Dheeraj Satya Sushant Viswanadham
(z5204820)

COMP3121 Assignment 2 – Question 5

5) We are required to find a sequence *x* satisfying the following:

$$x * < 1, 1, -1 > = < 1, 0, -1, 2, -1 >$$

The method we will follow to find the sequence *x* will be to first find the length of the sequence and then find the corresponding polynomials which correspond to the given sequences. We will then multiply the two polynomials together and then equate the coefficients to our product polynomial with the terms of the sequence $< 1, 0, -1, 2, -1 >$.

As seen here $< 1, 0, -1, 2, -1 >$, we can observe that it has 5 coefficients and hence is of degree 4. On the other hand, $< 1, 1, -1 >$ has 3 coefficients and is of degree 2. Hence we can surmise that our sequence *x* is also a degree 2 polynomial with three coefficients $< a, b, c >$ as $x^2 * x^2 = x^4$ or putting it another way - *x* is a sequence of length $5 + 1 - 3 = 3$.

We can then rewrite the above in polynomial form:

$$(a + bx + cx^2) * (1 + x - x^2) = 1 + (0x) - x^2 + 2x^3 - x^4$$

$$a + ax + bx - ax^2 + bx^2 + cx^2 - bx^3 + cx^3 - cx^4 = 1 - x^2 + 2x^3 - x^4$$

$$a + x(a + b) + x^2(b + c - a) + x^3(c - b) + x^4(-c) = 1 - x^2 + 2x^3 - x^4$$

Evaluating the above coefficients, we get:

$$a = 1, a + b = 0, b + c - a = -1, c - b = 2, -c = -1$$

Which is simple to evaluate → $\boldsymbol{a = 1}, \boldsymbol{b = -1}$ & $\boldsymbol{c = 1}$.

∴ **The sequence *x* we have found, corresponds to the coefficient vector form:**

$$\boldsymbol{< a, b, c > = < 1, -1, 1 >}$$

Alternatively, in polynomial form, it would be:

$$\boldsymbol{f(x) = 1 - x + x^2.}$$

End of Solution