LOMP 5821M Geometriz Processing Nov. 11, 2021 p.1 We need to understand smooth surfaces mathematically It's easiest to start with smooth curves & build up from there In a parameterised curve, the direction vector is the first derivative It Unfortunately, there are multiple equations describing any given curve IV. The solution to this is to pick one parameter Bation based on flee Sistance along  $\mathbf{V}_{\cdot}$ the curve - i.e. the Are Length Parameterization (ALP) M. Working with the ALP means you have no distortion in your description We used the derivatives to find the curvature of the curve Va VII. We want to do the same for surfaces We can study surfaces by looking at them as a set of curves - eg cross-secting VIII. When we do this, we look isoparametric curves - curves following u or v IX. 60 ross the surface. We discovered the Jacobian matrix J, and the first & second fundamental 文. forms I and I which capture forms of distortion We saw that these describe the relationship between a small circle in I and a small ellipse on the surface called the ellipse of anisotropy XI. XII We observed that ideally we would have a surface parameter 3 at it where the ellipse was always a circle. XIII. But it can't be done, so we ned to look at "intrinsis" properties of the surface instead.

We defined the two principal curvatures of the surface using a variation on the Boparametrie curves. K, Kz combine to give K, H, the XIV Gaussian & mean curvatures, of which K is intrinsiz.

Now we start simplifying & turn it into somethingue can use

Define an "operator" over the group of functions to be a mapping that takes a function of as a parameter and returns a new function of e.g. 1 1 is the absolute value operator Pass in f and it returns If)  $f: \mathbb{R} \rightarrow \mathbb{R}$  f(x)  $|f(x)| \neq |f(x)|$ The operator V takes a function of and returns semantically vector where each of = (Sf Sf Sf) a vector-valued function has as many comparents

The operator div takes a function f and returns  $\Rightarrow$  adds the components dN f = Sf + Sf + Sf <math>dN - dN ergence SX + Sy + SzDefine Of the Laplaces operator ("Lel f") to be Af = SNVf  $= S_{N}\left(\frac{\varepsilon f}{S_{X}}, \frac{\varepsilon f}{S_{Y}}, \frac{\varepsilon f}{S_{Z}}\right)$  $=\frac{S^2f}{Sx^2}+\frac{S^2f}{Sy^2}+\frac{S^2f}{Sz^2} \qquad \text{on Pythagoras} \qquad x^2+y^2+z^2$  On a surface S, defined by  $\mathbf{X}(u,v)$  [the position function  $\mathbf{X}:\Omega \to \mathbb{R}^3$ ]  $\Delta X = \frac{S^2 X}{S n^2} + \frac{J^2 X}{S v^2}$ = X um + X vv

Related to this is the "Laplace - Beltrami" operator asf = Sivs Vsg, but everyone is sloppy and writes

Of = div Of mstead

Note that the Laplace - Beltrami operator applies to any function of on the surface - e.g. colour position

so if we apply Laplace - Beltrami to the position function X, it has been proved that:

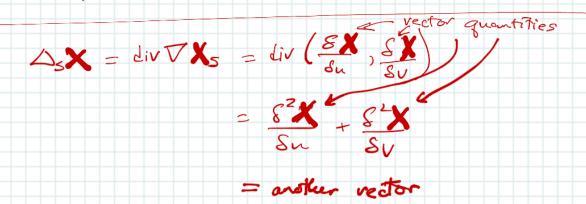
DSX = -2HM [stated without proof]

intrinsic curvature normal vector

we will estimate this

Now, in an ideal world, there would be a perfect parameterisation, and a side effect of that would be that the direction vectors of the Boparametriz curves would be Into each other, would be units, and the first fundamental firm I would be the identity matrix. But the normal vector would be length 1.

we cheat we assume that  $||\tilde{n}|| = 1$ , since we usually have a unit normal available. Then if we have approximated  $\Delta_s X$  and assumed  $||\tilde{n}|| = 1$ , we can solve for H.



All of this is for smooth surfaces.

We have triangulated meshes, so we have to do it again [more or less] to work out what this means in practice