1 problem.

Maximize $z(x_1, x_2, x_3) = x_1 + 2x_2 - x_3$

subject to

$-x_1 + 4x_2 - 2x_3 \leqslant 12$

$x_1 + x_2 + 2x_3 \leqslant 17$

$2x_1 - x_2 + 2x_3 = 4$

$x_1, x_2, x_3 \geq 0$

1) Equality form

Maximize $z(x_1, x_2, x_3) = x_1 + 2x_2 - x_3$

subject to

$-x_1 + 4x_2 - 2x_3 + x_4 = 12$ (1)

$x_1 + x_2 + 2x_3 + x_5 = 17$ (2)

$2x_1 - x_2 + 2x_3 + x_6 = 4$ (3)

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ (4)

ITERATION 1

2) Initial trial solution:

$$X_0 = (x_1, x_2, x_3, x_4, x_5, x_6) = (1, \frac{1}{2}, 1, 13, \frac{27}{2}, \frac{1}{2})$$

3) Fill D:

$$D = \begin{bmatrix} x_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & x_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & x_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 13 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{27}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

4) Put (1) – (4) in matrix A:

$$A = \begin{bmatrix} -1 & 4 & -2 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 2 & -1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

5) Fill C:

$$C = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

5) Calculate $\widetilde{A} = AD$ :

$$\tilde{A}=\begin{bmatrix}-1 & 4 & -2 & 1 & 0 & 0\\ 1 & 1 & 2 & 0 & 1 & 0\\ 2 & -1 & 2 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 13 & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{27}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}-1 & 2 & -2 & 13 & 0 & 0\\ 1 & 0.5 & 2 & 0 & 13.5 & 0\\ 2 & -0.5 & 2 & 0 & 0 & 0.5\end{bmatrix}$$

and $\tilde{C}=DC$ :

$$\tilde{C}=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 13 & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{27}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\end{bmatrix}\begin{bmatrix}1\\ 2\\ -1\\ 0\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}1\\ 1\\ -1\\ 0\\ 0\\ 0\end{bmatrix}$$

6) Calculate $P=I-\tilde{A}^T(\tilde{A}\tilde{A}^T)^{-1}\tilde{A}$

and calculate $C_p=P\tilde{C}$

$$C_p=\begin{bmatrix}\frac{-462719}{4300814}\\ \frac{112888}{2150407}\\ \frac{-265707}{2150407}\\ \frac{840411}{4300814}\\ \frac{-27324}{2150407}\\ \frac{-49253}{2150407}\end{bmatrix}=\begin{bmatrix}-0.1075887\\ 0.05249611\\ -0.12356126\\ 0.19540743\\ -0.01266458\\ -0.02290404\end{bmatrix}$$

7) Identify the negative component of $C_p$ having the largest absolute value, and set ν equal to this absolute value.

ν = 0.12356126

8) Calculate $\tilde{x}=\begin{bmatrix}1\\ 1\\ \dots\\ 1\end{bmatrix}+\frac{\alpha}{\nu}C_p$

$$\tilde{x}=\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}+\frac{0.5}{0.12356126}\begin{bmatrix}-0.1075887\\0.05249611\\-0.12356126\\0.19540743\\-0.01266458\\-0.02290404\end{bmatrix}=\begin{bmatrix}0.56463417\\1.21242949\\0.5\\1.79073097\\0.94875182\\0.90731707\end{bmatrix}$$

9) Calculate $X=D\tilde{x}$

$$X=\begin{bmatrix}1&0&0&0&0&0\\0&\frac{1}{2}&0&0&0&0\\0&0&1&0&0&0\\0&0&0&13&0&0\\0&0&0&0&\frac{27}{2}&0\\0&0&0&0&0&\frac{1}{2}\end{bmatrix}\begin{bmatrix}0.56463417\\1.21242949\\0.5\\1.79073097\\0.94875182\\0.90731707\end{bmatrix}=\begin{bmatrix}0.564634\\0.6062145\\0.5\\23.27949\\12.8081385\\0.4536585\end{bmatrix}$$

10) Check how different the new x is from the old one:

$$X_0-X=\begin{bmatrix}1\\0.5\\1\\13\\13.5\\0.5\end{bmatrix}-\begin{bmatrix}0.564634\\0.6062145\\0.5\\23.27949\\12.8081385\\0.4536585\end{bmatrix}=\begin{bmatrix}0.435366\\-0.1062145\\0.5\\-10.27949\\0.6918615\\0.0463415\end{bmatrix}$$

$$\|X_0-X\|=\left(0.435366^2+(-0.1062145)^2+(0.5)^2+(-10.27949)^2+(0.6918615)^2+(0.0463415)^2\right)^{\left(\frac{1}{2}\right)}=1.6828988$$

The result $>0.00001$ , therefore, X – initial trial solution for the next iteration.

Therefore, $X$ - initial trial solution for next iteration.

ITERATION 2

11) Fill D:

$$D=\begin{bmatrix}0.564634&0&0&0&0&0\\0&1.212429&0&0&0&0\\0&0&0.5&0&0&0\\0&0&0&1.79073097&0&0\\0&0&0&0&0.94875&0\\0&0&0&0&0&0.907317\end{bmatrix}$$

12)Calculate $\tilde{A}=AD$ and $\tilde{C}=DC$ :

13) Calculate $P=I-\tilde{A}^T(\tilde{A}\tilde{A}^T)^{-1}\tilde{A}$ and calculate $C_p=P\tilde{C}$

14) Identify the negative component of $C_p$ having the largest absolute value, and set ν equal to this absolute value.

15) Calculate $\tilde{x}=\begin{bmatrix}1\\1\\\dots\\1\end{bmatrix}+\frac{\alpha}{v}C_p$

16) Calculate $X=D\tilde{x}$

$$X = \begin{bmatrix} 2.31552523 \\ 1.58650358 \\ 0.25 \\ 8.46951092 \\ 12.59797119 \\ 0.45545312 \end{bmatrix}$$

17) Check how different the new x is from the old one. X changed enough, therefore, X- initial trial solution for next iteration.

ITERATION 3

$$X = \begin{bmatrix} 2.95799087 \\ 2.79198488 \\ 0.22235206 \\ 4.23475546 \\ 10.80532013 \\ 0.43129902 \end{bmatrix}$$

ITERATION 4

$$X = \begin{bmatrix} 3.28682646 \\ 3.39688477 \\ 0.20904518 \\ 2.11737773 \\ 9.8981984 \\ 0.40514149 \end{bmatrix}$$

…

ITERATION n

$$X = \begin{bmatrix} 3.99999558 \\ 3.99999901 \\ 0.00000224 \\ 0.00000403 \\ 9.00000093 \\ 0.00000336 \end{bmatrix}$$