Solution:

Part 1.

$$v_\pi(s) = E_\pi\left[R_{t+1} + \gamma G_{t+1} \mid s\right]$$

$$= \overline{\sum_a} R_s^a \, \pi(a|s) + \gamma \overline{\sum_a} \pi(a|s) \sum_{s'} P_{ss'}^a v(s')$$

Conservative policy: $\pi(a|s) = \begin{cases} 1 & s \text{ (slow)} \\ 0 & f \text{ (fast)} \end{cases}$

$$v_\pi(C) = 4 \times 1 + 0.9 \times 1 \times 1 \; v_\pi(C)$$

$$\longrightarrow \quad v_\pi(C) = 40 \; \textcolor{red}{\checkmark}$$

$$v_\pi(\omega) = 4 \times 1 + 0.9 \times 1 \times \left(0.5 \, v_\pi(C) + 0.5 \, v_\pi(\omega)\right)$$

$$\longrightarrow \quad v_\pi(\omega) = 40$$

Part 2: Recall that:

$$\pi(a|s) = \begin{cases} 1 & a = \arg\max_a q_\pi(s,a) \\ 0 & \text{otherwise} \end{cases}$$

and:

$$q_\pi(s,a) = R_s^a + \gamma \overline{\sum_{s'}} P_{ss'}^a v_\pi(s')$$

$$q(c,f) = 1 \times 10 + 0.9 \left(0.25 \, v_a(c) + 0.75 \, v_\pi(w) \right)$$

$$= 46$$

$$q(c,s) = 1 \times 4 + 0.9 \left(1 \times v_\pi(c) \right)$$

$$= 40$$

Since: $q(c,f) > q(c,s) \longrightarrow a_*(c) = f$

$$q(w,f) = 10 \times 1 + 0.9 \left(0.75 \, v_\pi(w) + 0.25 \, v_\pi(\underset{0}{\underline{off}}) \right)$$

$$= 37$$

$$q(w,s) = 4 \times 1 + 0.9 \left(0.5 \, v_\pi(w) + 0.5 \, v_a(c) \right)$$

$$= 40$$

$$\longrightarrow \quad a_*(w) = s$$

With the new policy:

$$v_\pi(c) = 1 \times 10 + 0.9 \left(0.75 \, v_\pi(w) + 0.25 \, v_\pi(c) \right)$$

$$v_\pi(w) = 1 \times 4 + 0.9 \left(0.5 \, v_\pi(w) + 0.5 \, v_\pi(c) \right)$$

$$\begin{bmatrix} \frac{31}{40} \, v(c) - \frac{27}{40} \, v(w) = 10 \\[2mm] -\frac{18}{40} \, v(c) + \frac{22}{40} \, v(w) = 4 \end{bmatrix} \longrightarrow \begin{bmatrix} v_\pi(c) = 66.9 \\[2mm] v_a(w) = 62.04 \end{bmatrix}$$

$$q(c,f) = 1 \times 10 + 0.9 \left(0.25 \times 66.9 + 0.75 \times 62.04 \right) = 66.9$$

$$q(c,s) = 1 \times 4 + 0.9 \left(1 \times 66.9 \right) = 64.21$$

$$q(c,f) > q(c,s) \longrightarrow \pi_*(c) = f$$

Similarly:

$$q(w,f) = 10 + 0.9 \left(\frac{3}{4} \times 62.04 \right) = 51.87$$

$$q(w,s) = 4 + 0.9 \left(0.5 \times 66.9 + 0.5 \times 62.04 \right) = 62.02$$

$$q(w,s) > q(w,f) \longrightarrow a_*(w) = s$$

Since π_* has not changed from the last iteration:

$$\begin{bmatrix} \pi_*(c) = f \\ \pi_*(w) = s \end{bmatrix} \checkmark$$