22: Review

1. Mathematics of Surfaces

- Function Notation
- Graphs of Functions
- Multi-dimensional Calculus
- Mathematics of Surfaces & Manifolds
- **Meshes & Attributes**
- **Planar Duality**
- The **Euler Formula**

2. Mesh Data Structures

- Requirements
- Operations
- Smooth Normal Calculation
- Trap Tables / POD Loophole
- Face, Indexed Face, Fans, Strips, Arrays, VBOs
- Winged Edge, Half Edge, **Directed Edge**

3. Mesh Operations

- Editing by Array Swapping
- Editing Operations in Directed Edge Structure
- **The 1-Ring**
- **Vertex Removal / Insertion**
- **(Half) Edge Collapse / (Restricted) Vertex Split**
- Face Collapse, Truncation, Stellation
- **Euler Operators & Manifold Preservation**

4. Differential Geometry of Curves

- **Parametric Curves**
- Derivative is **Direction Vector**
- **Arc-Length Integral**
- **Arc-Length Parameterisation**
- **(Unit) Principal Normal Vector**
- **Radius of Curvature**
- **Frénet Frame**

5. Interpolating Curves

- Continuity & Smoothness
- Least Squares & Lagrange Polynomials
- **Convex Hulls & Local Control**
- **Hermite Curves**
- **Bézier Curves** (Table & de Casteljau methods)
- Catmull-Rom Splines, NURBS & conversions
- **Subdivision Curves**

6. Differential Geometry of Surfaces

- Surfaces as Maps
- **Parameter Domain** & gluing
- **Isoparametric Curves**
- Tangent Plane and **Normals**
- **Jacobian Matrix & Distortion**
- First Fundamental Form
- Surface Area

7. Higher-Order Surfaces

- Bézier Triangles
- **Bézier Tensor Patches (Quads)**
- Direction Vectors & Normals
- **Surface Construction from Patches**
- Catmull-Clark Subdivision
- **Loop Subdivision**

8. Surface Anisotropy

- Anisotropy Ellipse
- Eigenvectors & Eigenvalues
- **No Canonical Surface Parameterisation**
  - (Parameterisation is not Intrinsic)

9. Surface Curvature

- Curves of Intersection
- Normal Curvature
- **Second Fundamental Form**
- **Principal Curvatures**
- **Euler Theorem** & Curvature Tensor
- **Mean & Gaussian Curvatures**
- **Gaussian Curvature is Intrinsic**

10. Laplace Operators

- Divergence & Gradient
- **Laplace Operator** is Divergence of Gradient
- **Laplace-Beltrami** is Laplace for Surfaces
  - **Intrinsic,** related to normal / mean curvature
  - Useful for computing mean curvature
- Discrete Differential Operators
  - **Weighted averages over 1-ring**

11-13. Discrete L-B Operators

- Definition of a discrete neighbourhood
- Defining Gradient on a Triangle
- Collapse to weighted sum of Edge Vectors
- **Gradient can be approximated on 1-Ring**
- **Use for computing Curvature**

14. Mesh Simplification

- **Greedy Decimation**
  - Using Mesh Operations
- Simulation of Simplicity
- Choice of Priority Measures
  - Curvature, Area, Degree
  - Delta Volume, Hausdorff Distance
  - Conservative Approximations

15. Smoothing

- Usually based on mean curvature
- Spherical / Manifold Harmonics
- Laplace-Beltrami Matrix
- **Diffusion Flow / Heat Maps**
  - **Conversion to Discrete Laplace-Beltrami**
  - Practical Computation

16. Mesh Repair

- The Rogues' Gallery
- Types of Input
- **Mesh Repair Pipeline**
- **Manifold Assembly & Repair**
- Stitching
- Connected Components

17. Isosurfaces

- Implicit & Blobby Surfaces
- 3D Meshes & Simplices
- Marching Tetrahedra / **Marching Cubes**
- Normals & Central Differencing
- Asymptotic Decider & Extra Cases
- **Distance Fields & Mesh Repair**

18. Voronoï Diagrams

- **Voronoï Diagrams**
- **Delaunay Triangulation** / Complex
- Empty Circle Property / **Triangle Quality**
- Algorithms for Construction, Render Hack
- **Surface Delaunay Triangulations**
- Geodesic Distance
- Medial Axis

19. Mesh Repair

- **Marching Cubes to get Manifold Surface**
- Postprocess to fix the problems
  - **Connected Components** to separate surfaces
  - **Greedy Delaunay Remesh** for new triangles
- **Mesh Simplification** for triangle count
- **Mesh Smoothing** if needed
- **Texture Parameterisation**
- **Texture Synthesis**

20. Texture Parameterisation

- Spherical Parameterisation & Cube Maps
- **Patch-Based Parameterisation**
- Tutte's Theorem
- **Floater's Algorithm**
- Least-Squares Conformal Mapping
- Deformation & Picking in 3D
- Morphing in Parameter Domain
- **Rasterising to Texture**