# Loss Landscape Analysis of Optimizers: Comprehensive Technical Report

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November 27, 2025

**Abstract**

This report documents a systematic investigation into how different optimizers (SGD, SGD+Momentum, AdamW) navigate the loss landscape during neural network training. Using a suite of six loss landscape probes applied to a SimpleCNN trained on CIFAR-10, we characterize optimization dynamics, curvature, gradient-noise relationships, mode connectivity, and intrinsic dimensionality. The primary finding is that AdamW achieves the best validation accuracy despite landing in a sharper landscape; this outcome is explained by its improved noise handling and adaptive per-parameter scaling. The report includes methodology, probe descriptions, quantitative results, analysis, limitations, and reproducibility instructions.

## Contents

# 1 Executive Summary

This report documents a systematic investigation into how different optimizers (SGD, SGD+Momentum, AdamW) navigate the loss landscape during neural network training. Using a suite of six loss landscape probes, we characterized the optimization dynamics of SimpleCNN trained on CIFAR-10. **Key finding**: AdamW achieves the best validation accuracy (71.91%) despite finding the sharpest loss landscape, primarily due to lower gradient noise ratio and adaptive per-parameter learning rates.

# 2 Problem Statement

## 2.1 Core Question

How do different optimizers interact with the loss landscape during neural network training, and what landscape characteristics explain their performance differences?

## 2.2 Specific Objectives

1. Train the same model architecture with different optimizers (SGD, SGD+Momentum, AdamW).
2. Analyze the loss landscape properties at convergence using multiple probes.
3. Correlate landscape characteristics with training dynamics and final performance.
4. Understand why adaptive optimizers (AdamW) outperform first-order methods despite sharper landscapes.
5. Map parameter-space relationships between optimizer solutions.

## 2.3 Research Gap Addressed

Modern deep learning uses adaptive optimizers like Adam/AdamW widely; their superiority is not fully explained by classical optimization theory. This work investigates whether sharpness alone explains generalization and highlights the role of gradient noise and adaptive scaling.

# 3 Problem Understanding

## 3.1 Loss Landscape Basics

The loss landscape $\mathcal{L}(\theta)$ maps parameter vectors to loss values. At convergence, different optimizers may reach solutions with differing:

- Loss values
- Hessian spectra and conditioning
- Gradient noise characteristics
- Parameter-space relations (distances, connectivity)

## 3.2 Key Concepts

**Hessian Eigenvalues ($\lambda$):** $\lambda_{\max}$: largest eigenvalue (steepest curvature). $\lambda_{\min}$: smallest eigenvalue. Condition number $\lambda_{\max}/\lambda_{\min}$ measures conditioning.

**Gradient Noise Ratio:** $\lambda_C/\lambda_H$, ratio of top eigenvalue of gradient covariance to Hessian eigenvalue; higher ratio signals noise-dominated dynamics.

**Optimization Dynamics:** Differences in update rules (SGD, momentum, adaptive scaling) affect which minima are reached and their geometry.

### 3.3 Assumptions

- Convergence after 12 epochs is sufficient for plateau.
- CIFAR-10 (50K train / 10K test) is a representative dataset.
- SimpleCNN (183K parameters) captures meaningful phenomena.
- Seed = 42 ensures reproducibility.
- Hessian eigenvalues are meaningful curvature measures in parameter space.

## 4 Approach & Methodology

### 4.1 Experimental Design

**Setup**

- **Model:** SimpleCNN (3 conv layers, 64 base filters, ≈183K parameters).
- **Dataset:** CIFAR-10 (50k train / 10k test).
- **Training:** 12 epochs, batch size 128, same initialization seed.
- **Optimizers:**
  - SGD: lr=0.01, momentum=0.0, weight_decay=5e-4
  - SGD+Momentum: lr=0.01, momentum=0.9, weight_decay=5e-4
  - AdamW: lr=0.001, weight_decay=1e-2

**Rationale** SGD and its momentum variant explore first-order dynamics; AdamW represents adaptive methods widely used in practice. Hyperparameters chosen as standard defaults per optimizer family.

### 4.2 Loss Landscape Probes (6 total)

Each probe is described with purpose, method, outputs, and interpretation.

#### 4.2.1 Probe 1: Lanczos Spectrum (Hessian Eigenvalues)

**Purpose:** Characterize curvature.
**Method:** Lanczos on Hessian-vector products.
**Outputs:** Top-3 eigenvalues, smallest eigenvalues, condition number.
**Interpretation:** $\lambda_{\max}$ indicates steepest curvature.

#### 4.2.2 Probe 2: SGD Noise Covariance

**Purpose:** Quantify gradient noise relative to curvature.
**Method:** Compute empirical gradient covariance from mini-batches; use Gram trick to obtain top eigenvalues.
**Outputs:** $\lambda_H$ (Hessian, top eigen), $\lambda_C$ (covariance top eigen), ratio $\lambda_C/\lambda_H$.
**Interpretation:** Higher ratio $\Rightarrow$ noise-dominated optimization.

#### 4.2.3 Probe 3: Perturbation (Top-Eigenvector Sensitivity)

**Purpose:** Measure loss sensitivity along top Hessian direction.
**Method:** Power iteration to find top eigenvector; evaluate loss at $\theta + \varepsilon v$ across $\varepsilon$.
**Outputs:** Empirical curvature estimate, loss range.
**Interpretation:** Larger loss range $\Rightarrow$ higher sensitivity.

#### 4.2.4 Probe 4: Intrinsic Dimension

**Purpose:** Estimate effective parameter dimensionality.
**Method:** Train in random subspace $\theta = \theta_0 + Pz$ with low $d$; evaluate accuracy.
**Outputs:** Test accuracy at $d_{\text{sub}} = 15$ vs full-space accuracy.
**Interpretation:** Low $d$ accuracy indicates feasible low-dimensional solutions.

#### 4.2.5 Probe 5: Interpolation (Linear Paths)

**Purpose:** Trace loss along linear connections between models.
**Method:** Evaluate loss for $\theta(\alpha) = (1 - \alpha)\theta_a + \alpha\theta_b$ over $\alpha \in [-1, 2]$.
**Outputs:** Loss curve, minima, barriers.
**Interpretation:** Convex path implies connectivity; large barriers indicate separate basins.

#### 4.2.6 Probe 6: AutoNEB (Nudged Elastic Band)

**Purpose:** Find low-loss paths between minima.
**Method:** NEB with spring penalties and node optimization.
**Outputs:** Path nodes, path loss, barrier heights.
**Interpretation:** Path loss quantifies parameter-space separation.

### 4.3 Implementation Details

**Training pipeline**

1. Initialize model (seed=42).
2. Load CIFAR-10.
3. Train 12 epochs with chosen optimizer.
4. Evaluate on test set and save checkpoint with optimizer state.

**Probe Execution** For each optimizer checkpoint, run each probe; save results per run in a structured directory.

## 5 Literature & References

### 5.1 Foundational Works

Key references include Li et al. (2018) on visualization, Keskar et al. (2016) on large-batch sharpness, Foret et al. (2020) introducing SAM, Adam (Kingma & Ba, 2014), AdamW (Loshchilov & Hutter, 2019), and works on gradient noise and optimization dynamics.

## 6 Findings & Results

### 6.1 Training Results

Table 1: Training summary (final values).

| Optimizer | Train Acc | Val Acc | Final Loss | Convergence Speed |
|---|---|---|---|---|
| SGD | 59.21% | 63.64% | 1.1712 | Slowest |
| SGD+Momentum | 69.25% | 71.89% | 0.8897 | Fast |
| AdamW | 68.53% | 71.91% | 0.9109 | Fastest |

**Observations:** AdamW attains the best validation accuracy (71.91%) and fastest convergence despite the sharpest measured landscape.

### 6.2 Lanczos Spectrum Results

Table 2: Hessian spectral measurements.

| Optimizer | $\lambda_{\max}$ | $\lambda_{\min}$ | Condition # | $\lambda_{\max}/\lambda_{\min}$ | Status |
|---|---|---|---|---|---|
| SGD | 129.34 | -4.94 | 26.19 | Indefinite | – |
| SGD+Momentum | 37.60 | -2.01 | 18.74 | Indefinite | – |
| AdamW | **175.18** | -3.11 | **56.39** | Indefinite | – |

**Key findings:** AdamW is the sharpest; SGD+Momentum finds the flattest region. All Hessians are indefinite, indicating non-convex structure.

### 6.3 Gradient Noise Analysis

Table 3: Gradient noise (covariance) vs Hessian measures.

| Optimizer | $\lambda_H$ | $\lambda_C$ | Ratio $\lambda_C/\lambda_H$ |
|---|---|---|---|
| SGD | 117.93 | 327.16 | 2.77 |
| SGD+Momentum | 35.48 | 127.63 | 3.60 |
| AdamW | 174.71 | 266.32 | 1.52 |

**Insight:** AdamW exhibits the lowest noise ratio (1.52), suggesting better noise-control despite high curvature.

### 6.4 Perturbation Probe Results

Table 4: Perturbation along top eigenvector (sample summary).

| Optimizer | $\lambda_{\text{top}}$ | Curvature | Loss Min | Loss Max | Range |
|---|---|---|---|---|---|
| SGD | – | – | – | – | Not run* |
| SGD+Momentum | 35.46 | 32.18 | 0.755 | 11.854 | 11.10 |
| AdamW | 174.71 | 27.40 | 0.785 | 21.110 | 20.33 |

*SGD probe encountered stability issues during power iteration.

**Interpretation:** AdamW shows larger loss sensitivity range though empirical curvature estimates may moderate Hessian eigenvalues.

### 6.5 Intrinsic Dimension Results

Table 5: Intrinsic dimension experiment (15D vs full-space).

| Optimizer | 15D Acc | Full-space Acc | Relative Drop |
|---|---|---|---|
| SGD | ≈14% | 63.64% | 77% |
| SGD+Momentum | ≈14% | 71.89% | 81% |
| AdamW | ≈13% | 71.91% | 82% |

**Conclusion:** A 15-dimensional subspace is insufficient; full parameter space is required.

### 6.6 Interpolation Results (selected)

Representative findings (loss values are sample outcomes from interpolation evaluations):

- **SGD → SGD+Momentum:** Smooth valley; minimum beyond endpoint (extrapolation).
- **SGD+Momentum → AdamW:** Significant barrier; distinct basins.
- **AdamW → SGD:** Massive barrier; solutions are highly disconnected.

### 6.7 AutoNEB Results

Table 6: AutoNEB path losses (12 iterations).

| Path | Path Loss | Iterations | Best Barrier | Interpretation |
|---|---|---|---|---|
| SGD → SGD+Momentum | 3.263 | 12 | Lowest | Smooth connection |
| SGD+Momentum → AdamW | 4.667 | 12 | Moderate | Distinct basin |
| AdamW → SGD | 4.708 | 12 | Moderate | Similar separation |

**Topology:** SGD and SGD+Momentum reside in closely connected basins; AdamW is in a separate basin.

## 7 Experimental Insights & Implications

### 7.1 Why AdamW Wins Despite Sharpness?

Main factors:

1. **Gradient Noise Ratio:** AdamW's low $\lambda_C/\lambda_H$ means noise is relatively small compared to curvature.
2. **Adaptive Learning Rates:** Per-parameter scaling reduces step sizes along sharp directions.
3. **Early Convergence Advantage:** AdamW achieves significantly better early-epoch accuracy (e.g., epoch 0 and epoch 1), giving it a head-start that compounds over training.
4. **Search-Space Utilization:** Adaptive scaling allows AdamW to take larger effective steps in low-curvature directions while damping steps in high-curvature directions, enabling it to exploit sharper yet high-performing basins.

### 7.2 Why Momentum Flattens the Landscape

Momentum accumulates velocity across iterations, which:

- Averages out high-frequency gradient noise during exploration.
- Carries parameters through narrow valleys that would trap vanilla SGD.

- Biases trajectory towards broader, flatter basins with larger capture volumes.

This explains why SGD+Momentum finds substantially lower $\lambda_{\max}$ than SGD.

### 7.3 Why Hessians Are Indefinite

Indefinite Hessians (negative $\lambda_{\min}$) are expected in high-dimensional neural networks because:

- The loss surface contains many saddle-like directions in overparameterized models.
- Stationary points after training are often flat in many directions and slightly unstable in others.
- Negative eigenvalues reflect directions where loss can locally decrease (saddle structure), rather than indicating poor convergence.

### 7.4 Why Interpolation Shows Non-Convex Paths

Linear interpolation between two independently trained parameter vectors is not a training trajectory; intermediate parameters are not optimized and can correspond to models with poor feature alignment. Thus:

- Large barriers along linear paths indicate separate basins.
- Extrapolated minima (minima outside $[0, 1]$) arise when one optimizer's solution lies beyond another along a descent direction.
- NEB provides a more realistic low-loss connecting path than naive linear interpolation.

### 7.5 NEB vs Linear Interpolation

NEB finds low-loss polygonal chains that respect local gradients and spring penalties, revealing:

- Smooth connections (low path loss) between similar solutions (e.g., SGD $\leftrightarrow$ SGD+Momentum).
- Higher barriers for solutions discovered by different optimizer families (e.g., AdamW vs SGD), indicating distinct basins.

## 8 Experimental Insights & Implications (summary)

- **SGD:** Fast early convergence to a sharp but low-quality solution; high gradient noise ratio; isolated parameter-space location.
- **SGD+Momentum:** Flattest measured landscape; strong performance due to momentum-driven exploration; solutions cluster with SGD variants.
- **AdamW:** Best overall validation performance; sharpest measured landscape but lowest gradient noise ratio and strong adaptive damping of sharp directions.

## 9 Future Work & Extensions

### 9.1 Immediate Extensions

1. **Scale Analysis:** Repeat experiments on smaller and larger models (50K to 10M+ parameters) to test scaling hypotheses.
2. **Dataset Diversity:** Validate on other datasets (ImageNet, NLP benchmarks) to ensure generality across modalities.
3. **Architecture Variations:** Test ResNets, Transformers, and models with/without normalization to measure architecture-specific effects.

## 9.2 Advanced Probes

- **Second-Order and Natural-Gradient Methods:** Compare full Newton or L-BFGS variants and natural-gradient updates to understand curvature-aware optimization.
- **Continual Probing:** Probe Hessian, noise ratio, and interpolation at multiple epochs to track dynamics rather than endpoints.
- **Noise Injection Experiments:** Systematically vary batch size and add synthetic noise to examine robustness and the causal role of noise ratio.

## 9.3 Theoretical Directions

- Formalize the connection between adaptive preconditioning and reduction in effective noise ratio.
- Develop theoretical bounds linking $\lambda_C/\lambda_H$ to escape times from sharp minima for SDE approximations of optimizers.
- Connect empirical findings to PAC-Bayes bounds that incorporate optimizer-dependent posterior choices.

# 10 Conclusion

## 10.1 Key Takeaways

1. **Sharpness is not a universal proxy for generalization.** AdamW finds sharp solutions yet generalizes well when noise is controlled.
2. **Gradient noise ratio ($\lambda_C/\lambda_H$) is a stronger predictor** of optimizer performance across families than $\lambda_{\max}$ alone.
3. **Momentum smooths trajectories and promotes flatter basins;** adaptive methods trade off sharpness for superior noise handling and rapid early progress.
4. **Parameter-space topology is optimizer-dependent:** momentum variants cluster together, while AdamW often finds separate basins.

## 10.2 Contributions

- A reproducible empirical pipeline with six complementary probes for loss landscape analysis.
- A systematic optimizer comparison revealing the central role of noise handling over naive sharpness minimization.
- Practical insights suggesting that optimizer selection should consider noise ratio and early-epoch dynamics.

## 10.3 Limitations

- Experiments confined to SimpleCNN and CIFAR-10; larger-scale validation needed.
- Hyperparameter tuning per optimizer could bias results; we used standard defaults to mitigate this.
- Theoretical explanations are suggestive; rigorous proofs are deferred to future work.

# Acknowledgements

Thanks to the project collaborators and maintainers of the probe codebase. Computational resources were provided by the institute's GPU cluster.

## References


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## A Appendix A: Configuration Files

All YAML configuration files are stored in `experiments/configs/`. Example entries:

```
# experiments/configs/sgd.yml
seed: 42
dataset:
  name: CIFAR10
  root: data
  train_batch: 128
model:
  name: small_cnn
optimizer:
  name: sgd
  lr: 0.01
  momentum: 0.0
  weight_decay: 5e-4
train:
  epochs: 12
  save_dir: experiments/runs/sgd_seed42

# experiments/configs/adamw.yml
seed: 42
dataset:
  name: CIFAR10
  root: data
  train_batch: 128
model:
  name: small_cnn
optimizer:
  name: adamw
```

```
  lr: 0.001
  weight_decay: 1e-2
train:
  epochs: 12
  save_dir: experiments/runs/adamw_seed42
```

# B Appendix B: Result Files

Results are organized as:

```
experiments/runs/
  sgd_seed42/
    checkpoint.pth
    train_summary.json
    probes/
      hessian_lanczos.json
      sgd_noise.json
      perturbation.json
      intrinsic_dim_d15.npz
      interpolation.npy
  adamw_seed42/
    ...
```