\delta}{X_{\mathrm{qT}}}.$$

Substitution of Eqs. 5.63 and 5.64 in Eq. 5.62 gives

$$\begin{aligned} P &= I_{\mathrm{d}} V_{\mathrm{d}} + I_{\mathrm{q}} V_{\mathrm{q}} \\ &= I_{\mathrm{d}} V_{\mathrm{EQ}} \sin\delta + I_{\mathrm{q}} V_{\mathrm{EQ}} \cos\delta \\ &= \left(\frac{E_{\mathrm{af}} - V_{\mathrm{EQ}} \cos\delta}{X_{\mathrm{dT}}} \right) V_{\mathrm{EQ}} \sin\delta + \frac{V_{\mathrm{EQ}} \sin\delta}{X_{\mathrm{qT}}} V_{\mathrm{EQ}} \cos\delta. \end{aligned}$$

Doing some tidying up of the aforementioned expression gives us the following equation.

$$P =$$

Equation 5.65 is directly analogous to Eq. 5.47 which applies to the case of a nonsalient machine. It gives the power per phase when E_{af} and V_{EQ} are expressed as line-neutral

voltages and the reactances are in Ω/phase, in which case the result must be multiplied by three to get three-phase power. Alternatively, expressing E_{af} and V_{EQ} as line-to-line voltages will result in three-phase power directly. Similarly, Eq. 5.65 can be applied directly if the various quantities are expressed in per unit.
The general form of this power-angle characteristic is shown in Fig. 5.27. The first term is the same as the expression obtained for a cylindrical-rotor machine (Eq. 5.47). The second term includes the effect of salient poles. It represents the fact that the air-gap flux wave creates torque, tending to align the field poles in the position of minimum

Example Round Rotor Steady-State Power 11

A 3 MW, 7 kV synchronous drive has a synchronous reactance (X_s) of 8.2 Ω per phase. The stator is connected in wye (Y), and the drive operates at full-load (3 MW) with a leading power factor of pf = 0.89. If the efficiency is η = 97 %, Please calculate the following parameters:

a. The apparent power,
b. The line current,
c. The internal voltage per phase with corresponding phasor diagram,
d. The power angle,
e. The total reactive power supplied to the system,
f. The approximate maximum power the motor can develop without pulling out.

Solution Round Rotor Steady-State Power

The first action we take is to calculate the power generated by the drive (this includes the unusable losses as well)

$$\eta = \frac{P_{\text{shaft}}}{P_{\text{in}}}$$

The real power generated by the drive (P_{in}) is equal to;

$$P_{\text{in}} = \frac{P_{\text{shaft}}}{\eta} =$$

The apparent power can be resolved by the following identity

$$P = |S| \cos\varphi$$
$$|S| = \frac{P}{\cos\varphi} =$$

The question ask of us the line current (not the phase current) and given the system is in wye/star formation (Y) the line current can be simply calculated to be:

$$S = \sqrt{3} \times V_a I_a$$
$$I_a = \ =$$

This is a star (Y) connection therefore the phase voltage is:

$$V_{\mathrm{a}} = \frac{V_{\mathrm{LL}}}{\sqrt{3}}$$

The power angle:

$$P_{\mathrm{in}} = \frac{V_{\mathrm{a}} E_{\mathrm{af}}}{X_{\mathrm{s}}} \sin \delta$$

The total reactive power supplied to the system:

$$Q = \sqrt{S^2 - P^2}$$

The approximate maximum power the drive can develop without pulling out of step

Chapter 6

Solid State Commutation Machines

6.1 Brushless DC Motors

A SynM with a Permanent Magnet (PM) in the rotor and operated in self-controlled mode, using a **rotor position sensor** and an **inverter** to control current in the stator windings, is generally known as a Brushless DC (BLDC) machine.

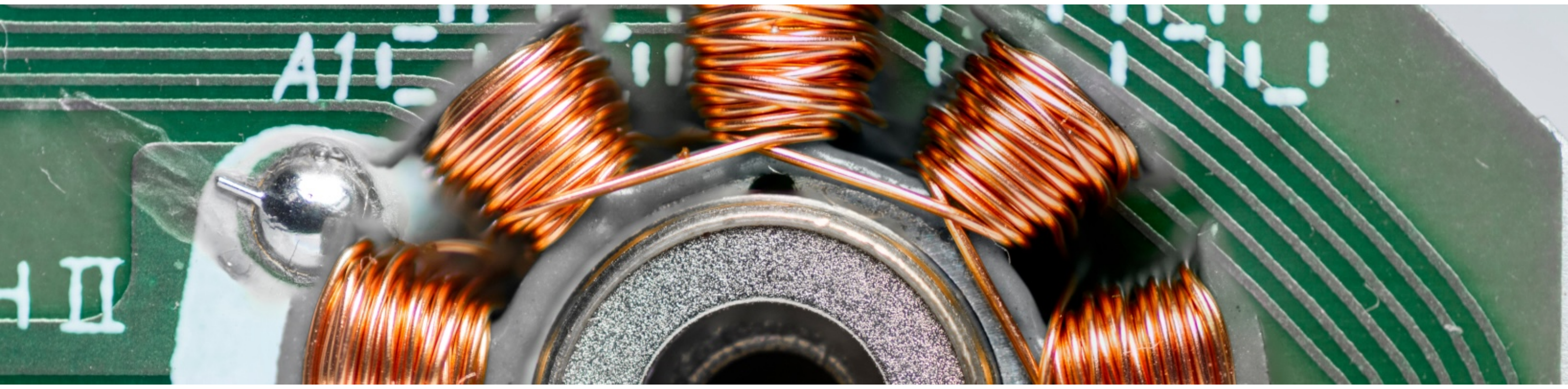


Figure 6.1: BLDC Machines are most common in places where maintenance are either.

The BLDC can be defined as **an inside-out brushed dc motor**, as the armature is in the stator and the magnets are in the rotor, and its operating characteristics are similar to a conventional brushed dc motor. This design is the more popular option and is known also as an **outrunner**. The opposite design is less used and is called an **inrunner** where the rotor is inside the stator.

The position sensor and the switches (i.e., MOSFETs, GaN) in the inverter perform the role of the brushes and the mechanical commutator of a conventional dc motor. The on–off state of these switches can be controlled by their gate signals, which are obtained from the rotor position sensors.

A typical transistor (BJT) inverter configuration fed from a DC source, as shown in 6.2, can be used for a BLDC drive system. The rotor position sensor will control the turn-on and turn-off instants for the switches such that the angle between the rotor field and the stator field is regulated at 90 degrees.

> The angle difference between the rotor and the stator field is similar to the conventional brushed DC motor.

The idealized waveforms of the stator currents are also shown in Figure 6.2. The switches of the inverter are turned on every 60 electrical degrees, and the switches are numbered in the sequence in which they are turned on. If the inverter is fed from a DC voltage source (voltage source inverter), pulse-width modulation of the individual switches can provide the regulation of the motor current.

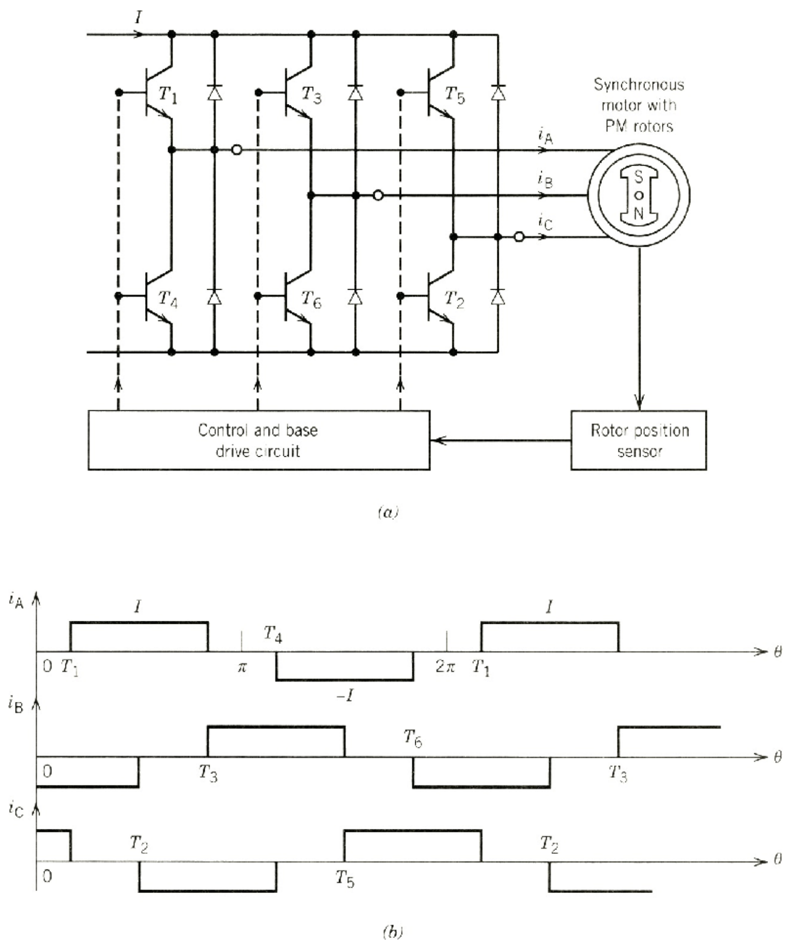


Figure 6.2: Brushless dc motor drive. (a) Cir- cuit. (b) Stator current waveforms.

There are two (2) basic types of BLDC motors:

- **Trapezoidal:** emf is of trapezoidal shape and, for ripple-free torque operation, the phase current required is a 120 quasi-square wave
- **Sinusoidal:** the back emf is sinusoidal and, for ripple-free torque operation, the phase current required is sinusoidal.

A trapezoidal back emf is generated by rotor PMs with a square air gap flux distribution and a concentrated stator winding. The 120 width of the phase current requires a low-resolution position sensor, such as a Hall effect sensor or an electro-optical sensor, to switch devices every 60 for commutation of the phase currents.

A sinusoidal back emf is generated by rotor PMs with an essentially sinu-soidal air gap flux distribution and a distributed stator winding.

> The winding can be short pitched to reduce the effect of the space-harmonic flux distribution.

The sinusoidal phase current requires an **absolute position sensor or resolver** for continuous position sensing to allow accurate synthesis of the sinusoidal current waveform. The current in a phase is a sinusoidal function of the rotor position. The ideal waveforms

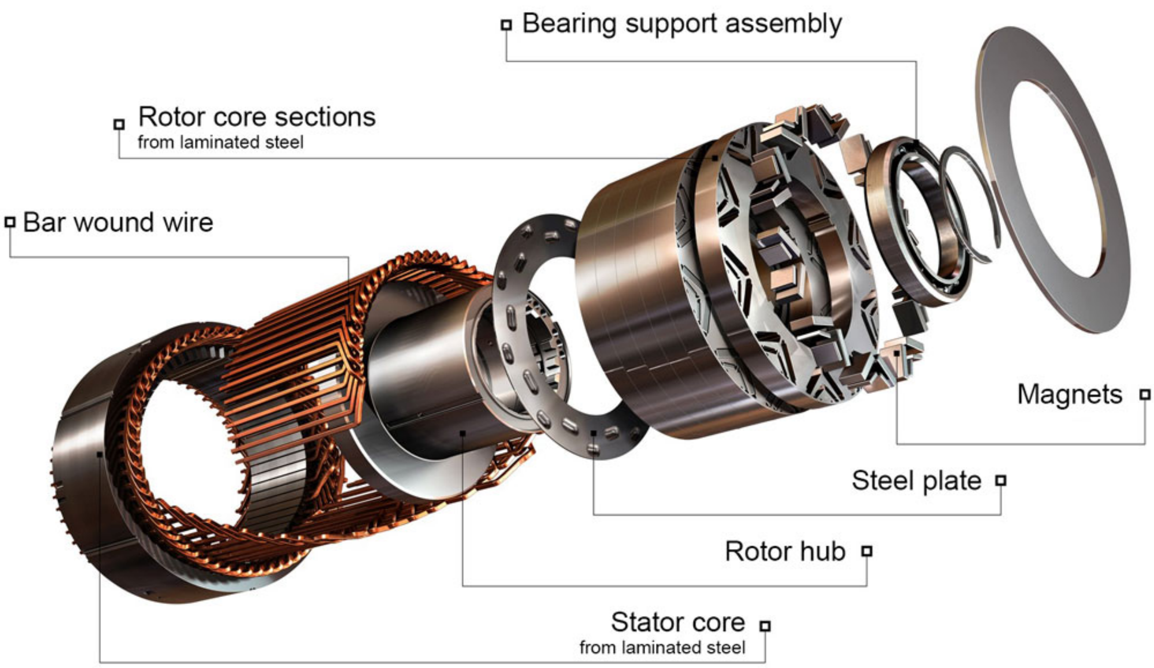


Figure 6.3: The exploded view of a interior PMSM.

(a) Concentrated winding **(b)** Distributed winding

Figure 6.4: Types of windings encountered in PMSM and BLDC.

of the various quantities in the two types of BLDC motors are shown in Fig. 6.5

In the trapezoidal-type motor, since the shapes of the back emf (trapezoidal) and the winding current (constant amplitude) are similar to those in a conventional dc machine, the motor is referred to as a brushless dc motor, whereas the sinusoidal type is referred to as a brushless synchronous motor.

The trapezoidal-type motor requires an inexpensive position sensor. Its commutation scheme is simple, and since only two switches are on at any time, the efficiency of the inverter is high. The overall cost will be lower. However, torque ripple and audible noise will be high if the voltage and current waveforms are nonideal. The sinusoidal-type motor requires a relatively more expensive position sensor. More hardware and software for signal processing are required. The inverter efficiency will be lower because three switches are on at the same time. The overall cost will be higher. However, torque ripple will be lower even with nonideal back emf.

Four types of PM rotor configurations are commonly used. These types are shown in Fig. 6.6. The four rotor types are characterized by surface-mounted magnets, inset magnets, and embedded or buried magnets, the latter having two variants.

Surface-Mounted Magnets

The radially magnetised PMs are mounted on a solid steel-core rotor structure as shown in Fig. 6.6 (a). As the relative permeability of the magnet material is near unity (around 1.05), it **acts like a large air gap**. The effective air gap is therefore large, making L_d low. The structure is magnetically non-salient (i.e., uniform air gap), which makes $L_d = L_q$.

This type of rotor structure is the most common topology, and is the industrial standard.

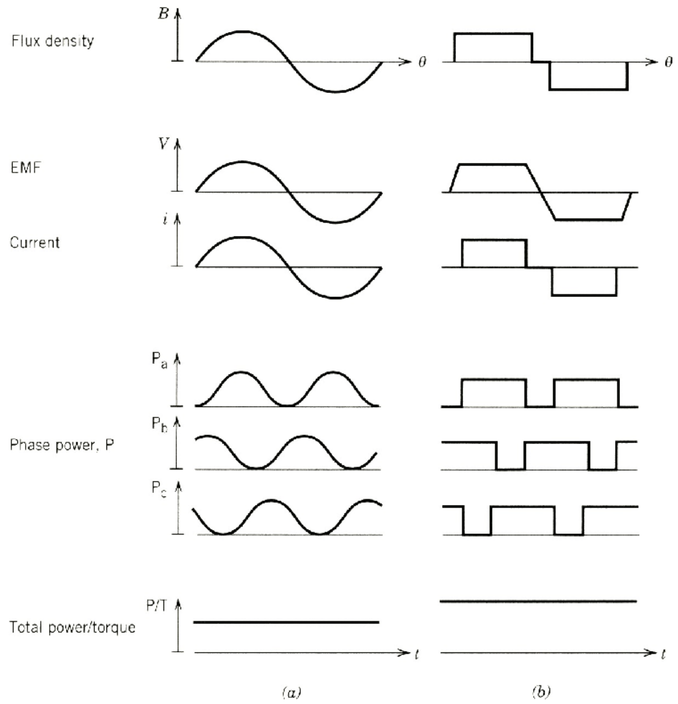


Figure 6.5: The idealised waveforms of both BLDC and PMSM.

The surface-mounted PM (SMPM) motor results in lower cogging torque due to the large effective air gap.

Cogging Torque

The torque due to the interaction between rotor PMs and stator slots of a PM machine. It is also known as detent or no-current torque. This torque is position dependent and its periodicity per revolution depends on the number of magnetic poles and the number of teeth on the stator. Cogging torque is an undesirable component for the operation of such a motor. It is especially prominent at lower speeds, with the symptom of jerkiness. Cogging torque results in torque as well as speed ripple; however, at high speed the motor moment of inertia filters out the effect of cogging torque.

SMPM motors are widely used in fans, blowers, robotics, servo drives such as vehicle electric power steering, machine tools, and so on, due to their low inertia.

In some high-power applications, the wound stator is placed in the center of the machine, while the magnets are mounted along the inner circumference of the outer rotating rotor. This allows a higher number of poles, as the rotor diameter is larger. In addition, the centrifugal forces exert a pressure on the PMs, leading to more robust and higher-speed operation.

This structure is also adapted to wind turbines, as the hub carrying the blades can be fixed directly to the outer rotor.

Inset Magnets

In the inset arrangement, the PMs are inserted in the steel rotor structure as shown in Fig. 6.6 (b). This construction provides a more secure magnet setting and ease of assembly.

In this configuration, Ld < Lq. For the same magnet size, the peak torque developed with inset magnets is higher than that for the surface-mounted magnets because of the **reluctance torque** developed with the former. To produce the same torque, the thickness required for the inset magnets is smaller and hence L_d is larger.

This type of rotor structure is more robust and has the same applications as the surface-mounted magnet type. They are suitable for high-power and high-speed applications, such as windmills.

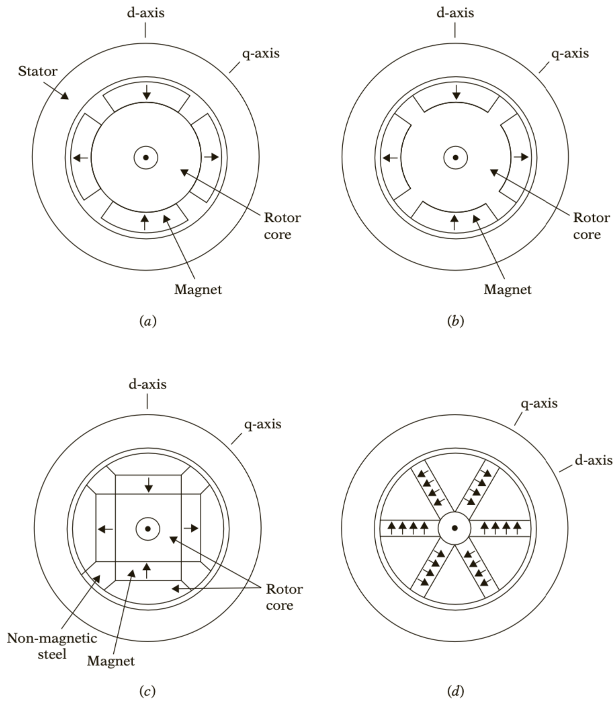


Figure 6.6: PM rotor configurations. (a) Surface-mounted magnets. (b) Inset magnets. (c) Interior (buried) magnets with radial magnetization. (d) Interior (buried) magnets with circumferential magnetization.

Interior (Buried) PM with Radial Magnetization

The magnets are **buried inside the rotor structure**, with radial magnetisation, as shown in Fig. 6.6 (c). The q-axis inductance is larger than the d-axis inductance Lq > Ld, and both Ld and Lq are larger than their corresponding values in surface-type and inset-type rotors.

Due to the additional reluctance torque capability and secure placement of magnets, IPM-type motors are used in electric vehicles and propulsion applications.

Interior (Buried) PM with Circumferential Magnetisation

This arrangement of rotor magnets is shown in Fig. 6.6 (d). Because of the flux-focusing effect, circumferential magnetization yields greater air gap flux than radial magnetisation.

Flux Focusing

The deliberate design of the PM geometry to increase the flux density of the rotor.

The d-axis inductance L_{d} is large, and the structure is magnetically salient, having Ld > Lq . Ferrite magnets have low flux density. The circumferential magnetization arrangement is particularly advantageous for ferrite magnets, because a substantial increase in flux density can be achieved.

This type of rotor arrangement can be used for low-cost, high-efficiency propulsion motors, machine tools, and appliances. The surface-mounted magnets, because of constant magnetic gap between the stator and rotor, can provide a square wave flux distribution. This type of magnet is normally used in trapezoidal-type motors. The inset and buried magnet arrangements are used primarily for sinusoidal-type motors, in which the sinusoidal flux distribution can easily be achieved by proper design of the rotor geometry. Because of sine commutation providing smooth and low audible noise, IPM motors are suitable for air compressors, refrigerators, washing machines, dishwashers, and electric power steering.

In BLDC motors, the rotor flux is essentially constant because of the PM. The flux weakening required to operate the motor beyond the base speed for constant-horsepower operation can be achieved by providing a negative d-axis stator current component ð–idÞ to oppose the rotor magnet flux. In the surface-mounted magnet rotor, since Ld is low, a very large id is required to provide effective field weakening. Thus, a limited speed range (up to 1:2nb, where nb is the base speed) can be achieved with the surface-mounted magnet rotor. This rotor configuration is therefore used primarily for constant-torque operation.

With the inset-magnet rotor, the speed range can be extended to 1:5nb. For the buried-magnet rotor with radial magnetization, the speed range can be extended to 2nb. For the circumferential-magnetization type of rotor, a speed range up to 3nb can be achieved.

6.1.1 Advantages v. Disadvantages

Compared to conventional brushed dc motors, they have the following advantages:

- Small rotor size and high-power density, because of the absence of mechanical commutators, brushes, and field windings.
- Lower inertia and faster dynamic response.
- Higher speed and torque capability, due to the absence of brushes and sparking.
- Lower maintenance cost.
- High torque/inertia ratio.
- Better heat dissipation, due to the stationary armature winding.
- Better overall reliability and life.

Compared to IM, they have the following advantages:

- High efficiency, because of no slip power losses.
- High power and torque density, because of high air gap flux density.
- Lower operating temperature, because of low losses.

BLDC motors have the following advantages compared with induction motors:
On the negative side, BLDC motor drive systems require rotor position sensors, complex power electronics (inverters), and complex controllers, resulting in a higher initial system cost.

There is a possibility that demagnetization of the PMs may occur due to accidental overload or fault. The advantages, generally, outweigh the disadvantages.

Improvements in PM materials and cost reduction in integrated circuits and electronic power switches have made BLDC motors—in particular the trapezoidal type—major contenders in the field of high-performance drives.

These motors are being extensively used in computer disk drives, servo drives, robotics, machine tools, electric vehicles, battery-powered applications, windmills, and many other applications.

6.2 Switched Reluctance Motors

Switched reluctance motors have saliency in both stator and rotor and are perhaps the simplest electric motors in construction. The stator consists of salient poles with excitation windings on them, and the rotor has salient poles with no windings. The torque is

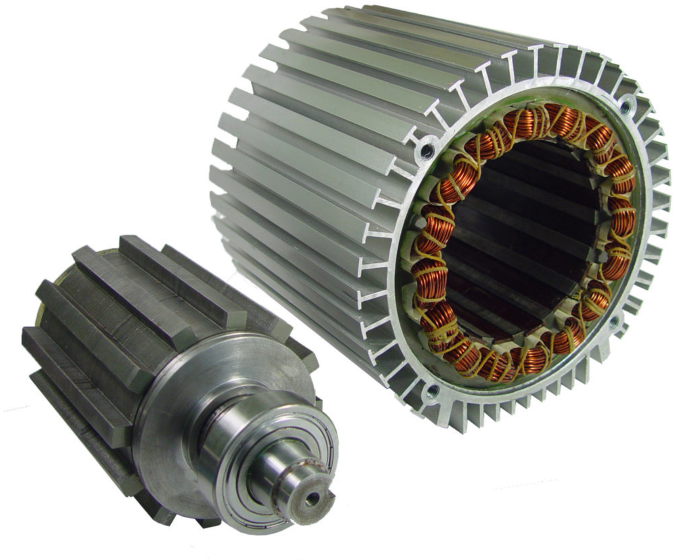


Figure 6.7: A switched reluctance motor. Notice the uneven air-gap of the rotor and the poles. This makes the motor to develop reluctance torque as it spins.

produced by the tendency of the rotor pole to align with the stator pole to maximize the stator flux linkage when the winding of the stator pole is excited by a current.

6.2.1 Operation Principle

Figure 6.8 shows a cross-sectional view of a switched reluctance motor with eight stator poles and six rotor poles. These motors have one pair of poles less on the rotor than on the stator. Diametrically opposite stator poles are excited simultaneously, as shown in Fig. 6.42. The SRM in Fig. 6.8 has four (4) stator phases. If phase A is excited, rotor poles marked a and a' will be aligned with stator poles marked A and AO. If phase B winding now is excited, rotor poles b and bO will be aligned with poles B and BO of stator phase B, and therefore the rotor will move clockwise. Thus, if the phases are excited in the sequence A, B, C, D, A . . ., the rotor will move in the clockwise direction. The rotor will move with some synchronism with the stator field, but the motions of the stator field and rotor poles are in opposite directions. Correct timing of the excitation of a phase winding depends on the position of the rotor. Therefore, for proper operation of the motor, a rotor position sensor is required. The excitation must be switched sequentially from phase to phase as

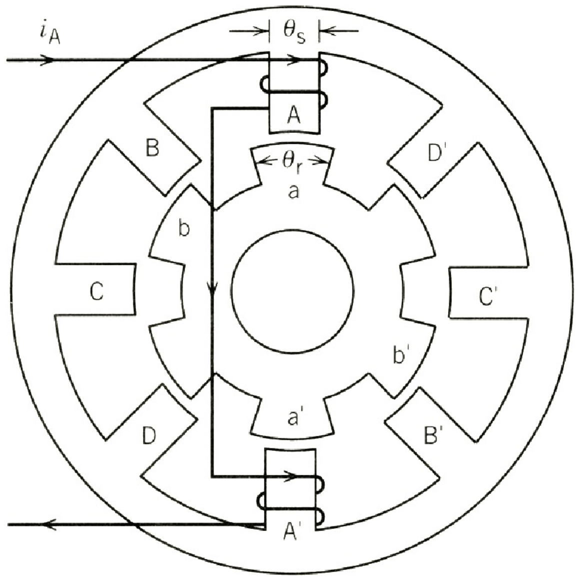


Figure 6.8: Cross section of a switched reluctance motor (SRM).

the rotor moves, hence the name switched reluctance motor

6.2.2 Modeling and Torque

A simplified model of the SRM is obtained on the basis of the following assumptions: 1. There is no mutual flux linkage between phase windings; that is, one phase is excited at a time. 2. The ferromagnetic materials in the machine have a linear **B**, **H** characteristic. With these assumptions, the flux linkage of a winding can be represented by an inductance. As the rotor changes position, the inductance of a phase winding will change. Various positions of the rotor are shown in Fig. 6.43.

Figure 6.44a shows the inductance profile of phase A. The torque produced by the excitation of the kth phase is Since the torque is proportional to the square of current, it is independent of the current direction. The torque produced for a constant current through the phase winding (Fig. 6.44b) is shown in Fig. 6.44c. The average torque produced is zero. If the current is applied only when the inductance is increasing (Fig. 6.44d), the negative torque is eliminated (Fig. 6.44e), and the motor will produce an average positive torque. It is important, therefore, for the production of an average positive torque that the current in a phase winding be applied when the rotor is in a particular position, and switched off at another predetermined rotor position. In a practical circuit the current in the phase winding will not have a square waveform. When a voltage is applied to a phase winding at rotor position θ2 (Fig. 6.45a), the current builds up with a finite time lag. The current will build up quickly, because the inductance is low. When the voltage is removed at rotor

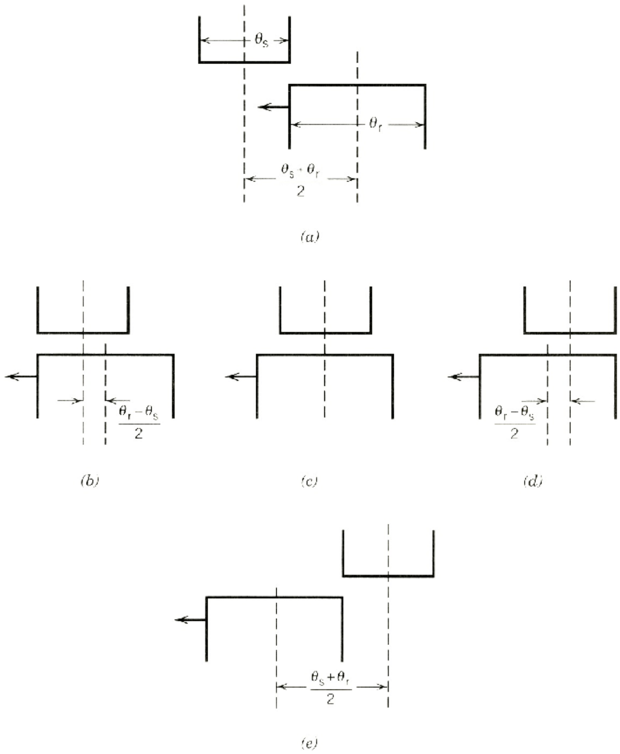


Figure 6.9: Rotor positions. (a) Inductance linearly increases from this position. (b–d) Maxi- mum inductance positions. (d) Inductance linearly decreases as rotor moves from this position. (e) Minimum inductance position.

position θ2, the current will decay at a slower rate, because the inductance is high. The current waveform and the corresponding torque waveform are also shown in Fig. 6.45a. If the negative torque is to be eliminated, the voltage from the phase winding would have to be removed earlier, as shown in Fig. 6.45b.
The voltage–current relationship of a phase winding can be expressed as follows:

6.2.3 Advantages v. Disadvantages

The SRM is of rugged construction, as there is no rotor winding. It is also reliable, because the power control circuit is simple, requiring only unipolar current, compared with the inverter used for IM and its simple design also allows it to have good power density. However, the SRM requires a rotor position sensor, and it is characterised by **torque ripple**, which causes acoustic, audible noise.

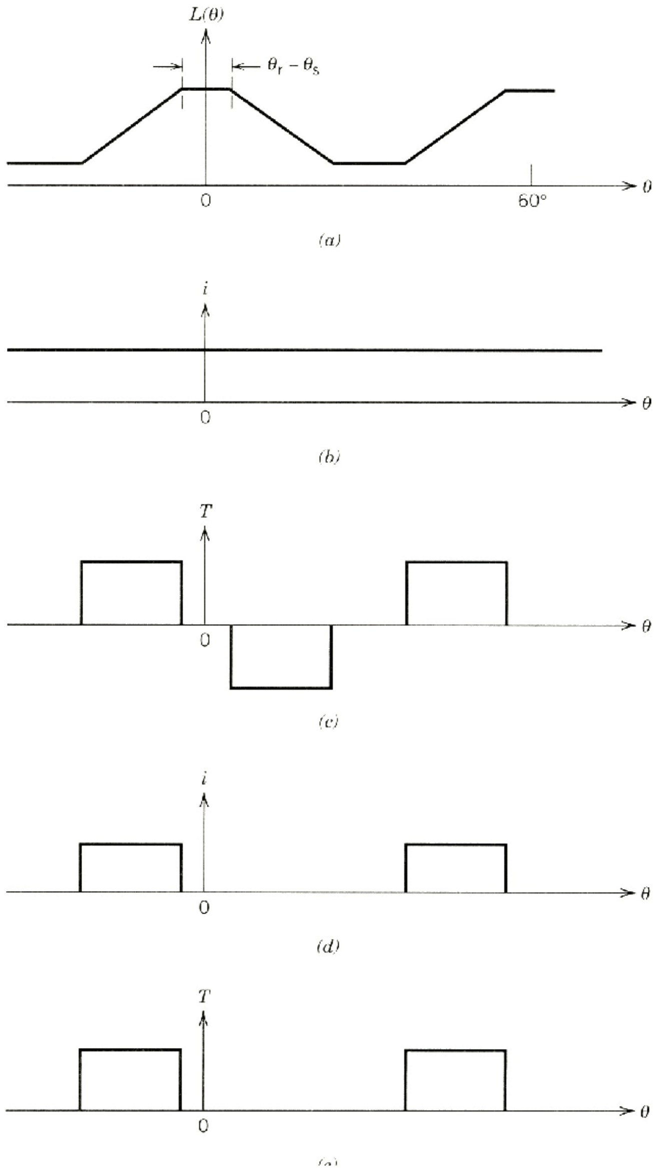


Figure 6.10: (a) Variation of induc- tance with rotor position. (b) Constant- current excitation. (c) T versus θ. (d) Pulsed-current excitation. (e) T versus θ.

6.2.4 Applications

The switched current of a phase winding does not have the sinusoidal shape associated with conventional AC machines. Although the machine's construction is simple, its control is somewhat complicated.

DC motors have dominated the field of variable-speed drives. In recent years, inverter-fed induction motors have challenged dc motors in the variable-speed drive applications. However, intensive academic and industrial research and development over the past 15 years have made the SRM drive a viable alternative to both dc and induction motor drives. Switched reluctance motors with power capabilities from 100 W to 100 kW are presently commercially available for various applications, ranging from low-power servo motors to high- power traction drives.