Question 1:-

Calculating initial entropy of a target
variable √taste.

$$\text{Entropy} = \sum_{i=1}^{n} -P_i \log_2 (P_i)$$

where P_i is the
probability of the target class.

$P(\text{Taste for Meh}) = 5/10 = 0.5$
$P(\text{Taste for yummy}) = 5/10 = 0.5$

Entropy for taste

$$E(T) = -\left(P_0 \log_2 (P_0) + P_1 \log_2 (P_1)\right)$$

$$= -\left(0.5 \cdot \log_2 (0.5) + 0.5 \cdot \log_2 (0.5)\right)$$

$$= -\left(\log_2 (0.5)\right) \quad \log_2(0.5) \text{ approxima}$$

$\log_2(0.5) = \text{approximately} -1$

$\frac{1}{2}, \frac{1}{2}$

$$= -(-1) = 1_{,,}$$

Entropy for taste $= \bullet 1_{,,}$

Question 2

if visual defects is root of a decision tree,
we need to calculate information gain

$$IG(T/A) = Entropy(T) - IT\ (information\ taken)$$
$$IG(T,A) = Entropy(T) - \sum_{v \in A} \frac{|Tv|}{T} \cdot Entropy(Tv)$$

where   T is the target variable
Here   Taste is target variable
   A is the variable we need to test.
Here   A is Visual defects.
   V is each value in A. that is
   some, many, none.

Visual defects for some ⇒ Meh=3, Yummy=0
Visual defects for Many ⇒ Meh=0, Yummy=3
Visual defects for None ⇒ Meh=2, Yummy=2
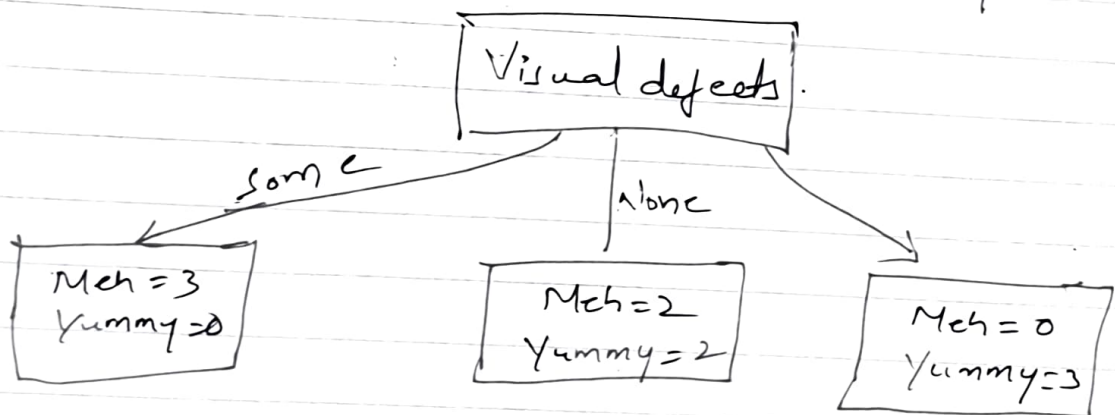
$$Entropy\left(Visual\ defects\ for\ some\right) = -\left(\frac{3}{3}\log_2 \frac{3}{3} + \frac{0}{3}\log_2\left(\frac{0}{3}\right)\right)$$
$$= -\log_2 1 \Rightarrow 0.$$

Entropy (visual defects when None)

$$= -\left(\tfrac{2}{4} \log\left(\tfrac{2}{4}\right) + \tfrac{2}{4} \log\left(\tfrac{2}{4}\right)\right)$$

$$= -\left(\log_2\left(\tfrac{1}{2}\right)\right)$$

$$= -(-1)$$

$$= 1$$

Entropy (visual defects when Many)

$$= -\left(\tfrac{0}{3} \log_2\left(\tfrac{0}{3}\right) + \tfrac{3}{3} \log\left(\tfrac{3}{3}\right)\right)$$

$$= -(0 + \log 1)$$

$$= 0$$

$$IG = E(T) - I_T$$

$$I_T = \frac{T_{some}}{T} \cdot Entropy(T_{some}) + \frac{T_{none}}{T} \cdot Entropy(T_{none}) +$$

$$\frac{T_{many}}{T} \cdot Entropy(T_{many})$$

$$= \tfrac{3}{10} \times 0 + \tfrac{4}{10}(1) + \tfrac{3}{10} \cdot 0$$

$$\Rightarrow 0.4$$

Information Gain $= E(T) - I_T$

E(T) is calculated in Question 1, which is

1

$$= 1 - 0.4 \Rightarrow 0.6$$

Information Gain $= 0.6$

Question 3

Entropy $H(Taste | visual\ defect = some)$

$$= -\left(\frac{3}{3} \log(\frac{3}{3}) + \frac{0}{3} \log(\frac{0}{3})\right)$$

$$= -(1 \log 1 + 0)$$

$$= 0$$

Entropy $H(Taste | visual\ defect = none)$

$$= -\left(\frac{2}{4} \log(\frac{2}{4}) + \frac{2}{4} \log(\frac{2}{4})\right)$$

$$= -(\log(\frac{1}{2}))$$

$$= -(-1) = 1\quad;$$