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<u>COMP3121 Assignment 2 – Question 4</u>

4A) In this question we have to convolute the following sequence $s = < 1, \overbrace{0, 0, \dots, 0}^{k}, 1 >$ with itself i.e. $s * s$. Note that the 0's in between are of length *k*. To compute this convolution, we can do the following:

The associated polynomial is $P(x) = 1 + x^{k+1}$ and thus to get the convolution of s with itself, it is of the sequence of the coefficients of the polynomial $P^2(x) = \left(1 + x^{k+1}\right)^2$.

This is of the form of $(a + b)^2 = a^2 + 2ab + b^2$ and hence we would get:

$$P^2(x) = 1 + 2 * 1 * x^{k+1} + x^{2(k+1)}$$
$$= 1 + 2x^{k+1} + x^{2k+2}$$

Hence, this is of the sequence $\hat{\boldsymbol{s}} = < \mathbf{1}, \overbrace{\mathbf{0}, \dots}^{k}, \mathbf{2}, \overbrace{\dots, \mathbf{0}}^{k}, \mathbf{1} >$ with *k* number of zeros between 1 and 2 exclusive on either side.

4B) Since $s = < 1, \overbrace{0, 0, \dots, 0}^{k}, 1 >$, in order to get the DFT of *s*, we need to evaluate it at all roots of unity of order *k + 2*. This can be done as shown below:

As shown in (a), the corresponding polynomial is $P(x) = 1 + x^{k+1}$ and hence,

$$\boldsymbol{DFT}(\boldsymbol{s}) = < P\left(\omega_{k+2}^{0}\right), P\left(\omega_{k+2}^{1}\right), \dots, P\left(\omega_{k+2}^{k+1}\right) >$$
$$= < 1 + \omega_{k+2}^{0*(k+1)}, 1 + \omega_{k+2}^{1*(k+1)}, \dots, 1 + \omega_{k+2}^{(k+1)*(k+1)} >$$
$$= < \mathbf{2}, \mathbf{1} + \boldsymbol{\omega}_{\boldsymbol{k+2}}^{\boldsymbol{k+1}}, \mathbf{1} + \boldsymbol{\omega}_{\boldsymbol{k+2}}^{\mathbf{2}(\boldsymbol{k+1})}, \dots, \mathbf{1} + \boldsymbol{\omega}_{\boldsymbol{k+2}}^{(\boldsymbol{k+1})^{\mathbf{2}}} >$$

End of Solution