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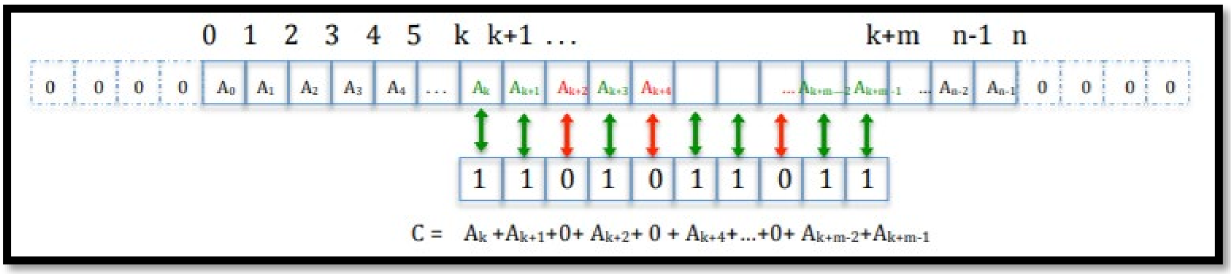
COMP3121 Assignment 2 – Question 3

3) We have a map of a straight seashore of length $100n$ metres as a sequence of $100n$ numbers such that A_i is the total number of fish between the i^{th} metre and $(i+1)^{th}$ metre of the shore, where $0 \leq i \leq (100n-1)$. Additionally, we also have a net of length n metres but it has holes in it. This net can be described as having an N sequence of *n* ones and zeros, where our zeros are the holes in the net. If we throw our net at *k* metres, then we will have our net ending at *k + n - 1* metres, where $0 \leq k + n \leq 100n$. Also note that we catch our fish in the 1 metre stretches of the net i.e. every 1 metre, our net will be 0 or 1 where 0 = holes = no fish can be caught and 1 = no holes = fish can be caught. To catch the largest number of fish in $\boldsymbol{O(nlog(n))}$ time, we must find the spot where we should place the left end of our net.

See given diagram as an overall visual example: (note this diagram is on the question sheet). Note that the right side should instead be $100n$ and not *n* shown on the diagram.



What we can do is let $N^{`}$ denote the net sequence $N$ in reverse order and by computing the sequence $A * N^{`}$ we can then look at our map and note where the largest value of this sequence is.

As seen in the diagram, let *C* be the convolution of $A * N^{`}$ and therefore we would get:

$$C = 1*(A_k) + 1*(A_{k+1}) + 0*(A_{k+2}) + 1*(A_{k+3}) + 0*(A_{k+4}) + 1*(A_{k+5}) + \cdots + 1*(A_{k+m-2}) + 1*(A_{k+m-1})$$

$$= (A_k) + (A_{k+1}) + 0 + (A_{k+3}) + 0 + (A_{k+5}) + \cdots + (A_{k+m-2}) + (A_{k+m-1})$$

$$where\ 0 < m < n$$

This multiplication takes $\boldsymbol{O(n)}$ time overall.

Next, we then look at our $N^{`}$ and "slide" it across the shore from left to right. By taking the inverse, we can find the value of the product of the polynomials using the coefficients obtained using this inverse which takes $\boldsymbol{O(nlog(n))}$ time and hence overall, it only takes $\boldsymbol{O(nlog(n))}$ time.

Thinking of it another way, our inverse net would have the left-most value being B_m for example, and right-most being B_0 and so as we slide it across our shore, our right-most side B_0 would encounter the first non-zero value of the shore's left-most side i.e. A_0. Taking the product of these, we would then have $C_0 = A_0B_0$. We can then shift it across every metre so then our next number of total fish would be $C_1 = A_1B_0 + A_0B_1$ and so on. Note that we do not stop when our B_0 reaches the right-most side of the shore A_{100n} but instead continue until our left-most side reaches A_{100n} and so our last non-zero value would be $C_{m+100n} = A_{100n}B_m$ which is exactly the last term of the product polynomial.

End of Solution