

Topic	Description
Module	Higher Mathematics II
Module Code	HMA
Semester	SS 2025
Lecturer	Daniel T. McGuiness, Ph.D
ECTS	4
SWS	3
Lecture Type	ILV
Teaching UE	45
Coursework Name	Individual Assignment
Work	Individual
Suggested Private Study	25 hours
Submission Format	Submission via SAKAI or Handwritten Calculations
Submission Deadline	27 <sup>th</sup> May 23:59
Late Submission	Not accepted
Resubmitting Opportunity	No re submission opportunity

No lecture time is exclusively devoted to the aforementioned assignment.

A portion of the mark for every assignment will be, where applicable, based on style. Style, in this context, refers to organisation, flow, sentence and paragraph structure, typographical accuracy, grammar, spelling, clarity of expression and use of correct IEEE style for citations and references. Students will find *The Elements of Style (3rd ed.)* (1979) by Strunk & White, published by Macmillan, useful with an alternative recommendation being *Economist Style Guide (12th ed.)* by Ann Wroe.

Question	Maximum Point	Result
Probability Theory	20	
Statistics	10	
Fourier Analysis	30	
Partial Differential Equations	40	
<b>Sum</b>	<b>100</b>	

Please answer all the questions below with sufficient detail. You can write your answers on a properly hand-written document or  $\text{\LaTeX}$  ..... or doc.

**[Q1] Probability Theory** \_\_\_\_\_ 20

1. A witness to a traffic accident told the police, the license number contained the letters RLH followed by 3 digits, the first of which was a 5.

If the witness cannot recall the last 2 digits, but is **certain** that all 3 digits are different, find the maximum number of automobile registrations that the police may have to check.

(10)

2. The total number of hours, measured in units of 100 hours, that a family runs a washing machine over a period of one year is a continuous random variable  $X$  that has the density function:

$$f(x) = \begin{cases} x & 0 < x < 1, \\ 2 - x & 1 \leq x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that over a period of one year, a family runs their washing machine:

- a. less than 120 hours, (5)

- b. between 50 and 100 hours. (5)

**[Q2] Statistics** \_\_\_\_\_ 10

1. The two samples:

70 80 30 70 60 80

and

140 120 130 120 120 130 120

are values of the differences of temperatures ( $^{\circ}\text{C}$ ) of iron at two stages of casting, taken from two different crucibles<sup>1</sup>.

Is the variance of the first population larger than that of the second? (Assume normality. Choose  $\alpha = 5\%$ .)

(20)

<sup>1</sup>a container in which metals or other substances may be melted or subjected to very high temperatures.

**[Q3] Fourier Analysis** 30

1. Find the Fourier series of the function obtained by passing the voltage

$$v(t) = V_o \cos 100\pi t$$

through a half-wave rectifier<sup>2</sup>.

(15)

2. Find the steady-state current ( $I(t)$ ) of an series connected RLC circuit where:

$$R = 100 \Omega \quad L = 10 \text{ H} \quad C = 1 \times 10^{-2} \text{ F}$$

and  $E(t)$  V as

$$E(t) = \begin{cases} 100(\pi t + t^2) & \text{if } -\pi < t < 0 \\ 100(\pi t - t^2) & \text{if } 0 < t < \pi \end{cases}$$

and periodic with period  $2\pi$ .

**Note:** the coefficients of the solution decrease rapidly.

**[Q4] Partial Differential Equations** 40

Find the temperature  $u(x, t)$  in a laterally insulated copper bar 80 cm long if the initial temperature  $100 \sin(\pi x/80)^\circ\text{C}$  and the ends are kept at  $0^\circ\text{C}$

How long will it take for the maximum temperature in the bar to drop to  $50^\circ\text{C}$ ?

(40)

**Properties of Copper:**

- density  $8.92 \text{ g cm}^{-3}$ ,
- specific heat  $0.092 \text{ cal g}^{-1}^\circ\text{C}^{-1}$ ,
- thermal conductivity  $0.95 \text{ cal cm}^{-1} \text{ s}^{-1}^\circ\text{C}^{-1}$

<sup>2</sup>Half-wave rectifiers transform AC voltage to DC voltage. A halfwave rectifier circuit uses only one diode for the transformation and defined as a type of rectifier that allows only one-half cycle of an AC voltage waveform to pass while blocking the other half cycle.