

COMP3121 Assignment 1 – Question 1

In order to determine if a number 'S' exists that satisfies the sum of the squares of two distinct numbers 'X' and 'Y' in two different ways, we first need to go through all of $\frac{n(n-1)}{2}$ possible pairs of distinct integers in array A and compute the sum of the squares which will then be stored in array B of size $\frac{n(n-1)}{2}$. From there, we can then observe that if S pops up again from another two distinct numbers.

1A) To get an algorithm that has at most has worst case performance of $n^2 \log(n)$ we can have the below implementation:

First implement two for loops where:

loop x = 0 through to the length of array A

loop y = (x + 1) through to the length of array A

In this way, we can avoid any possible duplicate pairs as $5^2 + 11^2 = 11^2 + 5^2$, regardless of the order.

We can then use a simple AVL tree (Adelson-Velsky and Landis) for insertion and searching if the sum S already exists in the tree for each iteration. These functions have worst-case time complexity of $O(\log(n))$ and as this needs to be done each time the arithmetic sum would be $n + (n - 1) + (n - 2) + \dots + 1 = O(n^2)$ for the nested loops and hence, it will take **$n^2 * \log(n)$ in the worst-case scenario.**

Example Pseudocode:

```
class AVLnode {
    # ... initialise ...
}

class AVLtree {
    private AVLnode key;

    # insert constructor

    # code to check if tree is empty

    # code to make the tree empty (to reset it)

    # code to insert data into the tree

    # code for getting the height of the node

    # code for LH and RH nodes

    # insert data recursively

    # code to rotate the tree with either left or right children

    # code to search given number/element

    # code for inorder/preorder/postorder traversal
}
```

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```
testAVL = AVLtree();  
  
# insert nested for loops e.g. as below:  
for x, X in array:  
    for y, Y in array[x + 1]:  
        sum = (X * X) + (Y * Y);  
        # check if the sum already exists in the AVL tree  
        if testAVL.contains(sum) already then:  
            return true;  
        # otherwise proceed with inserting the value into the AVL tree  
        testAVL.insert(sum);  
  
return false;  
  
# End
```

1B) We will follow the same approach as given in (a), however, we will now instead use a hash table or 'hash map' instead of an AVL tree as the hash table is a more efficient data structure operation in terms of average time complexity as each insertion and lookup takes $O(1)$ expected time as compared to the AVL tree's slower $O(\log(n))$ expected time. Since we still have the nested loops to get our values, and which have a time complexity of $O(n^2)$, then the **final expected time will evidently be $O(n^2) * O(1) = O(n^2)$.**

Example Pseudocode:

```
# create HashMap using java.util* library for example.  
  
HashMap <Integer, String> hm = new HashMap<Integer, String>  
  
for x, X in array:  
    for y, Y in array[x + 1]:  
        sum = (X * X) + (Y * Y);  
        # check if the sum already exists in the HashMap  
        if sum in hm already then:  
            return true;  
        # otherwise proceed with inserting the value into the HashMap  
        hm[sum] = (X, Y)  
  
return false;  
  
# End
```