Seminars 8 and 9 - 2025

1. A player tosses three coins into the air. He wins 1 dollar for the number of heads he will get. However, he will lose 8 dollars if neither coin is a head. (a) Calculate the expected value of this game.
(b) Is the game favorable for the player?
(c) What is the expected profit or loss after playing 10 times this game?

2. The function *numpy.random.rand* returns in Python, for each independent call, a random value from the interval $[0, 1]$, according to the uniform distribution $Unif[0, 1]$. Let $n \in \mathbb{N}^*$ and $p \in (0, 1)$. If the function is called n times, what is the expectation and variance of the number of values less than p?

3. The length of time for one individual to be served at a cafeteria is a random variable T having an exponential distribution with a mean of 5 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?

$$T \sim Exp\,(\lambda) \Longleftrightarrow f_T(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ \lambda e^{-t\lambda}, & \text{if } t > 0\,. \end{cases}$$

4. Peter has a bike combination lock with a 3 digit cipher code. He remembers that the first digit is 0 and the other two are even, but he forgot them. Compute the expected number of unsuccessful trials *before* opening the lock for the first time for the following cases:
(a) unsuccessful codes are not eliminated from further selections (Peter is not a well organized person);
(b) unsuccessful codes are eliminated (Peter likes systematical search and notes the already tried codes).

5. It is known that the density function of the random time T (in minutes) in which a device performs a certain task is $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \begin{cases} a(b-x), & \text{if } x \in (0, b) \\ \\ 0, & \text{otherwise,} \end{cases}$
where $a, b \in \mathbb{R}$ are parameters. Compute a and b if the expected time to complete the task is 1 minute.

6. A factory produces graphite-core wood pencils with the standard length of 190 mm and an error (in mm) which follows the normal distribution $N(1, 0.25)$. Compute the expectation and the variance of the length of a pencil.
$X \sim N(\mu, \sigma^2)$, then the density function of X is $f_X(x) = \dfrac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\dfrac{(x-\mu)^2}{2\sigma^2}\right\}, x \in \mathbb{R}$.

7. The life, in years, of a certain type of electrical switch has an exponential distribution with an average life of $\beta = 2$ years. If 100 of these switches function independently in a system, what is the expected value of the number of switches that fail during the first year?