Topic	Description
Module	Higher Mathematics II
Module Code	HMA
Semester	SS 2025
Lecturer	Daniel T. McGuiness, Ph.D
ECTS	4
SWS	3
Lecture Type	ILV
Teaching UE	45
Coursework Name	Individual Assignment
Work	Individual
Suggested Private Study	25 hours
Submission Format	Online via SAKAI
Submission Deadline	27th May 23:59
Late Submission	Not accepted
Resubmitting Opportunity	No re submission opportunity

No lecture time is exclusively devoted to the aforementioned assignment.

Please answer all the questions below with sufficient detail. You can write your answers on a properly hand-written document or LaTeX ...or doc.

A portion of the mark for every assignment will be, where applicable, based on style. Style, in this context, refers to organisation, flow, sentence and paragraph structure, typographical accuracy, grammar, spelling, clarity of expression and use of correct IEEE style for citations and references. Students will find *The Elements of Style (3rd ed.)* (1979) by Strunk & White, published by Macmillan, useful with an alternative recommendation being *Economist Style Guide (12th ed.)* by Ann Wroe.

Question	Maximum Point	Received Point
Licence Plates and Washing Machines	20	
Is it the Same Iron?	10	
Voltages and Waves	30	
Partial Differential Equations	40	
Sum	100	

[Q1] Licence Plates and Washing Machines 20

1. A witness to a traffic accident told the police, the license number contained the letters RLH followed by 3 digits, the first of which was a 5.

   If the witness cannot recall the last 2 digits, but is **certain** that all 3 digits are different, find the maximum number of automobile registrations that the police may have to check. (10)

2. The total number of hours, measured in units of 100 hours, that a family runs a washing machine over a period of one year is a continuous random variable X that has the density function:

$$f(x) = \begin{cases} x & 0 < x < 1, \\ 2 - x & 1 \leq x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

   Find the probability that over a period of one year, a family runs their washing machine:

   a. less than 120 hours, (5)

   b. between 50 and 100 hours. (5)

[A1] Licence Plates and Washing Machines 20

1. As the first digit is a 5, there are $n_1 = 9$ possibilities for the second digit and then $n_2 = 8$ possibilities for the third digit. Therefore, by the multiplication rule there are $n_1 n_2 = 9 \cdot 8 = 72$ registrations to be checked ■ (10)

2a. For less than 120 hours, (5)

$$P(X < 1.2) = \int_0^1 x \, dx + \int_1^{1.2} (2 - x) \, dx = \left.\frac{x^2}{2}\right|_0^1 + \left.\left(2x - \frac{x^2}{2}\right)\right|_1^{1.2} = 0.68 \quad ■$$

2b. For between 50 and 100 hours, (5)

$$P(0.5 < X < 1) = \int_{0.5}^1 x \, dx = \left.\frac{x^2}{2}\right|_{0.5}^1 = 0.375 \quad ■$$

[Q2] Is it the Same Iron? 10

The two samples:

$$70 \quad 80 \quad 30 \quad 70 \quad 60 \quad 80$$

and

$$140 \quad 120 \quad 130 \quad 120 \quad 120 \quad 130 \quad 120$$

are values of the differences of temperatures (°C) of iron at two stages of casting, taken from two different crucibles[1].
Is the variance of the first population larger than that of the second? (Assume normality. Choose $\alpha = 5\%$.) (10)

[A2] Is it the Same Iron? 10

We test the hypothesis $\sigma_x^2 = \sigma_y^2$ against the alternative $\sigma_x^2 > \sigma_y^2$. We proceed to calculate the variance of their given data as:

$$v_0 = \frac{s_x^2}{s_y^2} = \frac{350}{61.9} = 5.65.$$

For $\alpha = 5\%$ and $(5, 6)$ degrees of freedom, the reference Table gives the value 4.39.
As 5.65 is greater, we reject the hypothesis and assert that the variance of the first population is greater than that of the second ■ (10)

[Q3] Voltages and Waves 30

1. Find the Fourier series of the function obtained by passing the voltage:

$$v(t) = V_0 \cos 100\pi t$$

   through a half-wave rectifier[2]. (15)

2. Find the steady-state current $(I(t))$ of an series connected RLC circuit where:

$$R = 100\,\Omega \quad L = 10\,\text{H} \quad C = 1 \times 10^{-2}\,\text{F}$$

   and $E(t)$ V as

$$E(t) = \begin{cases} 100\left(\pi t + t^2\right) & \text{if} \quad -\pi < t < 0 \\ 100\left(\pi t - t^2\right) & \text{if} \quad 0 < t < \pi \end{cases}$$

   and periodic with period 2π. (15)

   Note: the coefficients of the solution decrease rapidly.

[1] a container in which metals or other substances may be melted or subjected to very high temperatures.
[2] Half-wave rectifiers transform AC voltage to DC voltage. A halfwave rectifier circuit uses only one diode for the transformation and defined as a type of rectifier that allows only one-half cycle of an AC voltage waveform to pass while blocking the other half cycle.

[A3] Voltages and Waves 30

1. The solution is as follows:

$$b_n = 0, \qquad \text{and} \qquad a_0 = \frac{V_0}{\pi}.$$

$$\begin{aligned}
a_n &= 100V_0 \int_{-1/200}^{1/200} \cos 100\pi t \cos 100 n\pi t \, dt \\
&= 50V_0 \int_{-1/200}^{1/200} \cos 100 (n+1) \pi t \, dt + 50V_0 \int_{-1/200}^{1/200} \cos 100 (n-1) \pi t \, dt \\
&= \frac{V_0}{\pi} + \frac{V_0}{2} \cos 100\pi t \\
&+ \frac{2V_0}{\pi} \left(\frac{1}{1 \cdot 3} \cos 200\pi t - \frac{1}{3 \cdot 5} \cos 400\pi t + \frac{1}{5 \cdot 7} \cos 600\pi t \cdots \right) \quad \blacksquare
\end{aligned}$$

2. The solution is as follows:

$$I = \sum_{n=1}^{\infty} (A_n \cos nt + B_n \sin nt),$$

where,

$$A_n = \frac{80 (10 - n^2)}{\pi n^2 D_n} \quad \text{and} \quad B_n = \frac{800}{n\pi D_n} \quad \text{(where } n \text{ is odd.)}$$

$$A_n = 0 \quad \text{and} \quad B_n = 0 \quad \text{(where } n \text{ is even.)}$$

and of course,

$$B_n = (10 - n^2)^2 + 100n^2$$

Which gives us the solution:

$$\begin{aligned}
I &= 1.266 \cos t + 1.406 \sin t + 0.003 \cos 3t + 0.094 \sin 3t \\
&- 0.006 \cos 5t + 0.019 \sin 5t - 0.003 \cos 7t + 0.006 \sin 7t \ldots \quad \blacksquare
\end{aligned}$$

[Q4] Partial Differential Equations 40

Find the temperature $u(x, t)$ in a laterally insulated copper bar 80 cm long if the initial temperature $100 \sin(\pi x/80)\,°C$ and the ends are kept at $0\,°C$

How long will it take for the maximum temperature in the bar to drop to $50\,°C$? (40)

Properties of Copper:

- density $8.92\ \mathrm{g\,cm^{-3}}$,
- specific heat $0.092\ \mathrm{cal\,g^{-1}\,°C^{-1}}$,
- thermal conductivity $0.95\ \mathrm{cal\,cm^{-1}\,s^{-1}\,°C^{-1}}$

[A4] Sinusoidal Initial Temperature 40

The initial condition gives:

$$u(x,\ 0) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{80} = f(x) = 100\frac{\pi x}{80}.$$

Therefore, by inspection or using the following:

$$u(x,\ t) = \sum_{n=1}^{\infty} u_n(x,\ t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L} e^{-\lambda_n^2 t}$$

we get:

$$B_1 = 100, B_2 = B_2 = \cdots = 0$$

We also need:

$$\lambda_1^2 = c^2\pi^2/L^2,$$

where:

$$c^2 = \frac{K}{\sigma\rho} = \frac{0.95}{0.092 \cdot 8.92} = 1.158\,\mathrm{cm}^2\,\mathrm{s}^{-1}$$

Therefore we obtain:

$$\lambda_1^2 = 1.158 \cdot \frac{9.870}{80^2} = 0.001\,785\,\mathrm{s}^{-1}$$

Using this we can obtain the solution:

$$u(x,\ t) = 100 \sin \frac{\pi x}{80} e^{-0.001875\ t}$$

In addition, $100e^{-0.001875\ t} = 50$ when:

$$t = \frac{\ln 0.5}{-0.0018785} = 388\,\mathrm{s} \approx 6.5\,\mathrm{min} \quad \blacksquare$$