Seminar 6

Theoretical part

The binomial probabilistic model

Repeated independent trials of an experiment such that there are only two possible outcomes for each trial - which we classify as either *success* or *failure* - and their probabilities remain the same throughout the trials are called **Bernoulli trials**. The binomial model describes the *number of successes* in a series of independent Bernoulli trials:

- *success* appears with probability p, *failure* with probability $1-p$;
- the experiment is repeated n times;
- the probability that success occurs k times in n trials for $k \in \mathbb{N}$, $k \in \{0,\dots,n\}$ is $\boldsymbol{C_n^k p^k (1-p)^{n-k}}$.

▶ $C_n^k p^k (1-p)^{n-k}$ represents the coefficient of x^k in the expansion $(px+1-p)^n$ for $k \in \{0,1,\dots,n\}$.

▶ This model corresponds to the binomial distribution $Bino(n,p), n \in \mathbb{N}^*, p \in (0,1)$.

▶ **Example:** A die is rolled 10 times. The probability that the number 6 shows up 3 times is $C_{10}^3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^7$.

The multinomial probabilistic model

Consider $n \in \mathbb{N}^*$ independent trials such that each trial can have several possible mutually exclusive outcomes $O_1,\dots,O_j$ ($j \in \mathbb{N}^*$) with $P(O_i) = p_i \in (0,1)$, $i \in \{1,\dots,j\}$. Obviously, $p_1 + \dots + p_j = 1$. The probability that O_i occurs n_i times in n trials for $n_i \in \mathbb{N}$, $i \in \{1,\dots,j\}$ and $n_1 + \dots + n_j = n$ is $\boldsymbol{\dfrac{n!}{n_1! n_2! \dots n_j!} p_1^{n_1} p_2^{n_2} \dots p_j^{n_j}}$.

▶ $\dfrac{n!}{n_1! n_2! \dots n_j!} p_1^{n_1} p_2^{n_2} \dots p_j^{n_j}$ represents the coefficient of $x_1^{n_1} \cdot \ldots \cdot x_j^{n_j}$ in the expansion of $(p_1 x_1 + \ldots + p_j x_j)^n$.

▶ This model corresponds to the multinomial distribution $Multino(n, p_1, \dots, p_j)$, $n \in \mathbb{N}^*$, $p_1,\dots,p_j \in (0,1)$, $p_1 + \ldots + p_j = 1$.

▶ **Example:** Suppose that an urn contains 2 red marbles, 1 yellow marble and 3 blue marbles. 7 marbles are drawn randomly with replacement from the urn (each drawn marble is put back into the urn). The probability that there are drawn 3 red marbles, 2 yellow marbles and 2 blue marble is $\dfrac{7!}{3!2!2!}\left(\dfrac{2}{6}\right)^3 \left(\dfrac{1}{6}\right)^2 \left(\dfrac{3}{6}\right)^2$.

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1. Let S be the set of all positive integers less or equal than 50, with exactly 2 digits such that one is an even digit and the other is an odd digit. A number is randomly extracted from S. Let X be the sum of its digits. Write the probability distribution of X.

2. The probability that a chipset is defective equals 0.06. A circuit board has 12 such independent chipsets and it's functional if at least 11 chipsets are operating. 4 independent such circuit boards are installed in a computer unit. Compute the probabilities of the following events:
B:"A circuit board is functional."
C:"Exactly two circuit boards are functional in the computer unit."
D:"At least a circuit board is functional in the computer unit."

3. Let (X,Y) be a discrete random vector with the joint probability distribution given by the following contingency table

$X \backslash Y$	-2	1	2
1	0.2	0.1	0.2
2	0.1	0.1	0.3

a) Find the probability distributions of X and Y.
b) Compute the probability that $|X - Y| = 1$, given that $Y > 0$.
c) Are the events $\{X = 2\}$ and $\{Y = 1\}$ independent?
d) Are the random variables X and Y independent?

4. It was found that the probability to log on to a computer from a remote terminal is 0.7. Let X denote the number of attempts, which are independent, that must be made to gain access to the computer:
a) Write the probability distribution of X.
b) Write the cumulative distribution function of X.
c) Compute the probability that at most 4 attempts must be made to gain access to the computer.
d) Compute the probability that at least 3 attempts must be made to gain access to the computer.