

Machine Learning Worksheet 05**Multivariate Normal**

Problem 1: We say that two random variables are pairwise independent, if $P(X_2 | X_1) = P(X_2)$ and hence

$$P(X_2, X_1) = P(X_1)P(X_2 | X_1) = P(X_1)p(X_2)$$

We say that n random variables are mutually independent if

$$P(X_i | X_S) = P(X_i) \quad \forall S \subseteq \{1, \dots, n\} \setminus \{i\}$$

and hence

$$P(X_{1:n}) = \prod_{i=1}^n P(X_i)$$

Show that *pairwise* independence between all pairs of variables however does not necessarily imply mutual independence. It suffices to give a counter example.

Problem 2: Let X and Y be two random variables. Express $Var[X + Y]$ in terms of $Var[X]$, $Var[Y]$ and $Cov[X, Y]$.

Problem 3: Let X be a random variable. Show that, if $Y = aX + b$ for some parameters $a > 0$ and b , then $\rho(X, Y) = 1$. Similarly show that if $a < 0$, then $\rho(X, Y) = -1$.

Problem 4: Let $Z = (X, Y)$ be a bivariate normal distributed random variable. Furthermore, let $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ and $Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$. Assume that $\rho(X, Y) = 0$. Show that in this case X and Y are independent.