Ep 5 - Combinatorics

Nov 30,2023

What is Combinatorics?

Combinatorics is the branch of mathematics that deals with counting, arranging, and selecting objects or elements. It provides tools and techniques to solve problems related to these tasks.

Fundamental Counting Principle

The fundamental counting principle states that if you have two independent tasks to perform, and task A can be done in m ways, while task B can be done in n ways, then there are $m * n$ ways to perform both tasks together.
Example: If you have 3 choices for a shirt and 4 choices for pants, you have $3 * 4 = 12$ outfit combinations.

Factorial

The factorial of a non-negative integer n, denoted as n!, is the product of all positive integers from 1 to n.
Formula: $n! = n * (n-1) * (n-2) * \ldots * 3 * 2 * 1$
Example: $5! = 5 * 4 * 3 * 2 * 1 = 120$

Permutations

A permutation of a set of objects is an arrangement of those objects in a specific order. The number of permutations of n distinct objects taken k at a time is denoted as $P(n, k)$.
Formula:

$$P(n, k) = \frac{n!}{(n-k)!}$$

Example: How many ways can you arrange 3 books out of 5 on a shelf? $P(5, 3)$= 60 ways.

Combinations

A combination of a set of objects is a selection of those objects without regard to the order. The number of combinations of n distinct objects taken k at a time is denoted as $C(n, k)$ or "n choose k."
Formula:

$$C(n, k) = \frac{n!}{k! * (n - k)!}$$

Example: How many ways can you select 2 cookies out of a jar with 5 different types of cookies? $C(5, 2)$ = 10 ways.

Binomial Coefficients

Binomial coefficients represent the number of ways to choose k elements from a set of n elements, which is also a combination. Binomial coefficients are often denoted as $C(n, k)$.
Example: In $(a + b)^n$ the coefficients of the terms represent binomial coefficients.