

Example 8

The following code:

```
(define (T3 N)
  (cond [(not (integer? N)) 'expect-an-integer]
        [(< N 1)           'expect-1-or-higher]
        [(= N 1) 1]
        [else (+ (fac N)
                  (T3 (- N 1)))])])
```

Has this recurrence relationship:

$$T(1) = 1$$

$$T(N) = 1 + N + T(N-1) \text{ for all } N > 1$$

Cheat sheet (rewritten versions of the recurrence relationship):

$$T(N-1) = N + T(N-2) \text{ \#equation 2}$$

$$T(N-2) = N-1 + T(N-3) \text{ \#equation 3}$$

$$T(N-3) = N-2 + T(N-4) \text{ \#equation 4}$$

Recurrence Relationship Unrolling

$$T(N) = 1 + N + T(N-1)$$

Plugging in equation 2:

$$T(N) = 2 + N + N-1 + T(N-2)$$

Plugging in equation 3:

$$T(N) = 3 + N + N-1 + N-2 + T(N-3)$$

Plugging in equations 4:

$$T(N) = 4 + N + N-1 + N-2 + N-3 + T(N-4)$$

We see a pattern here! We figure out the sum by plugging in a few small values for N to see what sum we should get. From that we can generalize to the following sum:

This will result in the following sum:

$$T(N) = (N - 1) + N + N-1 + N-2 + \dots + 4 + 3 + 2 + 1$$

The sum of that series is $N(N+1)/2 + (N-1)$

Therefore the Big-O runtime is:

$$O(N^2)$$