

hw3

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1 (27)

a.

induction

step1: prove $S(0)$ is odd

$S(0)=1$ which is odd

step2: assume $S(k)$ is odd which means $S(k)=2k+1$ ($k \geq 0$)

step3: prove $S(k+1)$ is odd

$S(k+1) = 2k+3$ ($(k \geq 0)$)

even + odd = odd

the statement that $S(k+1)$ is odd get proved.

so, $S(n)$ is an odd number for $n \geq 0$

b.

induction

step1: prove $S(4) < 6S(2)$

$17 < 18$

step2: assume $S(k) < 6S(k-2)$ $k \geq 4$

step3: prove $S(k+1) < 6S(k-1)$

$S(k+1) = 12S(k-2)+5S(k-3)$

$6S(k-1)=12S(k-2)+6S(k-3)$

$S(k+1) < 6S(k-1)$ get proved

so, $S(n) < 6S(n-2)$ for $n \geq 4$

2 (37)

$$\begin{aligned} S(0) &= a \\ S(n) &= S(n-1) + d \quad (n \geq 1) \end{aligned}$$

3 (11)

From 1200, 1800, 2700, 4050:
we could derive $S(n) = 1.5 * S(n-1)$ ($n \geq 2$)

from $S(n) = c * S(n-1) + g(n)$
we could derive $S(n) = 1200 * (1.5^{n-1})$

$$S(12) = 1200 * (1.5^{11}) = 103797$$

4 (16)

infection:

$$\begin{aligned} S_0 &= 14 \\ S_n &= 14 * 5^{n-1} \end{aligned}$$

$$T_n = -(3.5) * (1 - 5^n)$$

solution:

$$\begin{aligned} S_0 &= 1 \\ S_n &= 6^{n-1} \end{aligned}$$

$$T_n = -0.2 * (1 - 6^n)$$

$$\begin{aligned} T_{solution} &> T_{infection} \\ n &= 16 \end{aligned}$$

5 (additional problem)