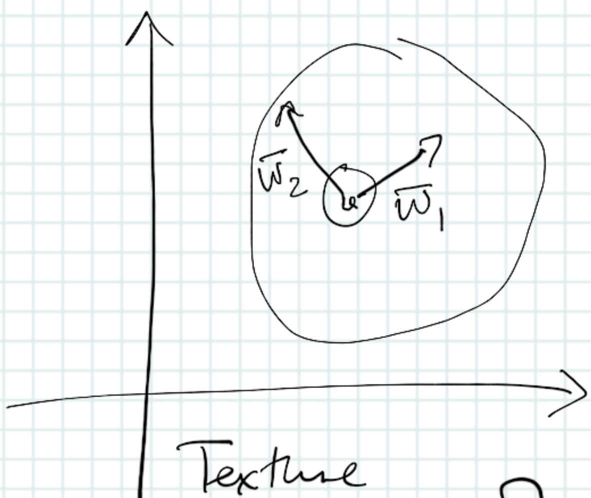
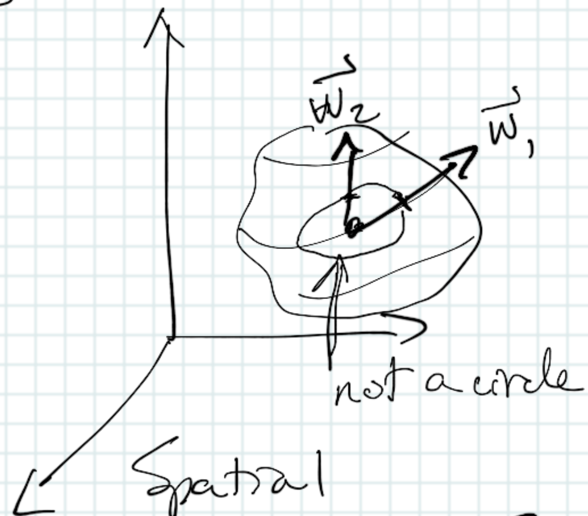
$$\mathrm{I} = \begin{bmatrix} \mathbf{x}_u \\ \mathbf{x}_v \end{bmatrix} \begin{bmatrix} \mathbf{x}_u & \mathbf{x}_v \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\delta x}{\delta u} & \frac{\delta x}{\delta v} \\ \frac{\delta y}{\delta u} & \frac{\delta y}{\delta v} \\ \frac{\delta z}{\delta u} & \frac{\delta z}{\delta v} \end{bmatrix}}_{J} \underbrace{\begin{bmatrix} \frac{\delta x}{\delta u} & \frac{\delta y}{\delta u} & \frac{\delta z}{\delta u} \\ \frac{\delta x}{\delta v} & \frac{\delta y}{\delta v} & \frac{\delta z}{\delta v} \end{bmatrix}}_{J^T}$$



Texture Domain Ω



Spatial Domain $\mathbb{R}^3$

Suppose $\vec{w}_1 \perp \vec{w}_2$
they are a basis for a coordinate system & we draw a small circle in it

What happens to the circle?

$\vec{w}_1 \not\perp \vec{w}_2$?

we map the circle to a small ellipse

an-iso-tropy
not equal shape

"ellipse of anisotropy"

Given a matrix M and a vector $\vec{v}$

$M\vec{v}$ is a vector. Can it be the same as $\vec{v}$?

Let $M = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$M\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$M = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad M\vec{v} = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\vec{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad M\vec{w} = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

So notice $M\vec{v} = \vec{v}$, but $M\vec{w}$ doesn't. So this is a property of a matrix <u>and</u> a vector. More generally, we are interested for a given matrix M, in vectors such that

$$M\vec{v} = \lambda\vec{v} \quad , \text{where } \lambda \neq 0$$

We refer to $\vec{v}$ as an eigenvector of M

and λ as the eigenvalue associated with $\vec{v}$

$$M\vec{v} - \lambda\vec{v} = \vec{0}$$

$$(M - \lambda I)\vec{v} = \vec{0}$$

identity, <u>not</u> the first fundamental form $\mathbf{I}$

$(M - \lambda I)\vec{v} = \vec{0}$

solve $M - \lambda I$:

$$\det \begin{bmatrix} m_{11}-\lambda & m_{21} & m_{31} \\ m_{12} & m_{22}-\lambda & m_{32} \\ m_{13} & m_{23} & m_{33}-\lambda \end{bmatrix} = p(\lambda)$$

polynomial in λ: $p(\lambda)$ – characteristic polynomial of M

We then solve for λ, and work backwards to get $\vec{v}$

→ If we take $\mathbf{I}$, which measures distortion, it's eigenvectors show us the principal & secondary directions of the distortion – i.e. the axes of our ellipse of anisotropy.

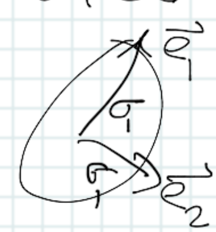
Given $\mathbf{I} = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$, the eigenvalues are:

$$\left.\begin{aligned} \sigma_1 &= \sqrt{\tfrac{1}{2}(E/G) + \sqrt{(E-G)^2 + 4F^2}} \\ \sigma_2 &= \sqrt{\tfrac{1}{2}(E/G) - \sqrt{(E-G)^2 + 4F^2}} \end{aligned}\right\} \text{sizes}$$

these are the two radii of the ellipse of anisotropy and

$\bar{e}_1$, $\bar{e}_2$ are the corresponding eigenvectors, with

$\vec{e}_1 = J\bar{e}_1$ and $\vec{e}_2 = J\bar{e}_2$ are the axes of the ellipse of anisotropy on our surface



For curves, we had ALP - a standard parameterization which meant that we had an <u>undistorted</u> description of the curve, where the rate of change was constant at all times (constant in magnitude)

For a surface, the equivalent "perfect" parameterization would need to have a uniform pair of eigenvalues/radii of the ellipse of anisotropy <u>everywhere</u> on the surface

<u>Bad news</u>: it doesn't exist, even for a sphere.

$\therefore$ there <u>is</u> no such equivalent for surfaces

$\wedge$

- we might be able to optimize to reduce distortion
- but there is <u>no</u> ALP for surfaces,
  which makes comparing them difficult
  describing them awkward
- so we instead focus on "intrinsic" properties
  that are independent of the actual parameterization
  - such as curvature