# Observation of Laughlin states made of light

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Much of the richness in nature emerges because simple constituents form an endless variety of ordered states [1]. While many such states are fully characterized by symmetries [2], interacting quantum systems can exhibit topological order, characterized instead by intricate patterns of entanglement [3, 4]. A paradigmatic example of topological order is the Laughlin state [5], which minimizes the interaction energy of charged particles in a magnetic field and underlies the fractional quantum Hall effect [6]. Broad efforts have arisen to enhance our understanding of topological order by forming Laughlin states in synthetic systems of ultracold atoms [7, 8] or photons [9–11]. Nonetheless, electron gases remain the only system where such topological states have been definitively observed [6, 12–14]. Here we create Laughlin-ordered photon pairs using a first-of-its-kind gas of strongly interacting, lowest Landau level polaritons as a photon collider. Initially uncorrelated photons enter a cavity and hybridize with atomic Rydberg excitations forming polaritons [15–17], quasi-particles that here behave like electrons in the lowest Landau level thanks to a synthetic magnetic field created by Floquet engineering [18] a twisted cavity [11, 19] and Rydberg-mediated interactions between them [16, 17, 20, 21]. Polariton pairs collide and self-organize to avoid each other while conserving angular momentum. Our finite-lifetime polaritons only weakly prefer such organization, so we harness the unique tunability of Floquet polaritons to distill high-fidelity Laughlin states of photons outside the cavity. Particle-resolved measurements demonstrate that these photons avoid each other and exhibit angular momentum correlations, the hallmarks of Laughlin physics. This work opens broad prospects for the study of topological quantum light [22].

The simplest recipe for realizing topological order is to place strongly-interacting particles in an effective magnetic field. The magnetic field quenches the kinetic energy of the particles so that they order themselves solely to minimize their interaction energy, resulting in intricate patterns of long-range entanglement [3]. These states exhibit fascinating properties largely unseen in other forms of matter; in addition to robust quantized edge transport which also appears without interactions [23], topologically ordered phases host excitations with fractional charge and anyonic statistics [24]. More exotic phases even host non-Abelian anyons, promising constituents for fault-tolerant quantum computers thanks to their insensitivity to local perturbations [4].

The scarcity of physical platforms hosting topological order has spurred great interest in elucidating its exotic properties using the wide tunability, particle-resolved control, and versatile detection capabilities afforded by synthetic quantum matter [7, 8, 10, 11]. The typical constituents of synthetic matter are atoms and photons, neither of which experiences a Lorentz force in ordinary magnetic fields, since they are charge neutral. Accordingly, the key challenge is to implement a synthetic magnetic field which creates an effective Lorentz force and is compatible with strong inter-particle interactions. A classic approach employed the Coriolis force in rotating ultracold atomic gases [25], and such systems approached the few-body fractional quantum Hall regime [26]. More recent efforts with ultracold atoms focused on Floquet engineering of synthetic magnetic fields [27] combined with strong atomic interactions thanks to tight optical lattice confinement [28]. Photonic systems have also demonstrated a variety of synthetic magnetic fields [11] compatible with strong interactions via coupling to superconducting qubits in the microwave domain [29, 30] and cold atoms [16, 17, 20, 21] or quantum dots [31] in the optical domain. These ingredients have yet to be effectively combined and scaled, so formation of topologically ordered synthetic matter has remained elusive.

In this work we assemble optical photon pairs into a Laughlin state. To achieve this, we construct a polaritonic system analogous to an electronic fractional quantum Hall fluid by combining two key ingredients: a synthetic magnetic field for light induced by a twisted optical cavity [19] and strong polaritonic interactions mediated by Rydberg atoms [17]. We inject photons possessing angular momentum into this system and observe that the resulting intra-cavity polaritons do indeed behave like fractional quantum Hall electrons, undergoing mode-changing, angular-momentum-conserving collisions to minimize their interactions with one another. Given the finite lifetime of our polaritons, this leads *towards* but not *to* an intracavity Laughlin state; we then distill a Laughlin state of photons outside the cavity by manipulating the optical leakage rate of each single-polariton state and compensating for a relative phase between photons and polaritons (SI A1). The distilled photon pairs exhibit angular momentum correlations and spatial avoidance characteristic of a Laughlin state.

A typical experiment begins by loading a pancake of $6(1) \times 10^3$ laser-cooled $^{87}$Rb atoms at the waist of an optical cavity (Fig. 1a; Methods C). We then continuously shine a weak probe laser beam on the cavity for 100 ms. Initially uncorrelated photons from the

beam entering the cavity are strongly coupled to a resonant atomic transition; with an additional Rydberg electromagnetically-induced transparency field they become polaritons, quasi-particles consisting of a superposition between a photon and a collective Rydberg excitation of the atomic gas (Fig. 1b; Methods B) [16, 17].

Polaritons inherit key properties from both their photonic and Rydberg components. The motion of individual polaritons is determined by the cavity modes accessible to their photonic part. The large energy spacing between longitudinal cavity manifolds restricts the polaritons to a single manifold, confining them to undergo two-dimensional motion among its transverse modes. We utilize a twisted optical cavity and Floquet engineering to form a set of degenerate orbital angular momentum modes equivalent to the lowest Landau level accessible to electrons in a magnetic field [18, 19] (Fig. 1c).

To form ordered states, polaritons must also interact with one another (Fig. 1d). Polaritons inherit the strong interactions of their Rydberg components, causing them to avoid each other. When confined to a single transverse mode, one polariton can only avoid another by blockading the cavity, preventing the second polariton from entering, akin to a zero-dimensional electronic quantum dot [17]. This phenomenon can be characterized via the two-photon correlation function $g^{(2)}(\tau)$ quantifying the likelihood that two photons emerge from the cavity separated by a time $\tau$ compared to uncorrelated photons (SI A3). Blockade manifests strikingly through anti-bunching; the $\tau = 0$ correlation $g^{(2)}(0)$ falls to zero because there are never two polaritons present in the cavity simultaneously.

When polaritons have access to multiple transverse modes, as in the lowest Landau level, new physics emerges. It becomes possible for two polaritons to enter the cavity simultaneously while avoiding one another: interactions need not lead to blockade, but can instead drive collisions between polaritons, causing them to move among the states of the Landau level and thereby reduce their interaction energy. To test for these collisions, we provide the polaritons access to exactly three states in the lowest Landau level – those with orbital angular momentum $l/\hbar = 3, 6$, and $9$ – using our Floquet scheme [18] (Methods D). Spectroscopy reveals the narrow Rydberg-polariton eigenstates that we employ for this work, alongside wider, non-interacting "bright" polaritons [32] that we can ignore (Fig. 2b). By utilizing a digital micromirror device for wavefront shaping, we inject photons with angular momentum $l = 6$. Photons entering the cavity become polaritons and can then collide and begin to order. When polaritons emerge from the cavity as photons through the output mirror, we count how many photons come from each angular momentum mode (Fig. 2a).

Despite the exotic nature of polaritonic quasiparticles, we find that they undergo collisions much like ordinary particles [22]: collisions between polaritons conserve total energy, as well as angular momentum thanks to the rotational symmetry of the Landau level and interactions. Accordingly, the only collision process which conserves angular momentum converts two input polaritons with $l = 6$ into one output polariton with $l = 3$ and another with $l = 9$. Similarly, as we tune the relative energies between the angular momentum states, we only detect photons with $l = 3$ or $9$ when the aforementioned collision process conserves energy (Fig. 2c, and SI B6). Moreover, increasing the probe beam power raises the likelihood of two polaritons being present simultaneously and thereby makes collisions more likely, a feature analogous to nonlinear reaction rates in chemistry. Indeed, we observe that the rate at which collision products appear $R_{3,9} \propto R_6^2$ is quadratic in the rate $R_6$ at which photons emerge in the initial angular momentum state (Fig. 2d).

To understand the ordering emerging from these collisions, in Fig. 3a & b we consider an idealized energy spectrum for zero, one, and two polaritons. We consider a "flat Landau level" where all single-polariton states $|l\rangle$ are degenerate, with energy $E_{pol}$. When only one single-particle eigenstate (for example, the $l = 6$ mode) is accessible, the interactions between polaritons that arise from their Rydberg components shift the energy and shorten the lifetime of the state $|66\rangle$ with two polaritons in that mode. Thus, a probe laser which resonantly excites $|6\rangle$ from the vacuum state $|vac\rangle$ does not subsequently excite the $|6\rangle \rightarrow |66\rangle$ transition, leading to blockade and precluding the formation of multi-polariton states.

In this idealized limit, providing the polaritons access to three single-particle eigenstates in the lowest Landau level leads to the emergence of a long-lived two-particle Laughlin state $|L\rangle$ (Fig. 3b). Our experiments offer a unique opportunity to connect the mathematical form of this Laughlin state to measurements of its microscopic structure. For the modes used in this work, the two-particle Laughlin wavefunction in real space is $\psi_L(z_1, z_2) \propto z_1^3 z_2^3 (z_1^3 - z_2^3)^2 \exp(-|z_1|^2/4 - |z_2|^2/4)$ where $z_k \equiv x_k + iy_k$ is a complex number reflecting the position $(x_k, y_k)$ of particle $k$. Expanding the polynomial prefactor lets us write this state in angular-momentum space as $|L\rangle = \frac{1}{\sqrt{3.1}}|66\rangle - \sqrt{\frac{2.1}{3.1}}|39\rangle$, where $|mn\rangle$ is the state with two polaritons of angular momenta $m\hbar$ and $n\hbar$ (SI B4). Because the wavefunction goes to zero when the particles are at the same position ($\psi_L(z_1 = z_2) = 0$), the Laughlin state enables two particles to avoid each other while remaining in the lowest Landau level. In the angular momentum basis, this avoidance arises from destructive interference between the $|66\rangle$ and $|39\rangle$ two-particle amplitudes for co-located particles. Similar two-particle Laughlin states can be formed in any set of three evenly spaced angular-momentum states (see SI B4).

The spatial anti-correlation of polaritons in the Laughlin state suppresses the interaction energy and interaction-induced decay present in other two-particle

states. Thus, for ideal polaritons, simply shining a laser into this atom-cavity system would cause a polaritonic Laughlin state to form inside, because all other two-polariton states are blockaded (including the "Anti-Laughlin" superposition state $|AL\rangle = \sqrt{\frac{2.1}{3.1}}\,|66\rangle + \frac{1}{\sqrt{3.1}}\,|39\rangle$).

The collisions observed in Fig. 2 are the first hint of such ordering. As detailed in Methods D, the finite optical depth of our polaritons prevents them from reaching a polaritonic Laughlin state before decaying. Surprisingly, we can distill a high purity photonic Laughlin state outside of the cavity from the polaritonic state produced within the cavity: this is achieved by choosing Floquet conditions to make the $l = 3$ and $l = 9$ polaritons more photon-like than the $l = 6$ polariton (apparent from their larger width in Fig. 2b), and correcting the phase imparted to the outgoing photons by the Floquet modulation (as detailed in SI A1).

To definitively test for the formation of these photonic Laughlin states we experimentally investigate the correlations of the emerging photon pairs [9]. First, when we detect all output photons regardless of their spatial mode, the "all-mode" correlations $g^{(2)}_{\text{all}}(\tau)$ reveal only weak blockade (Fig. 3c). This observation confirms that photon pairs can traverse the three-mode cavity whereas they were blocked in its single-mode counterpart; determining the structure of the pairs requires more detailed measurements.

We gain deeper insight by examining the correlations $g^{(2)}_{jk}$ between photons with angular momenta $l = j$ and $l = k$, again using the setup shown in Fig. 2a. The correlations $g^{(2)}_{66}$ between photons with $l = 6$ have a nonzero value $g^{(2)}_{66}(0) = 0.32(3)$ at zero time delay, indicating substantial population in the pair state $|66\rangle$ (Fig. 3d). However, their blockade is still stronger than that of the all-mode correlations $g^{(2)}_{\text{all}}$, indicating that the two-photon state has a large contribution from pairs not in $|66\rangle$. Most remaining pairs are accounted for by examining $g^{(2)}_{39}$, which exhibits a prominent peak at zero time (Fig. 3e). The peak height indicates that photons are $g^{(2)}_{39}(0) = 22(2)$ times more likely to appear in both modes simultaneously than expected for uncorrelated photons arriving with the same individual rates. This bunching arises because photons in these modes are produced together from polariton collisions and rarely injected independently. Unlike parametric collision processes observed in non-linear crystals requiring macroscopic mode populations [33], collisions in our system appear for *just two* intracavity polaritons.

We quantitatively compare the observed pair-photon state with the Laughlin state by calculating the two-particle populations $\rho_{jk}$ with angular momenta $j$ and $k$ from the coincidence rates of events where two photons are observed near-simultaneously (Fig. 3f; SI A3).

Coincidence events corresponding to $|66\rangle$ or $|39\rangle$ account for $85(15)\%$ of all observed photon pairs, consistent with angular momentum conservation. Moreover, the ratio $\rho_{39}/\rho_{66} = 1.5(5)$ of pair populations is near the intended ratio of 2.1 for the Laughlin state. Deviations from ideal populations could arise from the limited lifetime and interaction strength of our polaritons, slight drifts of system parameters between experiments, or because the polaritons with angular momentum $l = 3$ & $9$ are insufficiently photon-like.

We next test for the remaining essential physical feature of Laughlin states: that the photons avoid each other in real space. To measure in real-space we filter the output photons with a single-mode optical fiber (Fig. 4a) that only admits photons located at its tip, enabling us to count photons at that position. Since the average density in a state composed of $|39\rangle$ and $|66\rangle$ forms a smooth annulus, we translate the fiber to the radius with the highest density (Fig. 4b). A natural method for measuring angular correlations $g^{(2)}(\phi, \tau = 0)$ between photons separated by the angle $\phi$ around the annulus would be to use two fibers at different positions; our Floquet scheme induces a rapid rotation of the optical field, enabling an equivalent measurement with a single fiber.

While the average density of photon pairs leaving the cavity exhibits no angular structure, we find the photons are bunched in space (Fig. 4c) due to a Floquet-induced phase between photons and polaritons. After compensating this phase with linear optics (SI A3), the photons avoid each-other as expected for a Laughlin state (Fig. 4d). The spatial correlations should take the form $g^{(2)}_{\text{L}}(\phi) \propto |\psi_{\text{L}}(z_2 = e^{i\phi}z_1)|^2 \propto \sin^4(3\phi/2)$, which oscillates with the angle $\phi$ between the photons with a periodicity of $120°$ because only every third angular momentum state is accessible. Motivated by this expected form, we fit the observed correlations with $g^{(2)}_{\text{fit}}(\phi) = g_0 + g_1 \sin^4(3\phi/2)$ allowing for an offset $g_0$ from perfect spatial anti-bunching and a reduction of the oscillation amplitude $g_1$ due to imperfect state fidelity or detection. The fit yields a small offset $g_0 = 0.11(19)$ and an oscillation amplitude $g_1 = 0.77(36)$ with a significance of 2.1 standard deviations corresponding to a probability of 0.02 to arise from statistical fluctuations in a system with no dependence of correlation on angle. Combining with the uncompensated data further reduces this probability to $10^{-5}$ (SI A3).

On their own, the angular momentum mode populations are suggestive of Laughlin physics but insensitive to the phase and even the purity of the superposition between $|39\rangle$ and $|66\rangle$. However, the observed spatial anti-correlation only occurs for a coherent superposition with a minus sign. Combined, these results indicate that the photon pairs, in a virtual image plane at the twisted cavity mode waist (SI A7), have $76(18)\%$ overlap with a pure Laughlin state. This fidelity is limited primarily

by our conservative assumptions about the unmeasured momentum-non-conserving pair populations (SI A4).

This work establishes quantum many-body optics as a critical route to breakthroughs in quantum materials. Feasible technological upgrades, including novel state-preparation schemes, will enable our platform to support polaritonic Laughlin states and eventually explore the statistical phases of anyons (Methods A).

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## AUTHOR CONTRIBUTIONS

The experiment was designed and built by all authors. N.S. built the primary cavity. L.C., N.S. and C.B. collected the data. L.C. and N.S. analyzed the data. L.C. wrote, and all authors contributed to, the manuscript.

## AUTHOR INFORMATION

FIG. 1. **Ingredients for Laughlin States of Light. a,** Our experiment couples optical photons (red) with a gas of $^{87}$Rb atoms at the waist of a twisted, four mirror cavity. **b,** This coupling turns each photon entering the cavity into a *polariton*, a quasiparticle combining the photon with a collective Rydberg excitation of the atomic gas. Polaritons move around in the transverse modes available to their photonic component, and a pair of polaritons (depicted) strongly interacts because of their Rydberg components (blue spheres). Two key ingredients enable this system to explore topological order: **c,** First, we form a flat topological band of single-photon states using a twisted optical cavity, which hosts a set of degenerate photonic modes that are equivalent to the states in the lowest Landau level available to electrons in a strong magnetic field. Illustrations depict the origins of the fields in each system. **d,** Second, the strong polaritonic interactions are analogous to the Coulomb interactions between electrons in a traditional fractional quantum Hall system. Polaritons confined to a single cavity mode reveal their strong interactions via transport blockade, wherein a single polariton present in the cavity prevents a second photon from entering. Blockade results in anti-bunched correlations of photons exiting the cavity, shown here for $l = 0$.

FIG. 2. **Collisions between polaritons in the lowest Landau level. a,** Schematic: To test for mode-changing collisions between polaritons, we inject photons with orbital angular momentum $l = 6$ and then count the photons emerging from the cavity in other angular momentum modes (SI A2). **b,** We perform spectroscopy of the single-polariton eigenstates by weakly injecting $l = 3, 6, 9$ photons and scanning their energies. We observe Floquet Rydberg polariton resonances [18] (three narrow peaks) for each of the three accessible lowest Landau level eigenstates, separated by $\sim 73$ MHz. The $l = 3, 9$ polaritons are more photon-like (broader & taller) than the $l = 6$ polariton (see text). We ignore the broad "bright" polaritons [32], as they do not interact [17], instead probing collisions of photons injected on the Rydberg polariton resonances. **c,** When $l = 6$ photons are injected into the cavity at a higher rate, the total rate $R_{3,9}$ at which photons emerge with angular momentum $l = 3$ or 9 peaks when the collisions conserve energy (**inset**); $E_l$ is the angular momentum $l$ polariton's energy. Atomistic numerics (solid curve) capture key features of the data, including the FWHM of 1.1(1) MHz and slight energy mismatch 0.15(5) $h$·MHz of the peak due to the finite Rydberg interaction range (SI B5). **d,** At zero energy mismatch, varying the photon injection rate reveals that $R_{3,9}$ grows quadratically (solid curve) rather than linearly (dotted line) with the rate $R_6$ of injected $l = 6$ photons, consistent with $l = 3$ & 9 polaritons produced in two-body collisions between $l = 6$ polaritons. The probe power in **c** is the same as the $R_6 = 1.32(3)$ kHz datapoint in **d**; observed rates are incompatible due to mild $\nabla E$ broadening of **d**.

FIG. 3. **Laughlin state characterization in angular momentum space.** Schematic many-body energy spectrum for states containing polariton numbers $N_{\text{pol}} \leq 2$ with each particle able to explore only a single mode (**a**) or three modes of the lowest Landau level (**b**). In a single mode (here $l = 6$), the polaritons cannot avoid each other, so the two polariton state $|66\rangle$ has its energy shifted ($U_{\text{int}}$) and rapidly decays ($\Gamma_{\text{int}}$) due to interactions; these effects induce blockade. With three modes accessible, most two-polariton states are still shifted by interactions. However, a Laughlin state $|L\rangle$ arises in which the interference between $|66\rangle$ and $|39\rangle$ reduces the interaction energy by keeping the particles apart. This pair state is longer-lived and thus preferentially excited by the $l = 6$ probe laser (black arrows). In practice, we study and measure correlations in the related but inequivalent optical quantum field emitted from the cavity: **c,** When all emitted photons are included regardless of angular momentum, photons with access to all three modes exhibit weak blockade. **d,** Correlations between photons with $l = 6$ exhibit stronger blockade, with remnant nonzero correlation $g_{66}(0) = 0.32(3)$ at zero delay. **e,** Photons with angular momenta $l = 3$ and $l = 9$ exhibit a large positive cross-correlation because they are produced together in collisions. **f,** Relative populations in the two-photon manifold determined from coincidence events (bars) are comparable to the Laughlin state (squares) and an atomistic numerical model [18] (circles). Vertical bars in panel **c** indicate the portion of $g_{\text{all}}^{(2)}(0)$ accounted for by pairs in $|39\rangle$ (orange), $|66\rangle$ (green), and all other states (black), as detailed in SI A3. All error bars indicate standard error.

FIG. 4. **Spatial correlations of a photonic Laughlin puddle. a,** A single-mode fiber (purple) admits only photons at its location, enabling us to filter photons by their real-space position. A mode-dependent phase compensator counteracts the phase difference between polaritons and photons imprinted by our Floquet scheme (SI A3). **b,** The average density of photons forms a smooth annulus with no angular structure. We place the fiber off-center (solid circle) at the radius with the highest density and measure correlations as a function of the angle $\phi$ to a second location (dashed circle) around the annulus. **c,** The measured angular correlations (circles) for zero time delay exhibit a periodic structure. The maximum of correlation near $\phi = 0$ reveals that photons are *bunched* in space due to a Floquet-induced phase difference between photons and polaritons. **d,** After phase correction, the photons avoid one another. The solid curve is a fit to the function $g_{\text{fit}}^{(2)}(\phi) = g_0 + g_1 \sin^4(3\phi/2)$ (see text). Error bars indicate standard error.

# METHODS

## A. The near future of our platform

This work establishes quantum many-body optics in strongly interacting gases of topological polaritons as a critical route to breakthroughs in quantum materials. We bring to bear the unique microscopic control of our photonic platform, providing highly tunable system parameters, energy- and space- resolved particle injection, and the ability to measure correlations in almost any basis using simple linear optics. The present performance, combined with a cavity that can be made precisely degenerate in lieu of the Floquet scheme, should directly support polaritonic Laughlin states (SI A1). Looking ahead, novel state-preparation schemes [34, 35] such as dissipative stabilization [36–40] will enable the formation of larger topologically ordered states. Interestingly, the Laughlin state that we have assembled in this work already contains a quasi-hole at its center (SI B4), a precursor to directly measuring statistical phases of anyons [41–45] or even non-Abelian braiding in the Moore-Read state [46].

## B. Making polaritons with cavity Rydberg electromagnetically-induced transparency

Coupling atomic gases with multiple modes of optical resonators provides exciting opportunities for studying many-body physics [47–52]. Here, we use a multimode optical cavity to generate a synthetic gauge field for light [11, 19, 53–56]. Making photons interact strongly in this cavity enables us to study strongly correlated fractional quantum Hall states of light [9, 57–63].

In our system photonic interactions are mediated by Rydberg atoms [64] through a scheme called cavity Rydberg electromagnetically induced transparency [15–17, 32, 65–74]. First, we couple cavity photons to the $5S_{1/2} \rightarrow 5P_{3/2}$ transition of the gas of $^{87}$Rb atoms at the waist of our primary cavity, which we name the "science" cavity, with collective coupling strength $g$ (Fig. Extended Data 1a). These experiments operate in the regime of resonant collective strong coupling between the atoms and the cavity, in which the collective cooperativity $\eta_l = 4g_l^2/(\kappa\Gamma)$ in the relevant modes with angular momentum $l$, collective atom-cavity coupling $g_j$, cavity decay rate $\kappa$, and atomic excited state decay rate $\Gamma$ satisfies $\eta_l \gg 1$ [75]. The $5P_{3/2}$ state is subsequently coupled to a highly excited Rydberg level ($111D_{5/2}$) by an additional 480 nm field with coupling strength $\Omega$. As detailed below and in SI A6, we use a buildup cavity crossed with the science cavity in order to achieve sufficient intensity of that Rydberg coupling field whilst it covers a large area.

As a result of these couplings, photons no longer propagate in the cavity on their own as they would in vac-uum. Instead each cavity mode hosts three flavors of polaritons – quasiparticles composed of hybrids between a photon and a collective excitation of the atomic gas [32] (Fig. Extended Data 1b). The nature of these collective excitations is detailed in SI B1. The two "bright" polaritons are largely composed of a photon and a collective $5P_{3/2}$ excitation; we do not employ bright polaritons in this work because they are short lived due to rapid decay of the $5P_{3/2}$ state at $\Gamma = 2\pi \times 6$ MHz. We utilize the third "dark" polariton throughout the main text of this work. Dark polaritons are purely a superposition of a cavity photon and a collective Rydberg excitation. Because they do not have any $5P_{3/2}$ component, they are long-lived. Moreover, dark polaritons inherit strong interactions from their large collective Rydberg component [17, 64].

A typical scan of the polariton features corresponding to a single cavity mode is shown in Fig. Extended Data 1c. That spectrum, taken of the $l = 6$ cavity mode, directly reveals the narrow dark polariton excitation at frequency $f_6$ with a linewidth of 60 kHz, flanked by the two bright polariton resonances which are $\pm\sqrt{g^2 + \Omega^2}$ separated in frequency and have linewidths of 3.7(1) MHz. For the displayed data, the coupling strengths are $g = 2\pi \times 13$ MHz and $\Omega = 2\pi \times 1.3$ MHz and the effective Rydberg state linewidth (including decoherence effects) is observed to be $\Gamma_R = 2\pi \times 50$ kHz.

## C. Experiment Setup

The primary cavity used in our experiments, which couples with the $5S_{1/2} \rightarrow 5P_{3/2}$ transition, is the so-called "science" cavity, which consists of four mirrors in a twisted (non-planar) configuration [19, 76]. The cavity finesse is $\mathcal{F} = 1950$ yielding a linewidth of $\kappa = 2\pi \times 1.4$ MHz. The modes are approximately Laguerre-Gaussian in the lower waist where they intersect with the atomic cloud; the fundamental mode has a lower waist size of 19 $\mu$m. These parameters yield a peak coupling strength with a single atom of $g_{single} = 2\pi \times 0.58$ MHz, corresponding to a cooperativity of $\eta = 0.16$ per atom [75]. The science cavity is crossed with a buildup cavity for increasing the intensity of the Rydberg coupling beam at 480 nm; the buildup makes it possible to achieve a sufficient intensity over the wide area spanning the science cavity modes up to $l = 9$ in which we create polaritons. Note that the polarization eigenmodes of both cavities are circular. For more details on the cavity structure, see SI A6.

The science cavity was designed to be tuned to a length at which every third angular momentum state would be degenerate, forming a photonic Landau level [19]. However, as detailed in SI A6, we found that cubic intracavity aberrations arising from astigmatism destabilized the cavity modes when the length was tuned to this degen-

eracy point. Therefore, we instead operate at a cavity length far enough from degeneracy to create a 70 MHz splitting between every third angular momentum state and utilize the Floquet scheme described below in Methods D to create degenerate *polaritons* that are protected from the effects of intracavity aberrations (SI B2).

We mediate interactions between the photons in the science cavity modes using a gas of cold $^{87}$Rb atoms loaded at the lower waist of the cavity. There are $6(1) \times 10^3$ atoms distributed nearly uniformly over the region spanned by the first ten modes of the science cavity. The gas is cooled to a temperature below 1 $\mu$K and polarized into the lowest energy spin-state $|F = 2, m_F = -2\rangle$ within the hyperfine manifold $F$ using degenerate Raman sideband cooling [77]. For more details on the atom trapping configuration, atomic polarization, and experiment procedure, see SI A5.

### D. Forming the Landau level with Floquet polaritons

This section explains our scheme for forming the Landau level with Floquet polaritons, which provides a number of crucial features that made this work possible. Most importantly, Floquet polaritons helped us mitigate the effects of intracavity aberrations. As detailed in SI A6, aberrations prevented us from making the bare cavity modes with angular momenta $l = 3$, 6, & 9 degenerate. At the cavity length which would otherwise make them degenerate, aberrations mix the modes together, splitting them apart in energy and causing them to decay rapidly. Thankfully, Floquet polaritons are protected from the broadening caused by intracavity aberrations, as detailed in SI B2. The Floquet scheme also provides the ability to efficiently measure angular correlation functions as detailed in SI A3 and enables frequency discrimination to improve the selectivity of our mode sorters discussed in SI A2.

The essence of our Floquet scheme is depicted in Fig. Extended Data 2 and detailed in Ref. [18]. Briefly, a sinusoidal modulation of the energy of the intermediate $5P_{3/2}$ state splits it into three bands which exist at energies separated by the modulation frequency $f_{\mathrm{mod}}$. Cavity photons can excite the atom through any of these bands, thereby enabling the atoms to couple with modes whose energies are also split by $f_{\mathrm{mod}}$.

To implement the Floquet scheme, we first used temperature and piezo tuning to increase the length of the science cavity and induce a splitting of $f_{\mathrm{cav}} \approx$ 70 MHz between every third angular momentum mode (Fig. Extended Data 3a). Then, to enable the $l = 3$, 6, & 9 modes to simultaneously support dark polaritons, we fine-tuned the cavity length to make the $l = 6$ mode degenerate with the bare $5S_{1/2} \rightarrow 5P_{3/2}$ transition and modulated the intermediate state at $f_{\mathrm{mod}} \approx$ 70 MHz

to create sidebands resonant with the $l = 3$ & 9 states (Fig. Extended Data 3b). By making it possible for the atoms to strongly couple with each of the cavity modes, the Floquet scheme enables all three modes to host polaritons (Fig. Extended Data 3c). Note that the splitting $f_{\mathrm{cav}}$, and therefore the required modulation frequency $f_{\mathrm{mod}}$, varied over a couple of MHz from day to day due to variation in the cavity temperature.

Cavity spectroscopy reveals the polariton eigenmodes in all three modes simultaneously (Fig. Extended Data 4a). Because the cavity modes are separated by $f_{\mathrm{cav}} \approx$ 70 MHz, the photonic components of the polaritons also remain separated by approximately $f_{\mathrm{cav}}$. Therefore, to spectroscopically characterize the set of polaritons for mode $l$, we scan the probe frequency near the frequency $f_l$ of that mode and simultaneously use a digital micromirror device to spatially mode match the probe laser with the cavity mode [78], producing the spectra shown in Fig. Extended Data 4a. The asymmetric heights and frequency splittings of the bright polaritons relative to the dark polariton for $l = 3$ & 9 are caused by shifts due to coupling with off-resonant bands of the $5P_{3/2}$ state. Moreover, the smaller splitting of the bright polaritons with $l = 3$ & 9 (the "sideband" modes) relative to $l = 6$ (the "carrier" mode) results from smaller atom-cavity couplings $g_3 \approx g_9 = 0.35 \ g_6$ on those modes relative to the central mode. The relative strengths of these couplings are determined by the modulation amplitude; here, we chose to make the sideband couplings weaker in order to make the corresponding dark polaritons more photon-like, see SI A1 for details.

While the spectroscopic features are all clearly separated in the scan of the probe frequency $f$, the dark polaritons can be tuned to have equal quasi-frequency $\tilde{f}$, where $\tilde{f} = (f \mod f_{\mathrm{mod}})$ is the probe frequency modulo the modulation frequency. In a Floquet model the energy is only defined up to multiples of the modulation energy; thus, two states with the same quasi-energy (equivalently, quasi-frequency) behave as if they are degenerate [27]. In the example shown in Fig. Extended Data 4a, the dark polaritons are separated in probe frequency by an amount equal to the modulation frequency $f_{\mathrm{mod}} = 73.3$ MHz; thus, their quasi-frequencies are identical.

It is not trivial to find conditions at which the quasi-frequencies of the dark polaritons are equal. The primary challenges are the anharmonicity in the cavity mode spectrum and the off-resonant shifts caused by weak couplings to non-resonant Floquet bands. These effects typically prevent the dark polariton energies from matching under conditions which might naïvely seem suitable; in particular, when $f_{\mathrm{cav}} = f_{\mathrm{mod}}$, the quasi-frequency $\tilde{f}_9$ ($\tilde{f}_3$) of dark polaritons with $l = 9$ ($l = 3$) will typically be too large (small) due to the off-resonant shifts [18].

The dark polariton quasi-frequency $\tilde{f}_l = \tilde{\delta}_c^l \cos^2 \theta_l + \tilde{\delta}_r \sin^2 \theta_l$ in each mode is a weighted average of the cav-

ity mode quasi-frequency $\tilde{\delta}_c^l$ with the Rydberg quasi-frequency $\tilde{\delta}_r$; the weight is determined by the dark-state rotation angle $\theta_l$, which satisfies $\tan\theta_l \equiv g_l/\Omega$ (see SI B5 in Ref. [18]). A smaller ratio $g_l/\Omega$ increases the contribution from the cavity photon and thus makes the polariton more "photon-like"; in the opposite case, the polariton is more "Rydberg-like" [32].

We adjust the Rydberg quasi-frequency $\tilde{\delta}_r$ and the modulation frequency $f_{\mathrm{mod}}$ in order to tune the dark polaritons into degeneracy. The Rydberg quasi-frequency is controlled by the frequency of the Rydberg coupling laser. Because the $l = 6$ polaritons are more Rydberg-like than the $l = 3$ & 9 polaritons, the quasi-frequency $\tilde{f}_6$ of $l = 6$ polaritons increases more rapidly than the others with $\tilde{\delta}_r$. Thus, the quasi-frequencies $\tilde{f}_9$ and $\tilde{f}_3$ decrease relative to $\tilde{f}_6$ as $\tilde{\delta}_r$ increases, as depicted in Fig. Extended Data 4b. Note that, in Fig. 2c of the main text, we varied the Rydberg quasi-frequency $\tilde{\delta}_r$ in order to scan the energy mismatch. Adjusting the modulation frequency $f_{\mathrm{mod}}$ shifts the cavity mode quasi-frequencies $\tilde{\delta}_c^3$ and $\tilde{\delta}_c^9$ oppositely, because they couple to the atoms through opposite modulation sidebands of the $5P_{3/2}$ state (Fig. Extended Data 3c).

These two adjustment parameters are sufficient to tune the three dark polariton features into degeneracy, as demonstrated in Fig. Extended Data 4d. In practice, to reach degeneracy we repeatedly measure the quasi-frequency spectrum of the dark polaritons while adjusting parameters. We modify the frequency of the Rydberg laser until the average of the sideband quasi-frequencies matches the carrier, i.e. $(\tilde{f}_3 + \tilde{f}_6)/2 = \tilde{f}_6$. Similarly, we vary $f_{\mathrm{mod}}$ until $\tilde{f}_3 = \tilde{f}_9$. In the end, we are able to satisfy these conditions to an accuracy well within the linewidths of the dark polaritons.

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## I.  DATA AVAILABILITY

The experimental data presented in this manuscript is available from the corresponding authors upon request.

FIG. Extended Data 1. **Single mode polariton spectrum. a,** Photons at the waist of our twisted optical cavity (red) couple with strength $g$ to the $5S_{1/2} \rightarrow 5P_{3/2}$ transition of a gas of cold $^{87}$Rb atoms, which is subsequently coupled with strength $\Omega$ to the highly excited $111D_{5/2}$ Rydberg state using an additional laser (blue). **b,** These couplings cause excitations of the atom-cavity system to propagate as polaritons, quasiparticles combining photons with collective atomic excitations. **c,** The transmission spectrum of the cavity with atoms present directly reveals the narrow dark polariton flanked by two broad bright polariton peaks. The solid curve shows a fit of the cavity electromagnetically induced transparency spectrum to the measured transmission (see Ref. [32] Supplementary Information Eq. (7)).

FIG. Extended Data 2. **Essential features of the Floquet scheme. a,** Our Floquet scheme utilizes an additional laser beam (green) incident upon the atoms with a wavelength of $\lambda = 1529$ nm close to the $5P_{3/2} \rightarrow 4D$ transition. **b,** This beam induces a sinusoidally modulated AC Stark shift $E_p = \eta \sin(2\pi f_{\mathrm{mod}} t)$ of the $5P_{3/2}$ state with amplitude $\eta$ and frequency $f_{\mathrm{mod}}$. **c,** As a result of this modulation, the ordinary $5P_{3/2}$ state is split into three bands with energies separated by the modulation frequency. The additional bands enable the atoms to couple with cavity photons at frequencies shifted by $\pm f_{\mathrm{mod}}$ from the ordinary $5S_{1/2} \rightarrow 5P_{3/2}$ resonance frequency. For more details on the Floquet scheme see Ref. [18].

FIG. Extended Data 3. **Scheme for forming Landau level of Floquet polaritons. a,** The bare cavity modes are not degenerate in this work, but instead the length of the cavity is increased so that there is a $f_{\mathrm{cav}} \approx 70$ MHz splitting between every third angular momentum mode. **b,** In order to form polaritons in three modes, even though only the $l = 6$ mode is resonant with the un-modulated $5S_{1/2} \rightarrow 5P_{3/2}$ transition, we utilize the Floquet scheme depicted in Fig. Extended Data 2 [18]. Modulating the $5P_{3/2}$ state at $f_{\mathrm{mod}} \approx 70$ MHz splits it into three bands (gray), each of which is resonant with one of the three chosen cavity modes. The coupling strengths $g_l$ to each mode $l$ are controlled by the modulation amplitude; in this work, $g_3 = g_9 = 0.37(4)g_6$. Note that each mode couples to a unique collective atomic excitation, as depicted at the top (blue atoms are included in the corresponding collective excitation, while gray atoms are not). c, This scheme produces polaritons in the $l = 3, 6, \& 9$ modes. The dark polaritons can be made effectively degenerate (see Fig. Extended Data 4) without making the corresponding cavity modes degenerate, which protects the polaritons from intracavity aberrations (see SI B2).

FIG. Extended Data 4. **Understanding and controlling polariton spectra with the Floquet scheme. a,** Cavity transmission spectrum in the presence of the modulated atoms (see Fig. Extended Data 3), reproducing Main Text Fig. 2b. The spectrum was collected in three parts, corresponding to injection of photons into $l = 3$ (left, green), $l = 6$ (middle, black), and $l = 9$ (right, violet). Dark polaritons in the $l = 6$ mode are less photon-like than those in the other two modes, reducing their relative transmission (SI A1); we multiply the $l = 6$ transmission by four to improve visibility. The lower x-axis indicates the frequency $f$ of the probe laser relative to the $l = 6$ dark polariton resonance at $f_6$. The top x-axis indicates the quasi-frequency $\tilde{f}$, proportional to the quasi-energy of the polaritons from a treatment using Floquet theory; $\tilde{f}$ is equal to $f$ modulo the modulation frequency $f_{\mathrm{mod}}$. The solid curves show three independent fits of the cavity electromagnetically induced transparency spectrum to the measured transmission in each angular momentum mode (see Ref. [32] Supplementary Information Eq. (7)). **b & c,** Illustration of the theoretical dependence of the quasi-frequencies of the three dark polariton features on the Rydberg beam detuning (**b**) and the modulation frequency $f_{\mathrm{mod}}$ (**c**). **d,** Example transmission spectra for the $l = 6$ (black, lower), $l = 3$ (green, middle), and $l = 9$ (violet, upper) dark polaritons as a function of quasi-frequency. The scans are scaled to make their heights equal and have additional vertical offsets for clarity. Shortly before performing each of the experiments reported in the main text, we collect a sequence of plots similar to those displayed here and adjust the Rydberg detuning and modulation frequency to make all three dark polaritons have the same quasi-frequency (right-most plot). The only experiments reported in the main text which did not use this sequence are those shown in Fig. 2c, where instead we varied $\delta_r$ to intentionally vary the energy mismatch between the polaritons. Throughout this figure quasi-frequencies $\tilde{f}$ are reported relative to the $l = 6$ dark polariton resonance $\tilde{f}_6$. Solid curves provide a guide to the eye.