05.03.2025

Homework 1

A.

1.

f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
1	0	1	-7	20
1	1	-6	13	
2	-5	7		
-3	2			
-1				

2.

x	1	3	4	6	7
$f(x)$	3	0	5	7	1

x	f	$\mathcal{D} f$	$\mathcal{D}^2 f$	$\mathcal{D}^3 f$	$\mathcal{D}^4 f$
1	3	$\frac{0-3}{3-1} = -\frac{3}{2}$	$\frac{5+\frac{3}{2}}{4-1} = \frac{13}{6}$	$\frac{-\frac{4}{3}-\frac{13}{6}}{6-1} = \frac{-\cancel{21}^{7}}{\cancel{6}\cdot 5} = \frac{-7}{10}$	$\frac{-\frac{1}{4}+\frac{7}{10}}{7-1} = \frac{9}{206} = \frac{3}{40}$
3	0	$\frac{5-0}{4-3} = 5$	$\frac{1-5}{6-3} = -\frac{4}{3}$	$\frac{-\frac{7}{3}+\frac{4}{3}}{7-3} = \frac{-\cancel{3}}{\cancel{3}\cdot 4} = -\frac{1}{4}$	
4	5	$\frac{7-5}{6-4} = 1$	$\frac{-6-1}{7-4} = \frac{-7}{3}$		
6	7	$\frac{1-7}{7-6} = -6$			
7	1				

3. $f(x) = (1+x)^{\frac{1}{3}}$, $x_0 = 0$, $f(0) = 1$

$f'(x) = \frac{1}{3}(1+x)^{-\frac{2}{3}}$, $f'(0) = \frac{1}{3}$

$f''(x) = -\frac{2}{9}(1+x)^{\frac{-5}{3}}$, $f''(0) = -\frac{2}{9}$

$$P_2(x) = 1 + \frac{1}{3}x - \frac{2}{9}x^2$$

4. $f(x) = \sin x$, $x_0 = 0$, $[-\frac{\pi}{4}, \frac{\pi}{4}]$

a bound of the error for $T_5 f(x)$

Find $M > 0$ s.t. $|R_n f(x)| \leq M$

$R_n f(x) = \frac{(x - x_0)^{n+1}}{(n+1)!} \cdot f^{(n+1)}(\xi)$, ξ around x_0 and x

$R_5 f(x) = \frac{x^6}{6!} \cdot f^{VI}(\xi)$

$f(x) = \sin x$, $f'(x) = \cos x$, $f''(x) = -\sin x$, $f'''(x) = -\cos x$

$f^{IV}(x) = \sin x$, $f^{V}(x) = \cos x$, $f^{VI}(x) = -\sin x$

$|f^{VI}(\xi)| = |-\sin \xi| = \sin \xi \leq \frac{\sqrt{2}}{2}$

$|R_5 f(x)| \leq \left|\frac{f^{VI}(\xi) x^6}{6!}\right| = \frac{|f^{VI}(\xi)| |x^6|}{6!} \leq \frac{\sqrt{2} x^6}{2 \cdot 6!} \leq \frac{\sqrt{2} \cdot (\frac{\pi}{4})^6}{2 \cdot 6!}$

$-\frac{\pi}{4} < x < \frac{\pi}{4}$ $\uparrow^6$

$(-\frac{\pi}{4})^6 < x^6 < (\frac{\pi}{4})^6 \Rightarrow x^6 = (\frac{\pi}{4})^6$