Machine Learning Worksheet 10

Gaussian Process Regression

Let $\boldsymbol{X} = \begin{bmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\end{bmatrix} = \begin{pmatrix}0 & 1\\ 0.5 & 2\end{pmatrix}$ and $\boldsymbol{y} = \begin{pmatrix}y_1\\ y_2\end{pmatrix} = \begin{pmatrix}2\\ 5\end{pmatrix}$ be observations and $\boldsymbol{x}_* = \begin{pmatrix}0.5 & 1\end{pmatrix}$ be a datapoint with unknown function value $\boldsymbol{f}_* = f(\boldsymbol{x}_*)$.

A distribution over the function f is given by a Gaussian process $f \sim GP(m, K)$ with mean function $m(\boldsymbol{x}) = 0$ and covariance function $K(\boldsymbol{x}, \boldsymbol{x}') = \exp(-\frac{1}{2}(\boldsymbol{x} - \boldsymbol{x}')^T(\boldsymbol{x} - \boldsymbol{x}'))$.

Problem 1: Assuming a noise-free scenario, write down the joint distribution for $\boldsymbol{f}_{\text{jt}} = \begin{pmatrix}y_1\\ y_2\\ f(\boldsymbol{x}_*)\end{pmatrix}$.

Problem 2: Write down the conditional distribution $p(f_*|\boldsymbol{y}, \boldsymbol{X})$ using the rules for conditionals of an MVN (MVN/GP lecture slides).

Problem 3: Now, assume instead that the observations are disturbed by Gaussian noise with variance σ_n^2. Write down the joint distribution $p(\boldsymbol{y}, f(\boldsymbol{x}_1))$.