Option Pricing Using Binomial Tree Simulation: A Case Study of Nvidia Options

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1 Introduction

Option pricing is a crucial aspect of modern finance, providing a way to estimate the fair value of options based on the dynamics of the underlying asset. Options, which grant the holder the right but not the obligation to buy or sell an asset at a specified price, are highly valued in financial markets for their ability to hedge risks and speculate on price movements. However, accurately pricing options can be challenging, particularly when the underlying asset exhibits significant volatility.

In this report, we aim to determine the fair price of an option on Nvidia Corporation stock using the binomial tree simulation method. Nvidia, a prominent company in the semiconductor industry, is known for its high growth and innovation in areas like graphics processing and artificial intelligence. This, combined with its relatively high stock price volatility, makes Nvidia a fitting example for studying option pricing models.

The binomial tree model is chosen for this analysis due to its intuitive structure and flexibility in handling various option features. Unlike some closed-form models, such as the Black-Scholes formula, the binomial tree approach discretizes time into multiple steps, enabling a step-by-step simulation of potential price paths for Nvidia stock over the option's life. By calculating the potential payoff at each stage and then discounting these payoffs back to the present, we can estimate the option's fair price under the risk-neutral assumption.

The objective of this report is to illustrate how binomial tree simulation can be applied to price Nvidia options effectively, highlighting the method's usefulness for options where analytical solutions may be impractical. This approach provides valuable insights into option pricing mechanics and reinforces the importance of computational techniques in financial analysis.

2 Related literature

The study of option pricing has developed significantly since the pioneering work of Black and Scholes (1973) and Merton (1973), who introduced the now-famous Black-Scholes-Merton model. This closed-form solution allowed for the valuation of European options by assuming continuous price movements, constant volatility, and no dividends. The Black-Scholes-Merton model remains widely used due to its simplicity and analytical nature, but it has limitations in handling options with features such as early exercise or complex payoffs, and it may be less accurate for assets with high volatility, like Nvidia.

To address some of these limitations, Cox, Ross, and Rubinstein (1979) introduced the Binomial Tree Model, a discrete-time approach that provides more flexibility in option valuation. The binomial model allows for the valuation of both European and American options by simulating the underlying asset's price movements at each time step. At each node in the tree, the asset price can either go up by a factor or down by a factor and the option value is then determined by working backward from the expiration date to the present. This method is particularly useful for assets with high volatility and options that may require early exercise decisions. In practice, the binomial tree model provides a more accurate estimate of option prices when closed-form solutions are difficult to obtain, especially for American-style options and other path-dependent derivatives (Hull, 2009).

In recent years, researchers have extended the binomial model to account for various market conditions and underlying asset behaviors. For instance, Tian (1993) proposed modifications to the binomial model to better capture the volatility structure of underlying assets, making the model more accurate in situations where volatility is not constant. Additionally, Boyle (1986) and Broadie and Detemple (1996) explored further adaptations, such as the use of trinomial trees and higher-order approximations, to improve the model's convergence speed and precision.

Applications of the binomial model have also expanded beyond simple equity options. For example, Chiarella and Ziogas (2006) demonstrated its utility in valuing exotic options, while Leisen and Reimer (1996) refined the binomial approach by optimizing parameters to achieve faster convergence. This body of research illustrates the model's versatility and adaptability, especially for assets like Nvidia stock, where high volatility and rapid price fluctuations necessitate more robust simulation methods.

Several studies have highlighted the strengths and weaknesses of the binomial model in practice. While it is computationally more intensive than the Black-Scholes-Merton model, the binomial tree approach offers greater flexibility and accuracy when applied to complex financial derivatives. The model's ability to incorporate multiple steps allows it to approximate continuous-time processes more closely, a feature that is valuable for pricing options on volatile stocks such as Nvidia. The limitations of the binomial model, however, include increased computational time as the number of steps increases and potential inaccuracies when dealing with very high-frequency data.

3 Early exercise decisions in American-style options

One of the significant features of American-style options, as opposed to European-style options, is the possibility of early exercise. The binomial tree model inherently allows for the evaluation of this decision at each node, making it essential to consider potential early exercise opportunities when analyzing the value of the option.

In the context of a call option, early exercise may be advantageous under certain circumstances. One of the primary reasons to consider early exercise is the presence of dividend payments. If the underlying asset is expected to pay dividends before the option's expiration date, exercising the option early enables the holder to purchase the underlying stock and receive the dividend payout. This could enhance the overall return on investment. Additionally, when the option is deep in-the-money, meaning that the current stock price is significantly higher than the strike price, the intrinsic value becomes a crucial consideration. If the potential for further price appreciation is limited or if the time value of the option is minimal, exercising early may yield a higher payoff than holding onto the option.

Market conditions also play a significant role in the decision to exercise early. Factors such as market volatility, interest rates, and overall economic conditions can influence the attractiveness of early exercise. In a high-volatility environment, the potential for significant price movements may favor holding the option longer to capture further gains rather than exercising early.

The decision to exercise an option early can have several implications for its value. One immediate consequence is the reduction of the time value associated with the option. Since options gain value from the time remaining until expiration, exercising early means forgoing any potential future increases in option value due to market movements or volatility changes. This aspect is particularly relevant for options with considerable time until expiration. Moreover, the value of the option at any node in the binomial tree can be evaluated against the payoff from exercising the option early. If the immediate payoff from exercising exceeds the expected future value of holding the option, it suggests that early exercise may be the more profitable decision.

Early exercise decisions can also influence a trader's risk management strategy. By exercising early, the trader might lock in profits and hedge against potential adverse price movements, albeit at the cost of losing out on future gains. The model's structure allows for the simulation of various paths the underlying asset's price may take, and analyzing these paths can help identify the optimal timing for exercise, taking into account market conditions and potential future volatility.

Ultimately, the binomial tree model can be adjusted to reflect the expected value of holding the option versus exercising it. If holding the option is expected to provide greater value based on the underlying stock's volatility and market conditions, it may lead to a lower overall option price due to the higher likelihood of maintaining the position. In conclusion, while the binomial tree model provides a framework for evaluating potential early exercise decisions, it is crucial for option holders to weigh the immediate benefits against the potential for future gains. The decision to exercise early can significantly affect the option's value, and careful consideration of market conditions, intrinsic value, and time value is essential in making informed trading choices.

4 Methodology

The binomial tree model is used in this report to estimate the fair value of an option on Nvidia stock by constructing a discrete-time model that simulates potential price paths. This section outlines the steps taken to implement the binomial model, including the formulation of key parameters, construction of the binomial tree, and calculation of the option price through backward induction.

Model Parameters and Assumptions The binomial model divides the option's life span into n discrete time steps, where the price of Nvidia stock can move up or down in each step. The following parameters are essential for constructing the binomial tree:

- Initial Stock Price S_0: The current price of Nvidia stock at the time of valuation.
- Volatility σ: The annualized standard deviation of Nvidia stock returns, estimated from historical data.
- Risk-Free Rate r: The continuously compounded, risk-free interest rate over the life of the option.
- Time to Maturity T: The duration until the option's expiration, expressed in years.
- Number of Steps n: The number of discrete time intervals for the simulation. A higher n improves accuracy but increases computational intensity.

Calculating Up and Down Factors At each time step, the price of Nvidia stock can either increase by a factor u 0r decrease by a factor d. These factors are calculated based on the volatility and the length of each time interval $\Delta t = \frac{T}{n}$:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{1}$$

$$d = \frac{1}{u} = e^{-\sigma\sqrt{\Delta t}} \tag{2}$$

The use of $e^{\pm\sigma\sqrt{\Delta t}}$ ensures that the model captures the potential upward and downward movements of the stock price while maintaining a mean-reverting behavior over time.

Calculating the Risk-Neutral Probability Under the risk-neutral assumption, the expected return of the asset is the risk-free rate r. The risk-neutral probability p of an upward movement is calculated as:

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{3}$$

This probability is used to ensure that the expected value of the stock's price in the model aligns with the risk-free rate, in line with the principles of risk-neutral valuation.

Constructing the Binomial Tree The binomial tree is constructed by calculating the possible stock prices at each node. Starting from the initial stock price S_0 , we iterate over each time step to compute the possible prices at each node according to:

$$S_{i,j} = S_0 \cdot u^j \cdot d^{i-j} \tag{4}$$

where $S_{i,j}$ is the stock price at step i with j upward movements.

Calculating Option Payoffs at Expiration Once the binomial tree for stock prices is complete, we calculate the option payoffs at expiration. For a European call option, the payoff at each terminal node is given by:

$$\text{Payoff} = \max(S_{n,j} - K, 0) \tag{5}$$

where K is the strike price of the option.

Backward Induction for Option Pricing With the terminal payoffs calculated, we use backward induction to determine the option's value at each preceding node. At each node (i, j), the option value is given by:

$$\text{OptionValue}_{i,j} = e^{-r\Delta t}(p \cdot \text{OptionValue}_{i+1,j+1} + (1-p) \cdot \text{OptionValue}_{i+1,j}) \tag{6}$$

This process iterates from the last time step back to the present (initial node), where the calculated value is the estimated price of the option.

Simulation Process and Sensitivity Analysis To enhance the accuracy of the option price estimation, we conduct sensitivity analysis by varying parameters such as the volatility σ and the number of steps n . Sensitivity analysis provides insights into how changes in key parameters affect the option price, offering a better understanding of the impact of factors like volatility and interest rates on Nvidia's option pricing.

```python
import numpy as np

def binomial_option_pricing(S0, K, T, r, sigma, n, option_type='call'):

    dt = T / n  # Length of each time step
    u = np.exp(sigma * np.sqrt(dt))  # Up factor
    d = 1 / u  # Down factor
    p = (np.exp(r * dt) - d) / (u - d)  # Risk-neutral probability

    stock_prices = np.zeros(n + 1)
    option_values = np.zeros(n + 1)

    for i in range(n + 1):
        stock_prices[i] = S0 * (u ** i) * (d ** (n - i))

    if option_type == 'call':
        option_values = np.maximum(stock_prices - K, 0)
    elif option_type == 'put':
        option_values = np.maximum(K - stock_prices, 0)

    for j in range(n - 1, -1, -1):
        for i in range(j + 1):
            option_values[i] = np.exp(-r * dt) * (p * option_values[i + 1] -

    return option_values[0]

def calculate_confidence_interval(S0, K, T, r, sigma, n, option_type='call',
    prices = np.array([
        binomial_option_pricing(S0, K, T, r, np.random.normal(sigma, 0.05),
        for _ in range(num_simulations)
    ])

    mean_price = np.mean(prices)
    std_dev = np.std(prices)

    z_score = 1.96  # For a 95% confidence level
    margin_of_error = z_score * (std_dev / np.sqrt(num_simulations))

    lower_ci = mean_price - margin_of_error
    upper_ci = mean_price + margin_of_error

    return mean_price, lower_ci, upper_ci

# Example parameters
S0 = 450.0        # Initial stock price of Nvidia
K = 460.0         # Strike price
T = 1.0           # Time to maturity (1 year)
r = 0.05          # Risk-free interest rate (5% per annum)
sigma = 0.30      # Volatility (30% per annum)
n = 100           # Number of steps in the binomial tree
option_type = 'call'  # Type of option ('call' or 'put')
num_simulations = 1000  # Number of simulations to run

# Calculate the option price and confidence interval
mean_price, lower_ci, upper_ci = calculate_confidence_interval(S0, K, T, r,
print(f"The estimated price of the {option_type} option is: ${mean_price:.2
print(f"95% confidence interval: (${lower_ci:.2f}, ${upper_ci:.2f})")
```

```
The estimated price of the call option is: $59.91
95% confidence interval: ($59.38, $60.44)
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Confidence Interval and Error Estimation To assess the robustness of the binomial tree model for option pricing, we calculated a 95% confidence interval for the estimated price of a call option. The estimated price obtained from the simulations was \$59.91, with a 95% confidence interval of (\$59.38, \$60.44).

The central estimate of \$59.91 represents the most likely value for the call option under the specific parameters used in the model, which include the initial stock price, strike price, risk-free interest rate, volatility, and time to maturity. This estimate provides a quantitative assessment of the option's value based on the assumptions made in the pricing model.

The width of the confidence interval, approximately \$1.06, reflects the degree of uncertainty associated with the estimated price. A narrower interval would indicate greater certainty in the option price estimate, whereas a wider interval suggests a higher level of variability in the simulated prices. The moderate width of the interval in this case suggests a balanced degree of variability, emphasizing the importance of considering market fluctuations when evaluating option prices.

Introducing variability in the volatility parameter through random sampling allowed for a diverse range of simulated option prices, capturing the inherent uncertainty present in financial markets. This variability is crucial in financial modeling, as market conditions can lead to significant fluctuations in option pricing.

The resulting confidence interval provides a range of values within which the true price of the call option is likely to fall. If the market price of the option lies outside this interval, it may indicate that the option is either overvalued or undervalued compared to the model's predictions.

In future analyses, it would be beneficial to explore additional scenarios by varying other parameters, such as the risk-free rate or time to maturity, and by increasing the number of simulations. This approach could yield further insights into the sensitivity of the option pricing model to changes in market conditions, thereby enhancing the robustness of the findings.

Overall, the results underscore the significance of statistical measures like confidence intervals in the context of financial modeling, offering a framework for evaluating the reliability of option pricing estimates.

5 Financial implications

The simulation of option prices using the binomial tree model carries several important financial implications that can significantly impact investment strategies and risk management decisions. First and foremost, the estimated option price of \$59.91 derived from the simulations reflects a calculated fair value for the call option based on the current market conditions and the underlying asset's characteristics. This price serves as a benchmark for traders and investors, guiding their decisions regarding the purchase, sale, or holding of the option.

One of the primary financial implications of the price simulation is its role in identifying potential mispricing in the options market. If the market price of the call option deviates significantly from the estimated value, it could indicate an opportunity for arbitrage. Traders may seek to exploit these discrepancies by buying undervalued options or selling overvalued ones, thereby capitalizing on the potential for profit. This behavior contributes to market efficiency, as arbitrage activities tend to bring prices back in line with theoretical values over time.

Additionally, the calculated confidence interval of (\$59.38, \$60.44) introduces a layer of risk assessment to the decision-making process. Investors can utilize this interval to gauge the uncertainty associated with the option's price estimate. A wider confidence interval may signal higher volatility or unpredictability in the underlying asset, which could affect trading strategies. For instance, if the confidence interval is deemed too wide, investors might adopt a more conservative approach, adjusting their positions to mitigate potential losses.

The implications of price simulation extend to portfolio management as well. The estimated option price and its confidence interval can assist portfolio managers in optimizing their asset allocations. By incorporating options into their investment strategies, managers can hedge against potential downturns in the underlying stock or enhance returns through strategic positioning. The ability to simulate different price scenarios allows for more informed decision-making, enabling managers to adjust their strategies based on evolving market conditions and risk profiles.

Furthermore, the simulation results highlight the importance of considering market factors such as volatility, interest rates, and time to maturity. Changes in these parameters can significantly influence option pricing, making it essential for investors to stay informed about market dynamics. The model's flexibility allows for scenario analysis, enabling stakeholders to test the sensitivity of option prices to varying input values. This capability enhances strategic planning and aids in anticipating potential outcomes in different market environments.

In conclusion, the financial implications of the price simulation using the binomial tree model are profound. By providing a theoretical framework for estimating option prices and assessing associated risks, the simulation empowers investors and traders to make informed decisions, optimize their strategies, and effectively manage their portfolios. As such, the integration of price simulation into financial analysis serves as a vital tool for navigating the complexities of the options market.