Machine Learning from Data – IDC – 2022

HW5 – Theory + SVM

1. Kernels and mapping functions (25 pts)
   a. (20 pts) Let $K(x,y) = (x \cdot y + 1)^3$ be a function over $\mathbb{R}^2 \times \mathbb{R}^2$ (i.e., $x, y \in \mathbb{R}^2$).

      Find ψ for which $K$is a kernel. (It may help to first expand the above term on the right-hand side).

   b. (2 pts) What did we call the function ψ in class if we remove all coefficients?
   c. (3 pts) How many multiplication operations do we save by using $K(x,y)$ versus $\psi(x) \cdot \psi(y)$?

2. Lagrange multipliers (25 pts)

   Let $f(x,y) = 2x - y$. Find the minimum and the maximum points for f under the constraint $g(x,y) = \frac{x^2}{4} + y^2$.

3. PAC Learning (25 pts)

   Let $X = \mathbb{R}^2$. Let vectors $u = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), w = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right), v = (0,-1)$

   and $C = H = \left\{ h(r) = \left\{ (x_1, x_2) \middle| \begin{matrix} (x,y) \cdot u \leq r, \\ (x,y) \cdot v \leq r, \\ (x,y) \cdot w \leq r \end{matrix} \right\} \right\}$, for $r > 0$,

   the set of all origin-centered upright equilateral triangles.
   Describe a polynomial sample complexity algorithm L that learns C using H. State the time complexity and the sample complexity of your suggested algorithm. Prove all your steps.

4. (15 pts) A business manager at your ecommerce company asked you to make a model to predict whether a user is going to proceed to checkout or abandon their cart. You created the model using, and reported 20% error on your test set of size 1000 samples. In the business manager's presentation to upper management, he presented your

model and stated that the company can expect 20% error when deploying the model live on the website.

Luckily, you realize that this is a mistaken assumption, and you correct the statement to say that with 95% confidence, the true error they can expect is up to what percentage? (Just state the error percentage).

5. SVM (10 pts)

   See the notebook in the homework files and follow the instructions there.

   Take a **screenshot** of your resulting graph near the bottom of the notebook (titled "My Graph") and paste into your submission PDF along with your answers to the theoretical questions. Do **NOT** submit your code.