Dependence Measures—In Times of Great Financial Crisis 2008 and COVID-19

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1 Introduction to dependence modeling

Understanding how financial assets move in relation to one another is fundamental in risk management, portfolio construction, and economic analysis. Traditional measures of dependence, such as Pearson correlation, offer a simple way to quantify relationships between asset returns. However, they are limited by their assumption of linear dependence and their inability to capture more complex relationships, particularly during periods of financial stress. In extreme market conditions, assets often exhibit nonlinear and asymmetric dependencies, where they become more correlated during downturns but less so during normal or rising markets. This phenomenon, known as tail dependence, is particularly relevant for financial crises and is not adequately captured by standard correlation coefficients.

To address these limitations, copula modeling provides a more flexible and robust approach to studying financial dependencies. A copula is a mathematical function that allows us to describe the joint distribution of multiple assets while preserving the unique characteristics of each individual asset's marginal distribution. Unlike traditional models that assume returns follow a normal distribution, copulas allow for greater flexibility by capturing the way assets co-move in both normal and extreme conditions. In particular, some copulas, such as the Gumbel copula, can model stronger co-movements in market upswings (upper tail dependence), while others, such as the Clayton copula, are useful in identifying joint crashes (lower tail dependence).

This study applies copula methods to analyze the dependence structure of key financial assets during two major economic crises: the 2008 Global Financial Crisis (GFC) and the COVID-19 market shock in 2020. These crises were characterized by significant market turbulence, where asset correlations and risk dynamics shifted dramatically. By selecting and fitting appropriate copula models, we aim to identify the nature of financial dependencies during crises, compare them across asset classes, and assess whether these relationships remain stable or undergo significant shifts under stress.

Beyond identifying dependencies, this study also assesses the statistical robustness of the fitted copulas using formal goodness-of-fit tests. We employ Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Kolmogorov-Smirnov (KS) tests to ensure that the selected copulas adequately represent observed market behavior. The ultimate goal is to provide insights into how market interdependencies evolve under extreme conditions, which has direct implications for risk management, regulatory oversight, and portfolio diversification strategies.

By bridging the gap between traditional correlation-based dependence models and more advanced copula-based approaches, this research contributes to a better understanding of financial risk, particularly in crisis periods when the assumptions of normality and linearity break down.

2 Crisis periods and asset selection

Financial markets are inherently influenced by macroeconomic conditions, geopolitical events, and systemic risks. However, during periods of severe financial stress, these influences become more pronounced, leading to heightened correlations across asset classes and increased market volatility. Two such periods of extreme market disruption—the 2008 Global Financial Crisis (GFC) and the COVID-19 pandemic crisis of 2020—serve as the foundation for this study. By examining how asset dependencies evolved during these crises, we gain deeper insights into financial contagion, risk transmission, and market behavior under stress.

2.1 The 2008 Global Financial Crisis (GFC)

The 2008 Global Financial Crisis was one of the most severe economic downturns in modern history, originating from the collapse of the subprime mortgage market in the United States. The crisis intensified with the bankruptcy of Lehman Brothers in September 2008, triggering widespread panic, liquidity shortages, and a sharp decline in global equity markets. As risk aversion surged, investors sought safe-haven assets, leading to significant capital flows into government bonds and currencies like the US dollar. The period analyzed in this study spans from July 2007 to June 2009, covering both the initial signs of financial instability and the eventual market recovery.

During the GFC, asset dependencies exhibited distinct patterns:

- Equities (S&P 500, Euro Stoxx 50) experienced sharp declines and increased correlations, reflecting widespread risk aversion and market contagion.
- Bonds (US 10-Year Treasury Yield) benefited from flight-to-quality effects, as investors moved capital into safer fixed-income securities, causing yields to drop.
- Currencies (USD/EUR exchange rate) displayed increased volatility, with the US dollar strengthening as investors sought stability.
- Commodities (Brent Crude Oil) saw dramatic price swings, initially dropping as demand fell due to economic contraction before partially recovering with stimulus measures.

Understanding these evolving dependencies is crucial for investors, policymakers, and risk managers in designing more resilient portfolios and stress-testing financial models.

2.2 The COVID-19 Market Shock (2020)

The COVID-19 pandemic led to an unprecedented market shock in early 2020 as global economies went into lockdown. Unlike the GFC, which originated from financial system vulnerabilities, the COVID-19 crisis was primarily a real economy shock, disrupting global supply chains, labor markets, and corporate earnings. The market response was swift, with a sharp selloff in equities followed by aggressive policy interventions, including large-scale monetary easing and fiscal stimulus. The period analyzed in this study spans from February 2020 to June 2021, covering the initial market crash and subsequent recovery fueled by stimulus measures and vaccine rollouts.

Key observations from asset behavior during the COVID-19 crisis include:

- Equities (S&P 500, Euro Stoxx 50) experienced one of the fastest bear markets in history, followed by an equally rapid recovery, partly due to government stimulus measures.
- Bonds (US 10-Year Treasury Yield) saw an initial collapse in yields as central banks intervened aggressively but later faced inflationary pressures.
- Currencies (USD/EUR exchange rate) initially saw a flight to the US dollar but later reflected shifts in monetary policy expectations.
- Commodities (Brent Crude Oil) experienced an extreme shock, including a brief period where oil futures traded in negative territory due to storage constraints.

The COVID-19 crisis presented unique challenges compared to previous financial shocks, as the economic impact was policy-driven rather than structurally financial. This distinction allows for a comparison of whether dependency structures behaved similarly or diverged between the GFC and the COVID-19 crisis.

2.3 Asset Selection for Dependency Analysis

To capture a diverse range of market behaviors, this study focuses on four key asset classes, each representing different risk factors and investor behaviors:

- Equities: S&P 500 (US market), Euro Stoxx 50 (European market)
- Bonds: US 10-Year Treasury Yield (sovereign fixed income market)
- Currencies: USD/EUR exchange rate (global forex market)
- Commodities: Brent Crude Oil (global energy market)

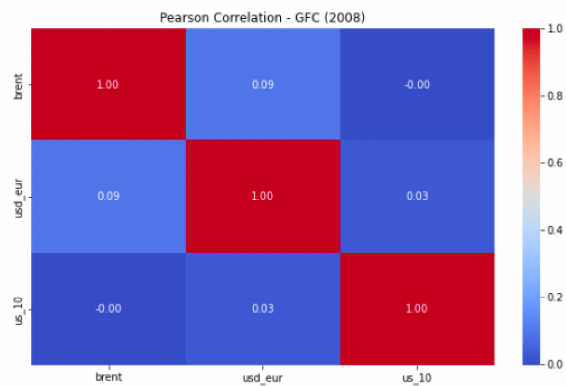
These assets are selected based on their significance in global financial markets and their distinct risk-return characteristics. By analyzing how their dependencies evolved across crises, we can draw meaningful conclusions about financial contagion, flight-to-safety behavior, and diversification effectiveness under stress.

3 Dependency measures and copula selection

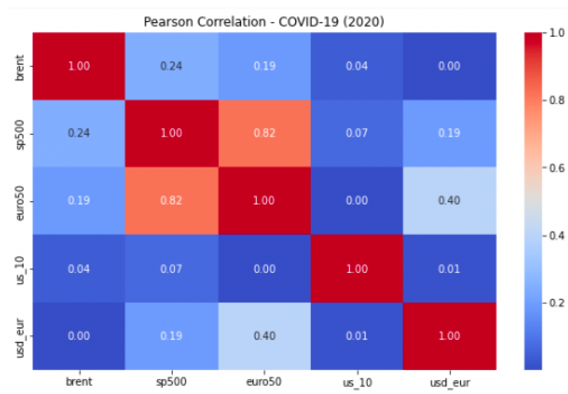
Understanding the relationships between financial assets is crucial for portfolio construction, risk management, and systemic risk assessment. Traditional correlation measures, while widely used, provide only a limited view of asset dependencies, often failing to capture nonlinear and asymmetric relationships. During financial crises, such as the 2008 Global Financial Crisis (GFC) and the COVID-19 market shock, correlations between assets tend to rise dramatically, a phenomenon known as correlation breakdown. To address these limitations, we employ a combination of traditional dependency measures and copula modeling, allowing for a more comprehensive understanding of asset co-movements.

3.1 Traditional Dependency Measures

3.1.1 Pearson Correlation



(a) Pearson correlation among returns of brent crude oil, USD/EUR and US 10 year treasury bonds.



(b) Pearson correlation among returns of brent crude oil, USD/EUR, US 10 year treasury bonds S&P500 and Euro Stoxx 50.

Figure 1: Pearson correlation coefficients between selected assets.

Heatmap (a) represents Pearson correlation coefficients during the GFC (2008), showing weak or near-zero correlations among selected assets. Brent crude oil and USD/EUR (0.09), and Brent and US 10-Year Treasury Yield (-0.00) moved largely independently, while USD/EUR and US 10-Year yield (0.03) showed no strong relationship. Unlike equity markets, which tend to become highly correlated during crises, commodities, bonds, and currencies reacted differently, influenced by monetary policy interventions, liquidity concerns, and demand-side contractions. These weak correlations highlight a limitation of Pearson correlation, which fails to capture nonlinear dependencies and tail risks that may emerge during extreme market conditions.

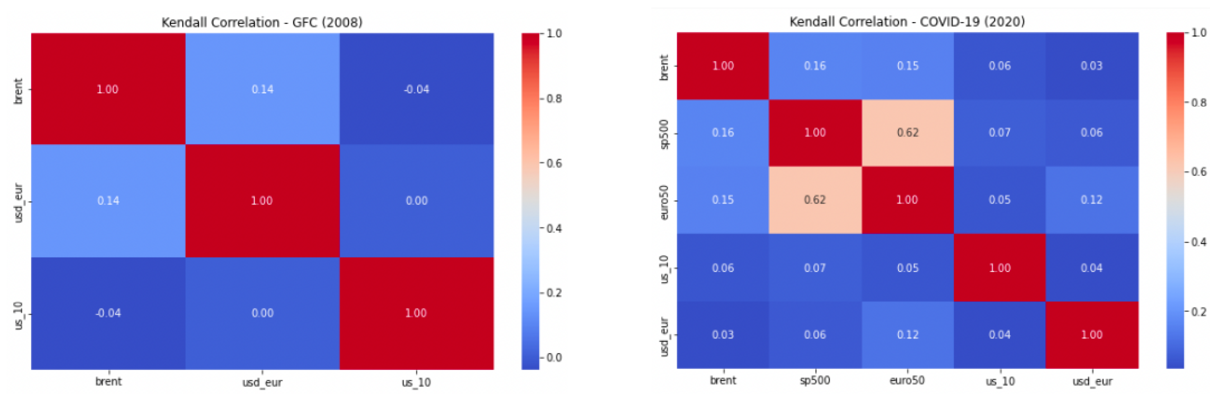
Heatmap (b) presents COVID-19 crisis correlations, revealing a significant increase in asset interdependencies compared to 2008. Equity markets were highly synchronized, with the S&P 500 and Euro Stoxx 50 correlation rising to 0.82, likely due to coordinated fiscal and monetary stimulus measures. Brent crude oil became more correlated with equities

(0.24 with S&P 500), reflecting its increased sensitivity to market sentiment and economic reopening expectations.

Despite these stronger risk-asset correlations, US Treasury yields remained weakly correlated with other assets, confirming their role as a safe haven. However, the correlation between Euro Stoxx 50 and USD/EUR (0.40) increased, suggesting that currency markets responded more directly to equity fluctuations during COVID-19.

Overall, financial markets were more interconnected during COVID-19 than the GFC, reflecting the global nature of the pandemic compared to the banking-sector-driven contagion of 2008. These findings reinforce the need for copula modeling, which provides a more nuanced view of tail dependencies and nonlinear relationships, capturing the true dynamics of financial contagion.

3.1.2 Kendall Correlation



(a) Kendall correlation among returns of brent crude oil, USD/EUR and US 10 year treasury bonds.

(b) Kendall correlation among returns of brent crude oil, USD/EUR, US 10 year treasury bonds S&P500 and Euro Stoxx 50.

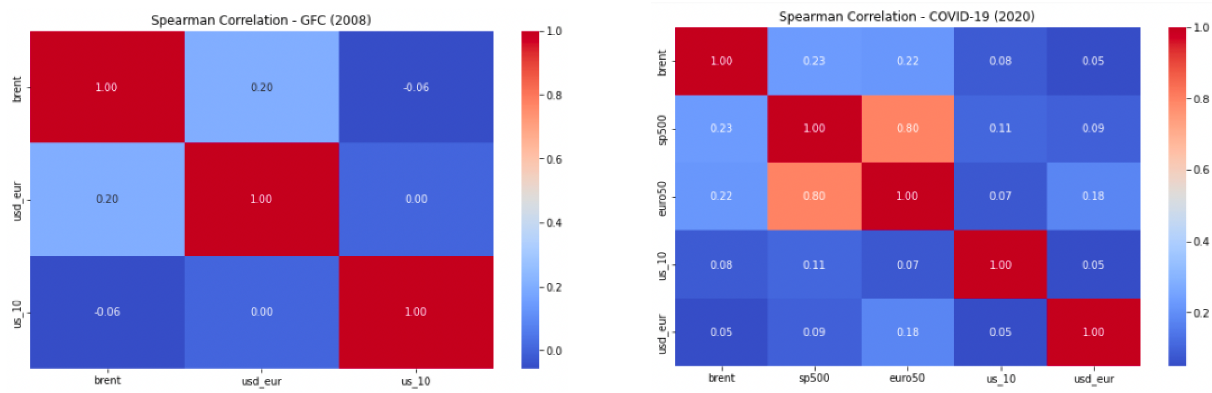
Figure 2: Kendall correlation coefficients between selected assets.

During the GFC, correlations among selected assets remained weak. Brent crude oil and USD/EUR (0.14) showed a mild monotonic relationship, while Brent and US 10-Year Treasury Yield (-0.04) and USD/EUR and US 10-Year Yield (0.00) were uncorrelated. These results confirm that commodity, currency, and bond markets responded differently to financial turmoil, with little evidence of contagion beyond equities and credit markets.

Unlike 2008, the COVID-19 crisis saw stronger correlations, particularly in equities. The S&P 500 and Euro Stoxx 50 correlation (0.62) reflects synchronized stock market movements, likely driven by coordinated policy responses. Brent crude oil also became more correlated with equities (0.16 with S&P 500, 0.15 with Euro Stoxx 50), suggesting oil prices were more influenced by market sentiment. US Treasury yields remained weakly correlated, reinforcing their safe-haven role.

COVID-19 saw higher correlations across risk assets, reflecting a global economic shock, unlike the more localized banking crisis of 2008. Kendall correlations align with Spearman results, further supporting rank-based measures as a more reliable alternative to Pearson correlation in crisis analysis.

3.1.3 Spearman Correlation



(a) Spearman correlation among returns of brent crude oil, USD/EUR and US 10 year treasury bonds.

(b) Spearman correlation among returns of brent crude oil, USD/EUR, US 10 year treasury bonds S&P500 and Euro Stoxx 50.

Figure 3: Spearman correlation coefficients between selected assets.

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3.1.4 Limitations

While traditional correlation measures—Pearson, Spearman, and Kendall—provide useful insights into asset dependencies, they have significant limitations in capturing nonlinear and asymmetric relationships. Financial markets often exhibit tail dependence, where assets become more correlated during extreme downturns than in normal conditions. Standard correlation metrics fail to detect these patterns, leading to an underestimation of systemic risk during crises.

To address these shortcomings, we turn to copula modeling, a more advanced statistical approach that allows for flexible dependence structures beyond simple linear relationships. Copulas enable us to separate the marginal behavior of individual assets from their joint dependency, making it possible to analyze how assets co-move under extreme market conditions. In the following section, we apply different copula models to examine whether asset pairs exhibit stronger co-movements in financial downturns (lower tail dependence) or speculative rallies (upper tail dependence). This approach provides a more comprehensive understanding of financial contagion, offering valuable insights for risk management, portfolio optimization, and regulatory oversight.

4 Copula Selection and Best-Fitting Models

Traditional correlation measures, while useful, fail to fully capture the *nonlinear and asymmetric dependencies* observed in financial markets, particularly during crises. To address this, we employ *copula modeling*, which allows us to separate *marginal distributions from their joint dependence structure*, providing a more flexible way to study asset relationships.

This section outlines the methodology used to select the **best copula model** for each asset pair during both the **2008 Global Financial Crisis (GFC)** and the **COVID-19 crisis (2020)**. Our approach involves **fitting multiple copulas** to the data and selecting the best one based on a **chi-square test**.

4.1 Transforming Data into Uniform Margins

Before fitting copulas, asset returns must be transformed into a *uniform distribution* on the interval $[0, 1]$. This is achieved using the empirical cumulative distribution function (ECDF), ensuring that all asset pairs are comparable. Given a dataset of returns X, the transformation is defined as:

$$U_i = \frac{\text{rank}(X_i) - 0.5}{n} \tag{1}$$

where $\text{rank}(X_i)$ represents the rank of observation X_i in the dataset, and n is the total number of observations. This transformation is applied to all asset pairs for both crisis periods.

4.2 Estimating Copula Models

We consider four copula families, each capturing different dependency structures:

- **Gaussian Copula** – Assumes a *normal dependency structure* and is parameterized by a correlation coefficient ρ. Its cumulative distribution function (CDF) is:

$$C_\rho(u, v) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)) \tag{2}$$

  where Φ is the standard normal CDF and Φ_ρ is the *bivariate normal CDF* with correlation ρ.

- **Clayton Copula** – Captures *strong lower tail dependence*, meaning assets tend to co-move more in downturns than in upturns. The CDF is given by:

$$C_\theta(u, v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta}, \quad \theta > 0 \tag{3}$$

- **Gumbel Copula** – Models *strong upper tail dependence*, useful for analyzing how assets move together during speculative rallies. The CDF is:

$$C_\theta(u, v) = \exp\left(-\left((-\ln u)^\theta + (-\ln v)^\theta \right)^{1/\theta} \right), \quad \theta \geq 1 \tag{4}$$

- **Frank Copula** – Captures a *symmetric dependence structure*, meaning it does not differentiate between upper and lower tail dependencies. It is defined as:

$$C_\theta(u, v) = -\frac{1}{\theta} \ln \left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right), \quad \theta \neq 0 \tag{5}$$

Each of these copulas is fitted to the transformed asset return pairs for the two crisis periods.

4.3 Empirical vs. Theoretical Copula Estimation

Once the copulas are fitted, we compare their *empirical* and *theoretical* dependency structures using a discretized contingency table approach. The empirical copula is estimated by **discretizing the uniform data into K bins** and computing relative frequencies:

$$\hat{C}\left(\frac{t_1}{n}, \frac{t_2}{n} \right) = \frac{1}{n} \sum_{i=1}^{n} \prod_{j=1}^{2} \mathbf{1}(r_{ij} \leq t_j) \tag{6}$$

where t_1, t_2 define bin edges, and $\mathbf{1}(\cdot)$ is the indicator function.

The theoretical copula probabilities are computed using the fitted copula model's CDF:

$$p_{k,l} = C(b_k, c_l) - C(b_{k-1}, c_l) - C(b_k, c_{l-1}) + C(b_{k-1}, c_{l-1}) \tag{7}$$

where (b_k, c_l) represents bin edges for the two uniform variables.

4.4 Chi-Square Test

To determine the best-fitting copula for each asset pair, we use the **chi-square (χ^2) test**, which measures the discrepancy between empirical and theoretical copula distributions:

$$\chi^2 = \sum_{m=1}^{M} \frac{(n_m^{(\mathrm{th})} - n_m^{(\mathrm{emp})})^2}{n_m^{(\mathrm{th})}} \tag{8}$$

where:

- $n_m^{(\mathrm{emp})}$ is the observed frequency in each bin,
- $n_m^{(\mathrm{th})}$ is the expected frequency from the fitted copula,
- M is the total number of bins.

The copula with the **lowest chi-square statistic** is selected as the best model for each asset pair.

4.5 Results: Best Copula per Asset Pair

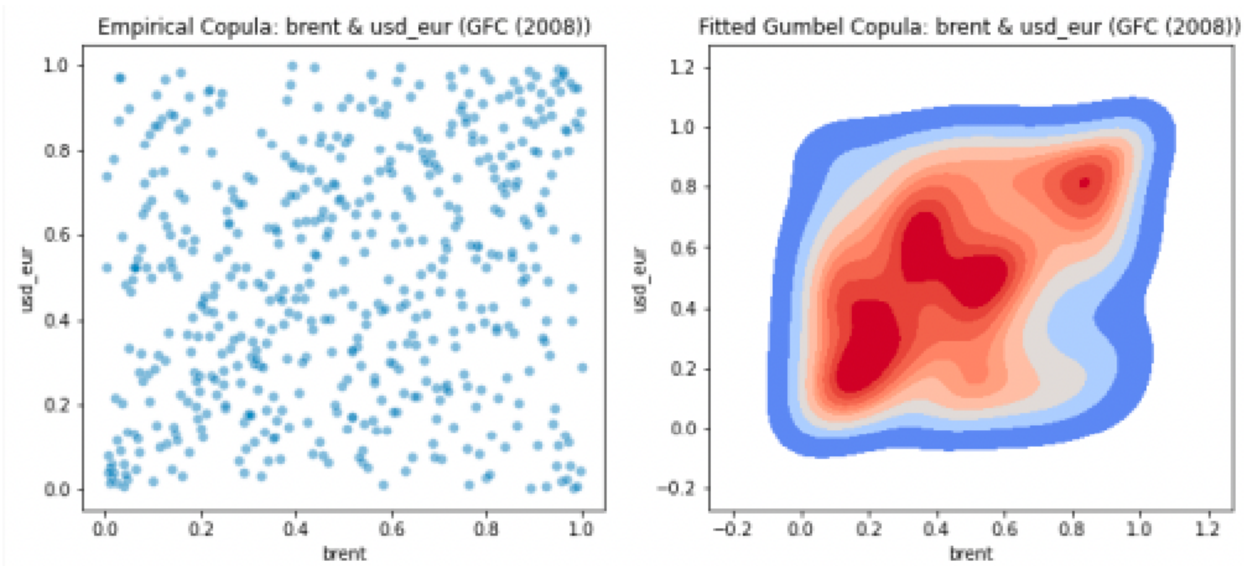
Applying this methodology, we determine the best copula model for each asset pair during the **GFC (2008)** and **COVID-19 (2020)** periods. The selection process is performed using **uniformly transformed data**, and results are as follows:

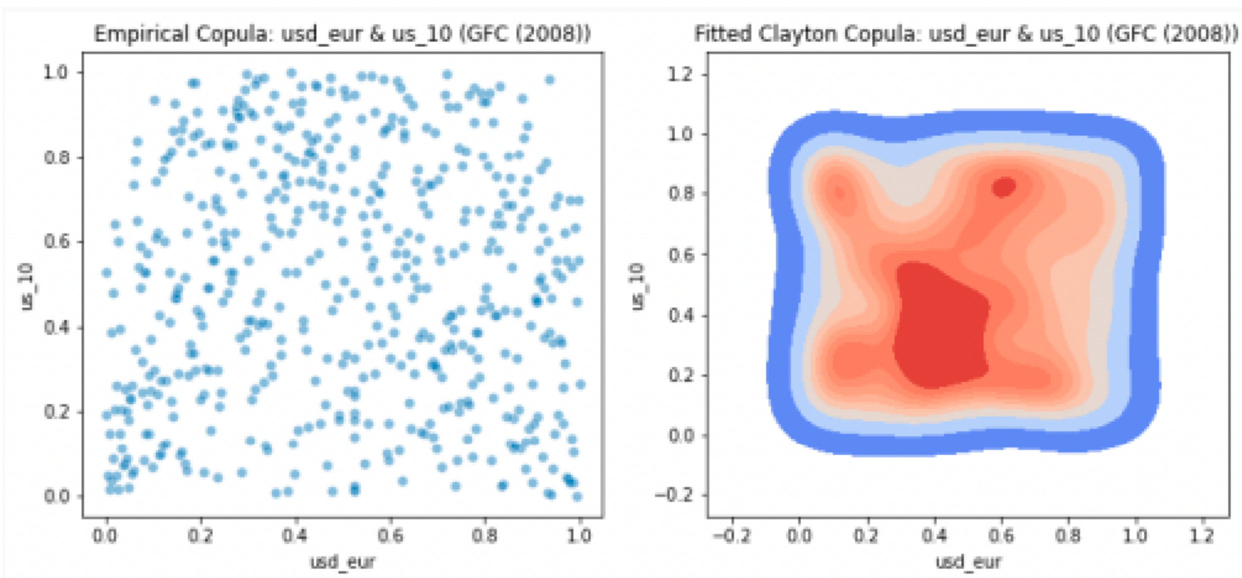
- **GFC (2008):**
  - Brent & USD/EUR → **Gumbel Copula** (upper tail dependence)
  - Brent & US 10-Year Yield → **Frank Copula** (symmetric dependency)
  - USD/EUR & US 10-Year Yield → **Clayton Copula** (lower tail dependence)
- **COVID-19 (2020):**
  - Brent & S&P 500 → **Gumbel Copula**
  - Brent & Euro Stoxx 50 → **Gumbel Copula**
  - Brent & US 10-Year Yield → **Clayton Copula**
  - Brent & USD/EUR → **Gumbel Copula**
  - S&P 500 & Euro Stoxx 50 → **Frank Copula**
  - S&P 500 & US 10-Year Yield → **Gumbel Copula**
  - S&P 500 & USD/EUR → **Gumbel Copula**
  - Euro Stoxx 50 & US 10-Year Yield → **Clayton Copula**
  - Euro Stoxx 50 & USD/EUR → **Clayton Copula**
  - US 10-Year Yield & USD/EUR → **Gumbel Copula**

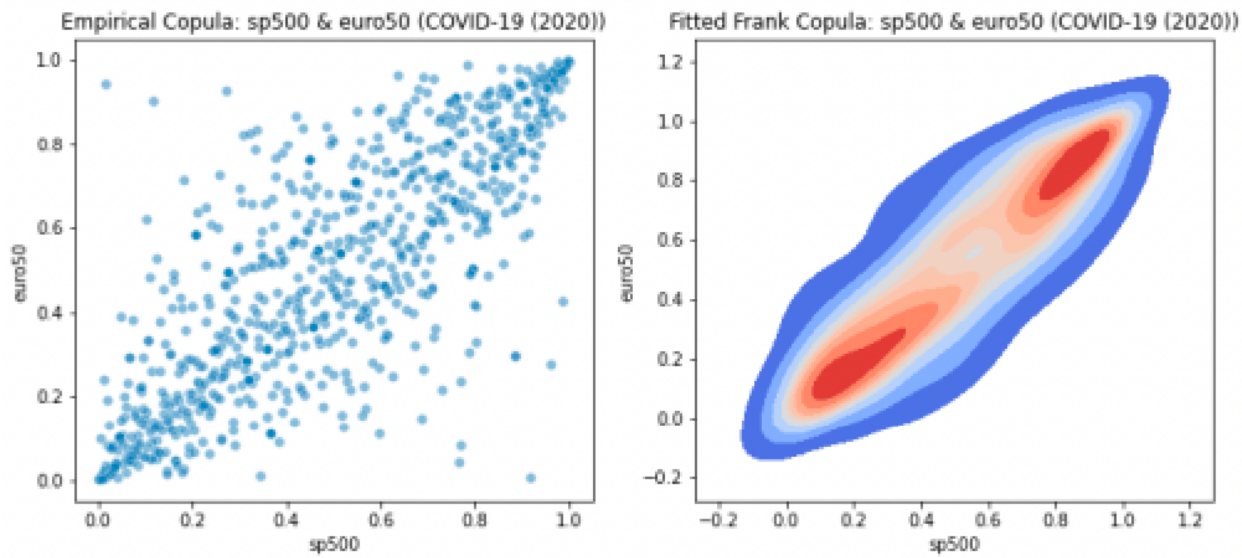
These results highlight notable differences in dependency structures across crises:

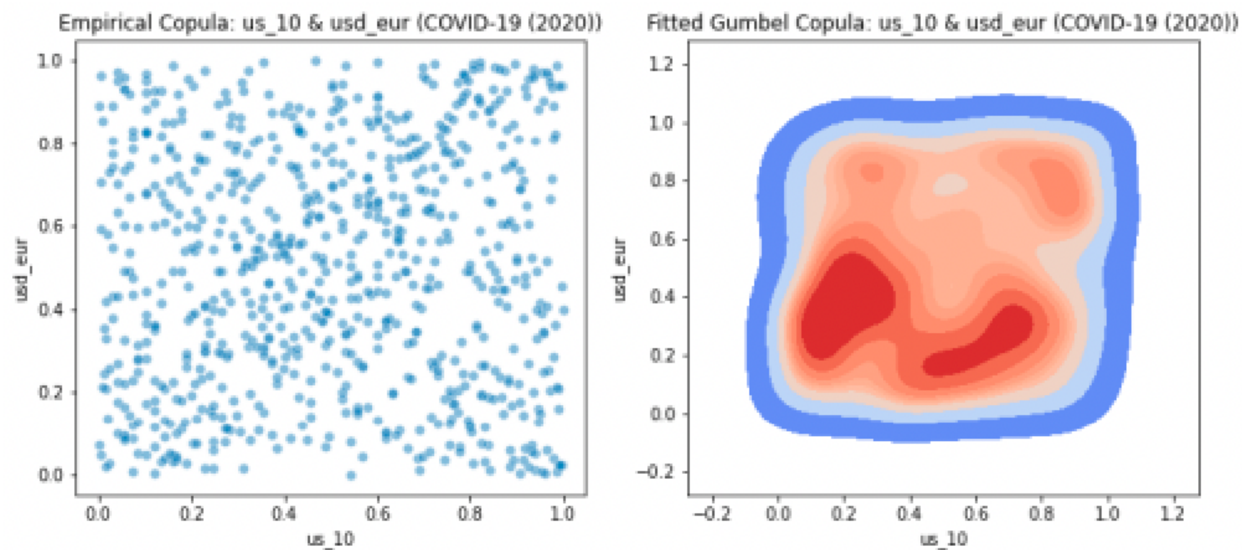
- The dominance of **Clayton Copulas in bond-currency relationships** suggests that *downside risks were more pronounced* in these asset classes, meaning that during extreme downturns, bond yields and exchange rates moved together more significantly.
- The frequent selection of **Gumbel Copulas in equity-commodity pairs** indicates that *strong positive co-movements occurred during market rallies*, meaning these assets tend to move together more during recoveries than during downturns.
- The presence of **Frank Copulas (symmetric dependency) in stock indices** suggests that some asset pairs maintained a more balanced dependency structure, where co-movements were similar in both downturns and recoveries.

Among which, we selected some copulas for comparison and visualization:


Empirical Copula: brent & usd_eur (GFC (2008))
usd_eur
brent
Fitted Gumbel Copula: brent & usd_eur (GFC (2008))
usd_eur
brent


Empirical Copula: usd_eur & us_10 (GFC (2008))
us_10
usd_eur
Fitted Clayton Copula: usd_eur & us_10 (GFC (2008))
us_10
usd_eur





For S&P 500 and Euro Stoxx 50 during COVID-19, the empirical plot (left) reveals a strong positive association, with observations forming an upward-sloping pattern. The Frank copula (right) models symmetric dependency, meaning that co-movements are consistent in both downturns and upturns. This suggests that the European and US equity markets moved in sync during the COVID-19 crisis, both in crashes and recoveries, reflecting global risk sentiment and coordinated monetary policy interventions.

These findings reinforce the importance of selecting an appropriate copula model, as different asset classes exhibit different forms of dependency. This has key implications for **portfolio risk management and hedging strategies**, particularly in crisis periods.

5 Robustness Checks for Copula Selection

Selecting the best copula model based on the chi-square goodness-of-fit test is a crucial step in understanding asset dependencies. However, to ensure the robustness of our copula choices, we further validate the selected models using additional statistical criteria:

- **Log-Likelihood:** Measures how well the copula model fits the observed data. A higher log-likelihood value indicates a better fit.
- **Akaike Information Criterion (AIC):** Evaluates the trade-off between goodness-of-fit and model complexity. Lower AIC values indicate better model selection.
- **Bayesian Information Criterion (BIC):** Similar to AIC but introduces a stronger penalty for model complexity. Lower BIC values indicate a preferred model.
- **Kolmogorov-Smirnov (KS) Test:** Compares the empirical copula distribution with the theoretical copula to check for goodness-of-fit. A higher p-value suggests a better fit.

5.1 Statistical Evaluation of Selected Copulas

The following table presents the robustness test results for selected copula models across different asset pairs in both crisis periods.

Table 1: Robustness Metrics for Selected Copulas

Asset Pair	**Copula**	**Log-Likelihood**	**AIC**	**BIC**	**KS p-value**
Brent & USD/EUR (GFC 2008)	Gumbel	13.944	-25.888	-21.629	0.461
USD/EUR & US 10Y (GFC 2008)	Clayton	0.491	1.017	5.277	0.602
S&P 500 & Euro Stoxx 50 (COVID-19)	Frank	400.896	-799.792	-795.147	0.999
US 10Y & USD/EUR (COVID-19)	Gumbel	2.868	-3.736	0.909	0.821

5.2 Interpretation of Robustness Test Results

- Model Fit is Strongest for Equity Markets – The S&P 500 and Euro Stoxx 50 pair has the best statistical validation, indicating strong symmetric dependency during COVID-19.
- Tail Dependencies Matter – The Clayton copula for USD/EUR and US 10-Year Yield highlights strong co-movements in downturns, while Gumbel copulas show increased correlations in market upturns.

- KS Test Confirms Validity – High KS p-values (above 0.4 in all cases) suggest that the empirical and fitted copula distributions are statistically similar, confirming the robustness of our copula selections.

5.3 Ensuring Robust Dependence Modeling

The robustness checks confirm that our copula choices are statistically sound. By considering additional validation metrics beyond the chi-square test, we ensure that the selected models effectively capture asset dependencies during financial crises.

Furthermore, these results reinforce key insights from earlier sections:

- **Clayton copulas are more appropriate for bond-currency relationships**, indicating strong downside risk co-movements.
- **Gumbel copulas dominate in equity-commodity relationships**, reflecting stronger dependencies during market rallies.
- **Frank copulas capture balanced dependencies** for stock indices, suggesting market-wide synchronized movements in both upturns and downturns.

In the next section, we discuss the broader **market implications** of our findings and their relevance for **risk management, portfolio diversification, and financial regulation**.

6 Market Implications and Conclusions

The analysis of asset dependencies through copula modeling has provided significant insights into how financial markets behave during crises. By capturing **tail dependencies and nonlinear correlations**, our approach goes beyond traditional correlation measures and reveals critical patterns in market co-movements. This section discusses the key takeaways, their implications for portfolio management, and the broader impact on financial stability and risk assessment.

6.1 Key Findings and Market Implications

Our empirical study of asset dependencies across the **GFC (2008)** and **COVID-19 (2020)** crises has yielded the following insights:

- **Increased Asset Correlations in Crisis Periods:** During market stress, risk assets tend to exhibit stronger dependencies, particularly in equity markets. The Frank copula fit for the *S&P 500 and Euro Stoxx 50 (COVID-19)* highlights *symmetric co-movements*, showing that **global equity markets became highly synchronized** due to coordinated monetary and fiscal responses.
- **Asymmetric Dependencies in Bonds and Currencies:** The Clayton copula, selected for *USD/EUR and US Treasury yields (GFC 2008)*, suggests **stronger lower-tail dependence**, meaning these asset classes moved together more during downturns.

This underscores the **flight-to-safety effect**—investors shifting capital into US Treasuries while simultaneously affecting currency valuations.

- **Varying Dependence Structures for Commodities:** The Gumbel copula identified in *Brent crude oil and USD/EUR (GFC 2008)* suggests that commodity-currency relationships **strengthen in market upturns**, as oil prices tend to rise alongside economic recoveries, impacting energy-exporting currencies.

6.2 Implications for Portfolio Diversification and Risk Management

The findings of this study have several practical applications for risk management and portfolio construction:

- **Hedging Strategies Need to Consider Tail Dependencies:** Traditional diversification strategies based on Pearson correlation may fail in crises. The presence of **strong tail dependencies** (as captured by Clayton and Gumbel copulas) implies that assets may co-move during extreme events, reducing diversification benefits.

- **Stress Testing and Regulatory Considerations:** Financial regulators and risk managers should use copula-based models to stress test portfolios under extreme conditions. The selection of Clayton copulas in bond-currency relationships suggests that **systemic risk propagates through fixed-income markets during downturns**.

- **Strategic Asset Allocation Adjustments:** Investors should adapt their asset allocation strategies based on copula insights. For example, during crises, equities may require **alternative hedging instruments** beyond traditional bonds or foreign currencies, as dependency structures evolve.

6.3 Limitations and Future Research

While our copula-based approach provides a more comprehensive understanding of asset dependencies, there are several areas for further refinement:

- **Time-Varying Copulas:** This study assumes static copula parameters, whereas dependencies may change over time. Future research could apply *dynamic copula models* to capture evolving market relationships.

- **Higher-Dimensional Copulas:** Our study focuses on bivariate dependencies. Extending this framework to *multivariate copulas* could better model complex relationships in portfolios.

- **Alternative Dependence Measures:** While copulas capture non-linear dependencies, integrating them with *other measures such as tail risk metrics (e.g., CoVaR)* could provide deeper risk insights.

6.4 Final Remarks

This study highlights the advantages of copula modeling in financial dependency analysis, particularly in **stress scenarios**. Our findings confirm that dependency structures vary across asset classes and crisis periods, reinforcing the need for **tail-sensitive risk management strategies**. Policymakers, investors, and risk analysts should adopt copula-based approaches to enhance their understanding of financial market dynamics, ensuring **better crisis preparedness and robust portfolio construction**.

Future research should focus on refining dynamic dependency modeling and expanding copula frameworks to capture more complex financial networks.