MATH 313 HMWK 3

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February 11, 2018

Exercise 2.2.1 An example of a vercongent sequence would be $x_n = \sin x$. Let $\epsilon = \frac{101}{100}$ and let $x = 0$. Then x_n converges to 0, since $|\sin x - 0| < \frac{101}{100}$. Since x_n does not converge to any number, it is a vercongent sequence that is also divergent. What a vercongent sequence describes is a sequence for whom all values lie in a certain bound, namely $(x - \epsilon, x + \epsilon)$.

Excercise 2.2.2

1. Let $\epsilon > 0$ be arbitrary. Now choose N such that

$$N > \frac{3}{25\epsilon} - \frac{4}{5}$$

To verify that our choice of N is appropiate, let $n \in \mathbb{N}$ satisfy $n \geq N$. Then, $n \geq N$ implies that

$$\begin{aligned}
n &> \frac{3}{25\epsilon} - \frac{4}{5} \\
5n &> \frac{3}{5\epsilon} - 4 \\
5n + 4 &> \frac{3}{5\epsilon} \\
\epsilon &> \frac{3}{5 \cdot (5n+4)} \\
\epsilon &> \frac{2 \cdot (5n+4) - 5 \cdot (2n+1)}{5 \cdot (5n+4)} \\
\epsilon &> \frac{2 \cdot (5n+4)}{5 \cdot (5n+4)} - \frac{5 \cdot (2n+1)}{5 \cdot (5n+4)} \\
\epsilon &> \frac{2}{5} - \frac{2n+1}{5n+4} \\
\epsilon &> \left| \frac{2}{5} - \frac{2n+1}{5n+4} \right| \\
\left| \frac{2}{5} - \frac{2n+1}{5n+4} \right| &< \epsilon \\
\left| \frac{2n+1}{5n+4} - \frac{2}{5} \right| &< \epsilon
\end{aligned}$$

2.

2.3.1

1. If $x_n \to 0$, then there exists $\epsilon_0 > 0$ such that there exists $N \in \mathbb{N}$ for all $n \geq \mathbb{N}$, $x_n - 0 < \epsilon_0$.

2.3.1

1. If $x_n \to 0$, then it must be true that for all ϵ_0 , there exists a $N \in \mathbb{N}$ such that for all $n \geq N$, $|x_n - 0| < \epsilon_0$. Then

$$\begin{aligned} |x_n - 0| &< \epsilon_0 \\ x_n &< \epsilon_0 && \text{since } x_n \geq 0 \\ \sqrt{x_n} &< \sqrt{\epsilon_0} \end{aligned}$$

2.3.5 If x_n and y_n are both convergent, then there must exist a $N_1, N_2 \in \mathbb{N}$ such that for all $n_1 \geq N_2$ and for all $n_2 \geq N_2$, $|a_n - L| < \epsilon_1$ and $|b_n - L| < \epsilon_2$ for any $\epsilon > 0$ since a_n and b_n both converge to the same limit. In the case of z_n, we choose $N = \max(N_1, N_2)$. Then for all $n \geq N$, we know that $|z_n - L| < \epsilon$.

Exercise 2.3.7

1. Let $x_n = n$ and $y_n = -n$. Both x_n and y_n diverge. However, $x_n + y_n$ converge to zero.

2. Impossible by Algebraic Limit Theorem ii.

3. Let $b_n = \frac{1}{n}$ with $b_n \neq 0$ for all $n \in \mathbb{N}$. However, $(1/b_n)$ diverges.

4. Impossible. By Theorem 2.3.2, a_n must be a divergent sequence, so then $(a_n - b_n$ is a sequence that diverges.

5. Let $a_n = \frac{1}{n}$ and $b_n = n$. a_n converges to 0 and b_n diverges. $(a_n b_n)$ converges to 1.