Ft. bine 😊

A.

1.

f	Δf	Δ²f	Δ³f	Δ⁴f
	0	1	-7	20
1				
1	1	-6	13	
2	-5	7		
-3	2			
-1				

✓

2.

x	1	3	4	6	7
f(x)	3	0	5	7	1

x	f	$\mathcal{D}f$	$\mathcal{D}^2 f$	$\mathcal{D}^3 f$	$\mathcal{D}^4 f$
1	3	$\frac{0-3}{3-1} = -\frac{3}{2}$	$\frac{5+\frac{3}{2}}{4-1} = \frac{13}{6}$	$\frac{-\frac{4}{3} - \frac{13}{6}}{6-1} = \frac{-21^{7}}{6\cdot5} = \frac{-7}{10}$	$\frac{-\frac{1}{4} + \frac{7}{10}}{7-1} = \frac{9}{20\cdot6} = \frac{3}{40}$
3	0	$\frac{5-0}{4-3} = 5$	$\frac{1-5}{6-3} = -\frac{4}{3}$	$\frac{-\frac{7}{3} + \frac{4}{3}}{7-3} = \frac{-8}{8\cdot4} = -\frac{1}{4}$	
4	5	$\frac{7-5}{6-4} = 1$	$\frac{-6-1}{7-4} = \frac{-7}{3}$	✓	✓
6	7	$\frac{1-7}{7-6} = -6$	✓		
7	1	✓			

3. $f(x) = (1+x)^{\frac{1}{3}}$, $x_0 = 0$, $f(0) = 1$

$f'(x) = \frac{1}{3}(1+x)^{-\frac{2}{3}}$, $f'(0) = \frac{1}{3}$

$f''(x) = -\frac{2}{9}(1+x)^{-\frac{5}{3}}$, $f''(0) = -\frac{2}{9}$ ✓

$P_2(x) = 1 + \frac{1}{3}x - \frac{2}{9}x^2$ ✓

4. $f(x) = \sin x$, $x_0 = 0$, $[-\frac{\pi}{4}, \frac{\pi}{4}]$

a bound of the error for $T_5 f(x)$

Find $M > 0$ st $|R_n f(x)| \leq M$

$R_n f(x) = \frac{(x-x_0)^{n+1}}{(n+1)!} f^{(n+1)}(\xi)$, ξ around x_0 and x

$R_5 f(x) = \frac{x^6}{6!} \cdot f^{vi}(\xi)$

$f(x) = \sin x$, $f'(x) = \cos x$, $f''(x) = -\sin x$, $f'''(x) = -\cos x$

$f^{iv}(x) = \sin x$, $f^{v}(x) = \cos x$, $f^{vi}(x) = -\sin x$

$|f^{iv}(\xi)| = |\sin \xi| = \sin \xi \leq \frac{\sqrt{2}}{2}$

$|R_5 f(x)| \leq \left| \frac{f^{vi}(\xi) x^6}{6!} \right| = \frac{|f^{vi}(\xi)||x^6|}{6!} \leq \frac{\sqrt{2} x^6}{2 \cdot 6!} \leq \frac{\sqrt{2} \cdot \left(\frac{\pi}{4}\right)^6}{2 \cdot 6!}$

$-\frac{\pi}{4} < x < \frac{\pi}{4} \quad \big\uparrow^6$

$\left(-\frac{\pi}{4}\right)^6 < x^6 < \left(\frac{\pi}{4}\right)^6 \implies x^6 = \left(\frac{\pi}{4}\right)^6$