Assignment 5 hints

NOTE: Due date August 7, 2020 at 8 PM sharp!

You have **three problems**, marked out of a total of 60 marks.

NOTE: Your solutions must be typed, machine readable .pdf files. **All submissions will be checked for plagiarism!**

1. Today was just a regular day for everyone in Krypton until a news flashed that a meteor is going to destroy Krypton in X days. Krypton has N cities connected together by bidirectional roads. It takes $t(i, j)$ days to travel from city i to city j. In each city C_i the Krypton Government built q_i pods to carry inhabitants in case of any calamity, which will transport them to Earth. City C_i has population p_i. As soon as the people hear this news they try to save themselves by acquiring these pods either at their own city or in other city before the meteor destroys everything. Note that a pod can carry only one person. Find the largest number of invaders the Earth will have to deal with. (20 pts)

   Hint: this is a typical Max Flow problem. Each inhabitant of Krypton can access the pods which are either in their own city or in cities which are less than X day's trip away from their city. To make the representation compact, make a bipartite graph with all the cities on the left side and all the cities on the right hand side; add a super source and a super sink and figure out what the capacities of edges should be.

2. You are given a usual 8×8 chess board with 2 white bishops on the board at the given cells (a, b) and (c, d) (of opposite colour), $(1 \leq a, b, c, d \leq 8)$. You have to determine the largest number of black rooks you can place on the board so that no two rooks are in the same row or in the same column or are under the attack of either of the two bishops (recall that bishops go diagonally).(20 pts)

   Hints: Make a bipartite graph with all the columns as vertices on the left hand side and all the rows as vertices on the right hand side. Each square s_{ij} can now be represented by and edge from column j to row i. Think which edges you allow.

3. There are N computers in a network, labelled $\{1, 2, 3, \ldots, N\}$. There are M one-directional links which connect pairs of computers. Computer 1 is trying to send a virus to computer N. This can happen as long as there is a path of links from computer 1 to computer N. To prevent this, you've decided to

remove some of the links from the network so that the two computers are no longer connected. For each link, you've calculated the cost of removing it. What is the minimum total cost to disconnect the computers as required, and which edges should be removed to achieve this minimum cost? (20 pts)

Hint: Again, a typical Max Flow - Min Cut problem. What should be the capacities of the edges be set to?