# Lecture Book
# M.Sc Electrodynamics

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# Chapter 1

# The Purpose of Electromagnetism

## 1.1 Mechanics in Four Different Views

Newtonian mechanics is usually enough for most purposes in **everyday life**, but for objects moving at high speeds (near the speed of light) it is incorrect, and must be replaced by special relativity (introduced by Einstein in 1905), for objects that are extremely small (near the size of atoms) it fails for different reasons, and is superseded by quantum mechanics (developed by Bohr, Schrödinger, Heisenberg, and many others, in the 1920's, mostly).

For objects that are both very fast and very small (as is common in modern particle physics), a mechanics that combines relativity and quantum principles is in order; this relativistic quantum mechanics is known as quantum field theory, but even today it cannot claim to be a completely satisfactory system.

In this lecture, we shall work exclusively in the domain of classical mechanics, although electrodynamics extends with ease to the other three realms.

> Interestingly, electromagnetism was one of the main catalyst for developing general relativity.

### 1.1.1 Four Kinds of Forces

Mechanics tells us how behaviour is when subjected to a given force. There are four (4) basic forces known to physics. Listing them in the order of decreasing strength:

- Strong
- Electromagnetic
- Weak

- Gravitational

The **strong force**, which hold protons and neutrons together in the atomic nucleus, have extremely short range, so we do not “feel” them, in spite of the fact that they are a hundred times more powerful than electrical forces.

The **weak force**, which account for certain kinds of radioactive decay, are also of short range, and they are far weaker than electromagnetic forces.

As for **Gravity**, it is very weak (compared to all of the others) that it is only in scale of huge mass concentrations (like the earth and the sun) that we ever notice it at all.

> For reference, the electrical repulsion between two electrons is $10^{42}$ times as large as their gravitational attraction

Not only are electromagnetic forces overwhelmingly dominant in everyday life, they are also, the only ones that are completely understood.

There is, of course, a classical theory of gravity and a relativistic one, but no entirely satisfactory quantum mechanical theory of gravity has been constructed. At the present time there is a very successful theory for the weak interactions, and a strikingly attractive candidate (called *chromodynamics*) for the strong interactions.

All these theories draw their inspiration from electrodynamics. None can claim conclusive experimental verification at this stage. So electrodynamics, a complete and successful theory, has become a pedestal for physicists: an ideal model that other theories emulate.

The laws of classical electrodynamics were discovered in bits and pieces by Franklin, Coulomb, Ampère, Faraday, and others, but the person who completed the job, and packaged it all in the compact and consistent form it has today, was James Clerk Maxwell. The theory is now about 150 years old.

### 1.1.2 Unifying Physical Theories

In the beginning, electricity and magnetism were entirely separate subjects. The one dealt with glass rods and cat’s fur, batteries, currents, electrolysis, and lightning, whereas the other with bar magnets, iron filings, compass needles, and the North Pole. But in 1820 Oersted noticed that an electric current could deflect a magnetic compass needle. Soon afterward, Ampère correctly postulated that all magnetic phenomena are due to electric charges in motion. Then, in 1831, Faraday discovered that a moving magnet generates an electric current. By the time Maxwell and Lorentz put the finishing touches on the theory, electricity and magnetism were inextricably intertwined.

They could no longer be regarded as separate subjects, but rather as two aspects of a single subject: electromagnetism.

Faraday speculated that light, too, is electrical in nature. Maxwell's theory provided justification for this hypothesis, and soon optics, which is the study of lenses, mirrors, prisms, interference, and diffraction, was incorporated into electromagnetism. Hertz, who presented the decisive experimental confirmation for Maxwell's theory in 1888, said:

> *The connection between light and electricity is now established . . . In every flame, in every luminous parti- cle, we see an electrical process . . . Thus, the domain of electricity extends over the whole of nature. It even affects ourselves intimately: we perceive that we possess . . . an electrical organ—the eye.*

By 1900, then, three great branches of physics: **electricity**, **magnetism**, and **optics** had merged into a single unified theory.

> it was soon apparent that visible light represents only a tiny "window" in the vast spectrum of electromagnetic radiation, from radio through microwaves, infrared and ultraviolet, to x-rays and gamma rays

Einstein worked on a further unification, which would combine gravity and electrodynamics, in much the same way as electricity and magnetism had been combined a century earlier. His unified field theory was not particularly successful, but in recent years the same impulse has spawned a hierarchy of increasingly ambitious unification schemes, beginning in the 1960s with the electroweak theory of Glashow, Weinberg, and Salam, which joins the weak and electromagnetic forces, and culminating in the 1980s with the superstring theory.

At each step in this hierarchy, the mathematical difficulties mount, and the gap between inspired conjecture and experimental test widens; nevertheless, it is clear the unification of forces initiated by electrodynamics has become a major theme in the progress of physics.

### 1.1.3 Fields of Electrodynamics

The essential problem the theory of electromagnetism hopes to solve is this:

*If there exists a bunch of electric charges here (and maybe shake them around), what happens to some other charge, over there?*

The classical solution takes the form of a **field theory**:

We say that the space around an electric charge is permeated by electric and magnetic fields. A second charge, in the presence of these fields, experiences a force. The fields, then, transmit the influence from one charge to the other—they "mediate" the interaction.

When a charge undergoes acceleration, a portion of the field detaches itself, in sense, and travels off at the speed of light, carrying with it energy, momentum, and angular momentum. We call this **electromagnetic radiation**. Its existence invites (if not compels) us to regard the fields as independent dynamical entities in their own right, every bit as real as atoms or baseballs.

Our interest accordingly shifts from the study of forces between charges to the theory of the fields themselves. But it takes a charge to produce an electromagnetic field, and it takes another charge to detect one, so we had best begin by reviewing the essential properties of electric charge.

### 1.1.4 Electric Charge

1. **Charge comes in two varieties:** We call **plus** and **minus**, as their effects tend to cancel (if you have $+q$ and $q$ at the same point, electrically it is the same as having no charge there at all). The interesting fact is that plus and minus charges occur in exactly equal amounts, to high precision, in bulk matter, so that their effects are almost completely neutralised.

2. **Charge is conserved:** It cannot be created or destroyed. What there is now has always been. A plus charge can **annihilate** an equal minus charge, but a plus charge cannot simply disappear by itself, something must pick up that electric charge. So the total charge of the universe is fixed for all time. This is called **global conservation of charge**. Charges also cannot disappear and then reappear in some other place, the charge, if moved must follow a continuous path. This is called **local conservation of charge**.

3. **Charge is quantised:** Although nothing in classical electrodynamics requires that it be so, electric charge comes only in integer multiples of the basic unit of charge. If we call the charge on the proton $+e$, then the electron carries charge $e$ and never 7.392e, or even 1/2e.