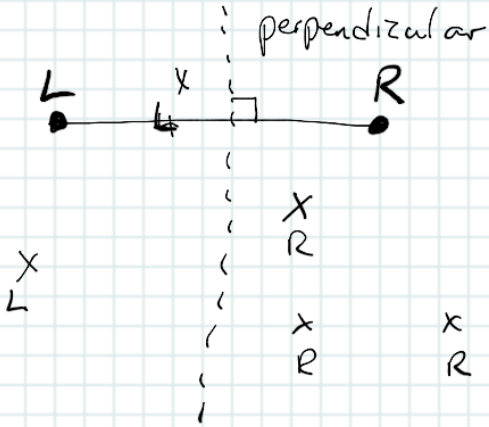
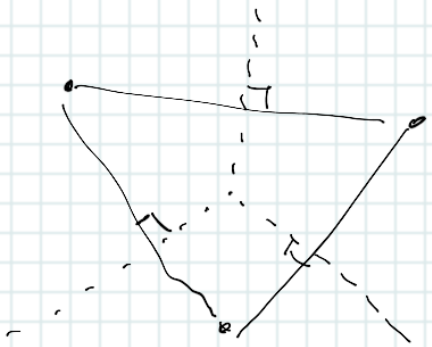
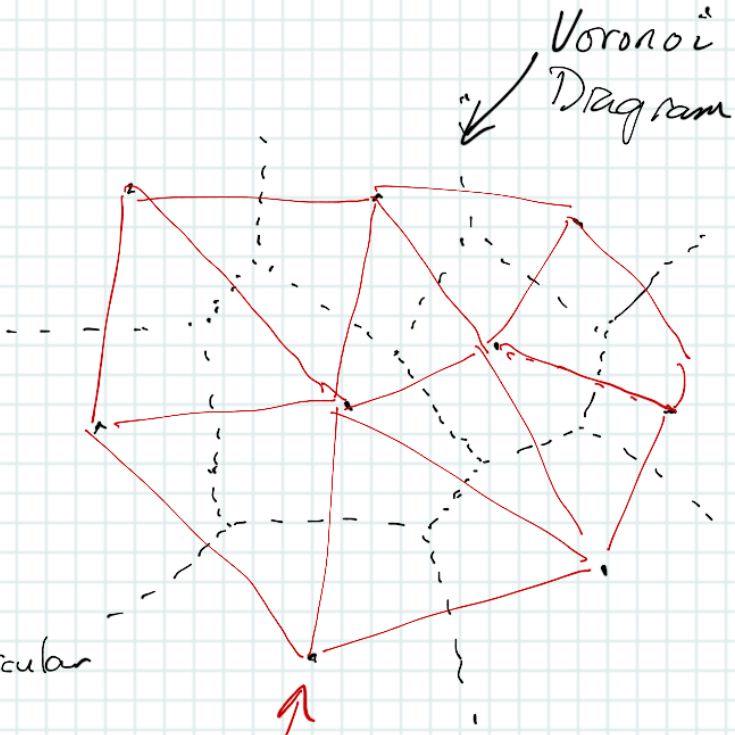
Voronoi Diagrams

Given a set of points $P = \{p_i\}$ and a manifold M

Define $Vor(i) = \{x \in M : dist(x, p_i) < d(x, p_j) \ \forall \ j \neq i\}$



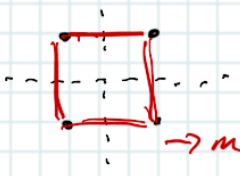




Notice these regions are roughly circular

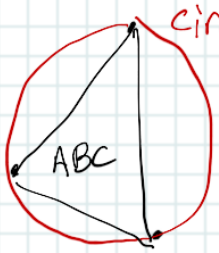
Voronoi Diagram is the planar dual graph of the Delaunay Triangulation!

Barring degenerate cases, the Delaunay is always a triangulation

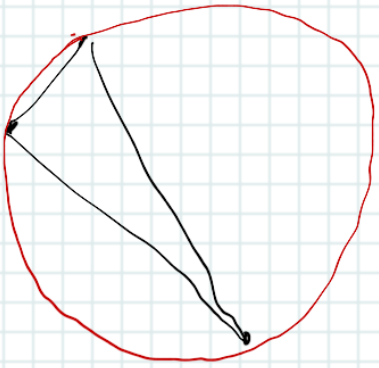


→ move a vertex a small random amount ⇒ Δ'n

Given a triangle ABC, the circumcircle is the circle that passes through A, B, and C. It is unique as long as ABC are not collinear & the closer the Δ is to equilateral, the smaller the circle is.



good triangle

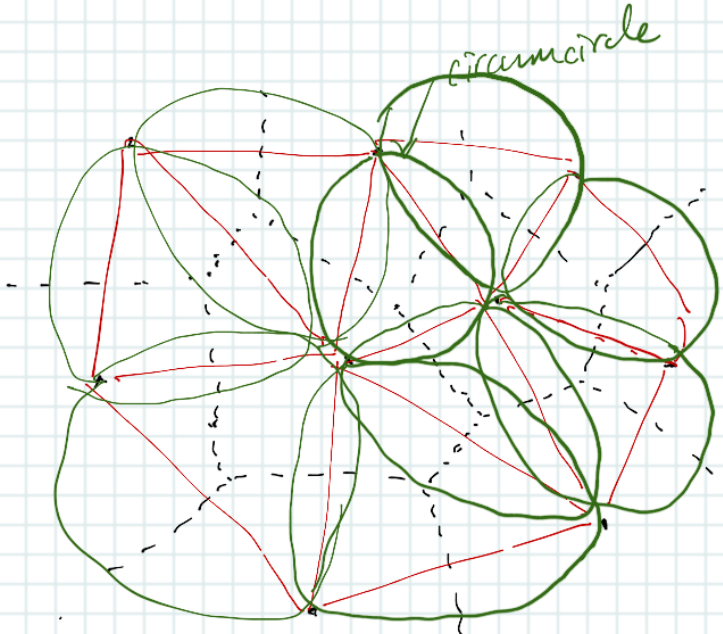


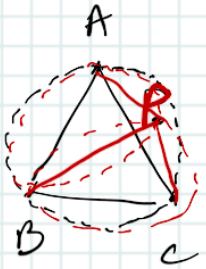
bad triangle

so circumcircle is related to triangle quality

Empty Circle Property:

If ΔABC is a Delaunay Triangle, then no other point p_j is in ΔABC's circumcircle

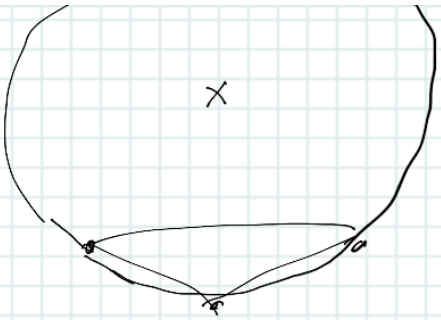




if P is inside, use ABP, PBC instead of ABC → better quality

Corollary: Delaunay Triangulations give you good quality meshes

- oh, this also works in 3D to generate good tet meshes
  → so many tet meshes are generated this way



at boundary, circumcircle is often outside

→ in the middle, it may be

Algorithms: They exist.

They're messy.

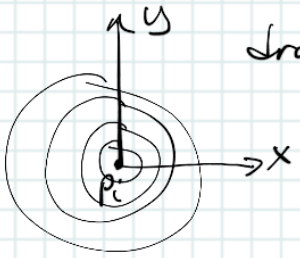
They're fragile. – they break easily in IEEE float

↳ entire complex may be incorrect

They're $O(n \lg n) - O(n^2)$

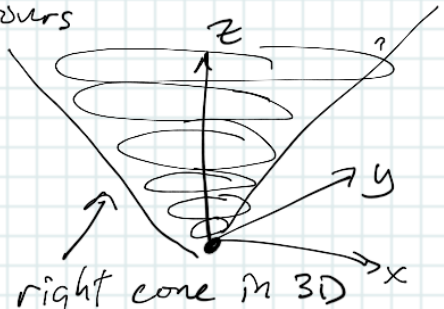
Conclusion: use a library (eg. CGAL)

cheap hack:



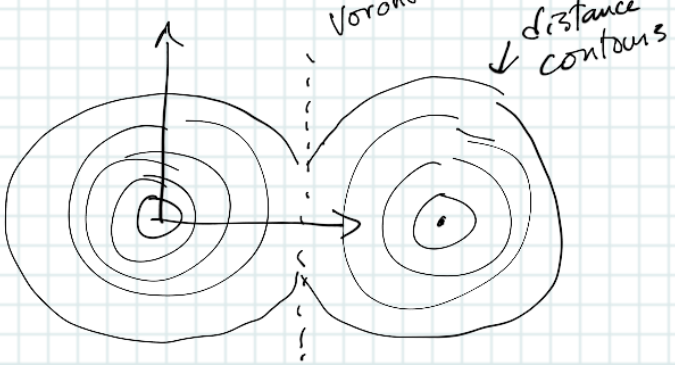
draw some circles around a point p_i

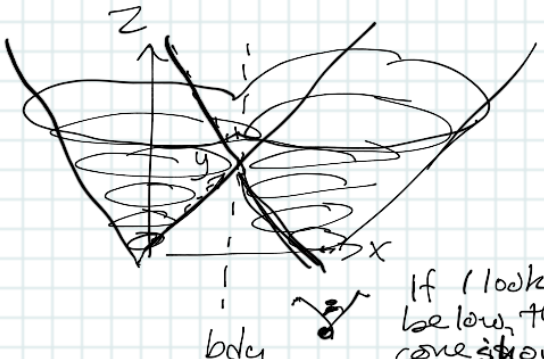
← those look like contours



right cone in 3D

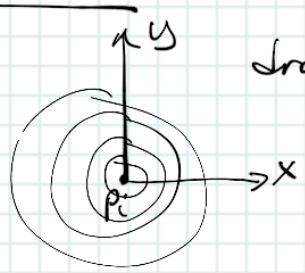
If I take 2 of these





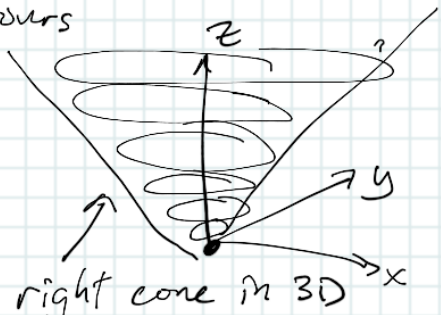
If I look from below, the closest cone shows the closest point

cheap hack:



draw some circles around a point p_i

← those look like contours



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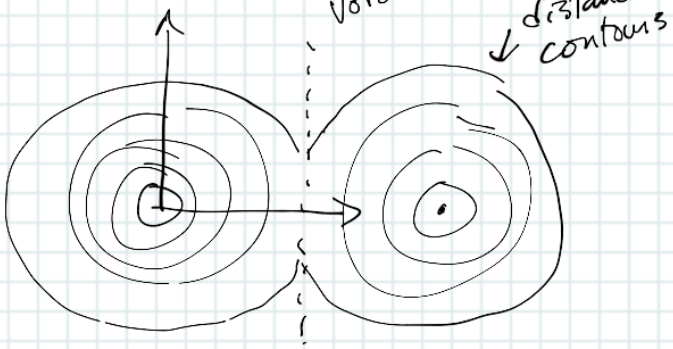

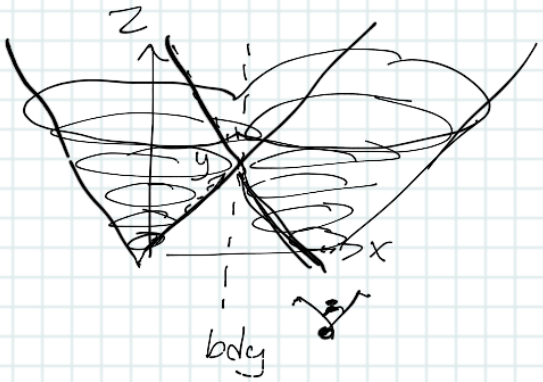

From below, the closest cone determines which point p_i a pixel belongs to

Hack: Render a cone at each point in different colours
Depth buffer takes care of finding lowest value/distance
Colour determines the Voronoi region
Work out the D.T. from there (Mng Lin)