Machine Learning Worksheet 12

Latent Variable Models

1 K-Means and MoG

Problem 1: Consider a mixture of K isotropic Gaussians, each with the same covariance $\boldsymbol{\Sigma} = \sigma^2 \boldsymbol{I}$. In the limit $\sigma^2 \to 0$ show that the EM algorithm for MoG converges to the K-Means algorithm.

Problem 2: Consider a mixture of K Gaussians

$$p(\boldsymbol{x}) = \sum_k \pi_k \mathcal{N}(\boldsymbol{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

Derive $E(\boldsymbol{x})$ and $Cov(\boldsymbol{x})$. It is helpful to remember the identity $Cov(\boldsymbol{x}) = E(\boldsymbol{x}\boldsymbol{x}^T) - E(\boldsymbol{x})E(\boldsymbol{x})^T$.