Tutorial Book

B.Sc Mobile Robotics

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List of Questions

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Statistical Methods

[Q1] Maximum Likelihood of Poisson Distribution

Consider a Poisson distribution with probability mass function:

$$f\left(x|\mu\right) = \frac{e^{-\mu}\mu^{x}}{x!} \qquad \text{where} \qquad x = 0, 1, 2, \ldots.$$

Suppose that a random sample $x_1, x_2, \ldots, x_n$ is taken from the distribution. What is the maximum likelihood estimate of μ?

[A1]

The likelihood function is

$$L\left(x_1, x_2, \ldots, x_n; \mu\right) = \prod_{i=1}^{n} f\left(x_i|\mu\right) = \frac{e^{-n\mu} \sum_{i=1}^{n} x_i}{\prod_{i=1}^{n} x_i!}.$$

Now consider is logarithmic representation:

$$\ln L\left(x_1, x_2, \ldots, x_n; \mu\right) = -n\mu + \sum_{i=1}^{n} x_i \ln \mu - \ln \prod_{i=1}^{n} x_i!$$

And taking its partial derivative to the parameter gives:

$$\frac{\partial \ln L\left(x_1, x_2, \ldots, x_n; \mu\right)}{\partial \mu} = -n + \sum_{i=1}^{n} \frac{x_i}{\mu}.$$

Solving for $\hat{\mu}$, the maximum likelihood estimator, involves setting the derivative to zero and solving for the parameter. Therefore,

$$\hat{\mu} = \sum_{i=1}^{n} \frac{x_i}{n} = \hat{x}$$

If you were to test it, the second derivative of the log-likelihood function is **negative**, which implies that the solution above indeed is a maximum. As μ is the mean of the Poisson distribution, the sample average would certainly seem like a reasonable estimator $\blacksquare$.

[Q2] Maximum Likelihood of Gaussian Distribution

Find maximum likelihood estimates for $\theta_1 = \mu$ and $\theta_2 = \sigma$ in the case of the normal distribution.

[A2]

We obtain the likelihood function:

$$L = \left(\frac{1}{\sqrt{2\pi}}\right)^n \left(\frac{1}{\sigma}\right)^n e^{-\mathrm{h}}$$

$$\text{where} \qquad h = \frac{1}{2\sigma^2}\sum_{j=1}^{n}\left(x_{\mathrm{j}} - \mu\right)^2.$$

Taking logarithms, we have

$$\ln L = -n \ln \sqrt{2\pi} - n \ln \sigma - h.$$

The first equation for the parameters is $\dfrac{\partial \ln L}{\partial \mu} = 0$, written out:

$$\frac{\partial \ln L}{\partial \mu} = -\frac{\partial h}{\partial \mu} = \frac{1}{\sigma^2}\sum_{j=1}^{n}\left(x_{\mathrm{j}} - \mu\right) = 0,$$

$$\text{therefore} \qquad \sum_{j=1}^{n} x_{\mathrm{j}} - n\mu = 0.$$

The solution is the desired estimate $\hat{\mu}$ for μ: we find

$$\hat{\mu} = \frac{1}{n}\sum_{j=1}^{n} x_{\mathrm{j}} = \overline{x}.$$

The second equation for the parameter is $\partial \ln L / \partial \sigma = 0$, written out

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\sigma} - \frac{\partial h}{\partial \sigma} = -\frac{1}{\sigma} + \frac{1}{\sigma^3}\sum_{j=1}^{n}\left(x_{\mathrm{j}} - \mu\right)^2 = 0.$$

Replacing μ by $\hat{\mu}$ and solving for σ^2, we obtain the estimate:

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{j=1}^{n}\left(x_{\mathrm{j}} - \overline{x}\right)^2 \qquad \blacksquare$$

[Q3] For Science

Suppose ten (10) rats are used in a biomedical study where they are injected with cancer cells and then given a cancer drug that is designed to increase their survival rate. The survival times, in months, are:

14 17 27 18 12 8 22 13 19 12

Assume exponential distribution applies which is given as:

$$f(x, \beta) = \begin{cases} \dfrac{1}{\beta} \exp \dfrac{x}{\beta}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Give a maximum likelihood estimate of the mean survival time.

[A3]

We know that the probability density function for the exponential random variable X. Therefore, the log-likelihood function for the data, given $n = 10$, is:

$$\ln L\left(x_1, x_2, \ldots, x_{10}; \beta\right) = -10 \ln \beta - \frac{1}{\beta} \sum_{i=1}^{10} x_{\mathrm{i}}.$$

Setting

$$\frac{\partial \ln L}{\partial \beta} = -\frac{10}{\beta} + \frac{1}{\beta^2} \sum_{i=1}^{10} x_{\mathrm{i}} = 0 \qquad \text{which means}$$

$$\hat{\beta} = \frac{1}{10} \sum_{i=1}^{10} x_{\mathrm{i}} = \hat{x} = 16.2 \quad \blacksquare$$

Evaluating the second derivative of the log-likelihood function at the value $\hat{\beta}$ above gives a negative value. As a result, the estimator of the parameter β, the population mean, is the sample average $\overline{x}$.

[Q4] Sampling the Population

It is known that a sample consisting of the values:

$$12 \quad 11.2 \quad 13.5 \quad 12.3 \quad 13.8 \quad 11.9$$

comes from a population with the density function:

$$f(x; \theta) = \begin{cases} \dfrac{\theta}{\theta^{\,x+1}}, & x > 1, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\theta > 0$. Find the maximum likelihood estimate of θ.

[A4]

The likelihood function of n observations from this population can be written as:

$$L\left(x_1, x_2, \ldots, x_{10}; \theta\right) = \prod_{i=1}^{n} \frac{\theta}{x_{\mathrm{i}}^{\theta+1}} = \frac{\theta^{\mathrm{n}}}{\left(\prod_{i=1}^{n} x_{\mathrm{i}}\right)^{\theta+1}},$$

which implies that

$$\ln L\left(x_1, x_2, \ldots, x_{10}; \theta\right) = n \ln \theta - (\theta + 1) \sum_{i=1}^{n} \ln x_{\mathrm{i}}.$$

Setting $0 = \partial \ln L / \partial \theta = {}^{n}\!/\!_{\theta} - \sum_{i=1}^{n} \ln(x_i)$ results in

$$\hat{\theta} = \frac{n}{\sum_{i=1}^{n} \ln x_{\mathrm{i}}} = 0.3970 \quad \blacksquare.$$

Since the second derivative of L is $-n/\theta^2$, which is always negative, the likelihood function does achieve its maximum value at $\hat{\theta}$.

[Q5] Confidence Interval for Mean with known Variance in Normal Distribution

Determine 95% confidence interval for the mean of a normal distribution with variance $\sigma^2 = 9$, using a sample of $n = 100$ values with mean $\overline{x} = 5$.

[A5]

1. First we define γ as 0.95 based on the 95% confidence.
2. Then looking at our reference table find the corresponding c which equals 1.960.
3. $\overline{x} = 5$ is given.
4. We need:

$$k = c\frac{\sigma}{\sqrt{n}} = 1.960\frac{3}{\sqrt{100}} = 0.588$$

Therefore

$$\overline{x} - k = 4.412 \qquad \text{and} \qquad \overline{x} + k = 5.588$$

and the confidence interval is:

$$\text{CONF}_{0.95}\{4.412 \leq \mu \leq 5.588\} \quad \blacksquare$$

[Q6] Sample Size Needed for a Confidence Interval of Prescribed Length

How large must n be in the Example **Confidence Interval for mean with known variance in Normal Distribution** to obtain a 95% confidence interval of length $L = 0.4$?

[A6]

The interval in Example **Confidence Interval for mean with known variance in Normal Distribution** has the length:

$$L = 2k = 2c\sigma/\sqrt{n}.$$

Solving for n, we obtain

$$n = \left(\frac{2c\sigma}{L}\right)^2$$

In the present case the answer is:

$$n = \left(\frac{2 \times 1.96 \times 3}{0.4}\right)^2 \approx 870 \quad \blacksquare$$

[Q7] Confidence Interval for Mean of Normal Distribution with Unknown Variance

The five (5) independent measurements of flash point of Diesel oil (D-2) gave the values (in °F):

144 147 146 142 144

If we assume normality, determine a 99% confidence interval for the mean.

[A7]

1. $\gamma = 0.99$ is required based on 99% confidence level.
2. $F(c) = \frac{1}{2}(1+\gamma) = 0.99$ and looking at the reference table with $n-1 = 4$ d.f., which gives $c = 4.60$.
3. Calculating the mean and the variance gives $\overline{x} = 144.6$ and $s = 3.8$,
4. $k = \sqrt{3.8} \times 4.60/\sqrt{5} = 4.01$. Therefore the confidence interval is:

$$\mathrm{CONF}_{0.99}\{140.5 \leq \mu \leq 148.7\} \quad \blacksquare$$

If the variance σ^2 were known and equal to the sample variance s^2, therefore $\sigma^2 = 3.8$, then the Reference Table would give:

$$k = \frac{c\sigma}{\sqrt{n}} = 2.576\frac{\sqrt{3.8}}{\sqrt{3}} = 2.25$$

and

$$\mathrm{CONF}_{0.99}\{142.35 \leq \mu \leq 146.85\} \quad \blacksquare$$

We see that the present interval is almost twice as long as that with a known variance $\sigma^2 = 3.8$.

[Q8] Confidence Interval for the Variance of the Normal Distribution

Determine a 95% confidence interval for the variance, based on the following sample (tensile strength of sheet steel in kg mm^{-2}, rounded to integer values)

89 84 87 81 89 86 91 90 78 89 87 99 83 89

[A8]

1. $\gamma = 0.95$ is required.
2. For $m = n - 1 = 13$ we find

$$c_1 = 5.01 \qquad \text{and} \qquad c_2 = 24.74.$$

3. $13s^2 = 326.9$
4. $13s^2/c_1 = 65.25$ and $13s^2/c_2 = 13.21$
5. This makes the confidence interval as:

$$\mathrm{CONF}_{0.09}\left\{13.21 \leq \sigma^2 \leq 65.25\right\} \quad \blacksquare$$

This is rather large, and for obtaining a more precise result, one would need a much larger sample.

[Q9] Test for the Mean of the Normal Distribution with Known Variance

Let X be a normal random variable with variance $\sigma^2 = 9$. Using a sample of size $n = 10$ with mean $\overline{x}$, test the hypothesis $\mu = \mu_0 = 24$ against the three (3) kinds of alternatives, namely,

$$\text{(a) } \mu > \mu_0 \qquad \text{(b) } \mu < \mu_0 \qquad \text{(c) } \mu \neq \mu_0$$

[A9]

We choose the significance level $\alpha = 0.05$ as it is customary at this point. An estimate of the mean will be obtained from:

$$\overline{X} = \frac{1}{n}\left(X_1 + \cdots + X_n\right).$$

If the hypothesis is true, $\overline{X}$ is normal with mean $\mu = 24$ and variance $\sigma^2/n = 0.9$. Therefore we may obtain the critical value c from X.

Right-Sided Test We determine c from

$$P\left(\overline{X} > c\right)_{\mu=24} = \alpha = 0.05$$

that is,

$$P\left(\hat{X} \leq c\right)_{\mu=24} = \Phi\left(\frac{c-24}{\sqrt{0.9}}\right) = 1 - \alpha = 0.95.$$

Reverse engineering **Table** 2.1 by looking for 0.95 percentile gives $(c-24)/\sqrt{0.9} = 1.645$, and $c = 25.56$, which is greater than μ_0. If $\overline{x} \leqq 25.56$, the hypothesis is accepted. If $\overline{x} > 25.56$, it is rejected ■

Left-Sided Test The critical value c is obtained from the equation

$$P\left(\hat{X} \leqq c\right)_{\mu-24} = \Phi\left(\frac{c-24}{\sqrt{0.9}}\right) = \alpha = 0.05.$$

Reverse engineering **Table** 2.1 by looking for 0.95 percentile gives $c = 24 - \sqrt{0.9} \times 1.645 = 22.44$. If $\hat{x} \geqq 22.44$, we accept the hypothesis. If $\hat{x} < 22.44$, we reject it ■

Two-Sided Test As the normal distribution is **symmetric**, we choose c_1 and c_2 equidistant from $\mu = 24$, say, $c_1 = 24 - k$ and $c_2 = 24 + k$, and determine k from:

$$P\left(24 - k \leqq \hat{X} \leqq 24 + k\right)_{\mu=24} = \Phi\left(\frac{k}{\sqrt{0.9}}\right) - \Phi\left(\frac{-k}{\sqrt{0.9}}\right) = 1 - \alpha = 0.95.$$

Looking for 0.975 in**Table** 2.1 gives $k/\sqrt{0.9} = 1.960$, therefore $k = 1.86$. This gives the values $c_1 = 24 - 1.86 = 22.14$ and $c_2 = 24 + 1.86 = 25.86$. If $\hat{x}$ is not smaller than c_1 and not greater than c_2, we accept the hypothesis. Otherwise, we reject it ■

[Q10] Test for the Mean of the Normal Distribution with Unknown Variance

The tensile strength of a sample of $n = 16$ manila ropes was measured. The sample mean was $\hat{x} = 4482$ kg, and the sample standard deviation was $s = 115$ kg. Assuming that the tensile strength is a normal random variable, test the hypothesis $\mu_0 = 4500$ kg against the alternative $\mu_1 = 4400$ kg. Here μ_0 may be a value given by the manufacturer, while μ_1 may result from previous experience.

[A10]

We choose the significance level a $\alpha = 5\%$. If the hypothesis is true, It follows that the random variable:

$$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}} = \frac{\overline{X} - 4500}{S/4}$$

has a t-distribution with $n - 1 = 15$ d.f. The test is left-sided. The critical value c is obtained from:

$$P(T < c)_{\mu_0} = \alpha = 0.05$$

Table 2.3 gives $c = -1.75$. As an observed value of T we obtain from the

$$t = \frac{4482 - 4500}{115/4} = -0.626.$$

We see that $t > c$ and accept the hypothesis ■

[Q11] Comparison of the Means of Two Normal Distributions

Using a sample $x_1, \cdots, x_n$ from a normal distribution with unknown mean μ_x and a sample $y_1, \cdots, y_n$ from another normal distribution with unknown mean μ_y, we want to test the hypothesis that the means are equal, $\mu_x = \mu_y$, against an alternative, $\mu_x \neq \mu_y$. The variances need not be known but are assumed to be equal.

105	108	86	103	103	107	124	105
89	92	84	97	103	107	111	97

[A11]

We find:

$$\overline{x} = 105.125 \qquad \overline{y} = 97.500 \qquad s_x^2 = 106.125 \qquad s_y^2 = 84.000.$$

We choose the significance level $\alpha = 5\%$. Using a two-sided test the cut-off points are 2.5% and 97.5%. The d.f. is calculated as:

$$n_1 + n_2 - 2 = 8 + 8 - 2 = 14$$

Using **Table** 2.3 with 14 d.f., the critical values are: $c_1 = -2.14$ and $c_2 = 2.14$. Using the following formula:

$$t_0 = \sqrt{n}\frac{\overline{x} - \overline{y}}{\sqrt{s_x^2 + s_y^2}}$$

Using $n_1 = n_2 = n = 8$ gives:

$$t_0 = \sqrt{8}\frac{7.625}{\sqrt{190.125}} = 1.56$$

Since $c_1 \leqq t_0 \leqq c_2$, we accept the hypothesis $\mu_x = \mu_y$ that under both conditions the mean output is the same.

[Q12] Printed Circuit Boards

The number of defects in printed circuit board is hypothesized to follow a Poisson distribution. A random sample of $n = 60$ printed boards have been collected, and following number of defects were observed

Number of Defects	Observed Frequency
0	32
1	15
2	9
3	4

[A12]

The mean of the assumed Poisson distribution in this example is unknown and must be estimated from the sample data. The estimate of the mean number of defects per board is the sample average, that is:

$$(32 \times 0 + 15 \times 1 + 9 \times 2 + 4 \times 3)\,/60 = 0.75$$

From the Poisson distribution with parameter 0.75, we may compute p_i, the theoretical, hypothesized probability associated with the ith class interval. Since each class interval corresponds to a particular number of defects, we may find the p_i as follows:

$$\begin{aligned}
p_1 &= P(X = 0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472 \\
p_2 &= P(X = 1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354 \\
p_3 &= P(X = 2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133 \\
p_4 &= P(X \geqq 3) = 1 - (p_1 + p_2 + p_3) = 0.041
\end{aligned}$$

The expected frequencies are computed by multiplying the sample size $n = 60$ times the probabilities p_i. That is, $e_i = np_i$. The expected frequencies follow:

Number of Defects	Probability	Expected Frequency
0	0.472	28.32
1	0.354	21.24
2	0.133	7.98
3 (or more)	0.041	2.46

Since the expected frequency in the last cell is less than 3, we combine the last two cells:

NOTE: Categories with expected frequency is combined because the Chi-square test would not work if the frequency is less than 5. If the sample size is too small the chi-square value is over-estimated and if it is too large chi-square value is under-estimated. Hency why we combine with the category with the lowest frequency.

Since the expected frequency in the last cell is less than 3, we combine the last two cells:

Number of Defects	Probability	Expected Frequency
0	32	28.32
1	15	21.24
2 (or more)	13	10.44

Now, the chi-square test will have $k - p - 1 = 3 - 1 - 1 = 1$ degree of freedom, because the mean of the Poisson distribution was estimated from the data.

The hypothesis-testing procedure may now be applied using $\alpha = 0.05$,

1. The variable of interest is the form of the distribution of defects in printed circuit boards.
2. θ_0 The form of the distribution of defects is Poisson.
3. θ_1 The form of the distribution of defects is not Poisson.
4. Test statistic is:
$$\chi_0^2 = \sum_{j=1}^{k} \frac{\left(b_{\mathrm{j}} - e_{\mathrm{j}}\right)^2}{e_{\mathrm{j}}}$$
5. Reject H_0 if $\chi_0^2 > \chi^2_{0.05,1} = 3.84$.
6. Time to calculate χ_0^2:
$$\chi_0^2 = \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} = 2.94$$
7. As $\chi_0^2 = 2.94 < \chi^2_{0.05,1} = 3.84$, we are unable to reject the null hypothesis that the distribution of defects in printed circuit boards is Poisson.

[Q13] Testing the Power Supply

A manufacturing engineer is testing a power supply used in a notebook computer and, using $\alpha = 0.05$, wishes to determine whether output voltage is adequately described by a normal distribution. Sample estimates of the mean and standard deviation of $\overline{x} = 5.04$ V and $s = 0.08$ V are obtained from a random sample of $n = 100$ units.

Is the data normally distributed?

[A13]

A common practice in constructing the class intervals for the frequency distribution used in the chi-square goodness-of-fit test is to choose the cell boundaries so that the expected frequencies $e_j = np_j$ are equal for all cells. To use this method, we want to choose the cell boundaries a_0, a_1,..., a_k for the k cells so that all the probabilities

$$p_{\mathrm{i}} = P\left(a_{\mathrm{j}-1} \leqq X \leqq a_{\mathrm{j}}\right) = \int_{a_{\mathrm{j}-1}}^{a_{\mathrm{j}}} f\left(x\right)\,\mathrm{d}x$$

are equal. Suppose we decide to use $k = 8$ cells. For the standard normal distribution, the intervals that divide the scale into eight equally likely segments are [0, 0.32), [0.32, 0.675) [0.675, 1.15), [1.15, ∞) and their four (4) "mirror image" intervals on the other side of zero. For each interval $p_i = 1/8 = 0.125$, so the expected cell frequencies are $e_j = np_j = 100(0.125) = 12.5$. The complete table of observed and expected frequencies is as follows:

Class Interval	Observed Frequency (b_{j})	Expected Frequency (e_{j})
$x < 4.948$	12	12.5
$4.948 \leqq x < 4.986$	14	12.5
$4.986 \leqq x < 5.014$	12	12.5
$5.014 \leqq x < 5.040$	13	12.5
$5.040 \leqq x < 5.066$	12	12.5
$5.066 \leqq x < 5.094$	11	12.5
$5.094 \leqq x < 5.132$	12	12.5
$5.132 \leqq x$	14	12.5

The boundary of the first class interval is $\overline{x} - 1.15s = 4,948$. The second class interval is $[\overline{x} - 1.15s, \overline{x} - 0.675s)$ and so forth. We may apply the hypothesis-testing procedure to this problem.

1. The variable of interest is the form of the distribution of power supply voltage.
2. θ_0 The form of the distribution is normal.
3. θ_1 The form of the distribution is not normal.
4. Test statistic is:

$$\chi_0^2 = \sum_{j=1}^{k} \frac{\left(b_{\mathrm{j}} - e_{\mathrm{j}}\right)^2}{e_{\mathrm{j}}}$$

5. Since two parameters in the normal distribution have been estimated, the chi-square statistic above will have $k - p - 1 = 8 - 2 - 1 = 5$ degrees of freedom. Therefore, we will reject χ_0 if $\chi_0^2 > \chi_{0.05,5}^2 = 11.07$.

6. Calculating χ_0^2:

$$\begin{aligned}\chi_0^2 &= \sum_{j=1}^{8} \frac{(b_\mathrm{j} - e_\mathrm{j})^2}{e_\mathrm{j}} \\ &= \frac{(12 - 12.5)^2}{12.5} + \frac{(14 - 12.5)^2}{12.5} + \cdots + \frac{(14 - 12.5)^2}{12.5} \\ &= 0.64\end{aligned}$$

7. Since $\chi_0^2 = 0.64 < \chi_{0.05,5}^2 = 11.07$, we are unable to reject θ_0, and there is no strong evidence to indicate that output voltage is not normally distributed ■

2

Tables

2.1 Introduction

The following are tables used in solving the exercises present in this book.

2.2 Special Functions

2.2.1 Bessel Function

2.3 CDF Normal Distribution

Table 2.1: Values of z for given values of the distribution function Φ(z) with $\Phi(-z) = 1 - \Phi(z)$.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.01	0.504	0.51	0.695	1.01	0.844	1.51	0.934	2.01	0.978	2.51	0.994
0.02	0.508	0.52	0.698	1.02	0.846	1.52	0.936	2.02	0.978	2.52	0.994
0.03	0.512	0.53	0.702	1.03	0.848	1.53	0.937	2.03	0.979	2.53	0.994
0.04	0.516	0.54	0.705	1.04	0.851	1.54	0.938	2.04	0.979	2.54	0.994
0.05	0.52	0.55	0.709	1.05	0.853	1.55	0.939	2.05	0.98	2.55	0.995
0.06	0.524	0.56	0.712	1.06	0.855	1.56	0.941	2.06	0.98	2.56	0.995
0.07	0.528	0.57	0.716	1.07	0.858	1.57	0.942	2.07	0.981	2.57	0.995
0.08	0.532	0.58	0.719	1.08	0.86	1.58	0.943	2.08	0.981	2.58	0.995
0.09	0.536	0.59	0.722	1.09	0.862	1.59	0.944	2.09	0.982	2.59	0.995
0.1	0.54	0.6	0.726	1.1	0.864	1.6	0.945	2.1	0.982	2.6	0.995
0.11	0.544	0.61	0.729	1.11	0.867	1.61	0.946	2.11	0.983	2.61	0.995
0.12	0.548	0.62	0.732	1.12	0.869	1.62	0.947	2.12	0.983	2.62	0.996

Continued on next page

Table 2.1: Values of z for given values of the distribution function Φ(z) with $\Phi(-z) = 1 - \Phi(z)$. (Continued)

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.13	0.552	0.63	0.736	1.13	0.871	1.63	0.948	2.13	0.983	2.63	0.996
0.14	0.556	0.64	0.739	1.14	0.873	1.64	0.949	2.14	0.984	2.64	0.996
0.15	0.56	0.65	0.742	1.15	0.875	1.65	0.951	2.15	0.984	2.65	0.996
0.16	0.564	0.66	0.745	1.16	0.877	1.66	0.952	2.16	0.985	2.66	0.996
0.17	0.567	0.67	0.749	1.17	0.879	1.67	0.953	2.17	0.985	2.67	0.996
0.18	0.571	0.68	0.752	1.18	0.881	1.68	0.954	2.18	0.985	2.68	0.996
0.19	0.575	0.69	0.755	1.19	0.883	1.69	0.954	2.19	0.986	2.69	0.996
0.2	0.579	0.7	0.758	1.2	0.885	1.7	0.955	2.2	0.986	2.7	0.997
0.21	0.583	0.71	0.761	1.21	0.887	1.71	0.956	2.21	0.986	2.71	0.997
0.22	0.587	0.72	0.764	1.22	0.889	1.72	0.957	2.22	0.987	2.72	0.997
0.23	0.591	0.73	0.767	1.23	0.891	1.73	0.958	2.23	0.987	2.73	0.997
0.24	0.595	0.74	0.77	1.24	0.893	1.74	0.959	2.24	0.987	2.74	0.997
0.25	0.599	0.75	0.773	1.25	0.894	1.75	0.96	2.25	0.988	2.75	0.997
0.26	0.603	0.76	0.776	1.26	0.896	1.76	0.961	2.26	0.988	2.76	0.997
0.27	0.606	0.77	0.779	1.27	0.898	1.77	0.962	2.27	0.988	2.77	0.997
0.28	0.61	0.78	0.782	1.28	0.9	1.78	0.962	2.28	0.989	2.78	0.997
0.29	0.614	0.79	0.785	1.29	0.901	1.79	0.963	2.29	0.989	2.79	0.997
0.3	0.618	0.8	0.788	1.3	0.903	1.8	0.964	2.3	0.989	2.8	0.997
0.31	0.622	0.81	0.791	1.31	0.905	1.81	0.965	2.31	0.99	2.81	0.998
0.32	0.626	0.82	0.794	1.32	0.907	1.82	0.966	2.32	0.99	2.82	0.998
0.33	0.629	0.83	0.797	1.33	0.908	1.83	0.966	2.33	0.99	2.83	0.998
0.34	0.633	0.84	0.8	1.34	0.91	1.84	0.967	2.34	0.99	2.84	0.998
0.35	0.637	0.85	0.802	1.35	0.911	1.85	0.968	2.35	0.991	2.85	0.998
0.36	0.641	0.86	0.805	1.36	0.913	1.86	0.969	2.36	0.991	2.86	0.998
0.37	0.644	0.87	0.808	1.37	0.915	1.87	0.969	2.37	0.991	2.87	0.998
0.38	0.648	0.88	0.811	1.38	0.916	1.88	0.97	2.38	0.991	2.88	0.998
0.39	0.652	0.89	0.813	1.39	0.918	1.89	0.971	2.39	0.992	2.89	0.998
0.4	0.655	0.9	0.816	1.4	0.919	1.9	0.971	2.4	0.992	2.9	0.998

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Table 2.1: Values of z for given values of the distribution function Φ(z) with $\Phi(-z) = 1-\Phi(z)$. (Continued)

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.41	0.659	0.91	0.819	1.41	0.921	1.91	0.972	2.41	0.992	2.91	0.998
0.42	0.663	0.92	0.821	1.42	0.922	1.92	0.973	2.42	0.992	2.92	0.998
0.43	0.666	0.93	0.824	1.43	0.924	1.93	0.973	2.43	0.992	2.93	0.998
0.44	0.67	0.94	0.826	1.44	0.925	1.94	0.974	2.44	0.993	2.94	0.998
0.45	0.674	0.95	0.829	1.45	0.926	1.95	0.974	2.45	0.993	2.95	0.998
0.46	0.677	0.96	0.831	1.46	0.928	1.96	0.975	2.46	0.993	2.96	0.998
0.47	0.681	0.97	0.834	1.47	0.929	1.97	0.976	2.47	0.993	2.97	0.999
0.48	0.684	0.98	0.836	1.48	0.931	1.98	0.976	2.48	0.993	2.98	0.999
0.49	0.688	0.99	0.839	1.49	0.932	1.99	0.977	2.49	0.994	2.99	0.999
0.5	0.691	1.0	0.841	1.5	0.933	2.0	0.977	2.5	0.994	3.0	0.999

2.4 Student-t Distribution

Table 2.2: Values of z for given values of the distribution function F(z) with $m = 1 - 10$.

	Degrees of Freedom									
$F(z)$	1	2	3	4	5	6	7	8	9	10
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	0.32	0.29	0.28	0.27	0.27	0.26	0.26	0.26	0.26	0.26
0.7	0.73	0.62	0.58	0.57	0.56	0.55	0.55	0.55	0.54	0.54
0.8	1.38	1.06	0.98	0.94	0.92	0.91	0.9	0.89	0.88	0.88
0.9	3.08	1.89	1.64	1.53	1.48	1.44	1.41	1.4	1.38	1.37
0.95	6.31	2.92	2.35	2.13	2.02	1.94	1.89	1.86	1.83	1.81
0.975	12.71	4.3	3.18	2.78	2.57	2.45	2.36	2.31	2.26	2.23
0.99	31.82	6.96	4.54	3.75	3.36	3.14	3.0	2.9	2.82	2.76
0.995	63.66	9.92	5.84	4.6	4.03	3.71	3.5	3.36	3.25	3.17
0.995	63.66	9.92	5.84	4.6	4.03	3.71	3.5	3.36	3.25	3.17
0.999	318.31	22.33	10.21	7.17	5.89	5.21	4.79	4.5	4.3	4.14

Table 2.3: Values of z for given values of the distribution function F(z) with $m = 11 - 20$.

	Degrees of Freedom									
$F(z)$	11	12	13	14	15	16	17	18	19	20
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
0.7	0.54	0.54	0.54	0.54	0.54	0.54	0.53	0.53	0.53	0.53
0.8	0.88	0.87	0.87	0.87	0.87	0.86	0.86	0.86	0.86	0.86
0.9	1.36	1.36	1.35	1.35	1.34	1.34	1.33	1.33	1.33	1.33
0.95	1.8	1.78	1.77	1.76	1.75	1.75	1.74	1.73	1.73	1.72
0.975	2.2	2.18	2.16	2.14	2.13	2.12	2.11	2.1	2.09	2.09
0.99	2.72	2.68	2.65	2.62	2.6	2.58	2.57	2.55	2.54	2.53
0.995	3.11	3.05	3.01	2.98	2.95	2.92	2.9	2.88	2.86	2.85
0.995	3.11	3.05	3.01	2.98	2.95	2.92	2.9	2.88	2.86	2.85
0.999	4.02	3.93	3.85	3.79	3.73	3.69	3.65	3.61	3.58	3.55

Table 2.4: Values of z for given values of the distribution function F(z) with $m = 21 - 30$.

	Degrees of Freedom (m)									
$F(z)$	21	22	23	24	25	26	27	28	29	30
0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
0.7	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
0.8	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.85	0.85	0.85
0.9	1.32	1.32	1.32	1.32	1.32	1.31	1.31	1.31	1.31	1.31
0.95	1.72	1.72	1.71	1.71	1.71	1.71	1.7	1.7	1.7	1.7
0.975	2.08	2.07	2.07	2.06	2.06	2.06	2.05	2.05	2.05	2.04
0.99	2.52	2.51	2.5	2.49	2.49	2.48	2.47	2.47	2.46	2.46
0.995	2.83	2.82	2.81	2.8	2.79	2.78	2.77	2.76	2.76	2.75
0.995	2.83	2.82	2.81	2.8	2.79	2.78	2.77	2.76	2.76	2.75
0.999	3.53	3.5	3.48	3.47	3.45	3.43	3.42	3.41	3.4	3.39

2.5 Chi-Square Distribution

Table 2.5: Values of z for given values of the distribution function F(z) with $m = 1 - 10$.

$F(z)$	Degrees of Freedom (m)									
	1	2	3	4	5	6	7	8	9	10
0.005	0.0	0.01	0.07	0.21	0.41	0.68	0.99	1.34	1.73	2.16
0.01	0.0	0.02	0.11	0.3	0.55	0.87	1.24	1.65	2.09	2.56
0.025	0.0	0.05	0.22	0.48	0.83	1.24	1.69	2.18	2.7	3.25
0.05	0.0	0.1	0.35	0.71	1.15	1.64	2.17	2.73	3.33	3.94
0.95	3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92	18.31
0.975	5.02	7.38	9.35	11.14	12.83	14.45	16.01	17.53	19.02	20.48
0.99	6.63	9.21	11.34	13.28	15.09	16.81	18.48	20.09	21.67	23.21
0.995	7.88	10.6	12.84	14.86	16.75	18.55	20.28	21.95	23.59	25.19

Table 2.6: Values of z for given values of the distribution function F(z) with $m = 11 - 20$.

$F(z)$	Degrees of Freedom (m)									
	11	12	13	14	15	16	17	18	19	20
0.005	2.6	3.07	3.57	4.07	4.6	5.14	5.7	6.26	6.84	7.43
0.01	3.05	3.57	4.11	4.66	5.23	5.81	6.41	7.01	7.63	8.26
0.025	3.82	4.4	5.01	5.63	6.26	6.91	7.56	8.23	8.91	9.59
0.05	4.57	5.23	5.89	6.57	7.26	7.96	8.67	9.39	10.12	10.85
0.95	19.68	21.03	22.36	23.68	25.0	26.3	27.59	28.87	30.14	31.41
0.975	21.92	23.34	24.74	26.12	27.49	28.85	30.19	31.53	32.85	34.17
0.99	24.72	26.22	27.69	29.14	30.58	32.0	33.41	34.81	36.19	37.57
0.995	26.76	28.3	29.82	31.32	32.8	34.27	35.72	37.16	38.58	40.0

Table 2.7: Values of z for given values of the distribution function F(z) with $m = 21 - 30$.

	Degrees of Freedom (m)									
$F(z)$	21	22	23	24	25	26	27	28	29	30
0.005	8.03	8.64	9.26	9.89	10.52	11.16	11.81	12.46	13.12	13.79
0.01	8.9	9.54	10.2	10.86	11.52	12.2	12.88	13.56	14.26	14.95
0.025	10.28	10.98	11.69	12.4	13.12	13.84	14.57	15.31	16.05	16.79
0.05	11.59	12.34	13.09	13.85	14.61	15.38	16.15	16.93	17.71	18.49
0.95	32.67	33.92	35.17	36.42	37.65	38.89	40.11	41.34	42.56	43.77
0.975	35.48	36.78	38.08	39.36	40.65	41.92	43.19	44.46	45.72	46.98
0.99	38.93	40.29	41.64	42.98	44.31	45.64	46.96	48.28	49.59	50.89
0.995	41.4	42.8	44.18	45.56	46.93	48.29	49.64	50.99	52.34	53.67

2.6 F-Distribution

Table 2.8: Values of z for which the distribution function F(z) has the value 0.95

	Degrees of Freedom (m)								
n	1	2	3	4	5	6	7	8	9
1	161.45	199.5	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.0	19.16	19.25	19.3	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.0
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.1
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.5	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.1	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.2	3.09	3.01	2.95	2.9
12	4.75	3.89	3.49	3.26	3.11	3.0	2.91	2.85	2.8
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71

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Table 2.8: Values of z for which the distribution function F(z) has the value 0.95 (Continued)

	Degrees of Freedom (m)								
n	1	2	3	4	5	6	7	8	9
14	4.6	3.74	3.34	3.11	2.96	2.85	2.76	2.7	2.65
15	4.54	3.68	3.29	3.06	2.9	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.2	2.96	2.81	2.7	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.9	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.1	2.87	2.71	2.6	2.51	2.45	2.39
22	4.3	3.44	3.05	2.82	2.66	2.55	2.46	2.4	2.34
24	4.26	3.4	3.01	2.78	2.62	2.51	2.42	2.36	2.3
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
28	4.2	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
32	4.15	3.29	2.9	2.67	2.51	2.4	2.31	2.24	2.19
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15
38	4.1	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.14
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
50	4.03	3.18	2.79	2.56	2.4	2.29	2.2	2.13	2.07
60	4.0	3.15	2.76	2.53	2.37	2.25	2.17	2.1	2.04
70	3.98	3.13	2.74	2.5	2.35	2.23	2.14	2.07	2.02
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.0
90	3.95	3.1	2.71	2.47	2.32	2.2	2.11	2.04	1.99
100	3.94	3.09	2.7	2.46	2.31	2.19	2.1	2.03	1.97
100	3.94	3.09	2.7	2.46	2.31	2.19	2.1	2.03	1.97
150	3.9	3.06	2.66	2.43	2.27	2.16	2.07	2.0	1.94
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
1000	3.85	3.0	2.61	2.38	2.22	2.11	2.02	1.95	1.89

Table 2.9: Values of z for which the distribution function F(z) has the value 0.95

	Degrees of Freedom (m)								
n	10	15	20	30	40	50	100	200	500
1	241.88	245.95	248.01	250.1	251.14	251.77	253.04	253.68	254.06
2	19.4	19.43	19.45	19.46	19.47	19.48	19.49	19.49	19.49
3	8.79	8.7	8.66	8.62	8.59	8.58	8.55	8.54	8.53
4	5.96	5.86	5.8	5.75	5.72	5.7	5.66	5.65	5.64
5	4.74	4.62	4.56	4.5	4.46	4.44	4.41	4.39	4.37
6	4.06	3.94	3.87	3.81	3.77	3.75	3.71	3.69	3.68
7	3.64	3.51	3.44	3.38	3.34	3.32	3.27	3.25	3.24
8	3.35	3.22	3.15	3.08	3.04	3.02	2.97	2.95	2.94
9	3.14	3.01	2.94	2.86	2.83	2.8	2.76	2.73	2.72
10	2.98	2.85	2.77	2.7	2.66	2.64	2.59	2.56	2.55
11	2.85	2.72	2.65	2.57	2.53	2.51	2.46	2.43	2.42
12	2.75	2.62	2.54	2.47	2.43	2.4	2.35	2.32	2.31
13	2.67	2.53	2.46	2.38	2.34	2.31	2.26	2.23	2.22
14	2.6	2.46	2.39	2.31	2.27	2.24	2.19	2.16	2.14
15	2.54	2.4	2.33	2.25	2.2	2.18	2.12	2.1	2.08
16	2.49	2.35	2.28	2.19	2.15	2.12	2.07	2.04	2.02
17	2.45	2.31	2.23	2.15	2.1	2.08	2.02	1.99	1.97
18	2.41	2.27	2.19	2.11	2.06	2.04	1.98	1.95	1.93
19	2.38	2.23	2.16	2.07	2.03	2.0	1.94	1.91	1.89
20	2.35	2.2	2.12	2.04	1.99	1.97	1.91	1.88	1.86
22	2.3	2.15	2.07	1.98	1.94	1.91	1.85	1.82	1.8
24	2.25	2.11	2.03	1.94	1.89	1.86	1.8	1.77	1.75
26	2.22	2.07	1.99	1.9	1.85	1.82	1.76	1.73	1.71
28	2.19	2.04	1.96	1.87	1.82	1.79	1.73	1.69	1.67
30	2.16	2.01	1.93	1.84	1.79	1.76	1.7	1.66	1.64
32	2.14	1.99	1.91	1.82	1.77	1.74	1.67	1.63	1.61
34	2.12	1.97	1.89	1.8	1.75	1.71	1.65	1.61	1.59

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Table 2.9: Values of z for which the distribution function F(z) has the value 0.95 (Continued)

	Degrees of Freedom (m)								
n	10	15	20	30	40	50	100	200	500
36	2.11	1.95	1.87	1.78	1.73	1.69	1.62	1.59	1.56
38	2.09	1.94	1.85	1.76	1.71	1.68	1.61	1.57	1.54
40	2.08	1.92	1.84	1.74	1.69	1.66	1.59	1.55	1.53
50	2.03	1.87	1.78	1.69	1.63	1.6	1.52	1.48	1.46
60	1.99	1.84	1.75	1.65	1.59	1.56	1.48	1.44	1.41
70	1.97	1.81	1.72	1.62	1.57	1.53	1.45	1.4	1.37
80	1.95	1.79	1.7	1.6	1.54	1.51	1.43	1.38	1.35
90	1.94	1.78	1.69	1.59	1.53	1.49	1.41	1.36	1.33
100	1.93	1.77	1.68	1.57	1.52	1.48	1.39	1.34	1.31
100	1.93	1.77	1.68	1.57	1.52	1.48	1.39	1.34	1.31
150	1.89	1.73	1.64	1.54	1.48	1.44	1.34	1.29	1.25
200	1.88	1.72	1.62	1.52	1.46	1.41	1.32	1.26	1.22
1000	1.84	1.68	1.58	1.47	1.41	1.36	1.26	1.19	1.13

Table 2.10: Values of z for which the distribution function F(z) has the value 0.99

	Degrees of Freedom (m)								
n	1	2	3	4	5	6	7	8	9
1	4052.18	4999.5	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47
2	98.5	99.0	99.17	99.25	99.3	99.33	99.36	99.37	99.39
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
4	21.2	18.0	16.69	15.98	15.52	15.21	14.98	14.8	14.66
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.1	7.98
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
9	10.56	8.02	6.99	6.42	6.06	5.8	5.61	5.47	5.35

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Table 2.10: Values of z for which the distribution function F(z) has the value 0.99 (Continued)

	Degrees of Freedom (m)								
n	1	2	3	4	5	6	7	8	9
10	10.04	7.56	6.55	5.99	5.64	5.39	5.2	5.06	4.94
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.5	4.39
13	9.07	6.7	5.74	5.21	4.86	4.62	4.44	4.3	4.19
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.0	3.89
16	8.53	6.23	5.29	4.77	4.44	4.2	4.03	3.89	3.78
17	8.4	6.11	5.18	4.67	4.34	4.1	3.93	3.79	3.68
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.6
19	8.18	5.93	5.01	4.5	4.17	3.94	3.77	3.63	3.52
20	8.1	5.85	4.94	4.43	4.1	3.87	3.7	3.56	3.46
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
24	7.82	5.61	4.72	4.22	3.9	3.67	3.5	3.36	3.26
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
30	7.56	5.39	4.51	4.02	3.7	3.47	3.3	3.17	3.07
32	7.5	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98
36	7.4	5.25	4.38	3.89	3.57	3.35	3.18	3.05	2.95
38	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.92
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
50	7.17	5.06	4.2	3.72	3.41	3.19	3.02	2.89	2.78
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
70	7.01	4.92	4.07	3.6	3.29	3.07	2.91	2.78	2.67
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61
100	6.9	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59

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Table 2.10: Values of z for which the distribution function F(z) has the value 0.99 (Continued)

	Degrees of Freedom (m)								
n	1	2	3	4	5	6	7	8	9
100	6.9	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.6	2.5
1000	6.66	4.63	3.8	3.34	3.04	2.82	2.66	2.53	2.43

Table 2.11: Values of z for which the distribution function F(z) has the value 0.99

	Degrees of Freedom (m)								
n	10	15	20	30	40	50	100	200	500
1	6055.85	6157.28	6208.73	6260.65	6286.78	6302.52	6334.11	6349.97	6359.5
2	99.4	99.43	99.45	99.47	99.47	99.48	99.49	99.49	99.5
3	27.23	26.87	26.69	26.5	26.41	26.35	26.24	26.18	26.15
4	14.55	14.2	14.02	13.84	13.75	13.69	13.58	13.52	13.49
5	10.05	9.72	9.55	9.38	9.29	9.24	9.13	9.08	9.04
6	7.87	7.56	7.4	7.23	7.14	7.09	6.99	6.93	6.9
7	6.62	6.31	6.16	5.99	5.91	5.86	5.75	5.7	5.67
8	5.81	5.52	5.36	5.2	5.12	5.07	4.96	4.91	4.88
9	5.26	4.96	4.81	4.65	4.57	4.52	4.41	4.36	4.33
10	4.85	4.56	4.41	4.25	4.17	4.12	4.01	3.96	3.93
11	4.54	4.25	4.1	3.94	3.86	3.81	3.71	3.66	3.62
12	4.3	4.01	3.86	3.7	3.62	3.57	3.47	3.41	3.38
13	4.1	3.82	3.66	3.51	3.43	3.38	3.27	3.22	3.19
14	3.94	3.66	3.51	3.35	3.27	3.22	3.11	3.06	3.03
15	3.8	3.52	3.37	3.21	3.13	3.08	2.98	2.92	2.89
16	3.69	3.41	3.26	3.1	3.02	2.97	2.86	2.81	2.78
17	3.59	3.31	3.16	3.0	2.92	2.87	2.76	2.71	2.68
18	3.51	3.23	3.08	2.92	2.84	2.78	2.68	2.62	2.59

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Table 2.11: Values of z for which the distribution function F(z) has the value 0.99 (Continued)

	Degrees of Freedom (m)								
n	10	15	20	30	40	50	100	200	500
19	3.43	3.15	3.0	2.84	2.76	2.71	2.6	2.55	2.51
20	3.37	3.09	2.94	2.78	2.69	2.64	2.54	2.48	2.44
22	3.26	2.98	2.83	2.67	2.58	2.53	2.42	2.36	2.33
24	3.17	2.89	2.74	2.58	2.49	2.44	2.33	2.27	2.24
26	3.09	2.81	2.66	2.5	2.42	2.36	2.25	2.19	2.16
28	3.03	2.75	2.6	2.44	2.35	2.3	2.19	2.13	2.09
30	2.98	2.7	2.55	2.39	2.3	2.25	2.13	2.07	2.03
32	2.93	2.65	2.5	2.34	2.25	2.2	2.08	2.02	1.98
34	2.89	2.61	2.46	2.3	2.21	2.16	2.04	1.98	1.94
36	2.86	2.58	2.43	2.26	2.18	2.12	2.0	1.94	1.9
38	2.83	2.55	2.4	2.23	2.14	2.09	1.97	1.9	1.86
40	2.8	2.52	2.37	2.2	2.11	2.06	1.94	1.87	1.83
50	2.7	2.42	2.27	2.1	2.01	1.95	1.82	1.76	1.71
60	2.63	2.35	2.2	2.03	1.94	1.88	1.75	1.68	1.63
70	2.59	2.31	2.15	1.98	1.89	1.83	1.7	1.62	1.57
80	2.55	2.27	2.12	1.94	1.85	1.79	1.65	1.58	1.53
90	2.52	2.24	2.09	1.92	1.82	1.76	1.62	1.55	1.49
100	2.5	2.22	2.07	1.89	1.8	1.74	1.6	1.52	1.47
100	2.5	2.22	2.07	1.89	1.8	1.74	1.6	1.52	1.47
150	2.44	2.16	2.0	1.83	1.73	1.66	1.52	1.43	1.38
200	2.41	2.13	1.97	1.79	1.69	1.63	1.48	1.39	1.33
1000	2.34	2.06	1.9	1.72	1.61	1.54	1.38	1.28	1.19