# **Uninformed Search**

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Lecture 2

Readings: RN 3.1, 3.2, 3.3, 3.4.1, 3.4.3, 3.4.4

### Outline

Learning Goals

Applications of Search

Formulating a Search Problem

Generic Search Algorithm

Uninformed Search Algorithms

Depth-First Search

Breadth-First Search

Iterative-Deepening Search

Learning Goals

### Learning goals

- Formulate a real-world problem as a search problem.
- ➤ Trace the execution of and implement uninformed search algorithms (Breadth-first search, Depth-first search, Iterative-deepening search).
- Given an uninformed search algorithm, explain its space complexity, time complexity, and whether it has any guarantees on the quality of the solution found.
- Given a scenario, explain whether and why it is appropriate to use an uninformed algorithm.

### Learning Goals

### Applications of Search

Formulating a Search Problem

Generic Search Algorithm

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Learning Goals

### Example: Robotics





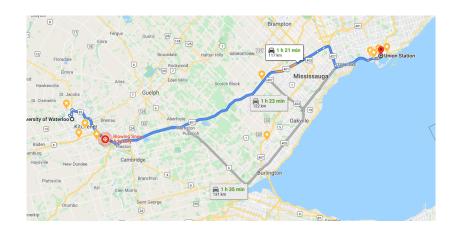


 $\rightarrow$  Letting Roomba vacuum the house.

Sending a robot to explore the planet Mars.

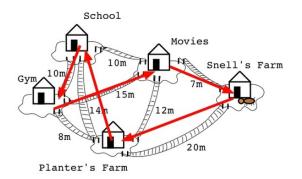
Letting robots handle search and rescue during natural disasters.

# Example: Route Planning



# Traveling Salesperson Problem

What is the shortest path that starts at city A, visits each city only once, and returns to A?



Applications: DNA sequencing. School bus routes. Parcel pickups. See this article for a recent breakthrough.

# Propositional Satisfiability

Propositional satisfiability: Given a formula, is there a way to assign true/false to the variables to make the formula true?

$$((((a \land b) \lor c) \land d) \lor (\neg e))$$

FCC spectrum auction: buy radio spectrums from TV broadcasters and sell them to the telecom companies

Check out this news article and this paper.

# 8-puzzle

#### Initial State

5	3	
8	7	6
2	4	1

### Goal State

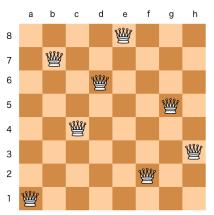
1	2	3
4	5	6
7	8	

# A River Crossing Puzzle



### N-Queens Problem

The n-queens problem: Place n queens on an  $n \times n$  board so that no pair of queens attacks each other.



http://yue-guo.com/wp-content/uploads/2019/02/N\_queen.png

Learning Goals

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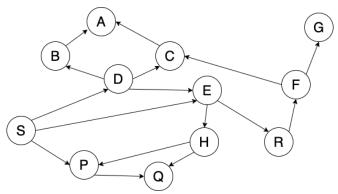
Uninformed Search Algorithms

Learning Goals

# Motivation for Search Algorithms

- Problems that involve sequential decision making
- ► Last resort: No efficient Algorithm (e.g. solve a linear equation)
- Easy to verify a solution, but to find a solution
- NP-hard problem
- Solve complex problems in Real-Life, trial-and-error
- On a computer, a search algorithm will explore all paths systematically

# **Graph Searching**



Any search problem can be visualized as a graph, where each node represents a state.

- ▶ S is the initial state, G is the goal state
- Search == Traversing the graph to find a path from S to G

### A Search Problem

### Definition (Search Problem)

A search problem is defined by:

- A set of states
- An initial state
- Goal states or a goal test
  - a boolean function which tells us whether a given state is a goal state
- A successor (neighbour) function
  - an action which takes us from one state to other states
- ▶ (Optionally) a **cost** associated with each action

A solution to this problem is a path from the start state to a goal state (optionally with the smallest total cost).

# Example: 8-Puzzle

Initial State

5	3	
8	7	6
2	4	1

Goal State

1	2	3
4	5	6
7	8	

► State:

► Initial state:

Goal states:

Successor function:

Cost function:

▶ State:  $x_{00}x_{01}x_{02}, x_{10}x_{11}x_{12}, x_{20}x_{21}x_{22}$   $x_{ij}$  is the number in row i and column j.  $i, j \in \{0, 1, 2\}$ .  $x_{ij} \in \{0, \dots, 8\}$ .  $x_{ij} = 0$  denotes the empty square.

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► Initial state: 530,876,241.

▶ State:  $x_{00}x_{01}x_{02}, x_{10}x_{11}x_{12}, x_{20}x_{21}x_{22}$   $x_{ij}$  is the number in row i and column j.  $i, j \in \{0, 1, 2\}$ .  $x_{ij} \in \{0, \dots, 8\}$ .  $x_{ij} = 0$  denotes the empty square.

▶ Initial state: 530,876,241.

► Goal states: 123, 456, 780.

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- ▶ Initial state: 530,876,241.
- ► Goal states: 123, 456, 780.
- ▶ Successor function: Consider the empty square as a tile. State B is a successor of state A if and only if we can convert A to B by moving the empty tile up, down, left, or right by one step.

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- ▶ Initial state: 530,876,241.
- ► Goal states: 123, 456, 780.
- ➤ Successor function: Consider the empty square as a tile. State B is a successor of state A if and only if we can convert A to B by moving the empty tile up, down, left, or right by one step.
- Cost function: Each move has a cost of 1.

### Q: The successor function

**Q:** Which of the following is a successor of 530,876,241?

#### Initial State

- (A) 350, 876, 241
- (B) 536, 870, 241
- (C) 537, 806, 241
- (D) 538, 076, 241

5	3	
8	7	6
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### Q: The successor function

**Q:** Which of the following is a successor of 530,876,241?

#### Initial State

(A)	350,	876,	241
-----	------	------	-----

- (B) 536, 870, 241
- (C) 537, 806, 241
- (D) 538, 076, 241

5	3	
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 $\rightarrow$  (B) is correct.



The state definition determines the nodes.
The successor function determines the directed edges.

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▶ Ideally, we want to minimize the number of nodes and edges in the graph.

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The successor function determines the directed edges.

▶ Ideally, we want to minimize the number of nodes and edges in the graph.

► Choosing a state definition may make it easier or harder to implement the successor function.

An alternative state definition for the 8-puzzle: A state is defined by 9 coordinates.  $(x_i,y_i)$  is the coordinates for tile i where  $0 \le i \le 8$ .

#### Initial State

5	3	
8	7	6
2	4	1

► State: (0, 2), (2, 2), (2, 0), (0, 1), (2, 1), (0, 0), (1, 2), (1, 1), (1, 0)

#### Initial State

5	3	
8	7	6
2	4	1

- ► State: (0, 2), (2, 2), (2, 0), (0, 1), (2, 1), (0, 0), (1, 2), (1, 1), (1, 0)
- ▶ Space Consumption will increase by 2x.

#### Initial State

5	3	
8	7	6
2	4	1

- ► State: (0, 2), (2, 2), (2, 0), (0, 1), (2, 1), (0, 0), (1, 2), (1, 1), (1, 0)
- ► Space Consumption will increase by 2x.
- Successor Function: Swap the 0 tile coordinate with another neighbor tile

Learning Goals

Applications of Search

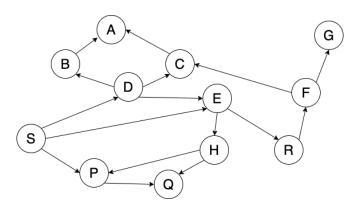
Formulating a Search Problem

Generic Search Algorithm

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Learning Goals

# The Search Graph



- normally no duplicate of nodes
- ► label each edge with its cost
- ▶ this lecture will assume each edge has an equal cost of 1

# Searching for a Solution

→ A formulation gives us enough info to generate the search graph. However, we often don't generate the search graph explicitly and store it. It may be large or infinite and it is not necessary.

Instead, we will generate a search tree as we explore the search graph.

Construct the search tree as we explore paths incrementally from the start node.

# Searching for a Solution

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- Construct the search tree as we explore paths incrementally from the start node.
- Maintain a frontier of paths from the start node.

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- Construct the search tree as we explore paths incrementally from the start node.
- Maintain a frontier of paths from the start node.
- Frontier contains all the paths available for expansion.

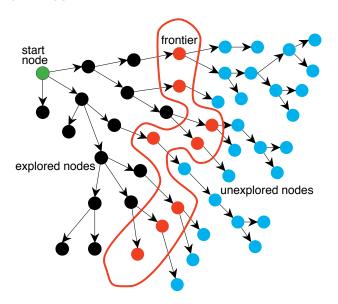
## Searching for a Solution

ightarrow A formulation gives us enough info to generate the search graph. However, we often don't generate the search graph explicitly and store it. It may be large or infinite and it is not necessary.

Instead, we will generate a search tree as we explore the search graph.

- Construct the search tree as we explore paths incrementally from the start node.
- Maintain a frontier of paths from the start node.
- Frontier contains all the paths available for expansion.
- Expanding a path: removing it from the frontier, generating all the neighbors of the last node, and adding the paths ending with each neighbor to the frontier.

#### The Search Tree



#### **Algorithm 1** A Generic Search Algorithm

1: **procedure** SEARCH(Graph, Start node s, Goal test goal(n))

#### Algorithm 2 A Generic Search Algorithm

- 1: **procedure** SEARCH(Graph, Start node s, Goal test goal(n))
- frontier :=  $\{\langle s \rangle\}$

#### Algorithm 3 A Generic Search Algorithm

- 1: **procedure** SEARCH(Graph, Start node s, Goal test goal(n))
- 2: frontier :=  $\{\langle s \rangle\}$
- 3: **while** frontier is not empty **do**

#### **Algorithm 4** A Generic Search Algorithm

- 1: **procedure** SEARCH(Graph, Start node s, Goal test goal(n))
- 2: frontier :=  $\{\langle s \rangle\}$
- 3: while frontier is not empty do
- 4: **select** and **remove** path  $\langle n_0, \ldots, n_k \rangle$  from frontier

#### **Algorithm 5** A Generic Search Algorithm

```
1: procedure SEARCH(Graph, Start node s, Goal test goal(n))
2: frontier := \{\langle s \rangle\}
3: while frontier is not empty do
4: select and remove path \langle n_0, \dots, n_k \rangle from frontier
5: if goal(n_k) then
6: return \langle n_0, \dots, n_k \rangle
```

#### **Algorithm 6** A Generic Search Algorithm

```
1: procedure SEARCH(Graph, Start node s, Goal test goal(n))
2: frontier := \{\langle s \rangle\}
3: while frontier is not empty do
4: select and remove path \langle n_0, \dots, n_k \rangle from frontier
5: if goal(n_k) then
6: return \langle n_0, \dots, n_k \rangle
7: for every neighbour n of n_k do
8: add \langle n_0, \dots, n_k, n \rangle to frontier
```

#### **Algorithm 7** A Generic Search Algorithm

```
1: procedure Search(Graph, Start node s, Goal test goal(n))
        frontier := \{\langle s \rangle\}
       while frontier is not empty do
3:
            select and remove path \langle n_0, \ldots, n_k \rangle from frontier
4:
5:
            if goal(n_k) then
                 return \langle n_0, \ldots, n_k \rangle
6:
7:
            for every neighbour n of n_k do
                 add \langle n_0, \ldots, n_k, n \rangle to frontier
8:
9:
        return no solution
```

Learning Goals

Applications of Search

Formulating a Search Problem

Generic Search Algorithm

#### Uninformed Search Algorithms

Depth-First Search

Breadth-First Search

Iterative-Deepening Search

Learning Goals

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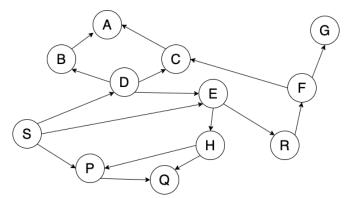
Learning Goals

### Depth-First Search

- ► Treats the frontier as a stack (LIFO).
- Expands the last/most recent node added to the frontier.
- $\rightarrow$  Search one path to completion before trying another path.

Backtracks to another alternative after it has explored all the paths from the first alternative.

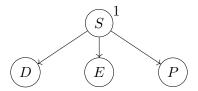
- ► Trace DFS on the search graph below, with *S* as the initial state.
- Add nodes to the frontier in alphabetical order.



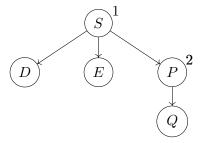
Frontier: (S):

 $\left(S
ight)$ 

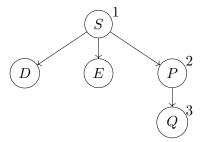
Frontier:  $(S) \rightarrow (D, E, P)$ :



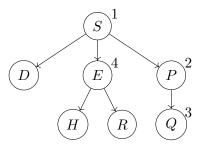
Frontier: (D, E, P)  $\rightarrow$  (D, E, Q):



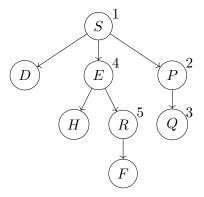
Frontier: (D, E, Q)  $\rightarrow$  (D, E):



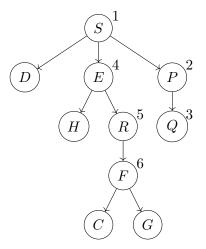
Frontier:  $(D, E) \rightarrow (D, H, R)$ :



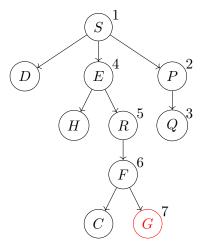
Frontier: (D, H, R)  $\rightarrow$  (D, H, F):



Frontier:  $(D, H, F) \rightarrow (D, H, C, G)$ :



Frontier: (D, H, C, G)  $\rightarrow$  (D, H, C):



## Properties of DFS

#### **Properties**

- Space Complexity: size of frontier in worst case
- Time Complexity: # nodes visited in worst case
- Completeness: does it find a solution when one exists?
- Optimality: if solution found, is it the one with the least cost?

#### Useful Quantities

- b is the branching factor.
- m is the maximum depth of the search tree.
- d is the depth of the shallowest goal node.

# Properties of DFS - Space Complexity

Space Complexity

# Properties of DFS - Space Complexity

Space Complexity

► *O*(*bm*)

b is the branching factor. m is the max depth of the search tree.

- Linear in m
- Remembers m nodes on current path and at most b siblings for each node.

# Properties of DFS - Time Complexity

Time Complexity

# Properties of DFS - Time Complexity

#### Time Complexity

- $ightharpoonup O(b^m)$
- ightharpoonup Exponential in m.
- ▶ Visit the entire search tree in the worst case.

## Properties of DFS - Completeness

Is DFS guaranteed to find a solution if a solution exists?

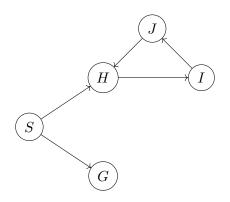
## Properties of DFS - Completeness

Is DFS guaranteed to find a solution if a solution exists?

- No.
- May get stuck in an infinite path.
- ► An infinite path may or may not be a cycle.

# Properties of DFS - Completeness

Is DFS guaranteed to find a solution if a solution exists?



- Solution G exists
- ▶ Loop:  $H \rightarrow I \rightarrow J \rightarrow H$ , cannot find the solution

# Properties of DFS - Optimality

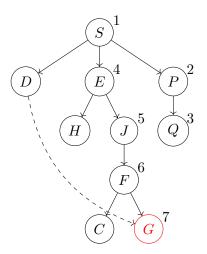
Is DFS guaranteed to return an optimal solution if it terminates?

# Properties of DFS - Optimality

Is DFS guaranteed to return an optimal solution if it terminates?

- No.
- Pays no attention to the costs and makes no guarantee on the solution's quality.

# Properties of DFS - Optimality



#### When should we use DFS?

#### DFS is useful when:

- Space is restricted.
- Many solutions exist, perhaps with long paths.

#### DFS is a poor method when:

- There are infinite paths.
- Solutions are shallow.
- There are multiple paths to a node.

Learning Goals

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#### Uninformed Search Algorithms

Depth-First Search

Breadth-First Search

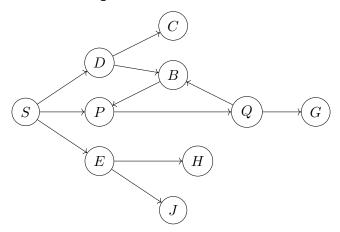
Iterative-Deepening Search

Learning Goals

#### Breadth-First Search

- ► Treats the frontier as a queue (FIFO).
- Expands the first/oldest node added to the frontier.
- $\rightarrow$  Select a path with the fewest arcs at each step.

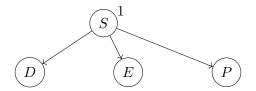
► Trace the BFS algorithm



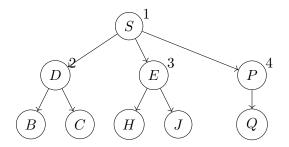
Frontier: (S)



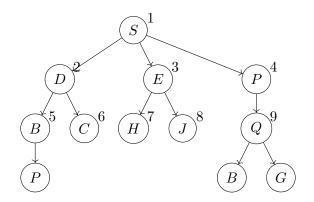
Frontier:  $(S) \rightarrow (D, E, P)$ 



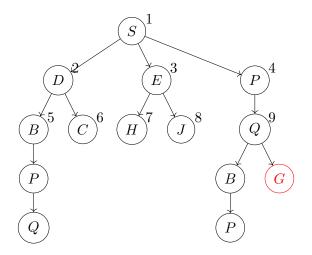
Frontier: (D, E, P)  $\rightarrow$  (B, C, H, J, Q)



Frontier: (B, C, H, J, Q)  $\rightarrow$  (P, B, G)



Frontier:  $(P, B, G) \rightarrow (G, Q, P)$ 



# Properties of BFS - Space Complexity

Space Complexity

# Properties of BFS - Space Complexity

#### Space Complexity

 $ightharpoonup O(b^d)$ 

b is the branching factor. d is the depth of the shallowest goal node.

- ightharpoonup Exponential in d.
- Must visit the top d levels.
  The size of the frontier is dominated by the size of level d.

# Properties of BFS - Time Complexity

Time Complexity

# Properties of BFS - Time Complexity

#### Time Complexity

- $ightharpoonup O(b^d)$
- $\triangleright$  Exponential in d.
- Visit the entire search tree in the worst case.

### Properties of BFS - Completeness

Is BFS guaranteed to find a solution if a solution exists?

### Properties of BFS - Completeness

Is BFS guaranteed to find a solution if a solution exists?

- Yes.
- Explores the tree level by level until it finds a goal.

# Properties of BFS - Optimality

Is BFS guaranteed to return an optimal solution if it terminates?

### Properties of BFS - Optimality

Is BFS guaranteed to return an optimal solution if it terminates?

- No.
- Guaranteed to find the shallowest goal node.

#### When should we use BFS?

#### BFS is useful when:

- Space is not a concern.
- Want a solution with the fewest arcs.

#### BFS is a poor method when:

- ▶ All the solutions are deep in the tree.
- The problem is large and the graph is dynamically generated.

**Q** #1: Suppose that memory is very limited. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

**Q** #1: Suppose that memory is very limited. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) **DFS** is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.
- $\rightarrow$  (B) DFS is better. BFS requires exponential memory. DFS requires linear memory.

**Q** #2: Suppose that all the solutions are deep in the search tree. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

**Q** #2: Suppose that all the solutions are deep in the search tree. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.
- $\rightarrow$  (B) DFS is better.

DFS will explore long paths first and find a solution faster. BFS will explore shallow nodes first and will likely require more time to find a solution.

**Q #3:** Suppose that the search graph contains cycles. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

**Q** #3: Suppose that the search graph contains cycles. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

ightarrow (A) BFS is better. DFS may get stuck in cycles. BFS has no problem with cycles.

**Q #4:** Suppose that the branching factor is large/infinite. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

**Q #4:** Suppose that the branching factor is large/infinite. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) **DFS** is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.
- $\rightarrow$  (B) DFS is better.

BFS will be slow for large b and will not terminate with infinite b. DFS requires linear memory.

**Q #5:** Suppose that we must find the shallowest goal node. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

**Q #5:** Suppose that we must find the shallowest goal node. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.
- $\rightarrow$  (A) BFS is better.

BFS is guaranteed to find the shallowest goal node. DFS will likely find a deep goal node first.

**Q #6:** Suppose that all the solutions are very shallow. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

**Q #6:** Suppose that all the solutions are very shallow. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

 $\rightarrow$  (A) BFS is better.

BFS is guaranteed to find the shallowest goal node. DFS will likely find a deep goal node first.

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#### Learning Goals

### Combining The Best of BFS and DFS

Can we create a search algorithm that combines the best of BFS and DFS?

BFS	DFS
$O(b^d)$ exponential space	O(bm) linear space
Guaranteed to find	May get stuck
a solution if one exists	on infinite paths

Can we bring the best of two together?

### Combining The Best of BFS and DFS

Can we create a search algorithm that combines the best of BFS and DFS?

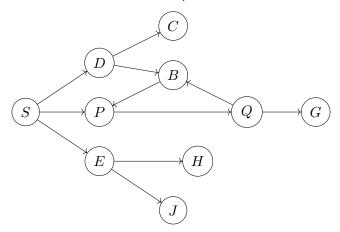
BFS	DFS
$O(b^d)$ exponential space	O(bm) linear space
Guaranteed to find	May get stuck
a solution if one exists	on infinite paths

Can we bring the best of two together?

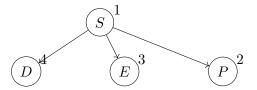
#### **Iterative-Deepening Search:**

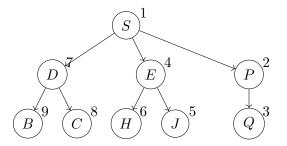
For every depth limit, perform depth-first search until the depth limit is reached.

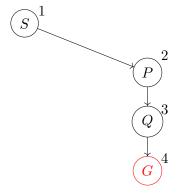
- ► Trace IDS on the search graph below.
- ▶ Add nodes to the frontier in alphabetical order.











# Properties of IDS - Space Complexity

Space Complexity

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#### Space Complexity

**▶** *O*(*bd*)

b is the branching factor. d is the depth of the shallowest goal node.

- Linear in d.
  Similar to DFS.
- Executes DFS for each depth limit. Guaranteed to terminate at depth d.

# Properties of IDS - Time Complexity

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#### Time Complexity

- $ightharpoonup O(b^d)$
- Exponential in d. Same as BFS.
- Visits all the nodes on the top d levels in the worst case.

### Properties of IDS - Completeness

Is IDS guaranteed to find a solution if a solution exists?

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Yes.
Same as BFS.

Explores the tree level by level until it finds a goal.

### Properties of IDS - Optimality

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Is IDS guaranteed to return an optimal solution if it terminates?

- No.
- Guaranteed to find the shallowest goal node. Same as BFS.

## A Summary of IDS Properties

Space Complexity:

O(bd), linear in d. Similar to DFS.

► Time Complexity:

 $O(b^d)$ , exponential in d. Same as BFS.

Completeness:

Yes. Same as BFS.

Optimality:

No, but guaranteed to find the shallowest goal node. Same as BFS.

### Learning goals

- Formulate a real-world problem as a search problem.
- ➤ Trace the execution of and implement uninformed search algorithms (Breadth-first search, Depth-first search, Iterative-deepening search).
- Given an uninformed search algorithm, explain its space complexity, time complexity, and whether it has any guarantees on the quality of the solution found.
- Given a scenario, explain whether and why it is appropriate to use an uninformed algorithm.