Engineers like to solve problems. If there are no problems handily available, they will create their own problems. - Scott AdamsTopics on Electrical Engineering

Questions

1. We have seen a three-phase rotating magnetic field (RMF) and how it was generated. For this exercise, here is a six-phase current waveform input. Please calculate the generated magnetic flux density ($\mathbf{B}_{\text{net}}$). The currents flowing through each phases (A, B, C, D, E, F) are as follows:

$$i_{\text{AA'}} = I_{\text{max}} \sin(\omega t), \qquad i_{\text{BB'}} = I_{\text{max}} \sin(\omega t - 60^\circ),$$
$$i_{\text{CC'}} = I_{\text{max}} \sin(\omega t - 120^\circ), \qquad i_{\text{DD'}} = I_{\text{max}} \sin(\omega t - 180^\circ),$$
$$i_{\text{EE'}} = I_{\text{max}} \sin(\omega t - 240^\circ), \qquad i_{\text{FF'}} = I_{\text{max}} \sin(\omega t - 300^\circ).$$

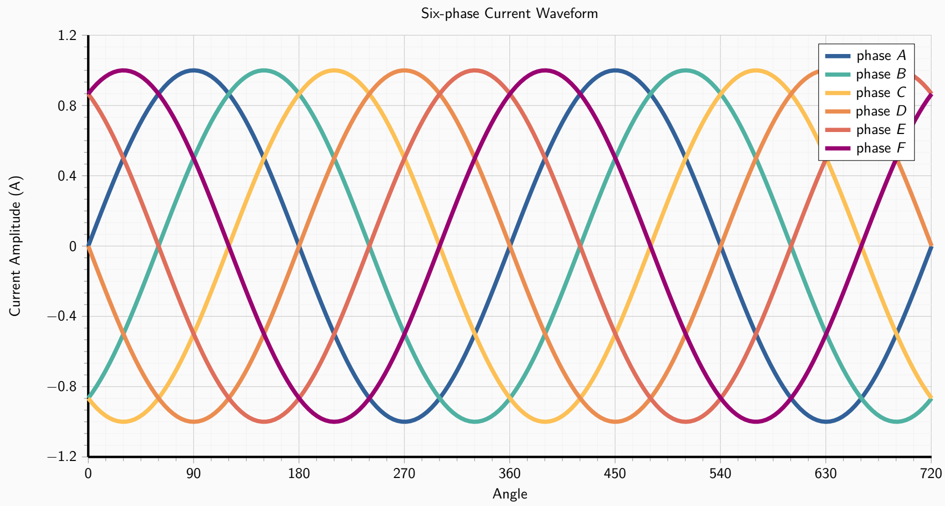


Figure 1: A graphic representation of a six-phase current waveform.

2. As an engineering consultant on electric machine design, you have been contacted to design a 10-phase rotating magnetic field for use in high power applications. For this exercise, show the diagram of how it should be wound.

3. An induction machine has the following parameters:

$$R_{\text{r}} = 0.183 \quad \Omega, \quad R_{\text{s}} = 0.277 \quad \Omega, \quad \text{Y-connected}, \quad V_{\text{in}} = 200 \quad \text{V}, \quad 2p = 4$$
$$L_{\text{r}} = 0.056 \quad \text{H}, \quad L_{\text{s}} = 0.0553 \quad \text{H}, \quad 3-\text{phase}, \quad f_{\text{in}} = 60 \quad \text{Hz} \quad a = 3.$$

where a is the effective stator to rotor turns ratio and p is the number of pole pairs. The motor is supplied with its rated and balanced voltages. Find the **q** and **d** axes steady-state voltages and currents and phase currents $\left(I_{\text{qrr}}, I_{\text{drr}}, I_{\alpha}, I_{\beta}\right)$ when the rotor is locked, Use the stator reference from model of the induction machine.

4. A three-phase wye (Y)-connected 220-V, 7.5-kW 60Hz six-pole induction motor has the following parameter values in Ω/phase referred to the stator:

$$R_1 = 0.294 \quad \Omega, \quad R_2 = 0.294 \quad \Omega,$$
$$X_1 = 0.503 \quad \Omega, \quad X_2 = 0.209 \quad \Omega, \quad X_{\text{m}} = 13.25 \quad \Omega.$$

The total friction, windage, and core losses are assumed to be constant at 403 W, independent of the load. For a slip of 2 percent, compute the speed, output torque and power, stator current, power factor, and efficiency when the motor is operated at rated voltage and frequency.

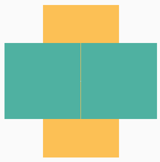
5. The three-phase, 230 V, 60 Hz, 12 kW, four-pole IM is to be driven by a field-oriented speed-control system at a speed of 1740 min^{-1} with following properties:

$$R_1 = 0.095 \quad \Omega, \quad R_2 = 0.200 \quad \Omega,$$

$$X_1 = 0.680 \quad \Omega, \quad X_2 = 0.672 \quad \Omega, \quad X_{\text{m}} = 18.7 \quad \Omega.$$

Assuming the controller is programmed to set the rotor flux linkages (λ_{dr}) to the machine rated peak value, find the RMS[1] amplitude of the input current (I_{in}), the applied electrical frequency (f_{e}), and the RMS terminal voltage, if the electromagnetic power is 9.7 kW and the motor is operating at a speed of 1680 min^{-1}.

[1] Root-Mean Square



Answers

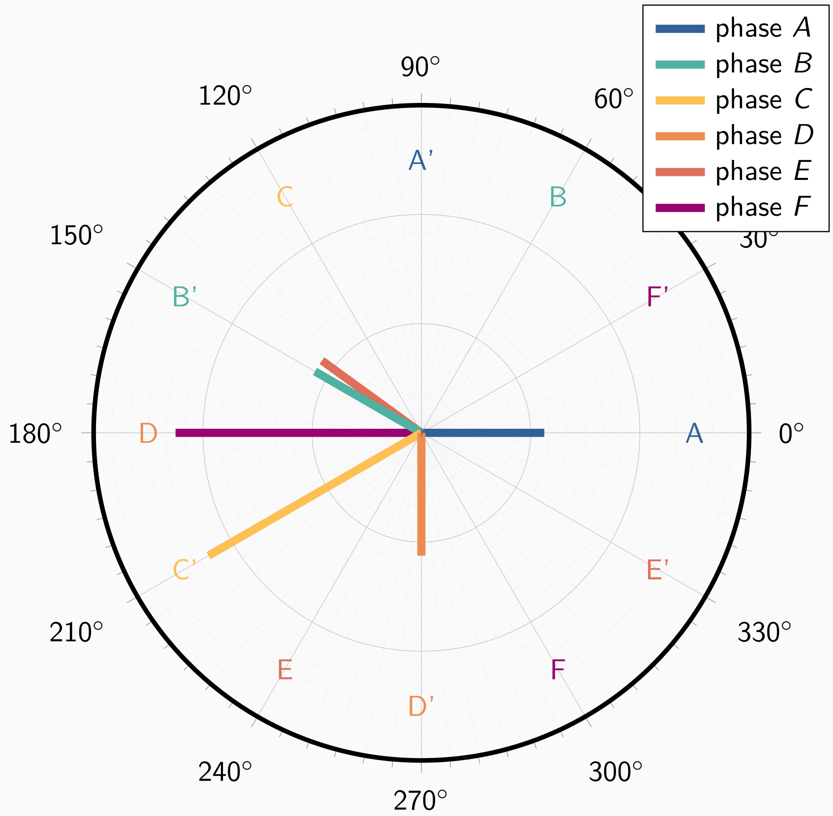


Figure 2: Polar coordinate form of the acting magnetic flux densities for the six phases at the electrical angle of $\theta_e = 30°$.

1. Let us first consider, what happens at the time instant of the voltage waveforms. Unlike a three-phase system, at the instant $\theta = 90°$, the voltage in phase **A** is positive maximum (1.0), phases **B** and **F** are half amplitude (0.5), **E** and **C** are also half amplitude but opposite sign (-0.5) and phase **D** is negative maximum (-1.0).

   These six (6) waveforms are represented by the following equations:

$$v_{\mathbf{AA'}} = V_{\max} \sin(\omega t)\,, \qquad v_{\mathbf{BB'}} = V_{\max} \sin(\omega\, t - 60°)\,, \tag{1a}$$

$$v_{\mathbf{CC'}} = V_{\max} \sin(\omega\, t - 120°)\,, \qquad v_{\mathbf{DD'}} = V_{\max} \sin(\omega\, t - 180°)\,, \tag{1b}$$

$$v_{\mathbf{EE'}} = V_{\max} \sin(\omega\, t - 240°)\,, \qquad v_{\mathbf{FF'}} = V_{\max} \sin(\omega\, t - 300°)\,. \tag{1c}$$

   where $\omega = 2\pi \quad \text{rad}\cdot\text{s}^{-1}$, $V_{\max}$ is the maximum value of the voltage or induced EMF in each phase and f is the supply frequency (Hz). If the flux densities (or fluxes (ϕ) as both are valid approaches) are added to find the resultant magnetic flux density in phasor form the magnitude is found as;

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= B_{\mathbf{AA'}} \sin(\omega t) \,\angle 0^\circ + B_{\mathbf{BB'}} \sin(\omega\, t - 60^\circ) \,\angle 60^\circ \\
&\quad + B_{\mathbf{CC'}} \sin(\omega\, t - 120^\circ) \,\angle 120^\circ + B_{\mathbf{DD'}} \sin(\omega\, t - 180^\circ) \,\angle 180^\circ \\
&\quad + B_{\mathbf{EE'}} \sin(\omega\, t - 240^\circ) \,\angle 240^\circ + B_{\mathbf{FF'}} \sin(\omega\, t - 300^\circ) \,\angle 300^\circ,
\end{aligned} \tag{2a}
$$

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= B_{\text{max}} \sin(\omega t) \left[\hat{\mathbf{x}} \cos(0^\circ) + \hat{\mathbf{y}} \sin(0)\right] \\
&\quad + B_{\text{max}} \sin(\omega\, t - 60^\circ) \left[\hat{\mathbf{x}} \cos(60^\circ) + \hat{\mathbf{y}} \sin(60^\circ)\right] \\
&\quad + B_{\text{max}} \sin(\omega\, t - 120^\circ) \left[\hat{\mathbf{x}} \cos(120^\circ) + \hat{\mathbf{y}} \sin(120^\circ)\right] \\
&\quad + B_{\text{max}} \sin(\omega\, t - 180^\circ) \left[\hat{\mathbf{x}} \cos(180^\circ) + \hat{\mathbf{y}} \sin(180^\circ)\right] \\
&\quad + B_{\text{max}} \sin(\omega\, t - 240^\circ) \left[\hat{\mathbf{x}} \cos(240^\circ) + \hat{\mathbf{y}} \sin(240^\circ)\right] \\
&\quad + B_{\text{max}} \sin(\omega\, t - 300^\circ) \left[\hat{\mathbf{x}} \cos(300^\circ) + \hat{\mathbf{y}} \sin(300^\circ)\right]
\end{aligned} \tag{2b}
$$

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= B_{\text{max}} \times \sin(\omega t) \times \left[1\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}}\right] \\
&\quad + B_{\text{max}} \times \sin(\omega\, t - 60^\circ) \times \left[\frac{1}{2}\,\hat{\mathbf{x}} + \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}}\right] \\
&\quad + B_{\text{max}} \times \sin(\omega\, t - 120^\circ) \times \left[-\frac{1}{2}\,\hat{\mathbf{x}} + \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}}\right] \\
&\quad + B_{\text{max}} \times \sin(\omega\, t - 180^\circ) \times \left[-1\,\hat{\mathbf{x}} + 0\,\hat{\mathbf{y}}\right] \\
&\quad + B_{\text{max}} \times \sin(\omega\, t - 240^\circ) \times \left[-\frac{1}{2}\,\hat{\mathbf{x}} - \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}}\right] \\
&\quad + B_{\text{max}} \times \sin(\omega\, t - 300^\circ) \times \left[\frac{1}{2}\,\hat{\mathbf{x}} - \frac{\sqrt{3}}{2}\,\hat{\mathbf{y}}\right]
\end{aligned} \tag{2c}
$$

Let's rearrange the equation by their cardinal components, namely $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$.

$$
\begin{aligned}
\mathbf{B}_{\text{net}} = \Big[& B_{\text{max}} \sin(\omega t) + \frac{1}{2} B_{\text{max}} \sin(\omega\, t - 60^\circ) \\
& - \frac{1}{2} B_{\text{max}} \sin(\omega\, t - 120^\circ) - 1 B_{\text{max}} \sin(\omega\, t - 180^\circ) \\
& - \frac{1}{2} B_{\text{max}} \sin(\omega\, t - 240^\circ) + \frac{1}{2} B_{\text{max}} \sin(\omega\, t - 300^\circ) \Big]\, \hat{\mathbf{x}} \\
& + \Big[\frac{\sqrt{3}}{2} B_{\text{max}} \sin(\omega\, t - 60^\circ) + \frac{\sqrt{3}}{2} B_{\text{max}} \sin(\omega\, t - 120^\circ) \\
& - \frac{\sqrt{3}}{2} B_{\text{max}} \sin(\omega\, t - 240^\circ) - \frac{\sqrt{3}}{2} B_{\text{max}} \sin(\omega\, t - 300^\circ) \Big]\, \hat{\mathbf{y}}
\end{aligned} \tag{3}
$$

We need to do the following trigonometric conversions if we want to simplify the aforementioned equation of a mess

These conversions are based on the following trigonometric identities:

$$\sin(x + 180^\circ) = -\sin(x), \tag{4a}$$

$$\sin(x + 360^\circ) = +\sin(x). \tag{4b}$$

$$\sin(\omega\, t + 300^\circ + 360^\circ) = +\sin(\omega\, t + 60^\circ)\,, \tag{5a}$$

$$\sin(\omega\, t - 240^\circ + 360^\circ) = +\sin(\omega\, t + 120^\circ)\,, \tag{5b}$$

$$\sin(\omega\, t - 180^\circ + 180^\circ) = -\sin(\omega t)\,. \tag{5c}$$

Inserting these equations to Eq. (3),

$$\begin{aligned}
\mathbf{B}_{\,\text{net}} = &\Big[B_{\text{max}}\sin(\omega t) + \frac{1}{2}B_{\text{max}}\sin(\omega\, t - 60^\circ) \\
&- \frac{1}{2}B_{\text{max}}\sin(\omega\, t - 120^\circ) + B_{\text{max}}\sin(\omega t) \\
&- \frac{1}{2}B_{\text{max}}\sin(\omega t + 120^\circ) + \frac{1}{2}B_{\text{max}}\sin(\omega\, t + 60^\circ)\Big]\,\hat{\mathbf{x}} \\
&+ \Big[\frac{\sqrt{3}}{2}B_{\text{max}}\sin(\omega\, t - 60^\circ) + \frac{\sqrt{3}}{2}B_{\text{max}}\sin(\omega\, t - 120^\circ) \\
&- \frac{\sqrt{3}}{2}B_{\text{max}}\sin(\omega t + 120^\circ) - \frac{\sqrt{3}}{2}B_{\text{max}}(\sin(\omega\, t + 60^\circ)\Big]\,\hat{\mathbf{y}}
\end{aligned} \tag{6}$$

Now we can do some tidying up and do some arrangements of these sine values;

$$\begin{aligned}
\mathbf{B}_{\,\text{net}} = &\Big[2B_{\text{max}}\sin(\omega t) + \frac{1}{2}B_{\text{max}}\Big(\sin(\omega\, t - 60^\circ) + \sin(\omega\, t + 60^\circ)\Big) \\
&- \frac{1}{2}B_{\text{max}}\Big(\sin(\omega\, t - 120^\circ) + \sin(\omega t + 120^\circ)\Big)\Big]\,\hat{\mathbf{x}} \\
&+ \Big[-\frac{\sqrt{3}}{2}B_{\text{max}}\Big(\sin(\omega\, t - 60^\circ) - \sin(\omega t + 60^\circ)\Big) \\
&- \frac{\sqrt{3}}{2}B_{\text{max}}\Big(\sin(\omega t + 120^\circ) - \sin(\omega\, t - 120^\circ)\Big)\Big]\,\hat{\mathbf{y}}
\end{aligned} \tag{7}$$

Now it is time to do some trigonometric manipulation

These conversions are based on the following trigonometric identities:

$$\sin(A + B) + \sin(A - B) = 2\sin(A)\cos(B), \tag{8}$$

$$\sin(A + B) - \sin(A - B) = 2\cos(A)\sin(B). \tag{9}$$

and come up with our final equation to explain this rotating magnetic field for a six-phase system.

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= \Big[2B_{\text{max}} \sin(\omega t) + \frac{1}{2}B_{\text{max}}\Big(2\sin(\omega t)\cos(60^\circ)\Big) \\
&\quad - \frac{1}{2}B_{\text{max}}\Big(2\sin(\omega t)\cos(120^\circ)\Big)\Big]\,\hat{\mathbf{x}} \\
&\quad + \Big[-\frac{\sqrt{3}}{2}B_{\text{max}}\Big(2\cos(\omega t)\sin(60^\circ)\Big) \\
&\quad - \frac{\sqrt{3}}{2}B_{\text{max}}\Big(2\sin(\omega t)\cos(120^\circ)\Big)\Big]\,\hat{\mathbf{y}}
\end{aligned} \tag{10a}
$$

$$
\begin{aligned}
\mathbf{B}_{\text{net}} &= B_{\text{max}}\Big[2B_{\text{max}}\sin(\omega t) + 0.5\sin(\omega t) + 0.5\sin(\omega t)\Big]\,\hat{\mathbf{x}} \\
&\quad + B_{\text{max}}\Big[-\frac{3}{2}\cos(\omega t) - \frac{3}{2}\cos(\omega t)\Big]\,\hat{\mathbf{y}}
\end{aligned} \tag{10b}
$$

$$
\mathbf{B}_{\text{net}} = 3B_{\text{max}}\sin(\omega t)\ \hat{\mathbf{x}} - 3B_{\text{max}}\cos(\omega t)\ \hat{\mathbf{y}}, \tag{11}
$$

$$
\mathbf{B}_{\text{net}} = \left(\frac{6}{2}\right) B_{\text{max}}\,[\sin(\omega t)\ \hat{\mathbf{x}} - \cos(\omega t)\ \hat{\mathbf{y}}] \quad \blacksquare \tag{12}
$$

Performance Advantages of Three-Phase Equipment

(i) In addition to saving on wiring, three-phase systems have notable performance advantages over a single-phase system.

(ii) For a given power rating, three-phase drives have superior efficiency than single-phase drives.

(iii) Three-phase drives also have a higher power factor, which means they draw less volt-amperes for a given load and efficiency. Some electricity tariffs include charges for deficient power factor, and three-phase drives can help reduce them.

(iv) Since single-phase systems deliver pulsating power, drives tend to experience more vibration, while the constant supply of three-phase systems results in a more stable operation.

(v) Single-phase drives cannot start by themselves, requiring external devices. On the other hand, a three-phase drives can start with the power supply alone, and it can even reverse direction if you switch two conductors with each other.

(vi) A three-phase system is also **more versatile** than a single-phase installation. If you need to run a single-phase device with a three-phase power supply, you can use only one of the three conductors.

> However, there opposite does not apply: three-phase appliances cannot be operated with single-phase power. Motors are an exception: you can run a three-phase drive with a single-phase power supply, but the drive's mechanical power is drastically reduced and its service life is shortened drastically.

-From NY Engineers

2. We first begin by analysing the current waveforms: The applied phase voltages are as follows:

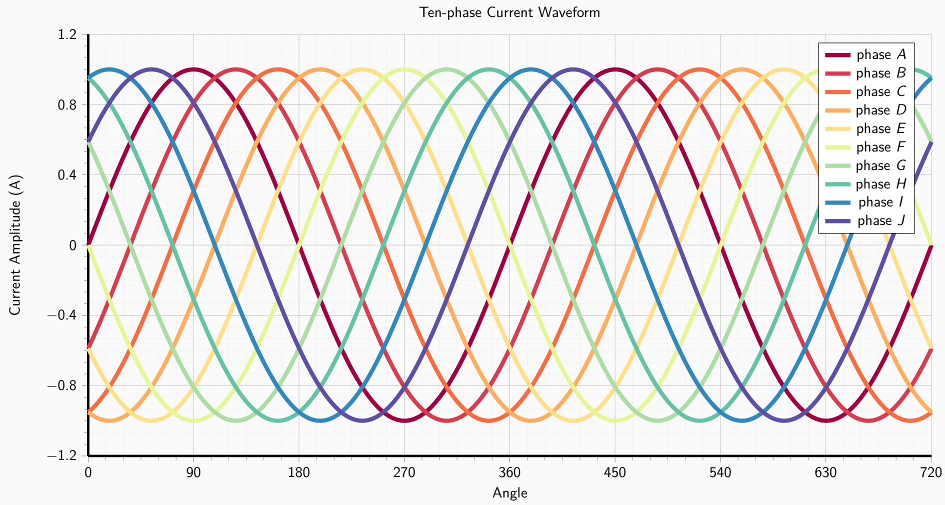


Figure 3: A graphic representation of a ten-phase current waveform.

As can be seen, the base phase (i.e, with a shift of 0) reaches maximum after the 10$^{\text{th}}$ phase.

1 2 3 4 5 6 7 8 9 10

And these phases are electrically 36° degrees apart (θ_e). The connection between the electrical and mechanical angle are as follows:

$$\theta_m = \frac{3}{2}\theta_e,$$

where based on this we can calculate the mechanical angle as $\theta_m = 54°$. Based on this we can draw the following diagram.

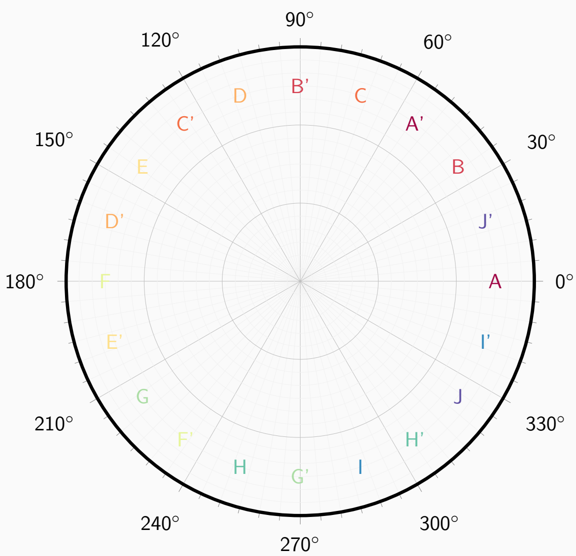


Figure 4: A graphic representation of a ten-phase current waveform.

3. The applied phase voltages are as follows:

$$v_{as} = \frac{200}{\sqrt{3}} \times \sqrt{2} \sin(\omega t) = 163.3 \sin(\omega\, ts) \quad \text{V},$$

$$v_{bs} = 163.3 \sin(\omega\, ts) \, [-120^\circ] \quad \text{V},$$

$$v_{cs} = 163.3 \sin(\omega\, ts) \, [+120^\circ] \quad \text{V}.$$

The **d** and **q** axes voltages are:

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_0 \end{bmatrix} = T^{s}_{abc} \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \quad \text{V}.$$

where,

$$\left[T^{s}_{abc}\right] = \frac{2}{3} \begin{bmatrix} \cos\theta_s & \cos\left(\theta_s - \frac{2\pi}{3}\right) & \cos\left(\theta_s + \frac{2\pi}{3}\right) \\ \sin\theta_s & \sin\left(\theta_s - \frac{2\pi}{3}\right) & \sin\left(\theta_s + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \text{where} \quad \theta_s = 0.$$

We can deduce the following statements based on the matrix multiplication.

$$v_{qs} = \frac{2}{3}\left[v_{as} - \frac{1}{2}(v_{bs} + v_{cs}) \right] \quad \text{V}.$$

For a balanced input the following statement is true:

$$v_{as} + v_{bs} + v_{cs} = 0 \quad \text{V}.$$

Substituting for v_{bs} and v_{cs} in terms of v_{as} presents the following:

$$v_{qs} = \frac{2}{3}\left[\frac{3}{2} v_{as}\right] = v_{as} \quad \text{V}.$$

Similarly:

$$v_{ds} = \frac{1}{\sqrt{3}}(v_{cs} - v_{bs}) \quad \text{V}.$$

and $v_{ds} = 0$,

$$v_{qs} = v_{as} = 163 \sin(\omega\, ts) = 163.3\underline{/0} = 163.3 \quad \text{V},$$

$$v_{ds} = \frac{1}{\sqrt{3}}(v_{cs} - v_{bs}) = 163.3 \cos(\omega_s t) = 163.3\underline{/90} = j163.3 \quad \text{V}.$$

The rotor is locked and therefore we can see:

$$\dot{\theta}_r = 0 \quad \text{rad} \cdot \text{s}^{-1}$$

For steady-state evaluation:

$$p_t = \mathbf{j}\omega_s = \mathbf{j}2\pi f_s = \mathbf{j}2\pi 60 = \mathbf{j}377 \quad \text{rad} \cdot \text{s}^{-1}$$

$$\left[\begin{array}{c} v_{qs} \\ v_{ds} \\ \hline 0 \\ 0 \end{array}\right] = \left[\begin{array}{cc|cc} R_s + L_s p_t & 0 & L_{sr} p_t & 0 \\ 0 & R_s + L_s p_t & 0 & L_{sr} p_t \\ \hline L_{sr} p_t & 0 & R_r + L_r p_t & 0 \\ 0 & L_{sr} p_t & 0 & R_r + L_r p_t \end{array}\right] \left[\begin{array}{c} i_{qs} \\ i_{ds} \\ \hline i_{qr} \\ i_{dr} \end{array}\right]$$

The rotor windings are short-circuited, and hence rotor voltages are zero.

Now we can put the numerical values for the parameters and solve the currents.

$$\begin{aligned} i_{qs} &= 35.37 - \mathbf{j}\,108.8 &&= 113.81\underline{/-71.9} \quad \text{A}, \\ i_{ds} &= 108.8 + \mathbf{j}\,35.37 &&= 113.81\underline{/18.1} \quad \text{A}, \\ i_{qr} &= -34.88 + \mathbf{j}\,103.63 &&= 109.34\underline{/108.6} \quad \text{A}, \\ i_{dr} &= -103.63 + \mathbf{j}\,34.88 &&= 109.34\underline{/-161.4} \quad \text{A}. \end{aligned}$$

The stator and rotor currents are displaced by 90 degrees among themselves as expected in a two-phase machine.

The zero-sequence currents are zero as it is non-existent with balanced supply voltages.

The phase currents are:

$$\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1/2 & -\sqrt{3}/2 & 1 \\ -1/2 & \sqrt{3}/2 & 1 \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix} = \begin{bmatrix} 113.9\underline{/-71.9} \\ 113.9\underline{/168.1} \\ 113.9\underline{/48.1} \end{bmatrix} \quad \text{A}.$$

The various rotor currents are:

$$\begin{aligned} i_{qrr} &= a i_{qr} = 328.02\underline{/108.6} \quad \text{A}, \\ i_{drr} &= a i_{dr} = 328.02\underline{/-161.4} \quad \text{A}. \end{aligned}$$

The α and β currents, assuming $\theta_r = 0$ are:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \sin\theta_r & -\cos\theta_r \end{bmatrix} \begin{bmatrix} i_{drr} \\ i_{qrr} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} i_{drr} \\ i_{qrr} \end{bmatrix} = \begin{bmatrix} 328.02\underline{/-161.4} \\ -328.02\underline{/108.6} \end{bmatrix} \quad \text{A} \quad \blacksquare$$

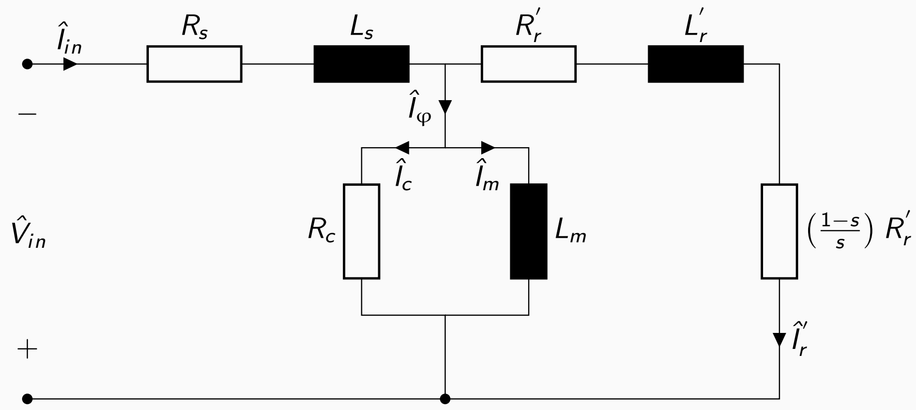


Figure 5: The steady-state equivalence circuit of an induction drive.

4. Let the impedance Z_{eq} define the per phase impedance presented to the stator by the magnetising reactance and the rotor. But first let's calculate the rotor impedance:

$$Z_{\text{r}} = \frac{R_{\text{r}}}{s} + \mathbf{j}X_{\text{r}} \quad \Omega,$$

We know to calculate the Z_{eq} we treat Z_{m} and Z_{r} as parallel impedances.

$$Z_{\text{m}} = \mathbf{j}X_{\text{m}} \qquad \text{and} \qquad Z_{\text{meq}} = \frac{Z_{\text{m}} \times Z_{\text{eq}}}{Z_{\text{m}} + Z_{\text{eq}}}.$$

Knowing these values, we can add the stator values to calculate the input impedance:

$$Z_{\text{in}} = R_{\text{s}} + \mathbf{j}X_{\text{s}} + Z_{\text{meq}} = 5.70 + \mathbf{j}\,3.61 = 6.75\underline{/32.3^\circ} \quad \Omega$$

Therefore, the input current can be calculate as:

$$V_{\text{ph}} = \frac{22}{\sqrt{3}} = 127 \quad \text{V}.$$

$$I_{\text{in}} = \frac{V_{\text{in}}}{Z_{\text{in}}} = \frac{127}{6.75\underline{/32.3^\circ}} = 18.8\underline{/\text{-}32.3^\circ} \quad \text{A}$$

The stator current is 18.8 A, therefore the power factor is equal to $\cos(\text{-}32.3^\circ) = 0.845$ lagging. Let us take our attention to speed calculations. To calculate the synchronous speed in min^{-1}:

$$n_{\text{s}} = \left(\frac{120}{\text{poles}}\right) f_{\text{s}} = \left(\frac{120}{6}\right) 60 = 1200 \quad \text{min}^{-1}.$$

And of course in $\text{rad} \cdot \text{s}^{-1}$, the speed is:

$$\omega_{\text{s}} = \frac{4\pi f_{\text{s}}}{\text{poles}} = 125.7 \quad \text{rad} \cdot \text{s}^{-1}.$$

The rotor speed is:

$$n_{\text{r}} = (1 - s)\, n_{\text{s}} = (0.98)\, 1200 \qquad = 1176 \quad \text{min}^{-1},$$

$$\omega_{\text{r}} = (1 - s)\, \omega_{\text{s}} = (0.98)\, 125.7 \qquad = 123.2 \quad \text{rad} \cdot \text{s}^{-1}.$$

For the power calculations, we start with the air-gap power:

$$P_{\text{gap}} = n_{\text{ph}} I_{\text{r}}^2 \left(\frac{R_2}{s}\right) \quad \text{W}$$

Note however that because the only resistance included in Z_{meq} is ${}^{R_2}/_{s}$ the power dissipated in Z_{meq} is equal to the power dissipated in ${}^{R_2}/_{s}$ and therefore we can write:

$$P_{\text{gap}} = n_{\text{ph}} I_s^2 R_{\text{meq}} = 3\,(18.8)^2\,(5.41) = 5740 \ \text{ W.}$$

From here, we can calculate the mechanical power and the shaft output power:

$$\begin{aligned} P_{\text{shaft}} &= P_{\text{mech}} - P_{\text{rot}} = (1-s)\,P_{\text{gap}} - P_{\text{rot}} \\ &= (0.98)\,5740 - 403 = 5220 \quad \text{W.} \end{aligned}$$

The shaft output torque can be calculated as:

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_{\text{m}}} = \frac{5220}{123.2} = 42.4 \quad \text{N} \cdot \text{m.}$$

The efficiency is calculated as the ratio of shaft output power to stator input power. The input power is given by:

$$\begin{aligned} P_{\text{in}} &= n_{\text{ph}} \mathbf{Re}\,[V_{\text{in}} I_{\text{in}}] = 3\mathbf{Re}\,[127\,(18.8\underline{/32.3^\circ})] \\ &= 3 \times 127 \times 18.8 \cos(32.2^\circ) = 6060 \quad \text{W.} \end{aligned}$$

Therefore, the efficiency (η) is equal to:

$$\eta = \frac{P_{\text{shaft}}}{P_{\text{in}}} = 0.861 = 86.1 \ \% \quad \blacksquare$$

5. We must first determine the parameters for this machine:

$$L_{\mathrm{m}} = \frac{X_{\mathrm{m0}}}{\omega_{\mathrm{e0}}} = \frac{18.7}{120\pi} = 49.6 \quad \mathrm{mH},$$

$$L_{\mathrm{s}} = L_{\mathrm{m}} + \frac{X_{\mathrm{s0}}}{\omega_{\mathrm{e0}}} = 49.6 + \frac{0.680}{120\pi} = 51.41 \quad \mathrm{mH},$$

$$L_{\mathrm{r}} = L_{\mathrm{m}} + \frac{X_{\mathrm{r0}}}{\omega_{\mathrm{e0}}} = 49.6 + \frac{0.672}{120\pi} = 51.39 \quad \mathrm{mH}.$$

The rated rms line-to-neutral terminal voltage for this machine is $230/\sqrt{3} = 132.8$ and therefore the peak rated flux for this machine is:

$$(\lambda_{\mathrm{rated}})_{\mathrm{peak}} = \frac{\sqrt{2}\,(V_{\mathrm{in}})_{\mathrm{rated}}}{\omega_{\mathrm{e}}} = \frac{\sqrt{2}\times 132.8}{120\pi} = 0.498 \quad \mathrm{Wb}.$$

For the specified operating condition:

$$\omega_{\mathrm{r}} = n_{\mathrm{r}}\left(\frac{\pi}{30}\right) = 1680\left(\frac{\pi}{30}\right) = 176 \quad \mathrm{rad}\cdot\mathrm{s}^{-1},$$

and the mechanical torque is:

$$T_{\mathrm{mech}} = \frac{P_{\mathrm{mech}}}{\omega_{\mathrm{s}}} = \frac{9.7\times 10^{3}}{176} = 55.1 \quad \mathrm{N}\cdot\mathrm{m}^{-1}.$$

We can re-use the torque equation to isolate the quadrature current:

$$T_{\mathrm{mech}} = \frac{3}{2}\left(\frac{p}{2}\right)\left(\frac{L_{\mathrm{m}}}{L_{\mathrm{r}}}\right)\left(\lambda_{\mathrm{dr}} i_{\mathrm{q}} - \lambda_{\mathrm{qr}} i_{\mathrm{d}}\right),$$

$$i_{\mathrm{q}} = \frac{2}{3}\left(\frac{2}{p}\right)\left(\frac{L_{\mathrm{r}}}{L_{\mathrm{m}}}\right)\left(\frac{T_{\mathrm{mech}}}{\lambda_{\mathrm{dr}}}\right),$$

$$= \frac{2}{3}\left(\frac{2}{4}\right)\left(\frac{51.39\times 10^{-3}}{49.6\times 10^{-3}}\right)\left(\frac{55.1}{0.498}\right) = 38.2 \quad \mathrm{A}.$$

Using $\lambda_{\mathrm{dr}} = L_{\mathrm{m}} i_{\mathrm{d}}$, we can deduce:

$$i_{\mathrm{d}} = \frac{\lambda_{\mathrm{dr}}}{L_{\mathrm{m}}} = \frac{0.498}{49.6\times 10^{-3}} = 10.0 \quad \mathrm{A}$$

The rms input current is:

$$I_{\mathrm{in}} = \sqrt{\frac{i_{\mathrm{d}}^2 + i_{\mathrm{q}}^2}{2}} = \sqrt{\frac{10.0^2 + 38.2^2}{2}} = 27.9 \quad \mathrm{A}$$

The electrical frequency can be found from:

$$\omega_{\mathrm{e}} = \omega_{\mathrm{me}} + \frac{R_{\mathrm{r}}}{L_{\mathrm{r}}}\left(\frac{i_{\mathrm{q}}}{i_{\mathrm{d}}}\right)$$

With:

$$\omega_{\mathrm{me}} = (p/2)\,\omega_{\mathrm{m}} = 2\times 176 = 352 \quad \mathrm{rad}\cdot\mathrm{s}^{-1},$$

$$\omega_{\mathrm{e}} = 352 + \left(\frac{0.2}{51.39\times 10^{-3}}\right)\left(\frac{38.2}{10.0}\right) = 367 \quad \mathrm{rad}\cdot\mathrm{s}^{-1}$$

and $f_{\mathrm{e}} = \omega_{\mathrm{e}}/(2\pi) = 58.4$ Hz. Finally we can calculate the rms line-to-neutral terminal voltage as:

$$V_{\mathrm{a}} = \sqrt{\frac{\left(R_{\mathrm{s}} i_{\mathrm{d}} - \omega_{\mathrm{e}} \left(L_{\mathrm{s}} - {}^{L_{\mathrm{m}}^2}\!/_{L_{\mathrm{r}}}\right) i_{\mathrm{q}}\right)^2 + \left(R_{\mathrm{s}} i_{\mathrm{q}} + \omega_{\mathrm{e}} L_{\mathrm{s}} i_{\mathrm{d}}\right)^2}{2}},$$
$$= 140.6 \quad \text{V line-to-neutral} = 243.6 \quad \text{V line-to-line} \quad \blacksquare$$