From January 2020

1) Compute the matrix resulting from the convolution between the filter and the image in the figure, with a stride of 1 and no padding.

Image

0	1	0	0
0	1	0	0
0	1	1	1
0	1	0	0

Filter

0	1
0	1

class

x_1 x_2

2) Build a Multi-Layer Perceptron able to classify the points: <1, 2, 0>, <3, 0, 0>, <1, 0, 1>, <2, 1, 1>, where the first two elements of each vectors are the feature values, and the last element is the class.

From January 2019

3) Given the following dataset: {<0,-2,1>, <1,0,1>, <-1,3,1>, <2,3,0>, <-1,2,0>}, where the last element of each vector is its class, construct a multi-layer perceptron that classifies the dataset, and draw its diagram (nodes and edges with corresponding weights).

4) Derive the update rule of the *output* neurons of a multi-layer perceptron according to the algorithm Backpropagation of Errors.

Tutorial 2

1)

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} p_1 & p_2 & p_3 \\ p_4 & p_5 & p_6 \\ p_7 & p_8 & p_9 \end{bmatrix}$$

$$p_1 = 0\times0 + 0\times1 + 1\times0 + 1\times1 = 1$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$p_2 = 0\times1 + 0\times0 + 1\times1 + 1\times0 = 1$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$p_3 = 0\times0 + 0\times0 + 0\times1 + 0\times0 = 0$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$p_4 = 0\times0 + 0\times1 + 1\times0 + 1\times1 = 1$$

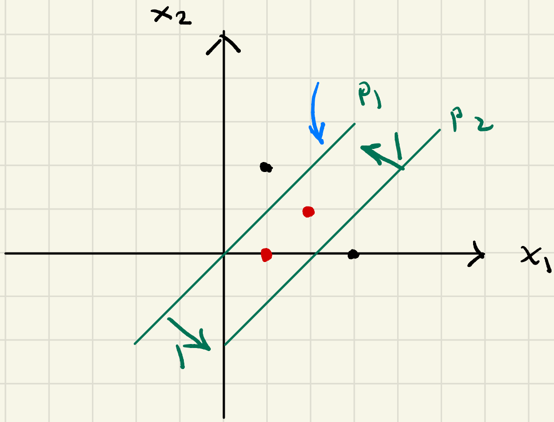
⋮

$x \rightarrow$

$y \downarrow$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{conv}} 4\times4$$

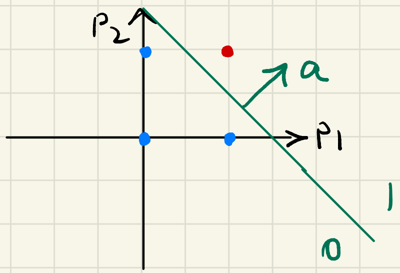
2)

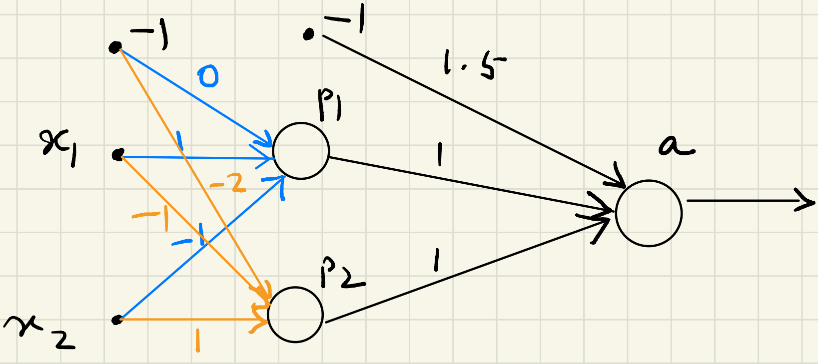


$p_1: \quad +1x_1 - 1x_2 + 0 = 0$

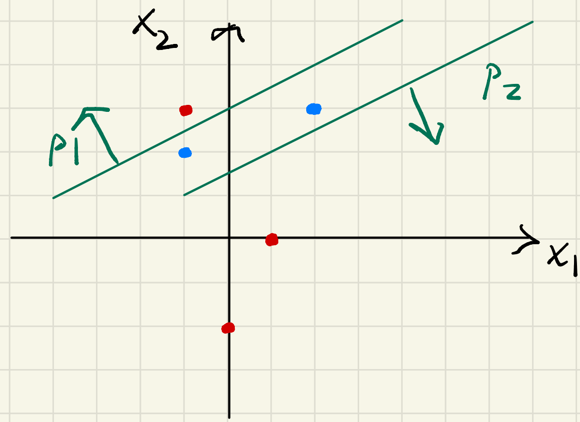
$p_2: \quad -1x_1 + 1x_2 + 2 = 0$

$a: \quad +1p_1 + 1p_2 - 1.5 = 0 \qquad (a = p_1 \wedge p_2)$





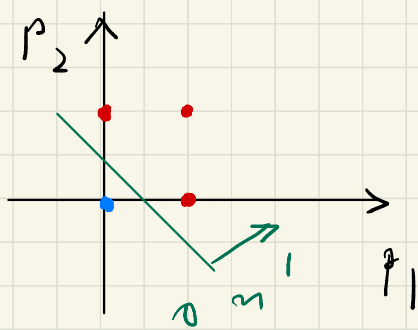
3)



$p_1: \; -2x_1 + 1x_2 - 3 = 0$ ✓

$p_2: \; +2x_1 - x_2 + 3/2 = 0$ ✓

$n: \; -p_1 - p_2 + 0.5 = 0 \quad \left(n = p_1 \overset{\text{or}}{\vee} p_2\right)$



4)
$$E = \frac{1}{2} | y - t |^2 = \frac{1}{2} \sum_k (y_k - t_k)^2$$

$$y_k = \sigma(a_k);$$

$$a_k = \sum_\ell w_{\ell k} z_\ell$$

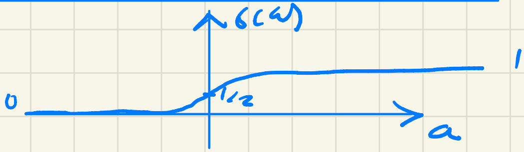
$$\frac{\partial E}{\partial w_{nk}} = \frac{\partial E}{\partial y_k} \cdot \frac{\partial y_k}{\partial a_k} \cdot \frac{\partial a_k}{\partial w_{nk}}$$

$$= \underbrace{(y_k - t_k) \cdot \sigma(a_k)(1 - \sigma(a_k))}_{\delta_k} \cdot z_n$$

$$= \delta_k \cdot z_n$$

$$w_{nk} \longleftarrow w_{nk} - \eta \cdot \frac{\partial E}{\partial w_{nk}}$$

$$\sigma(a) = \frac{1}{1 + e^{-a}}$$



$$\sigma'(a) = \frac{+ e^{-a}}{(1 + e^{-a})^2} = \frac{1}{1+e^{-a}} \times \left(1 - \frac{1}{1+e^{-a}}\right)$$

$$= \sigma(a)(1 - \sigma(a))$$