DCM workshop

Part 2

- Second-level PEB model

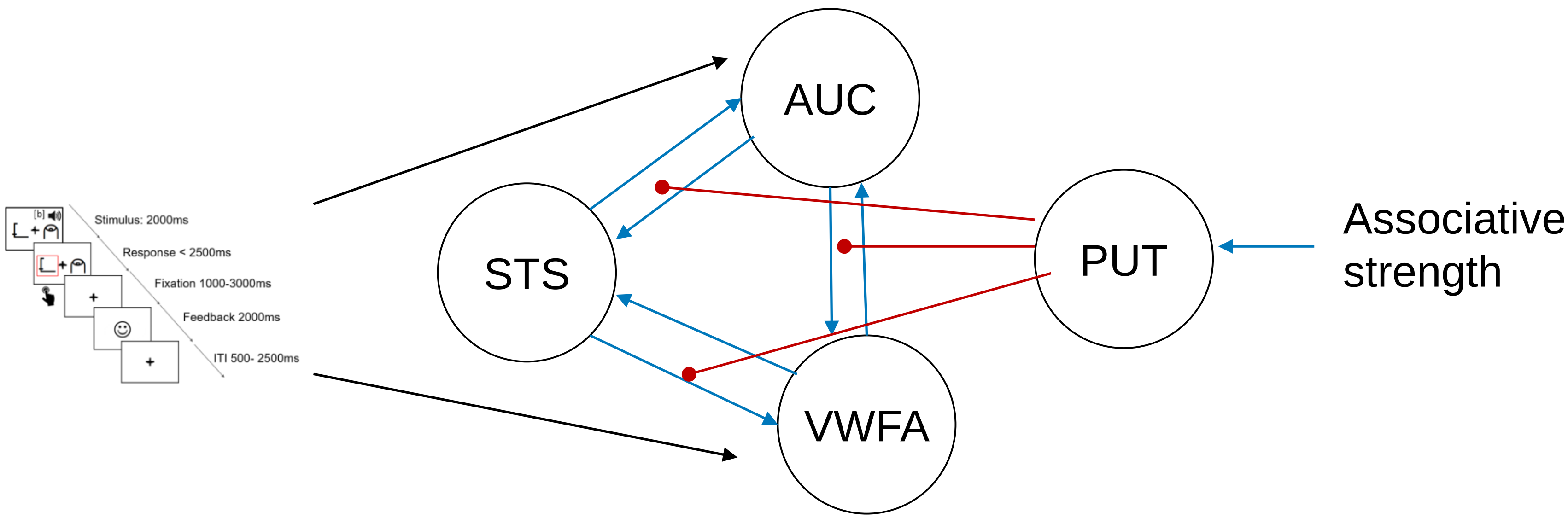
- Assessment of results

- Questions

David Willinger

DCM example

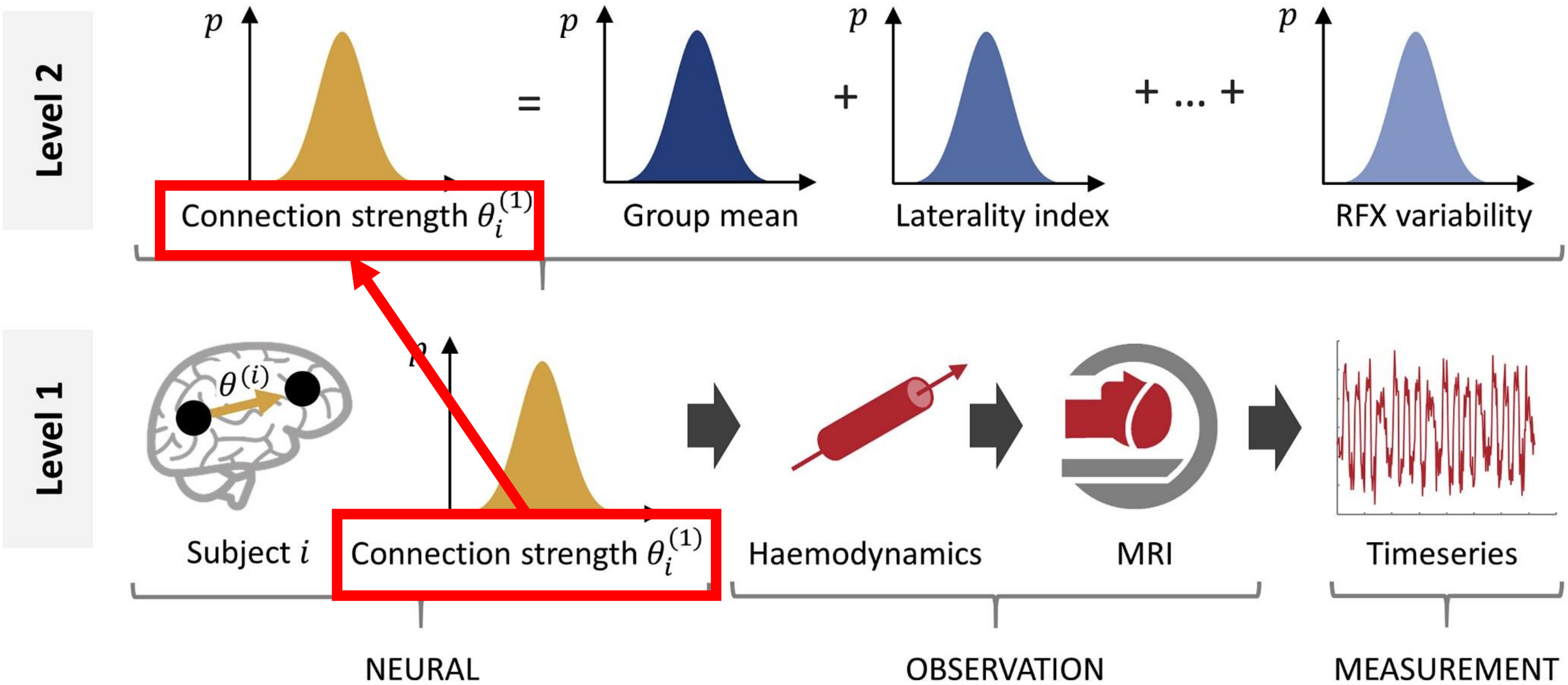
– Where we are: estimated first-level



DCM: group study

- We will follow the workflow described in Zeidman et al. (2019), using the Parametric Empirical Bayes (PEB) framework
- What are the commonalities in effective connectivity across subjects within a group or differences between groups?
- Which combination of connections best predicts a group difference (or covariate)?

Group analaysis in the PEB framework



Zeidman et al. (2019), *NeuroImage*

DCM with PEB

First-level

$$Y_i = \Gamma_i\left(\theta_i^{(1)}\right) + X_0\beta_i + \varepsilon_i^{(1)}$$

Full model - Inversion with VB
Nested models – BMR

BOLD timeseries DCM Effects of no interest (e.g. mean)

Second-level

Group effects

Between-subject error

$$\theta_i^{(1)} = X\theta^{(2)} + \varepsilon^{(2)}$$

DCM param.
Expected values
Covariance matrix

Group-level design matrix

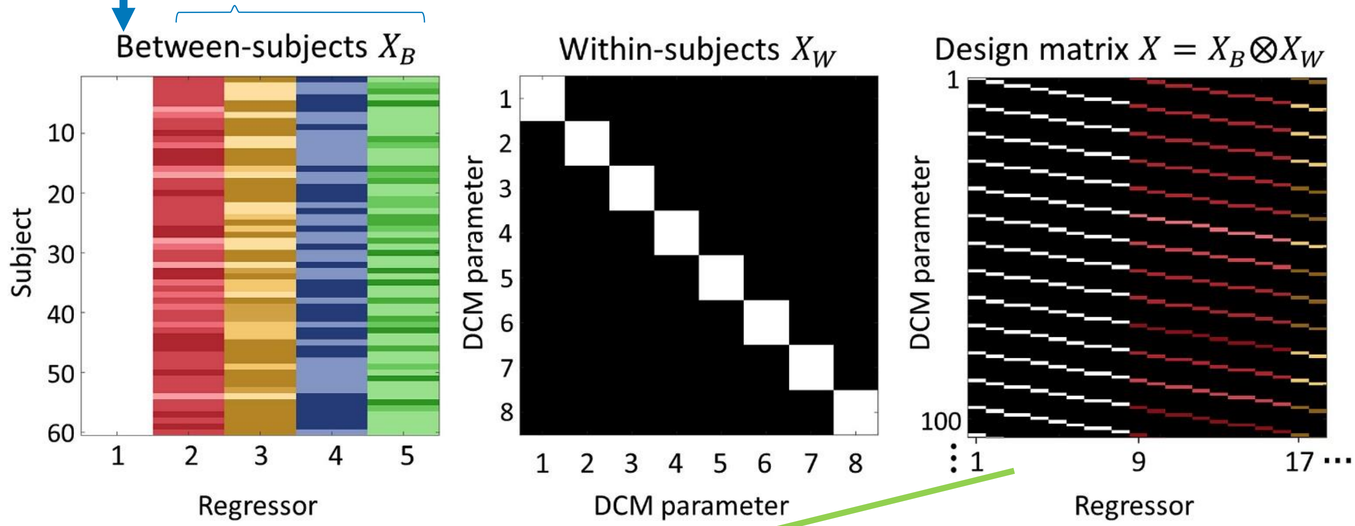
$$\theta^{(2)} = \eta + \varepsilon^{(3)}$$

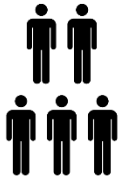
Fixed group priors

PEB: Design Matrix Specification

Mean

Covariates (e.g. age, sex, behavioural scores)





$$\theta_i^{(1)} = X\theta^{(2)} + \varepsilon^{(2)}$$

Output: One estimate for each covariate and each parameter

PEB is implemented in the SPM12 Toolbox

Table 1
SPM software functions in the PEB framework.

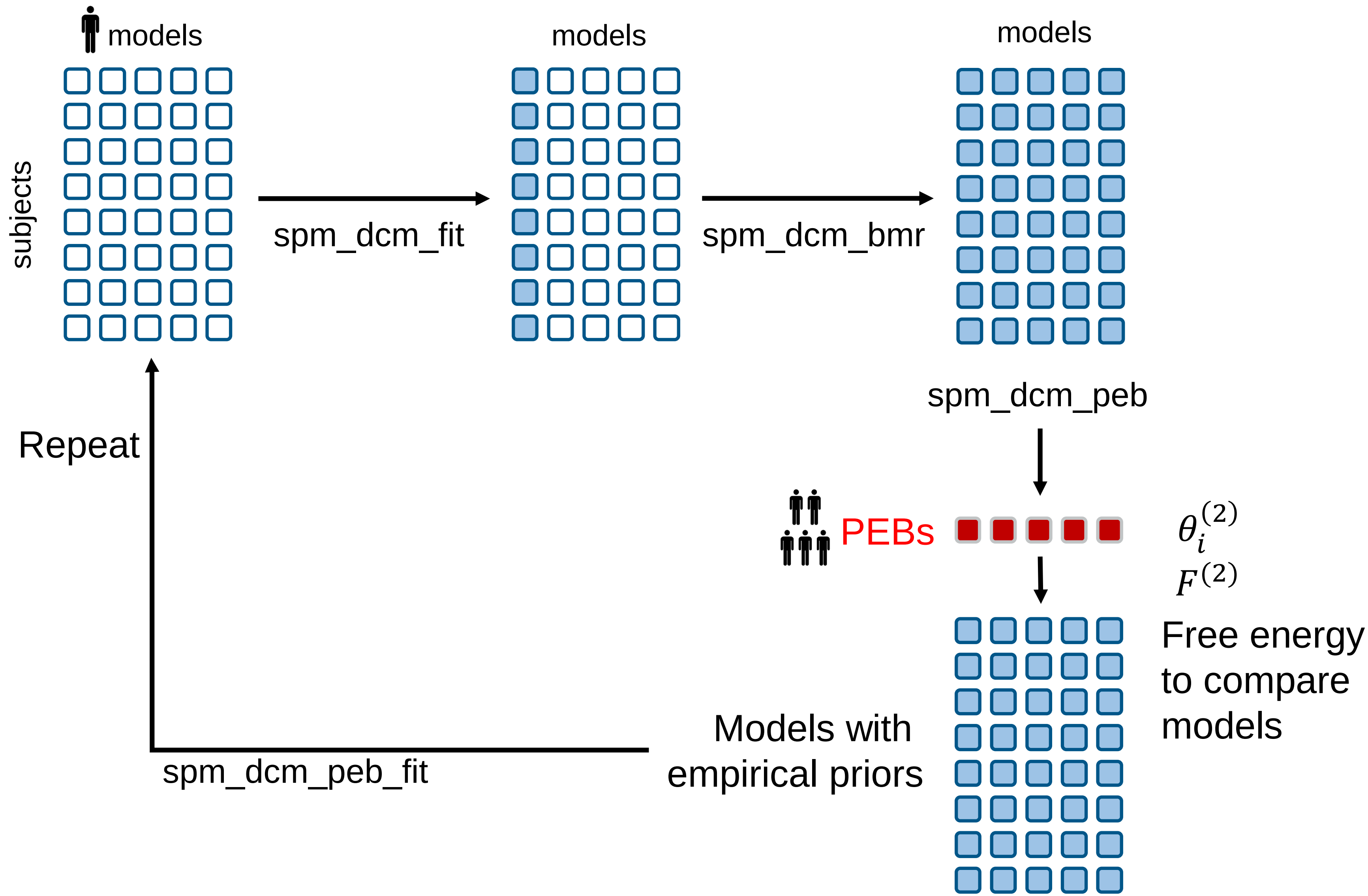
Function name	Analysis level	Description
spm_dcm_fit	1st	Fits first level DCMs, arranged in a GCM array*, to the data using variational Laplace.
spm_dcm_peb_fit	1st	Iteratively re-estimates each subject's first level DCM using the group average parameter estimates as empirical priors.
spm_dcm_bmc	2nd	Performs fixed effects and random effects Bayesian model selection (RFX BMS) on DCMs (this is not required for PEB analysis).
spm_dcm_peb	≥2nd	Specifies and estimates a PEB model.
spm_dcm_peb_bmc	≥2nd	Compares an estimated PEB model against reduced models where particular combinations of parameters (relating to particular connections) are switched off. In the absence of pre-specified alternative models, an automatic search over reduced models is performed. (To compare different mixtures of covariates and pre-specified first level models, instead use spm_dcm_bmc_peb.)
spm_dcm_peb_bmc_fam	≥2nd	Performs family-wise model comparison on the output of spm_dcm_peb_bmc, to enable groups of pre-defined models to be compared.
spm_dcm_peb_review	≥2nd	Graphical user interface for reviewing a PEB model or model comparison result.
spm_dcm_loo	1st and 2nd	Performs leave-one-out cross validation to assess the predictive validity of DCM parameters.

First-level estimation with empirical priors

Gives us - **model evidence** (*quality*)
- group parameters

Performs greedy search over entire model space

Put the «E» in PEB



DCM example: PEB group analysis

1. Inference based on **Bayesian model selection**
   - What is the model architecture that describes my data best?
   - Compares different hypotheses about model structure
   - You can also test *families* of models

2. Inference based on **Bayesian model averaging**
   - If there is no clear winner…
   - Average models weighted by their posterior probability
   - Look at connectivity estimates between regions
   - Uses a “greedy search” algorithm

Metric? – The *free energy*

Free energy as approximated model evidence

$$\underbrace{p(\theta|y,m)}_{\text{posterior}} = \frac{\overbrace{p(y|\theta,m)}^{\text{likelihood}}\overbrace{p(\theta|m)}^{\text{prior}}}{\underbrace{p(y|m)}_{\text{model evidence}}}$$

- Model evidence is the marginal likelihood across the entire parameter space
- „Given all possible parameter values, how well can the model describe the data?“
- For complex models (like DCM) often impossible to derive analytically (= no formula available) → need for an approximation

Free energy as approximated model evidence

Log model evidence = balance between fit and complexity

$$\log p(y|m) = accuracy(m) - complexity(m)$$
$$= \log p(y|\theta, m) - complexity(m)$$

Akaike Information Criterion: $AIC = \log p(y|\theta, m) - \boxed{p}$ number of parameters

Bayesian Information Criterion: $BIC = \log p(y|\theta, m) - \frac{p}{2} \log \boxed{N}$ number of data points

Penny et al., 2004, *NeuroImage*

Free Energy:

$$F = \langle \log p(y|\theta, m) \rangle_q - KL[q(\theta) \| p(\theta|m)]$$

$$KL[q(\theta) \| p(\theta|m)] = \frac{1}{2} \ln|C_\theta| - \frac{1}{2} \ln|C_{\theta|y}| + \boxed{\frac{1}{2} (\mu_{\theta|y} - \mu_\theta)^T C_\theta^{-1} (\mu_{\theta|y} - \mu_\theta)}$$

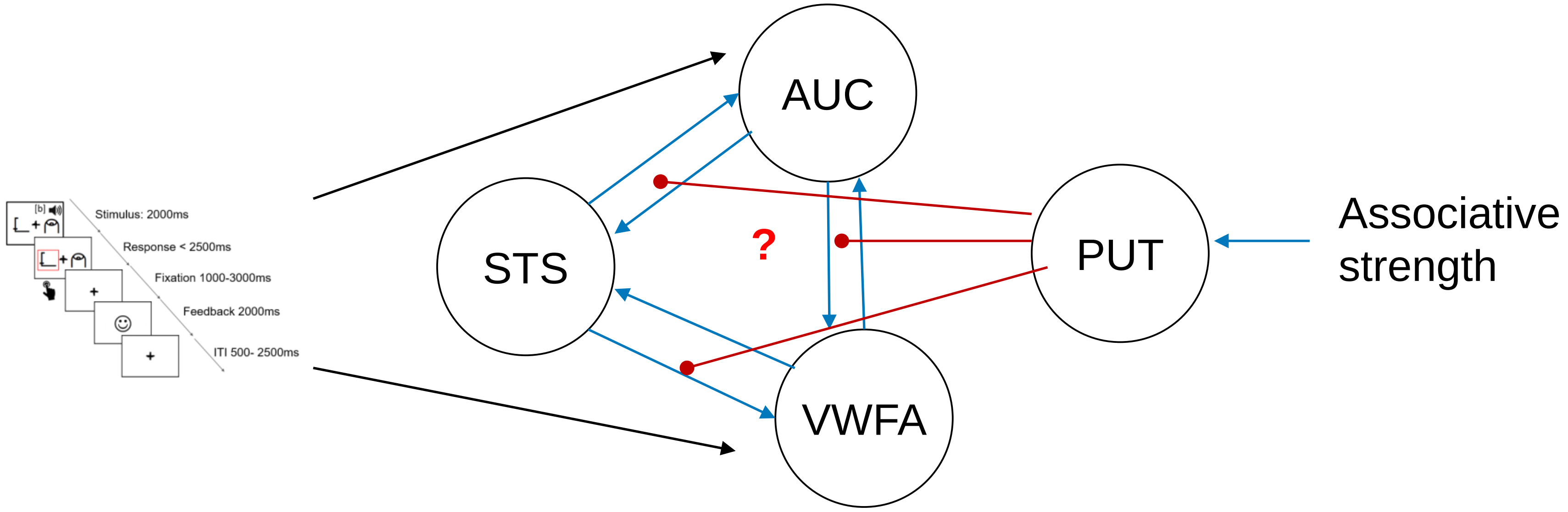
complexity **higher** the more posterior deviates from prior mean

Bishop, 2006; Friston et al., 2007, *NeuroImage*

Stefan Frässle

DCM example: PEB group analysis

– Build a model to test the overall connectivity changes across the entire group during the task



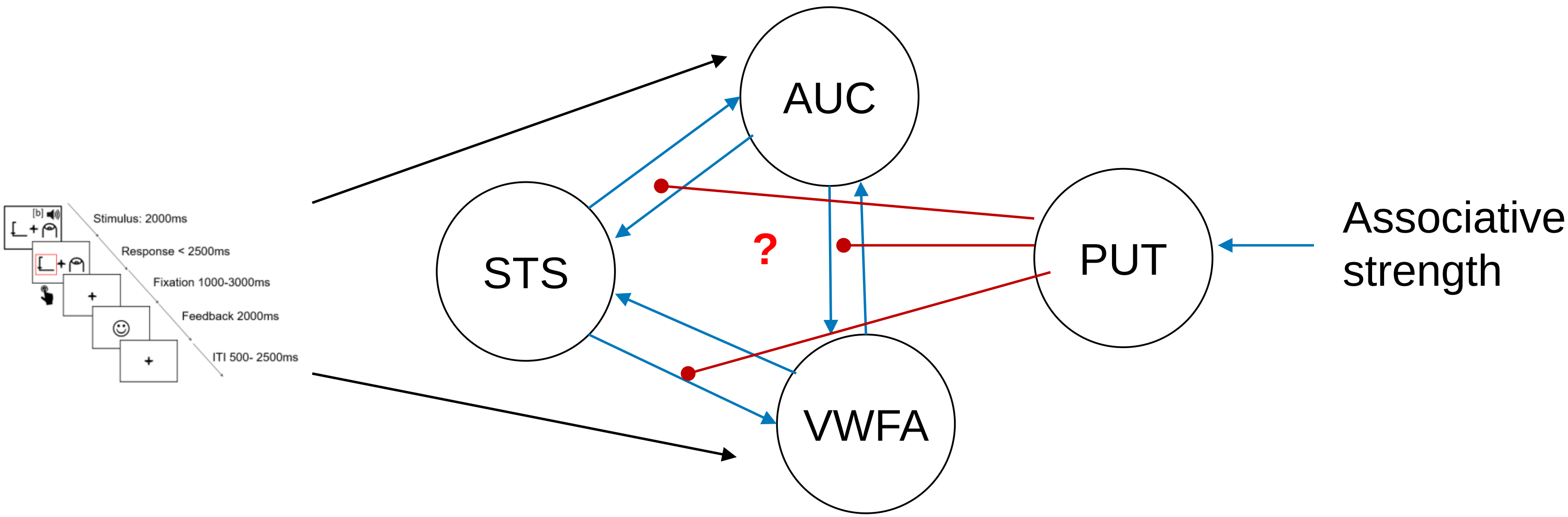
DCM example: 1. PEB Bayesian model selection

Steps:

1. Create a PEB model
   a. Specify 2nd-level design matrix
   b. Specify model space

2. Invert model

3. Perform model comparison



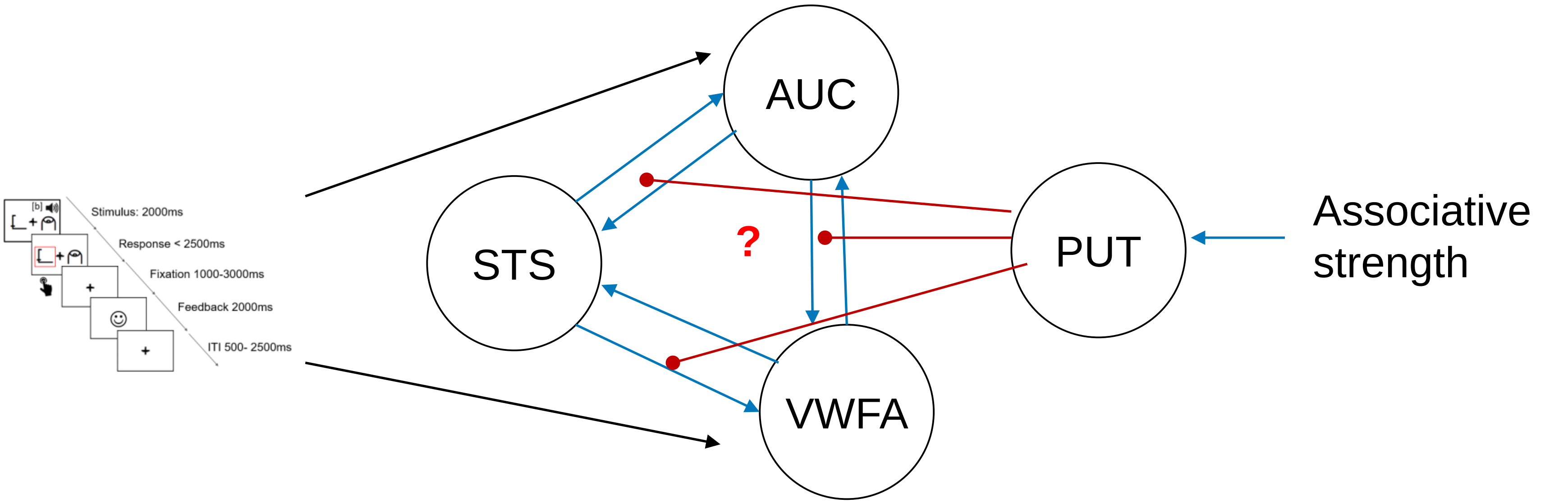
DCM example: 2. PEB Bayesian model averaging with greedy search

Steps:

1. Create a PEB model
   a. Specify 2nd-level design matrix
   ~~b. Specify model space~~

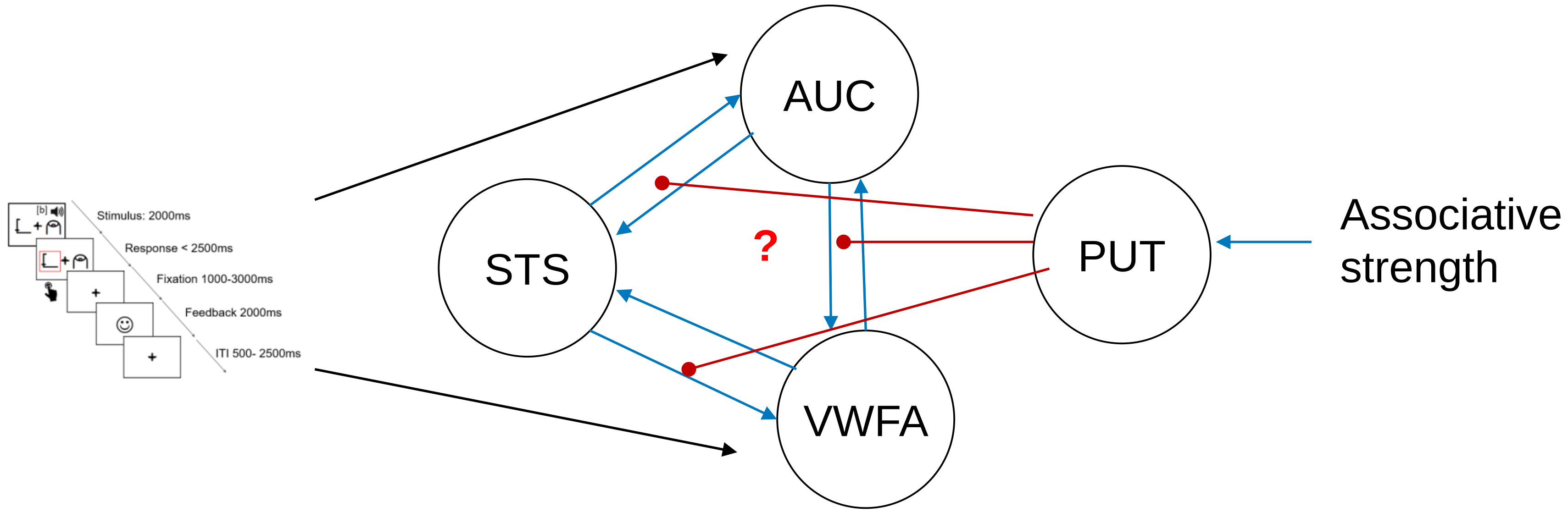
2. Invert model

3. Perform model comparison and remove parameters (FE with vs without)

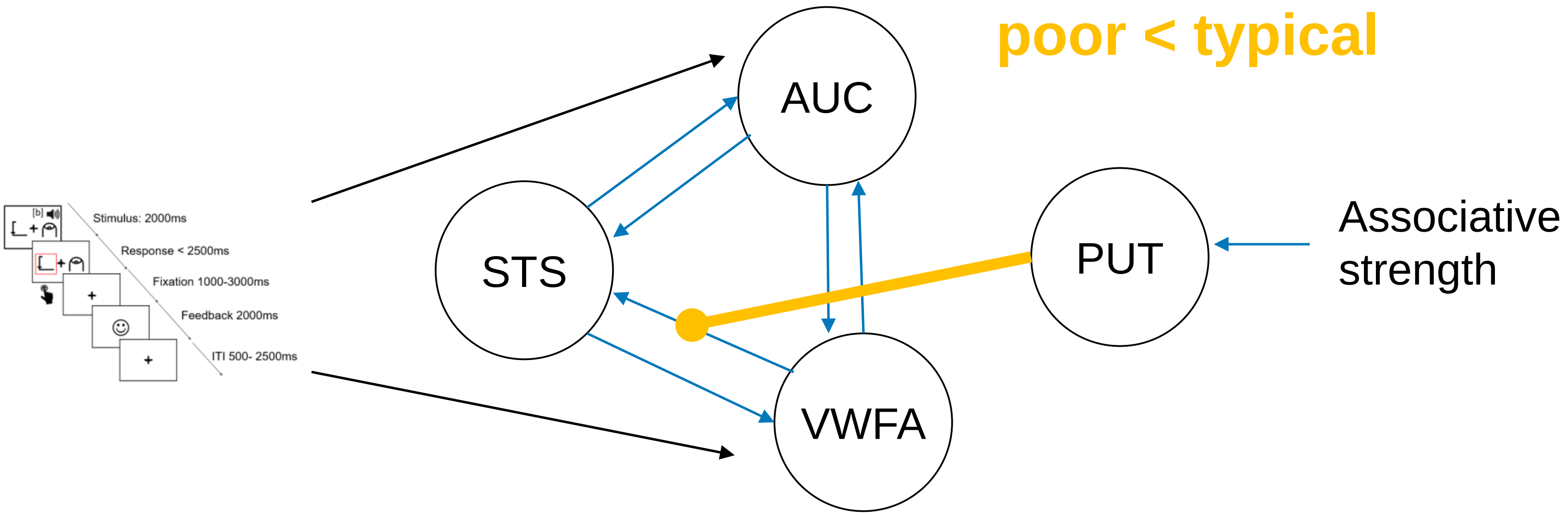


DCM example: Add a covariate to your design matrix

- To test for associations with a covariate or for group differences we can add regressors to the design matrix
- Should be limited to a reasonable amount …
- Are there group differences between typical and poor readers during the learning task?



DCM example: Group differences



- Poor readers show a decreased coupling between VWFA and STS compared to typical readers

DCM example: Leave-one-out cross-validation

- Refits the model n-1 times and excludes one subject at a time
- Can we predict the variable? (e.g. group membership)
- Is the effect large enough to be meaningful?

Questions?