

COMP3121 Assignment 1 – Question 4

4) To start off with, we would need to first obtain the co-ordinates of the four corners of the $4n$ by $4n$ square. Then, starting with the top left corner, which we mark as our origin point $(0, 0)$ we can then go through each square and store the number of trees into an array $A[x][y]$ where x and y are the co-ordinates of the larger square from the origin to (x, y) . This takes a time complexity of $O(n^2)$ as we are going through each square in the matrix/orchard. Additionally, if we already have overlapping rectangles, then we can simply find our sum by subtracting the overlapping rectangle from the other neighbouring rectangles. Once our array $A[x][y]$ has been generated, we can then find out how many trees are in a given square given its co-ordinates (which has time complexity of $O(1)$). Essentially, we are going through each square and finding the number of trees in it by using its neighbouring rectangle values.

E.g. given size n square;

$A[x][y]$ = large square

$A[x - n][y]$ = right neighbouring rectangle

$A[x][y - n]$ = left neighbouring rectangle

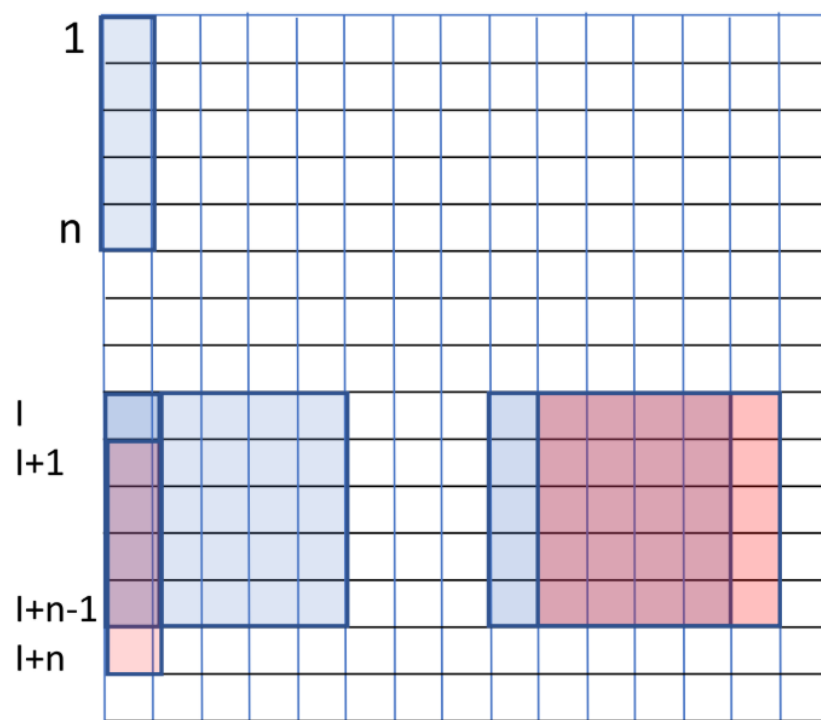
$A[x - n][y - n]$ = overlapping square

Hence: $A[x][y] - A[x - n][y] - A[x][y - n] + A[x - n][y - n] = \text{our square of size } n \text{ by } n.$

Ultimately, we will have time complexity of $O(n^2)$ from the above algorithm.

Note: 'rectangle' is used above as a rectangle is a square but not vice-versa and the neighbouring rectangles do not have to be a square in order to find the sum of the n by n square.

See given diagram as an overall visual example: (note this diagram was emailed by Aleks as a hint)



Example Pseudocode:

Overall runtime of $O(n^2)$:

`total[x][y]` will be our total number of trees present between the origin of `matrix[0][0]` and `matrix[x][y]`

`width = length(matrix);`

for loop to determine whether x is in the range of the length

 # for loop to determine whether y is in the range of the length

 # then we simply do our calculations as explained above i.e.

`total[x][y] = matrix[x][y];`

 # if $x > 0$ then simply add it:

 # e.g. `total[x][y] += total[x - 1][y];`

 # if $y > 0$ then simply add it:

 # e.g. `total[x][y] += total[x][y - 1];`

 # make sure to also subtract any overlapping squares if both x & $y > 0$:

 # e.g. `total[x][y] -= total[x - 1][y - 1];`

Then loop through the matrix/orchard again to find the number of trees of n square such as below:

for loop to determine whether x is in the range of the length

 # for loop to determine whether y is in the range of the length

 # then we simply do our calculations as explained above given size n square i.e.

`final[x][y] = total[x][y];`

 # if $x - n \geq 0$ then simply remove right side rectangle:

 # e.g. `final[x][y] -= total[x - n][y];`

 # if $y - n \geq 0$ then simply remove left side rectangle:

 # e.g. `final[x][y] -= total[x][y - n];`

 # make sure to add the overlapping squares if both $(x - n)$ & $(y - n) \geq 0$

 # e.g. `final[x][y] += total[x - n][y - n];`

End